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Optimal SVC Placement in Electric Power Systems Using a Genetic Algorithms Based Method

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***Abstract*--The problem of improving the voltage profile and reducing power losses in electrical networks is a task that must be solved in an optimal manner. At present time, this optimality can be achieved by efficient usage of existing facilities alongside with installing FACTS devices. The Static VAr Compensator (SVC) was chosen for study as its maturity and acceptable costs make it more usable in practical applications than other FACTS devices This paper proposes a genetic algorithm that tries to identify the optimal location and size of an SVC. A multi-criteria function is developed, comprising of both operational objectives and investment costs. The computer program is run on a 13 nodes test system, assessing improvements in voltage profile and reducing power losses. The purpose of this study is to validate the solution method in order for it to be adapted for systems of higher dimensionality.**

***Index Terms*--**FACTS, SVC optimal placement, genetic algorithms.

I. INTRODUCTION

CONTINUOUS studies and developments have been carried out since the first appearance of FACTS devices, from. pioneering concepts to present mature devices.

The advantages of installing FACTS devices in a network are nowadays clear and concern improving the power flow control and voltage support, increasing network stability and oscillations damping [1].

The most widely used shunt FACTS devices within power networks are the static VAr compensators (SVC), as their costs are smaller but nevertheless with significant system enhancements. Appeared about two decades ago, the SVC is mainly installed for voltage support and, furthermore, when installed in a proper location, it can also reduce power losses.

Identifying the best location for SVCs implies calculating steady-state regimes for the network; as the load flow equations are nonlinear, the problem proves to be very complex, and extensive investigations have been undertaken in order to solve it.

A node is said to be "best location" for an SVC if after installing it in that node the improvements are better in comparison to other locations.

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Many solution methods and approaches have been reported during time, but no complete algorithm has been found so far. Extensive literature reviews can be found in [2, 3].

For simplicity, this paper only addresses the placement of one SVC device, and the improvements are assessed with respect to voltages deviations and power losses reductions in the network.

The proposed solution method uses genetic algorithms and can easily be extended to support multiple devices and to include more refined optimization criteria.

II. SVC MODEL

In order to present the problem formulation, a brief view of the SVC model and the way it influences the network is given in this section.

The SVC is modeled by a shunt variable admittance and can be placed either at the terminal bus of a transmission line or in the middle of a long line [5, 6]. Considering the SVC without losses, the admittance only has its imaginary component and it can take values in a specified range (usually between 0 and the maximum SVC capacity studied, here 500 MVAr). This is denoted by:

$$\underline{y}_{SVC} = jb_{SVC} \tag{1}$$

This paper considers the case of a SVC installed in a node (Fig. 1) with a continuously variable set point.

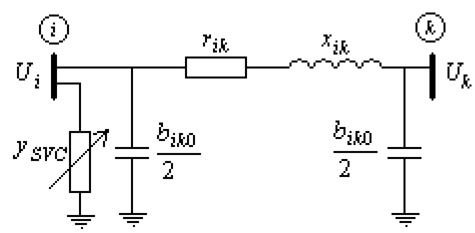

Fig. 1. Equivalent circuit of an SVC connected to a bus terminal

In this case, only one term of the nodal admittances matrix is modified, corresponding to the node where the SVC is connected:

$$\underline{Y}_{ii}' = \underline{Y}_{ii} + \underline{y}_{SVC} \tag{2}$$

The matrix is therefore modified as follows:

$$[\underline{Y}'_{nn}] = \begin{pmatrix} \underline{y}_{ik} + \dfrac{\underline{y}_{ik0}}{2} + \underline{y}_{SVC} & -\underline{y}_{ik} \\ -\underline{y}_{ik} & \underline{y}_{ik} + \dfrac{\underline{y}_{ik0}}{2} \end{pmatrix} \quad (3)$$

III. Problem Formulation

Consider a transmission network represented by its nodal admittance matrix $[\underline{Y}_{nn}]$ and the vector of nodal powers $[\underline{S}]$. Let *Sv* be the vector of state variables (voltage phase and magnitude) and let *Cv* be the set of control variables (location, size, reference SVC values, the domain of variable *location* – consisting in a set of nodes where the SVC placement study is carried out).

The problem lays in determining *Sv* and *Cv* so as to minimize or maximize a certain objective function $f(Sv, Cv)$ while verifying the following two types of constraints:

$$g(Sv, Cv) = 0 \ \text{(Kirchhoff's law)} \quad (4)$$

$$h(Sv, Cv) \le 0 \ \text{(security constraints).} \quad (5)$$

The domains of definition for the variables are also set as inequality constraints.

The objective function when searching for optimal SVC locations can include several optimization criteria. This paper proposes a multi-objective function, searching for a solution consisting of both the SVC location and SVC size that minimizes the voltage deviations, active power losses and installation costs.

The multi-objective function and the constraints are used to form the fitness function within the genetic algorithm, presented in section IV.

A. The objective function

The objective function consists of three objectives, two of which are technical and one economical, as follows:

- Minimize the active power losses:

$$O_1 = \sum_{l=1}^{b} R_l I_l^2 = \\ = \sum_{l=1}^{b} [V_i^2 + V_j^2 - 2V_i V_j \cos(\delta_i - \delta_j)] Y_{ij} \cos\varphi_{ij} \quad (6)$$

where b is the number of branches, R_l is the resistance of line l, I_l is the current through line l, V_i, δ_i are the voltage magnitude and angle from node i and Y_{ij}, φ_{ij} are the magnitude and angle of the i-j line admittance.

- Minimize the voltage deviations

$$O_2 = \sum_{i=1}^{n} \left(\frac{U_{iref} - U_i}{U_{iref}} \right)^2 \quad (7)$$

where n is the number of buses, U_{iref} is the reference voltage at bus i and U_i is the actual voltage at bus i.

- Minimize the investment costs.

The SVC costs in US$/kVAr installed are given as [4]:

$$O_3 = 0.0003Q^2 - 0.3051Q + 127.38 \quad (8)$$

where Q is the SVC installed reactive power, in MVAr.

B. Operational constraints

- Power flow balance equations. The balance of active and reactive powers must be satisfied in each node:

$$P_{Gi} - P_{Li} = \\ = U_i \sum_{k=1}^{n} [U_k [G'_{ik} \cos(\theta_i - \theta_k) + B'_{ik} \sin(\theta_i - \theta_k)]] \\ Q_{Gi} - Q_{Li} = \\ = U_i \sum_{k=1}^{n} [U_k [G'_{ik} \sin(\theta_i - \theta_k) - G'_{ik} \sin(\theta_i - \theta_k)]] \quad (9)$$

where the conductance G'_{ik} and susceptance B'_{ik} represent the real and imaginary components of element $\underline{Y}'_{ik}$ of the $[\underline{Y}'_{nn}]$ matrix, obtained by modifying the initial nodal admittances matrix when introducing the SVC (from equation (3)).

- Power flow limits. The apparent power that is transmitted through a branch l must not exceed a limit value, $S_{l\max}$, which represents the thermal limit of the line or transformer in steady-state operation:

$$S_l \le S_{l\max} \quad (10)$$

- Bus voltages. For several reasons (stability, power quality, etc.), the bus voltages must be maintained around the nominal value:

$$U_{i\min} \le U_{inom} \le U_{i\max} \quad (11)$$

In practice, the accepted deviations can reach up to 10% of the nominal values.

C. SVC reference value

The size of an SVC is expressed as an amount of reactive power connected to a bus of voltage 1 p.u.

Sign conventions: a positive value indicates the fact that the SVC generates reactive power and injects it into the network through the node to which it is connected (capacitive state); a negative value characterizes the inductive state, where the SVC absorbs reactive power from the network.

The SVC size is a variable that can take *nv* discrete values from the interval:

$$-Q_{L\max} \le Q_{SVC} \le Q_{C\max} \quad (12)$$

IV. Problem Modelling With Genetic Algorithms

Genetic Algorithms are a way of solving problems by emulating the mechanism of evolution as found in natural processes. They use the same principles of selection, recombination and mutation to evolve a set of solutions toward a "best" one.

Before using any of the GA models, the problem must be represented in a suitable format that allows the application of genetic operators. GAs optimize a single variable, the fitness function. Hence, the objective function and some of the constraints of the problem at hand must be transformed into some measure of fitness.

Encodings. The first feature that should be defined is the type of representation to be used, so that an individual represents one and only one of the candidate solutions. A candidate solution (or chromosome) designed in this paper for the problem of finding the optimal location of an SVC device is a two-component vector (fig. 2). The first component represents the location, the node in which the SVC should be connected, and can take values from 1 to the number of buses in the network. The second component represents the SVC size and can take values from 0 to 500 MVAr. A population of possible solution will be evolved from one generation to another, in order to obtain a very well fitted individual.

Position(node number)	Size(max. 500 MVAr)

Fig. 2. Chromosome encoding

Fitness Function. This function measures the quality of chromosomes and it is closely related to the objective function. The objective function for this paper is computed using equations (6)-(8). The constraints of this particular problem do not explicitly contain the variables (the genes in this case) and therefore the effect of the constraints must be included in the value of the fitness function. The constraints are checked separately and the violations are handled using a penalty function approach. Because the three objectives have different natures, it would be impossible to incorporate all of them in the same mathematical function. Each objective function is normalized in a comparative manner with the base case (the system without SVC). But taking into consideration the fact that the cost objective is less important than the first two, we use corresponding coefficients for each objective. The overall fitness function designed during this study is:

$$f(x) = 0.4\frac{O_1}{\sum \Delta Loss_{Base}} + 0.4\frac{O_2}{\sum \Delta V_{Base}} + 0.2\frac{O_3}{C_{\max}} - \\ -\xi \cdot \sum_{i=1}^{nr} bal_i - \zeta \cdot \sum_{k=1}^{n} thermal_k - \vartheta \cdot \sum_{k=1}^{n} voltage_k \tag{13}$$

where $\sum \Delta Loss_{base}$ is the total base case active power losses in the network, $\sum \Delta V_{base}$ represents the total base case voltage deviation, $C_{\max}$ is the maximum investment cost, computed with equation (8) for Q=500 MVAr, and the following are the penalty functions. The element bal_i is a factor equal to 0 if the powers balance constraint at bus i is not violated and 1 otherwise. The sum of these violations represents the total number of buses in the network that do not follow constraint (9) and it is multiplied by a penalty factor meant to increase the fitness function up to an unacceptable figure and therefore to discard the unfeasible solution. The second and third sums in the fitness function represent the total number of violations of constraints (10) and (11) respectively and they are also multiplied by cost factors. The last three sums in this fitness function are a measure of unfeasibility for each candidate solution *x*. The penalty factors used in this study were set to 100.

Without the cost integrated in the fitness function, the algorithm would tend to set the SVC size very close to its superior limit, as this would improve the voltage profile. The cost function, on the other hand, tries to keep the SVC size closer to its inferior limit. As a result of these two contradictory objectives, the solution method finds an optimum that satisfies both of them.

The constraint expressed in equation (12) is satisfied each run, as the limits for each individual are set within the main computer routine: the first component (location) varies between 0 and the number of buses and the second component (size) varies accordingly to eq. (12).

The genetic algorithm proposed for solving the optimal SVC placement under the above-described problem formulation can be written in the following simplified form:

```
Begin
 Read network data
 Run power flow and store results for base case
 Encode network data
 Set genetic parameters
 Create initial population
While <stopping condition not met> execute
  For each individual in current generation
     Run power flow and evaluate fitness
  EndFor
  Select(current_generation, population_size)
  Crossover(selected_parents, crossover_rate)
  Mutation(current_generation, mutation_rate)
  current_generation++
EndWhile
Show solution
End.
```

Selection Methods. The selection methods specify how the genetic algorithm chooses parents for the next generation. In this study, two selection methods were tested. The first method was Roulette Wheel Selection, which chooses parents by simulating a roulette wheel with different sized slots, proportional to the individuals' fitness. The second method tested was Tournament Selection and it proved to work better for the SVC optimal location problem. Each parent is chosen as the best individual from a random selection of *k* individuals, where *k* is a preset number – here 5 proved to be a suited tournament size.

Crossover Mechanism. The one-point and scattered crossover mechanisms were tested in this study. The one-point crossover exchanges the genetic information found after a random position in the two selected parents. The scattered crossover mechanism is described in the following.

For each pair of selected parents, the algorithm generates a set of binary components. The number of components is equal to the number of genes in an individual. This is a mask that will guide the crossover: if the mask value for the i^{th} gene is 0, then this gene of the offspring will inherit the i^{th} gene from the first parent. Otherwise, the i^{th} gene of the offspring will be the i^{th} gene from the second parent. This mechanism is applied for each gene. For example, if the number of genes is set to 4, then a possible mask would be 0110. Let ABCD and XYZW be the two selected parents. The scattered crossover would lead in this case to the following two offspring: AYZD and XBCW.

The scattered crossover proved to work better for the problem at hand.

The crossover is applied in each successive generation with a certain probability (here 0.8), known as the crossover fraction or rate. A large crossover rate decreases the population diversity, but in this problem a higher exchange of genetic material is needed.

Mutation Mechanisms. This mechanism is very important from the genetic diversity point of view, and it prevents landing a local, sub-optimal solution. The mutation rate is highly connected with the crossover fraction. The mutation mechanism used in this study implies generating a random gene number and flipping the bit found at that position. The mutation rate was set to 0.2.

Initial Population. Genetic Algorithms are theoretically able to find global optimum solutions, but the initial population must contain individuals with good genetic material for the problem at hand. This paper uses an initial population randomly generated, with individuals within the bounds set for each independent variable of the problem. The unfeasible solutions are discarded by penalizing the fitness function.

V. CASE STUDY

The proposed solution method was tested on 220 kV 13 nodes test system, shown in Figure 3. The network consists of 5 generators, of which one is slack node, 7 consumers and 15 lines, and their data are given in tables 1 and 2.

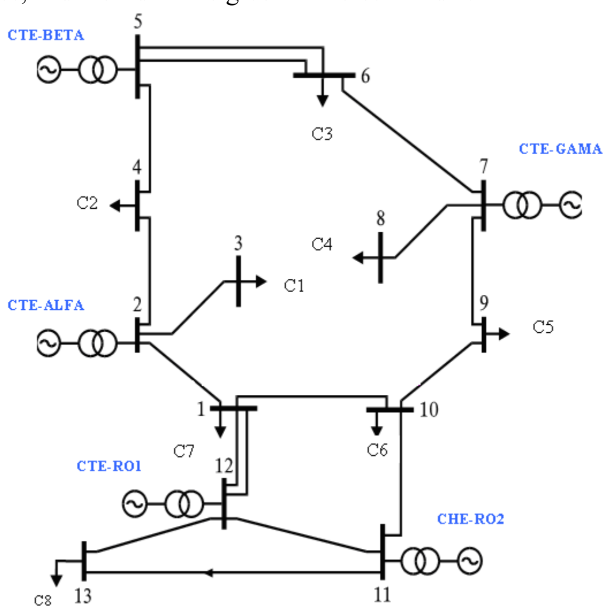

Fig. 3. Single line diagram of the test network

The GA was run 100 independent times, starting from a different initial population at each simulation. The results are synthesized in Table 3. As it can be observed, all solutions indicate bus number 10 as the most suitable location for the SVC, and 63% of the results suggest an SVC installed reactive power of 143 MVAr.

TABLE 1
LINE DATA

From	To	No. of circuits	Length [km]	r0 [Ω/km]	x0 [Ω/km]	b0 [μS/km]
1	2	1	150	0.066	0.404	2.773
2	3	1	80	0.06625	0.40375	2.775
2	4	1	112	0.063393	0.403571	2.777
5	6	2	100	0.066	0.418	2.72
6	7	1	90	0.073889	0.421111	2.7
7	8	1	70	0.065714	0.404286	2.786
7	9	1	60	0.066667	0.403333	2.783
9	10	1	80	0.06625	0.40375	2.775
1	10	1	75	0.073333	0.42	2.693
10	11	1	120	0.065833	0.404167	2.775
11	13	1	80	0.06625	0.40375	2.775
11	12	1	55	0.065454	0.403636	2.782
12	13	1	83	0.066265	0.403614	2.771
1	12	2	150	0.065333	0.418667	2.707
4	5	1	60	0.066667	0.403333	2.783333

TABLE 2
BUS DATA

Bus No.	Type	Pc [MW]	Qc [MVAr]	Pg [MW]	Usch [kV]	Qmin [MVAr]	Qmax [MVAr]
1	C	250	155	0	220	0	0
2	G	0	0	255	230	0	190
3	C	60	35	0	220	0	0
4	C	190	130	0	220	0	0
5	G	0	0	240	225	0	180
6	C	220	135	0	220	0	0
7	G	0	0	240	225	0	180
8	C	65	35	0	220	0	0
9	C	130	70	0	220	0	0
10	C	200	140	0	220	0	0
11	G	0	0	165	233	-60	160
12	S	0	0	395	235	0	300
13	C	150	90	0	220	0	0

TABLE 3
RESULTS

Bus number	Ratio [%]	Size [MVAr]	Max. voltage deviation
10	6	142	0.002364
10	63	143	0.002355
10	19	144	0.002347
10	5	145	0.002344
10	5	146	0.002337
10	2	147	0.002331

Different solutions were obtained each run, as the initial population, which gives the first genetic material, is randomly generated. Furthermore, the entire algorithm is based on random processes. Nevertheless, because of the high ratio of similar results indicating bus number 10 with 143 MVAr as best location, one can accept this result as accurate.

Figure 4 shows the performance of the GA for one of the runs. The initial population was randomly generated. The graph shows a good convergence of the fitness value with the generations. Both the best fitness value and the mean value drop with generations, which shows that GAs are suitable for solving SVC optimal location problem.

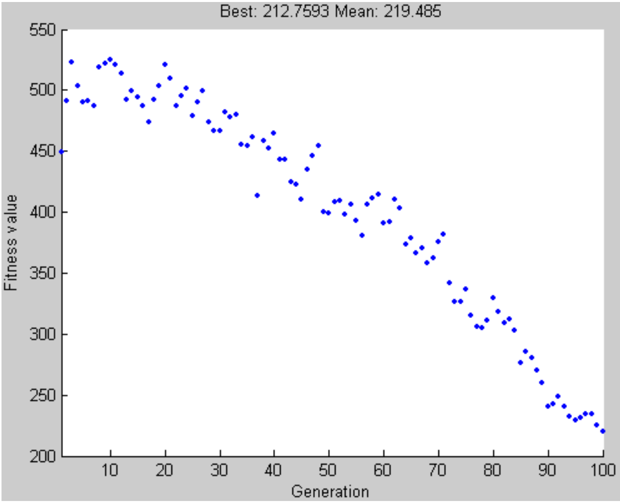

Fig. 4. GA performance

For a more detailed analysis of the results, consider Table 4, containing the total active power losses and total voltage deviations for each SVC size.

TABLE 4
RESULTS

Q [MVAr]	142	143	144	145	146	147	No SVC
Losses	22.4461	22.432	22.4179	22.404	22.3903	22.3767	25.8862
Dev.	0.002364	0.002355	0.002347	0.002344	0.002337	0.002331	0.007764

As it can be seen, the minimum total voltage deviation is achieved for a SVC capacity of 143 MVAr. Excepting this point, the active power losses drop with the SVC capacity, but the improvements are not significant and therefore the investment difference would not be accounted for. The results are very close to each other and the algorithm has chosen more frequently the capacity of 143 because of the economical point of view. The voltage profiles and total voltage deviations in all resulted scenarios are presented in figures 5 and 6 respectively.

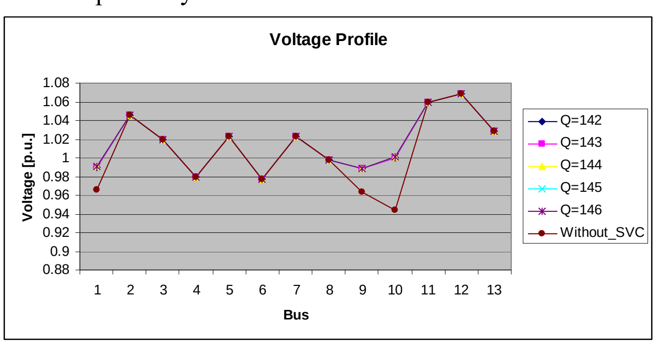

Fig. 5. Voltage profiles for each resulted scenario and base case

An improvement with respect to the base case can be observed.

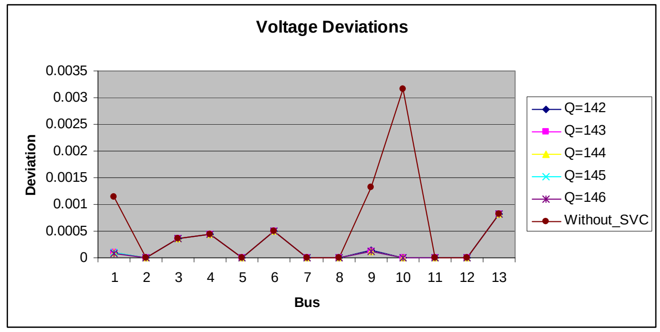

Fig. 6. Total voltage deviations

As figure 6 shows, the maximum voltage deviation was obtained for bus number 10 in the base scenario (without SVC), and the genetic algorithm minimizes this deviation.

Looking at the GA performance from the computational time point of view, the proposed solution method proves to be slow. The power flow routine is run for each individual in order to compute its fitness. There are 100 individuals in each generation, and the number of generations also reaches 100. Even with CPU-optimized power flow routines, the overall computational time reaches an average of 23.00762 seconds per run. This, however, was improved by avoiding computing the admittance matrix each run, which led to an average computational time of 14.3036 seconds per run.

VI. CONCLUSIONS

An application of genetic algorithms was presented for finding the best location of an SVC within a power network, with the objective of reducing power losses, reducing voltage deviations and costs. Results show that the proposed approach is a suitable and promising technique in solving the problem, but the best solution found by GAs and computational time can be improved by tuning the parameters (crossover rate, population size and others) and improving the mathematical model of the problem (taking into consideration stability margin, system loadability etc).

Because of the flexibility of Genetic Algorithms, further modeling requirements can be included in the fitness function to further improve the optimization design. For example, some of the initial simplifications can be eluded from the design, transforming the problem into a more realistic one. Future work will focus on increasing the number of devices to be installed, which implies finding a more suitable encoding method for the chromosomes.

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VII. BIOGRAPHIES

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