Optimal Utilization of Transmission Capacity to Reduce Congestion with Distributed FACTS

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Abstract-- **The Flexible AC Transmission System (FACTS) device has been applied to enhance the controllability of power systems. A new generation of FACTS device called Distributed FACTS such as distributed series impedance or distributed static series compensator has recently received increasing interests for power system control and are expected to be broadly deployed. When technology of D-FACTS is being further advanced, there is an increasing interest to find the optimal locations of these devices. This paper presents a detailed formulation and algorithm to find the best location and size of D-FACTS to achieve the optimal utilization of transmission capacity to mitigate congestion. Simulation results are presented with the PJM 5-bus system.**

Index Terms—**D-FACTS; distributed series impedance; distributed static series compensator; optimal placement; OPF.**

I. INTRODUCTION

THE Flexible AC Transmission System (FACTS) device has been applied to enhance the controllability of power systems. However, the high cost and reliability concern restrict the wide deployment of the conventional FACTS devices. The emerging low-cost distributed FACTS (D-FACTS) supplies an alternative solution for the flexible control of the power systems. The typical D-FACTS device like distributed series impedance (DSI) or distributed static series compensator (DSSC), which can be easily deployed on the existing power lines to change the impedance of line, receives increasing interests and are expected to be broadly deployed to mitigate transmission stress [1]. When technology of D-FACTS is being further advanced, there is an increasing interest to find the optimal locations of these devices.

The techniques to find the good location and size of FACTS can be classified as sensitivity-based or optimization-based. The sensitivity-based approach typically use some indices based on sensitivity of a technical measure, such as a modal controllability index to damp inter-area oscillations [2] or the real power flow performance index with respect to control parameters [3-5]. The optimization-based approaches

are more systematic because interaction of different FACTS device can be simultaneously considered. The objective functions in optimization model are typically represented by minimization of generation cost [6, 7], minimization of losses [3], maximization of Available Transfer Capability or Total Transfer Capability [5, 8, 9]. With the optimization model, different solution techniques can be applied. For instance, References [7] apply linearized network model for DC power flow with mixed integer linear programming technique. Reference [10] uses genetic algorithm to determine the location as well as the parameters of TCSC simultaneously. Reference [9] applies Particle Swamp Optimization (PSO) to achieve maximum system loadability with minimum cost of installation of FACTS devices.

Different from the previous optimization model that optimizes the installation of conventional FACTS, this work optimizes the new, simply structured, low-cost D-FACTS devices like DSI or DSSC. And, the technical goal of installing these D-FACTS is to mitigate transmission stress. So, we use the system-wide line flow margin as the goal to formulate our objective function. Also, linearized DC model is applied to easily model power flow as constraints since a large number of independent, line-flow-affecting variables must be considered due to the expected large-scale deployment of D-FACTS. In addition, compensators are modeled with susceptance to be easily included in the optimization. These are the unique features of this work.

This paper is organized as follows. Section II gives the generic problem formulation. Section III presents the solution techniques using deepest descent algorithm, including the case with inequality constraints. Section IV presents test results, and Section V concludes the paper.

II. PROBLEM FORMULATION

This section will formulate the proposed optimization model. In the discussion below, it is assumed that generation dispatch and load are known from typical or historical data. The problem here is for the transmission owners or operators to identify the best location and size of series compensators. This can be interpreted as solving this problem: *Given one or more typical patterns of generation dispatch, where and how much should the series compensators be placed?* Ideally, more than one typical patterns (such as peak load, shoulder load, or valley load) should be addressed. However, this paper considers only a single pattern (like peak load case) for easy

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illustration. A combination of multiple patterns may be addressed in the future work.

The power flow is modeled with linearized DC model, which stems from the fast decoupled power flow and assumes voltage is 1.0 across the system and there is no real power loss in power lines. The reason for using linearized DC model is that it is robust and easy to solve, while using an AC-based optimization model may suffer from convergence problem and be sensitive to initial input. This is especially important when considering many distributed FACTS devices to be deployed in a large power system.

With the linearized DC power flow model, line flow at the *kth* line (connecting buses *i* and *j*) can be calculated using the following equation:

$$F_k = F_{ij} = \frac{\delta_{ij}}{x_{ij}} = \frac{\delta_i - \delta_j}{x_{ij}} = b_{ij} \cdot (\delta_i - \delta_j) \tag{1}$$

where

F_k = the line flow at Line k, which connects two buses, *i* and *j*;

δ_i, δ_j = the voltage angles at Bus *i* and Bus *j*, respectively;

x_{ij} = the reactance of line *k*;

b_{ij} = the susceptance of the (*i,j*) entry in Y_{bus}, which is equal to $\frac{1}{x_{ij}}$ in value.

The optimization can be written in an easy-to-understand formulation as follows:

$$\text{Min} \sum_{k=1}^{M} \left(F_k^2 - F_k^{\max 2} \right)^2 \tag{2}$$

Subject to:
$$[\mathbf{B}] \cdot \mathbf{\delta} = \mathbf{P} \tag{3}$$

$$F_k^2 = F_{ij}^2 = \left(b_{ij} \cdot (\delta_i - \delta_j) \right)^2 \leq F_k^{\max 2} \tag{4}$$

$$b_{ij} = b_{ij,0} + b_{ij,c} \tag{5}$$

$$b_{ij,c}^{\min} \leq b_{ij,c} \leq b_{ij,c}^{\max} \tag{6}$$

where
N = the number of buses;
M = the number of lines;
F_k = the power flow through line k;
F_{kmax} = the flow limit at line *k*;
[**B**] = the nodal admittance matrix;
$\mathbf{\delta}$ = the vector of bus voltage angle, i.e., $[\delta_1, \delta_2, ..., \delta_N]^T$;
P = the vector of net nodal injection, i.e, $[P_1, P_2, ..., P_n]^T$;
$b_{ij,0}$ = the susceptance of the original (uncompensated) line;
$b_{ij,c}$ = the equivalent susceptance of series compensator.

In the above formulation, the independent variables (unknowns) are line susceptance, b_{ij}, which is the reciprocal of line reactance (or impedance in value since resistance is ignored). The susceptance is used instead of reactance to make the optimization model easy to solve. Since the uncompensated line has a fixed susceptance $b_{ij,0}$, the independence variables are essentially $b_{ij,c}$, i.e., the susceptance of series compensators. As shown in [1], the distributed series compensators consist of a set of switchable inductors or capacitors. So, it has minimum and maximum bounds as modeled in (6). It should be noted that here b_{ij} is represented as a continuous variable even though it is discrete in theory due to the possible step change characteristics of series compensators. This should be reasonable, especially at the planning stage. Since the series compensator may consist of many small inductors and capacitors, it can be roughly considered as a continuous variable. Also, the linearized DC model is suitable here to easily model line flows, because many independent, line-flow-affecting variables ($b_{ij,c}$) must be considered due to the expected large-scale deployment of D-FACTS.

The goal in this formulation is to achieve the lowest congestion with installation of D-FACTS. To do so, an expression of absolute values may be used, but that will be more difficult to solve. Hence, Eq. (2) uses the squared sum of squared capacity margins, which is to address the possible direction problem, of all transmission lines. The choice of $b_{ij,c}$ impacts the voltage angles, which determine the line flows. The solution shall give the best combination of $b_{ij,c}$ to achieve the optimization objective function.

In equation (3), the nodal injection is obtained as the product of a row in [**B**] and the vector **δ**. Different from a conventional OPF formulation for generation dispatch where [**B**] is known and **P** is unknown, this model considers **P** is known and [**B**] is unknown. The reason is that the generation dispatch is assumed to be known as a typical case like the peak-load case, as previously mentioned. Here, [**B**] is unknown since the susceptance of each compensator is an independent unknown variable. Hence, (2)-(6) can be rewritten as follows, after putting (4) into (2) and combining (5) and (6).

$$\text{Min} \sum_{k=1}^{M} \left[\left(b_{ij} \cdot (\delta_i - \delta_j) \right)^2 - \left(F_k^{\max} \right)^2 \right]^2 \tag{7}$$

Subject to:

$$\left(\sum_{j=1}^{N} b_{ij} \right) \delta_i - b_{i1} \cdot \delta_1 - b_{i2} \cdot \delta_2 - ... - b_{i,N} \cdot \delta_N = P_i$$
$$\text{for } i = 1, 2, ..., N. \tag{8}$$

$$F_k^2 = F_{ij}^2 = \left(b_{ij} \cdot (\delta_i - \delta_j) \right)^2 \leq F_k^{\max 2} \tag{9}$$

$$b_{ij,0} + b_{ij,c}^{\min} \leq b_{ij} \leq b_{ij,0} + b_{ij,c}^{\max} \tag{10}$$

III. SOLUTIONS WITH STEEPEST DESCENT ALGORITHM

A. Solution techniques – unconstrained case

To solve the above formulation in (7-10), we first formulate the solution procedures with the steepest descent algorithm [11] by ignoring (9) and (10), i.e., considering unconstrained case only. To do so, it is necessary to formulate the augmented objective function as follows:

$$C^* = f(\mathbf{b}, \boldsymbol{\delta}) - \boldsymbol{\lambda}^T \times g(\mathbf{b}, \boldsymbol{\delta}) \tag{11}$$

$$f(\mathbf{b}, \boldsymbol{\delta}) = \sum_{k=1}^{M} \left(\left(b_{ij} \cdot (\delta_i - \delta_j) \right)^2 - \left(F_k^{\max} \right)^2 \right)^2 ,$$
$$k \in \{all\ lines\} \tag{12}$$

$$g_i(\mathbf{b}, \boldsymbol{\delta}) = \left(\sum_{j=1}^{N} b_{ij} \right) \delta_i - b_{i1} \cdot \delta_1 - b_{i2} \cdot \delta_2 - ... - b_{i,N} \cdot \delta_N - P_i$$
$$i \in \{all\ buses\} \tag{13}$$

where

b = the vector of all M lines (vector of b_{ij}, not a row or column in **[B]**), which are the independent variables;

δ = the vector of all N bus angles (vector of δ_i), which are dependent state variables;

λ = the vector of all N Lagrangian multipliers for $g(\mathbf{b}, \boldsymbol{\delta})$.

If we apply steepest descent algorithm, we have

$$\nabla C^* = \left[\frac{\partial C^*}{\partial b} \right] = \left[\frac{\partial f}{\partial b} \right] - \left[\frac{\partial g}{\partial b} \right]^T \left[\left[\frac{\partial g}{\partial \delta} \right]^T \right]^{-1} \left[\frac{\partial f}{\partial \delta} \right] \tag{14}$$

In (14), we have

$$\frac{\partial f}{\partial b_{ij}} = 4 b_{ij} (\delta_i - \delta_j)^2 (b_{ij}^2 (\delta_i - \delta_j)^2 - F_{ij}^{\max 2}) \tag{15}$$

$$\frac{\partial f}{\partial \delta_i} = \sum_{j=1}^{M_i} 4 b_{ij}^{\,2} (\delta_i - \delta_j)(b_{ij}^{\,2} (\delta_i - \delta_j)^2 - F_{ij}^{\max 2}) \tag{16}$$

where

M_i = number of branches connected with bus i.

Also in (14), we have

$$\frac{\partial g}{\partial \delta} = B \tag{17}$$

$$\frac{\partial g}{\partial b} = \begin{bmatrix} \dfrac{\partial g_1}{\partial b_{line-1}} & \dfrac{\partial g_1}{\partial b_{line-2}} & \cdots & \dfrac{\partial g_1}{\partial b_{line-M}} \\ \dfrac{\partial g_2}{\partial b_{line-1}} & \dfrac{\partial g_2}{\partial b_{line-2}} & \cdots & \dfrac{\partial g_2}{\partial b_{line-M}} \\ \vdots & \vdots & \vdots & \vdots \\ \dfrac{\partial g_N}{\partial b_{line-1}} & \dfrac{\partial g_N}{\partial b_{line-2}} & \cdots & \dfrac{\partial g_N}{\partial b_{line-M}} \end{bmatrix} \tag{18}$$

Assume Line k connects Buses i and j. Then, $\dfrac{\partial g_l}{\partial b_{line-k}}$ in (18) is equal to 0 if $l \neq i$ and $l \neq j$; $\delta_i - \delta_j$ if $l=i$; and $\delta_j - \delta_i$ if $l=j$.

Then common iterative approach of steepest descent algorithm can be applied to solve the unconstrained case. The procedure is as follows:

1. Let k=0, start from an initial vector $\mathbf{b}^k = \mathbf{b}^0$.
2. Given $\mathbf{b}^k$, solve the DC power flow ($g(\mathbf{b}, \boldsymbol{\delta}) = 0$) to obtain $\boldsymbol{\delta}$, the angle vector of the buses.
3. Calculate C^{*k+1} and ∇C^{*k+1}. If $\left\| C^{*k+1} - C^{*k} \right\|$ is less than the predefined tolerance, Stop and the optimal solution is found. Otherwise go to 4.
4. Update $\mathbf{b}^{k+1} = \mathbf{b}^k - \gamma \cdot \nabla C^{*k}$, here γ is user defined step size. Go to Step 2 to continue the next iteration.

B. Solution techniques – considering capacity limits constraints cases

The system has two inequality constraints: inequality constraints on independent variables as (10), which represent the capacity limit of DSSC, and inequality constraints on dependent variables as (9), which represent the thermal capacity limit of the power lines. For the former one, we can set the independent variables to the active limit when the limit is violated and update the other variables. For the latter constraints, a quadratic penalty function method is applied.

$$p(h_i(b, \delta)) = s \cdot (h_i(b, \delta))^2 \tag{19}$$

$$s = \begin{cases} 0, & when\ h_i(b, \delta) < -t \\ p, & when\ h_i(b, \delta) \ge -t \end{cases} \tag{20}$$

$$h_i(b, \delta) = b_{ij}^2 (\delta_i - \delta_j)^2 - F_{ij}^{\max 2} \tag{21}$$

where

t is user defined tolerance from the limit;
p is the predefined penalty parameter.

Therefore, the augmented cost function can be written as:

$$C^* = \sum_{k=1}^{M} \left(b_{ij}^{\,2} \cdot (\delta_i - \delta_j)^2 - F_k^{\max 2} \right)^2$$
$$+ \sum_{t=1}^{L} s \cdot \left(b_{ij}^{\,2} \cdot (\delta_i - \delta_j)^2 - F_t^{\max 2} \right)^2 \tag{22}$$
$$- \sum_{i=1}^{n} \lambda_i \left[\left(\sum_{j=1}^{N} b_{ij} \right) \delta_i - b_{i1} \cdot \delta_1 - b_{i2} \cdot \delta_2 - ... - b_{i,N} \cdot \delta_N - P_i \right]$$

where L is the number of branches whose line flow is very close to or exceeds the limit.

Compared (22) with (11)-(13), it is apparent that only $f(\mathbf{b}, \boldsymbol{\delta})$ has been changed, while $g(\mathbf{b}, \boldsymbol{\delta})$ keeps unchanged. So, applying steepest descent algorithm, we have the same

$\dfrac{\partial g}{\partial b}$ and $\dfrac{\partial g}{\partial \delta}$; while only $\dfrac{\partial f}{\partial b}$ and $\dfrac{\partial f}{\partial \delta}$ need to be changed as follows:

$$\frac{\partial f}{\partial b_{ij}} = 4(1+s)b_{ij}(\delta_i - \delta_j)^2(b_{ij}^2(\delta_i - \delta_j)^2 - F_{ij}^{\max 2}) \quad (23)$$

$$\frac{\partial f}{\partial \delta_i} = \sum_{j=1}^{M_i} 4b_{ij}^{\,2}(\delta_i - \delta_j)(b_{ij}^{\,2}(\delta_i - \delta_j)^2 - F_{ij}^{\max 2}) +$$
$$\sum_{k}^{M_i} 4s\, b_{ik}^{\,2}(\delta_i - \delta_k)(b_{ik}^{\,2}(\delta_i - \delta_k)^2 - F_{ik}^{\max 2}) \quad (24)$$

The iteration procedure is very similar to what is presented in Section III(A).

It should be noted that since this is based on the linearized DC power flow model, the line flow is impacted by line susceptance only. Hence, there could be multiple solutions, especially if shunt admittances are all ignored. For instance, we have a solution with all line susceptance to be 1.0 p. u. Then, if all line susceptances are doubled, then the voltage angles will be only 50% of the values in the original solution. Since line flow is the voltage angle difference multiplied by line admittance, the line flow will be unchanged because we have $2b_{ij} \times 0.5(\delta_i - \delta_j) = b_{ij} \times (\delta_i - \delta_j)$. Fortunately, if we start with all b_{ij} as the base case b_{ij} (no series compensation at all), the solution should be the one closest to this initial condition. Therefore, it is avoided that the optimization model gives a large deviation in line susceptance which means unnecessary over series compensation on many lines.

IV. TEST RESULTS

The tests are carried out on the PJM-5-bus system, which is shown in Fig. 1. The uncompensated system data and the line flow are shown in Table 1. Two cases are tested. In Case 1, the only binding limits are the distributed series compensators' capacity limits. In Case 2, both the distributed series compensators' capacity limits and line thermal capacity limits are binding. It should be noted that the line flow limits in Case 3 are carefully set to lead a few binding transmission limits for testing purpose. Results as well as the input data are shown in Table 2 and Table 3.
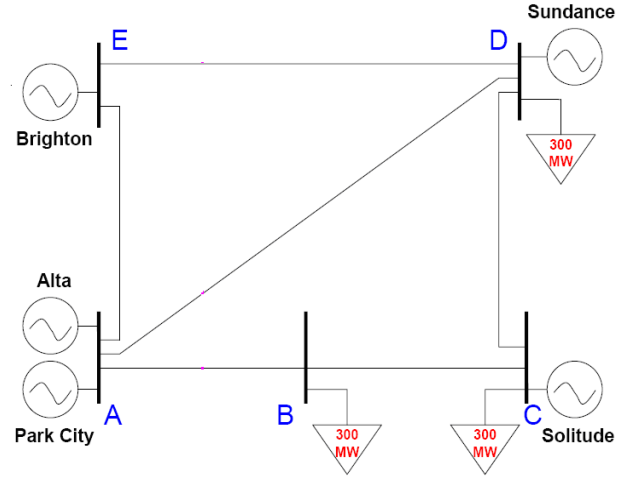

Fig.1. The PJM-5-bus system.

Table 1. Uncompensated system data

Line #	From Bus #	To Bus #	$b_{initial}$(p.u.)	Line Flow (MW)
1	A	B	35.587	377.32
2	A	D	32.894	158.68
3	A	E	156.250	-360.00
4	B	C	92.592	77.32
5	C	D	33.670	-222.68
6	D	E	33.670	-240.00

Table 2. The data and results of Case 1

Line #	b_{min} (p.u.)	b_{max} (p.u.)	Max Flow (MW)	b (p.u.) at Optimum	Line flow (MW) at Optimum
1	28.469	35.587	500	**28.469**	356.80
2	26.315	32.894	500	**32.894**	194.87
3	125.000	156.250	500	**156.250**	-375.67
4	74.074	92.592	500	92.427	56.80
5	26.936	33.670	500	**33.670**	-243.20
6	26.936	33.670	500	**26.936**	-224.33

Table 3. The data and results of Case 2

Line #	b_{min} (p.u.)	b_{max} (p.u.)	Max Flow (MW)	b (p.u.) at Optimum	Line flow (MW) at Optimum
1	28.469	35.587	380	**35.587**	**380**
2	26.315	32.894	160	32.749	**160**
3	125.000	156.250	380	**156.250**	-364
4	74.074	92.592	100	**92.593**	80
5	26.936	33.670	250	33.052	-220
6	26.936	33.670	240	32.707	-236

Table 2 and Table 3 show that the algorithm can effectively identify both types of binding inequality constraints in (9) and (10), shown in bold in the two tables.

The test results, especially in Case 1, demonstrate that the proposed formulation and algorithm tends to have each line

flow away from their individual limit as much as possible to have a good mitigation of the overall congestion problem.

In addition, the optimum solution for the susceptance of each line is in the reasonable region without unnecessary over compensation problem.

V. CONCLUSIONS

This research work presents an optimization model that can achieve the best mitigation of transmission congestion with D-FACTS (such as distributed series impedances or distributed static series compensators), which are expected to be deployed in large scales. The proposed model is shown to be effective. Future work may lie in modeling the accurate AC power flow constraints and a comparison with this work.

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