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Geometrical Approaches for Gross Errors Analysis in Power Systems State Estimation

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Abstract- In this paper, a geometrical based approach is used to define the undetectability index (UI) that gives the distance of a measurement from the range of the jacobian matrix. The higher the value of this index the closer this measurement will be to the range of that matrix; the error in measurements with high UI is not reflected in their residues. A critical measurement has infinite UI, belongs to the range of the Jacobian matrix, and its error is totally masked. Using the UI, it is shown measurements not classified as leverage points, and having large masked gross errors in the state estimation process. As example to illustrate the way that UI index works, a two bus power system will be used; to test the index efficiency in identifying the measurements that have the gross errors masked by the state estimation process the IEEE-14 bus system will be used.

Index Terms–State Estimation, Orthogonal Projections, Gross Errors Analysis, Recovering Errors, Undetectability Index.

I. INTRODUCTION

THE ability to detect and identify gross errors is one of the most important attributes of the state estimation process in power systems. This characteristic to deal with gross errors makes the results of the state estimation process preferable if compared to the SCADA raw data [1].

In order to mitigate the influence of gross errors on the state estimation results, some robust estimators have been introduced in power systems [2]. One of the first robust state estimators applied to power system was the Weighted Least Squares (WLS) Estimator endowed with the largest normalized residual test [3] for gross error detection and identification. However, this combination is not robust in the presence of single and multiple non-interacting gross errors [2]. Also, this estimator is not able to reliably identify multiple interacting gross errors, especially when the errors are conforming and/or occur in Leverage Points [4], which are highly influential measurements that “attract” the state estimation solution towards them [5], [6].

Other alternative estimators, which are more robust than the WLS Estimator, have been proposed. The Weighted Least Absolute Value Estimator (WLAV), for example, can deal better with multiple gross errors but it is prone to fail in the presence of a single gross error at a Leverage Point [2], [4].

The Least Median of Squares Estimator (LMS) [7, 8] is another estimator alternative. This estimator is a member of the family of estimators known collectively as high-breakdown point estimators. It is inherently resistant to outliers in Leverage Points and can handle multiple interacting gross errors, even when they are conforming. However it requires excessive computing time for on-line applications [1].

Despite the existence of more robust estimators, the WLS Estimator is the most used estimator in power systems. This is due to its simplicity and velocity.

The idea of exploring geometry to detect gross errors in power systems is not new. Based on a geometric interpretation of the residue estimation for single gross error, a method for detection/identification of multiple gross errors was developed in [10]. The authors claim that their method is able to determine whether the residue vector lies in a subspace defined by the suspect measurements (suspect in terms of having gross errors) and whether any portion of that subspace is orthogonal to the residual vector. They also claim to find the suspect measurements of the measurement set; however, the algorithm to find them is complicated and difficult to be implemented.

In this paper, using the WLS Estimator, more insights related to detectability of gross errors in power system state estimation, using geometrical approaches are provided. This is achieved decomposing the measurement error into two parts: the undetectable part and the detectable one. The ratio between the norms of those quantities, the undetectability index (*UI*), gives the distance of a measurement from the range of the jacobian matrix. In another words, the UI is a measure of how difficult is to detect errors in those measurements. That index presents a more comprehensive picture of the problem of gross error detection in power system state estimation than critical measurements and leverage points.

II. BACKGROUNDS

Consider a power system with n buses and m measurements. The power system is modeled, for state estimation purposes, as a set of nonlinear algebraic equations as:

$$z = h(x) + e\,, \tag{1}$$

where $z \in R^m$ is the measurement vector, $x \in R^N$ is the state vector, $h\colon R^N \to R^m$ $(m > N)$ is a continuously nonlinear differentiable function, “e” is the $(mx1)$ measurement error vector with zero mean and Gaussian probability distribution, and $N=2n-1$ is the number of unknown state variables to be estimated. Since the number m of measurements is higher than the number N, a common solution to estimate the states variables is the well known WLS method that searches for the state x that minimizes the functional $J(x) = (z - h(x))^T W(z - h(x))$, where W is a symmetric and positive definite real matrix. In power system state estimation, the weight matrix W is usually chosen as the inverse of the measurement covariance matrix. The functional J is a norm in the measurement vector space R^m that is induced by the inner product $<u,v>=u^TWv$, that is:

$$J(x) = \|z-h(x)\|^2_W = \prec z-h(x), z-h(x) \succ = \\ = [z-h(x)]^T W [z-h(x)]. \quad (2)$$

Let $\hat{x}$ be the estimated state, that is, the solution of the aforementioned minimization problem, and define the estimated measurement vector as $\hat{z} = h(\hat{x})$. The residue vector is defined as the difference between z and $\hat{z}$, that is, $r = z - \hat{z}$.

The linearization of equation (1), at a certain point x^*, gives:

$$\Delta z = H\Delta x + e, \quad (3)$$

where $H = \partial h / \partial x$ is the Jacobian of h calculated at x^*, $\Delta z = z - h(x^*) = z - z^*$ is the measurement vector mismatch, and $\Delta x = x - x^*$.

If system (2) is observable, that is, $rank(H)=N$, then the vector space of the measurements R^m can be decomposed into a direct sum of two vector subspaces, that is, $R^m = \Re(H) \oplus (\Re(H))^\perp$ in which the range of H, $\Re(H)$, is a N-dimensional vector subspace into R^m and $\Re(H)^\perp$ is its orthogonal complement, that is, if $u \in \Re(H)$ and $v \in \Re(H)^\perp$, then $<u,v>=u^TWv=0$.

In the linear state estimation formulation, equation (3), the solution can be interpreted as a projection of the measurement vector mismatch Δz onto the $\Re(H)$. Let P be the linear operator which projects vector Δz onto $\Re(H)$, that is, $\Delta\hat{z} = P\Delta z$ and $r = \Delta z - \Delta\hat{z}$ be the residue vector for the linearized model. The projection operator P that minimizes the norm J is the one that projects Δz orthogonally onto $\Re(H)$ in the sense of the inner product $<u,v> = u^TWv$, that is, the vector $\Delta\hat{z} = H\Delta\hat{x}$ is orthogonal to the residue vector. More precisely:

$$\langle \Delta\hat{z}, r \rangle = (H\Delta\hat{x})^T W (\Delta z - H\Delta\hat{x}) = 0. \quad (4)$$

Solving this equation for $\Delta\hat{x}$, one obtains: $\Delta\hat{x} = (H^TWH)^{-1} H^TW\Delta z$.

As $\Delta\hat{z} = H\Delta\hat{x}$, the projection matrix P will be the idempotent matrix:

$$P = H(H^TWH)^{-1} H^TW. \quad (5)$$

Obs.: if W is a diagonal matrix given by $W=cI$, where $c>0$ is a real number, and I is the identity matrix, then $P=P^T$ and P is said to be an orthogonal projection.

In power system literature, matrix P is usually called the Hat matrix and is also known as the K matrix [11]. The residue vector is found to be:

$$r = \Delta z - \Delta\hat{z} = \Delta z - P\Delta z = (I-P)\Delta z, \quad (6)$$

where the idempotent matrix $(I-P)$ is an operator that projects Δz onto $\Re(H)^\perp$. Matrix $(I-P)$ is given by:

$$(I-P) = I - H(H^TWH)^{-1} H^TW, \quad (7)$$

and is usually called the residual sensitivity matrix; it is also known as the S matrix [11] in power system literature.

Figure 1 illustrates operator P acting on the vector Δz.

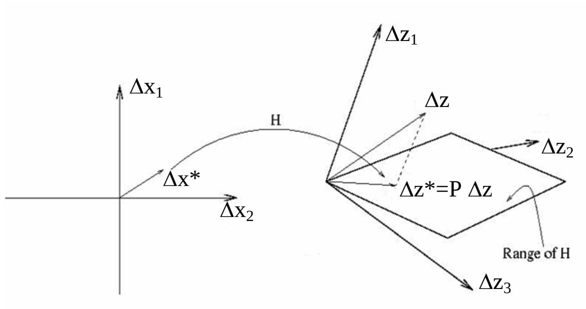

Figure 1- Geometric interpretation of operator P acting on vector Δz

III. UNDETECTABLE ERRORS

In this section, the decomposition of the measurement vector space into a direct sum of $\Re(H)$ and $\Re(H)^\perp$ will be used to decompose the measurement error vector e into two parts: the detectable and the undetectable components. For that purpose, let x_{true} be the vector of the true unknown states and define $z_{true} = h(x_{true})$. Consider the linearized model (3), where $\Delta x_{true} = x_{true} - x^*$, and $\Delta z_{true} = H\Delta x_{true}$. Assuming the available jacobian matrix H is close to the one obtained with measurements without gross errors, then z_{true} is close to $\Re(H)$ and $\Delta z_{true} \approx z_{true} - z^*$. The measurement error vector is given by $e = z - z_{true} \approx \Delta z - \Delta z_{true}$, and can be written as: $e = Pe + (I-P)e$. Denominating e_U and e_D as:

$$e_U := Pe, \quad (8)$$

$$e_D := (I-P)e, \quad (9)$$

respectively, the undetectable and detectable components of "e", one has $e = e_U + e_D$. It is easy to see that $e_U \in \Re(H)$ while $e_D \in \Re(H)^\perp$ and as a consequence:

$$\|e\|_W^2 = \|e_D\|_W^2 + \|e_U\|_W^2. \quad (10)$$

The next proposition shows that the undetectable part of the error does not contribute to the residue vector.

<u>**Proposition 1**</u>: The residue vector, r, of the WLS state estimation is not affected at all by the undetectable component e_U of the measurement error vector.

Proof: The measurement vector mismatch can always be written as:

$\Delta z \approx \Delta z_{true} + e \approx \Delta z_{true} + e_U + e_D$.

As a consequence:

$$r = (I-P)\Delta z \approx (I-P)\Delta z_{true} + (I-P)e \approx \\ \approx (I-P)\Delta z_{true} + (I-P)e_U + (I-P)e_D. \quad (11)$$

Since $\Delta z_{true} \in \Re(H)$ and $e_U \in \Re(H)$, one has $(I-P)\Delta z_{true} = 0$ and $(I-P)e_U = 0$. Consequently, equation (11) becomes: $r = (I-P)\Delta z \approx (I-P)e_D \approx e_D$ ■

The previous proposition gives an indication that the difficulty, or even the impossibility, of detecting gross errors in some measurements is due to the fact that these errors have a significant undetectable component as compared to the detectable one. As a consequence, their residuals are small even when those measurements are contaminated with gross errors. The largest normalized residual test is often employed to check the existence of gross errors in the measurement set. If the random error vector *e* has a normal distribution with expected value equal to zero, then it is well known that the normalized residue r_i^N of the *i-th* measurement is a random variable with normal distribution, expected value equal to zero, and variance equal to one. Given a snapshot of measurements, one can calculate the normalized residue of each measurement and check the existence of gross errors. More precisely, if the largest normalized residue r_i^N is larger than a threshold value we can affirm, with a certain degree of confidence, the existence of gross error in the *i-th* measurement. However, the operator *(I-P)*, is not invertible and, as a consequence, the backward implication is not true, that is, small normalized residues do not imply small errors in the measurements due to the presence of undetectable components of the errors. More precisely, the residue does not change if one adds up, in the measurement vector, an extra error that lies in the vector space $\Re(H)$. In other words, when the largest normalized residue r_i^N is not larger than a threshold value, one can affirm, with a certain degree of confidence, only that the **detectable components** of the errors in all the available measurements are small. As a consequence, the largest normalized residue test can often fail in the detection of gross errors for the measurements that have a large **undetectable component**. However, if additional information is provided, one can say more about the relationship between residue and the components of the errors in the measurements.

IV. UNDETECTABILITY INDEX (*UI*) PROPOSITION AND COMPUTATION

In this section an index, UI, based on the detectable and undetectable component of the measurement error vector will be proposed. Also an algorithm to calculate the *UI* for each available measurement will be presented.

For that purpose, suppose the existence of a single error in the *i-th* measurement, that is, $e_i = b\delta_i$ with $\delta_i = [0 \ \dots \ \overset{i}{1} \ \dots \ 0]^T$, and using equations (8) and (9) define the corresponding undetectable $e_{U_i} = P(b\delta_i)$ and detectable $e_{D_i} = (I-P)(b\delta_i)$ components of that error.

With that in mind, the following definition of error undetectability index for the *i-th* measurement is proposed:

$$UI_i = \frac{\left\| e_{U_i} \right\|_W}{\left\| e_{D_i} \right\|_W}. \qquad (12)$$

Remark 2: As can be seen from the previous equation, UI_i does not depend on the error magnitude *b* but only depends on the projection matrices *P*.

A large *UI* for a measurement indicates a large component of the measurement error will be masked in the state estimation process and will not contribute to its residue. In particular, if the measurement error $e_i \in \Re(H)$, then $e_{Di} = 0$, and as a consequence the associated *UI* will tend to infinite. That is the case of critical measurements. The *UI* of a measurement will allow one to identify measurements that, although do not belong to the range of *H*, are close to that range, making, as a consequence, the gross error detection/identification less reliable in the largest normalized residue test. This geometrical view of undetectable errors presents a more comprehensive picture of this problem than the concept of leverage points.
In the following an algorithm to compute the UI index is proposed.

Algorithm:

1. For each measurement "*i*", suppose the existence of an error vector given by $e_i = b\delta_i$. Repeat this step for each measurement *i*, *i* from 1 to *m*.
2. Compute the undetectable (e_{Ui}) and detectable (e_{Di}) parts of each error vector e_i as previously defined. With these vectors in hands compute the measurement i undetectability index, using equation (12).

The algorithm is processed using the estimated state vector obtained from the available measurement set

V. EXAMPLES OF MEASUREMENTS WITH *UI* VERY HIGH AND VERY LOW

In this section two examples will be used in order to show the position relative to the $\Re(H)$, of measurements that may hide errors (very high *UI*), and measurements that do not hide errors (very low *UI*).

Remark 3: In the next two examples, the detection test of gross errors is made by means of both the largest normalized residual test (considering a threshold value $\lambda = 3$) and the $J(\hat{x})$-test. The threshold value "C" for the $J(\hat{x})$-test is obtained via Chi-square distribution table for a 2.5% probability of false alarm with: $J(\hat{x}) = (z - H\hat{x})^T W (z - H\hat{x})$ (See Appendix)

Example 1: Consider the system of two buses (*n* = 1) connected through two lines, as shown in Figure 2. Taking bus 2 as a reference of angle ($\delta_2 = 0$), the Jacobian matrix for that system becomes $H = \begin{bmatrix} 2 \\ 10 \end{bmatrix}$. Considering that the standard deviations of z_1 and z_2 are equal to 1p.u.

($\sigma_1 = \sigma_2 = 1$), that is, "$W = I$", the projection matrix P will be $P = \begin{bmatrix} 0.038 & 0.192 \\ 0.192 & 0.961 \end{bmatrix}$.

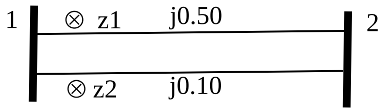

Figure 2 - Two-bus-system, ⊗ indicates power flow measurements

Let $x_{true} = 0.175(\delta_{1true} = 0.175)$ be the vector of the unknown states and $z_{true} = Hx_{true} = [0.35 \quad 1.75]^T$ be the true unknown measurement vector. Suppose the existence of a gross error of magnitude $9\sigma_{z2}$ in the measurement z_2, that is: $z = [0.35 \quad 10.75]^T$. Solving the linear WLS state estimation equations, with $W = I$, one obtains: $r = \begin{bmatrix} -1.73 \\ 0.346 \end{bmatrix}, |r^N| = \begin{bmatrix} 1.765 \\ 1.765 \end{bmatrix}$ and $J(\hat{x}) = (z - H\hat{x})^T W(z - H\hat{x}) = 3.11$. The threshold value $C = 5.02$ is obtained via Chi-square distribution table for $\chi^2_{m-n,1-\alpha}$ (see Appendix).

As a consequence, the hypothesis of error existence is erroneously rejected. This situation is depicted in Figure 3. In order to understand the reason for that wrong decision let us compute the *UI* for those measurements, whose values are shown in Table I.

Table I – *UI* – Example 3

	z_1	z_2
UI	0.1997	5.0073

As one can see in Table I, the *UI* of measurement z_2 is very high, and the *UI* of measurement z_2 is very low. As it can be seen in Figure 3, z_2 is closer to the range of *H* than z_1. In this way, z_2 behaves more similar to a critical measurement than z_1, and to detect an error in z_2 that error has to be very high.

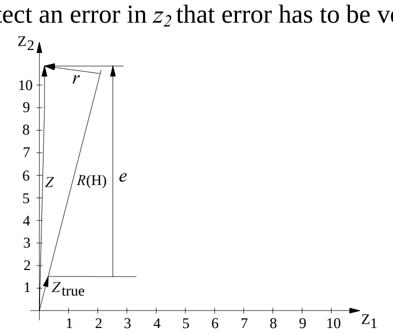

Figure 3 - Gross error in measurement z_2 (Example 1)

Example 2: Using the same system considered in Example 1 (Figure 2), but now with a gross error of magnitude $9\sigma_{z1}$ in the measurement z_1, that is, $z = [9.35 \quad 1.75]^T$, solving the linear WLS state estimation equations one obtains: $r = \begin{bmatrix} 8.65 \\ -1.73 \end{bmatrix}, |r^N| = \begin{bmatrix} 8.825 \\ 8.825 \end{bmatrix}$ and $J(x) = 77.88$ (see Figure 4).

In this case the error is detected ($\lambda = 3$ and $C = 5.024$) because it occurred in the measurement z_1, with low *UI*.

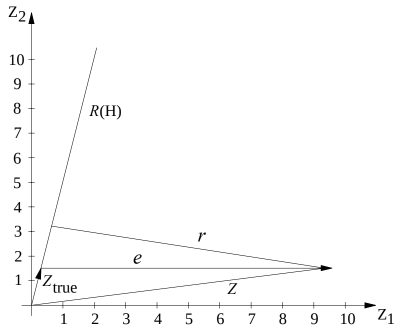

Figure 4 - Gross errors in measurement z_1 (Example 2)

VI. NUMERICAL TESTS

For the numerical tests, the IEEE -14 bus system will be used (Figure 5). The parameters of this system are available in www.ee.washington.edu/research/pstca/.

The measurements values used in the tests were obtained from a load flow solution (z^{lf}) to which normally distributed noise was added. The measurement noise was assumed to have zero mean and a standard deviation (STD) "σ" given by: $\sigma = \frac{pr * |z^{lf}|}{3}$; with *pr*, the meter precision, equal to 3%.

Scenario-I: In this measurement scenario the measurement set shown in Table II will be used.

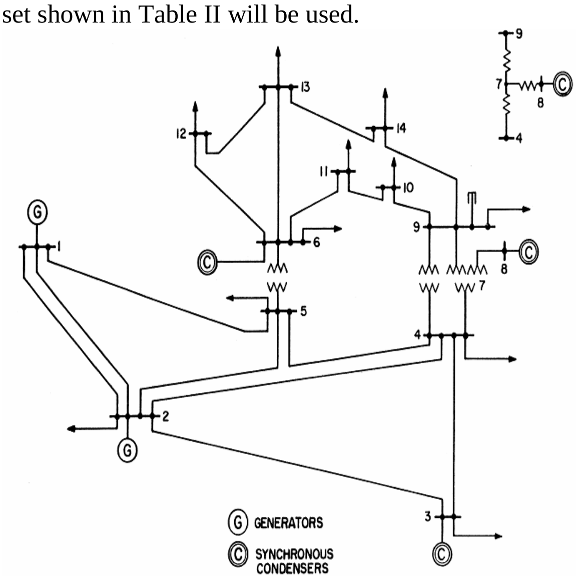

Figure 5 – IEEE-14-bus system

Remark 4: For the next Tables the following nomenclature is used: Ia – injection measurement at bus "a";

Fa-b – flow measurement from bus "a" to bus "b"; and Va – voltage magnitude measurement at bus "a". All the numbers in these Tables are average values **(AV),** with very small STD, corresponding to thirty different cases.

Table II – Measurements Scenario –I: Gross Error Detection Level (AV)

	Active Power	*UI* (a)	Detec. Level (STD)	React Power	*UI* (r)	Detec. Level (STD)
I:3	-0.942	0.56	3.8	0.099	1.40	5.3
I:8	-0.150	0.70	3.9	0.308	0.72	3.9
I:9	-0.295	0.84	3.5	-0.166	0.78	4.5
I:10	-0.090	3.52	9.7	-0.058	2.77	12.5
I:11	-0.035	4.21	15.4	-0.018	3.34	9.9
I:13	-0.135	5.89	23.6	-0.058	5.45	25.7
I:14	-0.149	1.50	4.5	-0.050	1.66	3.3
F:1-2	1.656	0.75	3.4	-0.223	1.50	5.2
F:2-1	-1.601	0.74	3.4	0.311	0.89	3.9
F:1-5	0.852	0.34	3.2	0.058	3.31	10.8
F:5-1	-0.817	0.34	3.2	0.035	3.74	11.8
F:3-4	-0.174	3.17	9.2	0.061	1.05	4.3
F:4-5	-0.712	0.96	4.4	0.164	0.88	4.0
F:5-4	0.7165	0.98	4.6	-0.142	1.14	4.5
F:4-7	0.4929	0.49	3.7	-0.119	0.8	3.8
F:4-9	0.2189	0.39	3.6	0.021	1.14	3.6
F:9-4	-0.219	0.39	3.0	0.004	8.48	18.9
F:11-6	-0.121	0.64	3.0	-0.051	0.62	3.0
F:6-12	0.085	9.73	36.4	0.027	12.81	17.4
F:7-8	0.149	0.71	3.9	-0.291	0.69	3.7
F:8-7	-0.149	0.71	3.7	0.309	0.72	3.5
F:7-9	0.143	2.26	6.7	0.019	11.26	38.9
F:11-10	0.087	1.04	3.5	0.033	1.14	6.1
F:13-14	0.089	0.87	3.5	0.028	1.02	3.0
	Magnitude		***UI***		**Detection Level**	
V:1	1.060		0.21		3.0	
V:3	1.010		0.21		3.0	
V:8	1.090		0.21		3.0	
V:13	1.048		0.21		3.0	
The *UI* indexes were obtained using the estimated states.						

In Table II, using the paper proposition, the *UI* for each measurement was calculated, as well as the minimum gross error value required, detection level, in order that the error can be detected by means of the largest normalized residue test, considering a threshold value of $\lambda = 3$.

As it can be seen in that table, there is a close relation between the detection level and the measurement undetectability index. Measurements with small *UIs*, in general less than one, have gross errors detected /identified, using the largest normalized residue test with the detection level very close to 3.0STD.

On the other hand, measurements with high *UI* require very large gross errors in order to be detected/identified that is, the masked error due to the estimating process is high. As a consequence, the largest normalized residual test may fail, in a significant way, when the gross errors are in those measurements.

To solve this kind of problem, the literature on gross error detection/identification establishes the solution as increasing the measurement set redundancy level.

However, using a very high redundant measurement set scenario this paper's results do not confirm that affirmative.

Scenario-II: In this measurement scenario, the measurement set with a very high redundancy level, as shown in Table III, is used.

Obs.: Although voltage magnitude measurements are used in this measurement set scenario (V:1, V:3, V:8, and V:13), they do not appear in the corresponding tables due to space limitation.

In Table III it can be seen that although the gross error detection/identification results were improved, when compared to results of Table II, some measurements still present high *UIs* and require very high gross errors level in order to be detected/identified. As a consequence, the usual gross error detection/identification test on those measurements will have a high chance of failing.

An example of the measurement redundancy level action on the UI of a measurement it can be seen at the active power flow F3-4: in the Measurement Scenario I its UI was 3.17 while in the Measurement Scenario II its UI is now 0.58.

An example of measurement of high UI is the active and reactive power injections I:5, shown in Table III.

VII. MEASUREMENTS OF HIGH UI VERSUS LEVERAGE POINTS

In this section, with the Measurement sets Scenarios **I** an **II,** and the available literature for classification of leverage points [13], it will be shown that the UI index presents a more wide and comprehensive picture of the problem of gross error detection in power system state estimation than critical measurements and leverage points.

In the Measurement set Scenario I, Table II, and applying the formulation as in [13], pg. 131, do not exist any measurement classified as suspected of being leverage point. However many measurements of that Table present a very high gross error detection level. That is, they are measurements whose gross error detection/identification test may fail in a significant way.

Again but now using the Measurement set Scenario II, and with the formulation of [13] the following measurements are classified as leverage points:

(i) The active powers: I5, I6, I7, I11, F9-10, and F10-9

(ii) The reactive powers: I4, I5, I7, I11, I12, F2-3, F2-4, F9-4, F7-9, F9-7.

Using the UI index, as can be seen in Table III, the following power measurements are not measurements having a high detection level:

(i) Active measurements: I7, F9-10, F10-9

(ii) Reactive measurements: In this case all the results matched with the paper proposition.

In other hand, are measurements of high gross error detection level and not suspected of being leverage points:

(i) Active measurements: I2

(ii) Reactive measurements: No one measurement found.

As conclusion one can say that for the measurement set of high redundancy level the results using the paper proposition matched reasonably well with that of reference [13]. However for the measurement set of low redundancy level the results are completely different, but the paper proposition is in accord with performed tests.

Table III – Measurements Scenario –II: Gross Error Detection Level (AV)

	Active Power	*UI* (a)	Detec. Level (STD)	React Power	*UI* (r)	Detec. Level (STD)
I:1	2.501	0.19	3.4	-0.166	0.77	5.4
I:2	0.384	1.62	7.8	0.358	0.27	3.6
I:3	-0.942	0.19	3.1	0.099	0.51	4.3
I:4	-0.477	0.77	3.1	0.039	1.98	3.5
I:5	-0.076	5.79	24.3	-0.016	4.19	13.7
I:6	-0.112	1.36	3.6	0.118	0.46	4.1
I:7	-0.199	0.99	3.5	-0.100	1.67	6.7
I:8	-0.150	0.66	3.7	0.308	0.33	3.6
I:9	-0.295	0.40	3.0	-0.166	0.25	3.2
I:10	-0.090	0.40	3.5	-0.058	0.43	3.6
I:11	-0.035	2.47	8.4	-0.018	2.14	6.5
I:12	-0.061	0.63	3.0	-0.016	0.98	5.9
I:13	-0.135	0.67	3.8	-0.058	0.60	3.0
I:14	-0.149	0.45	3.2	-0.050	0.50	3.1
F:1-2	1.657	0.22	3.0	-0.223	0.38	3.5
F:2-1	-1.601	0.22	3.0	0.311	0.29	3.0
F:1-5	0.852	0.15	3.3	0.058	0.54	3.6
F:5-1	-0.817	0.15	3.0	0.035	0.76	5.5
F:2-3	0.794	0.15	3.2	0.030	1.26	5.7
F:3-2	-0.768	0.15	3.2	0.039	0.77	3.0
F:2-4	0.684	0.15	3.0	-0.002	9.10	5.9
F:4-2	-0.655	0.15	3.0	0.042	0.34	3.0
F:2-5	0.512	0.15	3.4	0.021	0.28	5.4
F:5-2	-0.498	0.15	3.0	-0.016	0.73	4.2
F:3-4	-0.174	0.58	3.3	0.061	0.40	3.4
F:4-3	0.178	0.58	3.0	-0.068	0.32	3.1
F:4-5	-0.712	0.22	3.3	0.164	0.18	3.0
F:5-4	0.716	0.22	3.3	-0.142	0.18	3.3
F:4-7	0.493	0.21	3.3	-0.119	0.25	3.0
F:7-4	-0.493	0.21	3.0	0.172	0.15	3.2
F:4-9	0.219	0.19	3.3	0.021	0.44	3.0
F:9-4	-0.219	0.19	3.0	0.004	2.95	13.0
F:5-6	0.524	0.16	3.2	0.107	0.28	3.0
F:6-5	-0.525	0.16	3.2	-0.046	0.71	5.7
F:6-11	0.124	0.31	3.0	0.055	0.30	3.0
F:11-6	-0.121	0.31	3.0	-0.051	0.33	3.0
F:6-12	0.085	0.33	3.1	0.027	0.40	3.0
F:12-6	-0.084	0.33	3.0	-0.025	0.42	4.1
F:6-13	0.203	0.27	3.4	0.083	0.28	3.2
F:13-6	-0.201	0.26	3.2	-0.077	0.29	3.0
F:7-8	0.149	0.66	3.9	-0.291	0.32	3.0
F:8-7	-0.149	0.66	3.5	0.309	0.33	3.3
F:7-9	0.143	0.70	3.6	0.019	0.89	4.5
F:9-7	-0.143	0.70	3.6	-0.017	1.10	5.4
F:9-10	0.004	0.99	4.8	0.026	0.88	4.2
F:10-9	-0.004	0.99	4.8	-0.026	0.87	4.2
F:9-14	0.062	0.77	4.5	0.026	0.70	5.2
F:14-9	-0.062	0.75	4.5	-0.025	0.72	3.0
F:10-11	-0.086	0.42	3.4	-0.032	0.49	3.9
F:11-10	0.087	0.42	3.0	0.033	0.47	4.8
F:12-13	0.023	0.78	3.7	0.009	0.77	3.3
F:13-12	-0.023	0.77	4.9	-0.009	0.77	6.3
F:13-14	0.089	0.47	3.0	0.028	0.53	3.0
F:14-13	-0.087	0.46	3.0	-0.025	0.61	4.2

The *UI* indexes were obtained using the estimated states.

VIII. CONCLUSIONS

In this paper, more insights related to detectability of gross errors in power system state estimation, using geometrical approaches, are provided. This is achieved decomposing the measurement error vector into two parts: the undetectable part and the detectable one. The ratio between the w-norm of those quantities, the undetectability index, gives the distance of a measurement from the range of the jacobian matrix. In other words, the UI is a measure of how difficult is to detect errors in those measurements. That index does not depend on any classification, in terms of gross errors detection, those measurements might have. The paper's results lead to the conclusion that the higher a UI of a measurement is, in general, the higher the masked error will be. It is shown that the UI index presents a more comprehensive picture of the problem of gross error detection in power system state estimation than critical measurements and leverage points. Even more, for the measurement set scenario of low redundancy level, there exist a big difference between the conclusion about gross error detection level test when using the paper proposition and the one of reference [13].

IX. REFERENCES

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APPENDIX: DETECTION AND IDENTIFICATION OF GROSS ERRORS

Given a measurement vector *z*, the estimated state $\hat{x}$ will depend on the projection operator *P*, which depends only on the inner product choice *(W)* and matrix *H*.

Matrix "*W*" is given by the inverse of the measurement covariance matrix. After finding $\hat{x}$, it is interesting to check the existence of gross errors in the measurements. Therefore a routine for error detection is required. At this point only statistic concepts are required.

Assuming that measurement errors have a normal distribution, it is easy to show that index $J(\hat{x})$, i.e., the function to be minimized in (7), has a Chi-square distribution (χ^2) with *(m-n)* degrees of freedom. Choosing a probability "1-α" of false alarm and being "α" the significance level of the test, a number "*C*" is obtained (via Chi-square distribution table for $\chi^2_{m-n,1-\alpha}$) such that, in the presence of gross errors $J(\hat{x})$ > *C*.

Another way to detect the presence of gross errors is by the largest normalized residue test. Based on the same assumption about measurement errors, the vector of residuals *r* is normalized and subjected to a validation test: $r(k)^N = \left|r(k)\right| / \sigma_r(k) \leq \lambda$ (threshold value), where $r(k)^N$ is the largest among all $r(i)^N, i = 1,\ldots,m$; $\sigma_r(k) = \sqrt{\Omega(k,k)}$ being the Standard Deviation of the k^{th} component of the residuals vector, and Ω the residue covariance matrix given by $\Omega = W^{-1} - H(H^T W H)^{-1} H^T$.

If $r(k)^N > \lambda$, the measurement with gross error is detected and the k^{th} measurement will be the one with gross error (usually $\lambda = 3$ [13]).

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