













Introduction	
Fuzzy Rule	
Fuzzy systems are essentially rule-based expert systems which consist of a set of linguistic rules in the form of "IF-THEN".),
Fuzzy IF-THEN rules:	
R^i : IF x_1 is F_1^i and and x_r is F_r^i , THEN y_1 is G_1^i and and y_s is G_s^i	
Takagi-Sugeno-Kang model:	
R^i : IF x_1 is F_1^i andand x_r is F_r^i , THEN $y^i = \alpha_0^i + \alpha_1^i x_1 + \cdots + \alpha_r^i x_r$	
Advantages of TSK model:	
Computational efficiency	
Works well with linear techniques	
Works well with optimization and adaptive techniques	
Guaranteed continuity of the output surface	
Better suited to mathematical analysis	8















Introduction								
Comparison								
Skills Fuzzy System Neural Networks								
Knowledge	Input	human expert	sample sets					
acquisition	Tools	interaction	algorithm					
l la conte inte	Information	quantitative and qualitative	quantitative					
Uncertainty	Cognition	decision making	perception					
Reasoning	Mechanism	heuristic search	parallel computation					
ricussining	Speed	high	low					
	Fault-tolerance	low	very high					
Adaptation	Learning	induction	adjusting weights					
Natural	Implementation	explicit	implicit					
language	Flexibility	high	low					



















Dynamic Fuzzy Neural Network							
Learning Algorithm of the D-FNN							
Rule Generation	Allocation of RBF Unit Parameters	Weight Adjustment	Rule Pruning				
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Dynamic Fuzzy Neural Network							
Learning Algorithm of the D-FNN							
Rule Generation	Allocation of RBF Unit Parameters	Weight Adjustment	Rule Pruning				
Error criterion	For observation	$ig(X_i,t_iig)$, define					
	$\left\ \boldsymbol{e}_{i}\right\ = \left\ \boldsymbol{t}_{i} - \boldsymbol{y}_{i}\right\ \geq$	$> k_e?$ An RBF consider	unit should be ed				
Accommodation	on criterion:						
$d_i(j) = X_i - C_i , j = 1, 2, \cdots, u$							
If $d_{\min} = \arg \min$	If $d_{\min} = \arg\min(d_i(j)) > k_d$, an RBF unit should be considered.						
Hierarchical learning: Accommodation boundary is adjusted							
dynamically	$k_e = \max[e_{\max} \times k_d = \max[d_{\max} \times k_d]$	$egin{array}{c} eta^i, e_{\min} \ \gamma^i, d_{\min} \ \end{bmatrix}$	se learning to le learning 27				

Dynamic Fuzzy Neural Network								
Learning Algorithm of the D-FNN								
Rule Generation	Allocation of RBF Unit Parameters	Weight Adjustment	Rule Pruning					
Allocation of p	remise paramet	er:						
$C_i = X$	$\kappa_i \qquad \sigma_i =$	$k \times d_{\min}$						
Case 1: $ e_i $	$> k_e, d_{\min} > k_d$	Rule generati	on					
Case 2: $ e_i \le k_e$, $d_{\min} \le k_d$ \square Do nothing or only update consequent parameters								
Case 3: $ e_i \le k_e$, $d_{\min} > k_d$ \square Only adjust consequent parameters								
Case 4: $ e_i $	$> k_e, \ d_{\min} \le k_d$	Update the w RBF node an	idth of the nearest d all the weights					
For the <i>k</i> th nea	rest RBF unit:	$\sigma_k^i = k_w \times \sigma_k^{i-1}$	1 28					

















Enhanced Dynamic Fuzzy Neural Network **ED-FIND Learning Algorithms** Adaptation of the Output Linear Weights: Recursive Linear Least Square (RLLS) estimator is used to adjust the weights RLLS: $s_0 = (\Psi_{i-1}\Psi_{i-1}^T)^{-1}$ $s_i = s_{i-1} - \frac{S_{i-1}\Psi_i\Psi_i^TS_{i-1}}{1+S_{i-1}}$ $W_i = W_{i-1} + S_{i-1}\Psi_i(T_i - \Psi_i^TW_{i-1}) \qquad i = 1, 2, \cdots, n$ where S_i is the error covariance matrix for the *i*th observation.















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Generalized Dynamic Fuzzy Neural Networks

Premise Parameter Estimation

Definition (Semi-Closed Fuzzy Sets): Let U = [a, b] be the universe of discourse of input x. If each fuzzy set $A_i = \{(x, \mu_i(x)) | x \in U\}$ ($i = 1, 2, \dots, m$) is represented as the Gaussian function $\mu_i(x) = \exp\left[-\frac{(x-c_i)}{\sigma_i^2}\right]$ for all $x \in U$, and satisfies the following boundary conditions simultaneously:

$$\mu_i(x) = \exp\left[-\frac{(x-a)}{\sigma_i^2}\right] \quad \text{if } |c_i - a| \le \delta$$
$$\mu_i(x) = \exp\left[-\frac{(x-b)}{\sigma_i^2}\right] \quad \text{if } |c_i - b| \le \delta$$
$$\mu_i(x) = \exp\left[-\frac{(x-c_i)}{\sigma_i^2}\right] \quad \text{if } |c_i - a| > \delta \text{ and } |c_i - b|$$

where δ is a tolerable small value, and the widths of Gaussian functions are selected as

 $> \delta$

 $\sigma_i(x) = \frac{\max\{|c_i - c_{i-1}|, |c_i - c_{i+1}|\}}{\sqrt{\ln(1/\varepsilon)}}, \ i = 1, 2, \cdots, m$

where c_{i-1} and c_{i+1} are the two centers of neighboring MFs of *i*th membership function. Then, we call A_i semi-closed fuzzy sets. 47

Generalized Dynamic Fuzzy Neural Networks

Premise Parameter Estimation

Theorem: The semi-closed fuzzy set A_i satisfy ε -completeness of fuzzy rules, i.e., for all $x \in U$, there exists $i \in \{1, 2, \dots, m\}$, such that $\mu_i(x) \ge \varepsilon$.

Calculate E-distance:

$$ed_i(j) = |x_i^k - \Phi_i(j)| \quad j = 1, 2, \cdots, u+2$$

where $\Phi_i \in \{x_{imin}, c_{i1}, c_{i2}, \dots, c_{iu}, x_{imax}\}$ and find

$$j_n = \arg\min_{j=1,2,\cdots,u+2} (ed_i(j))$$

If $ed_i(j_n) \leq k_{mf}$, x_i^k can be completely represented by the existing fuzzy set $A_{ij_n}(c_{ij_n}, \sigma_{ij_n})$ without generating a new MF. Otherwise, a new Gaussian MF is allocated with its width and center respectively being determined by

$$\sigma_i(x) = \frac{\max\{|c_i - c_{i-1}|, |c_i - c_{i+1}|\}}{\sqrt{\ln(1/\varepsilon)}}, \qquad c_{i(u+1)} = x_i^k$$

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Generalized Dynamic Fuzzy Neural Networks Sensitivity of Input Variables Define $B_j = \sum_{k=2}^{r+1} \rho_j(k)$ $j = 1, 2, \dots, u$ $B_{ij} = \frac{err_{ij}}{B_j}$ $i = 1, 2, \dots, r$ B_j -- total ERR related to input variables in *j*th rule err_{ij} -- ERR corresponding to *i*th input variable in *j*th rule B_{ij} -- The significance of *i*th input variable in *j*th rule B_{ij} is small, *i*th input variable is not sensitive to the system output, and consequently, width of the hyperellipsoidal region in *i*th input variable could be reduced without significant effect of system performance.















Mackey—Glass Time-Series Prediction Comparisons of structure and performance of different algorithms (Training case: n = 500, p = 6)								
	Technique	Number of rules	Number of parameters	RMSE of training	RMSE of testing			
	D-FNN	10	100	0.0082	0.0083			
	ANFIS	16	104	0.0016	0.0015			
	OLS	35	211	0.0087	0.0089			
	RBF-AFS	21	210	0.0107	0.0128			

Application 1: Mackey—Glass Time-Series Prediction								
eneralizatio	n comparison	between the	D-FNN and the	ANFIS for p=8				
Technique	Pattern	Number of	Number of	NDEI				
	length	rules	parameters					
D-FNN	500	14	140	0.3375				
ANFIS	500	16	104	0.036				
ANFIS500161040.036Remark: It is shown that the performance of the D-FNN is not so good as that of the ANFIS, even the D-FNN has more adjustable parameters. The reason is that the ANFIS is trained by iterative learning so that an overall optimal 								

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Gene RAN	eralization for <i>p</i> =50	ackey—G comparison	lass Time	-Series Pi IN and RAN, I	rediction RANEKF and M-
Т	echnique	Pattern	Number of	Number of	NDEI
		length	rules	parameters	
	D-FNN	2000	25	250	0.0544
	RAN	5000	50	301	0.071
F	RANEKF	5000	56	337	0.056
	M-RAN	5000	28	169	*









Applications Application 2: Prediction of Machine Health Condition									
Performance	Performance comparisons of all prediction methods								
Case Achieve a good trade-			t _{com}	$arphi_{train}$	$arphi_{pred}$				
Lineaccurac	cy and		8.2471	99.52	99.31				
mod	ational cost	I	1.3841	99.23	98.28				
	RRLS IN		5.2681	99.43	99.34				
	OD-FNN		1.6833	99.31	99.18				
Bea Best on	ine prediction		4.0210	98.38	96.01				
a	curacy	I	0.6765	97.31	95.35				
	RRL NN		2.5951	97.34	95.89				
	OD-FNN		1.6833	97.94	97.54				
t _{com} total	t_{com} total computational time φ_{train} online training accuracy φ_{nred} online prediction accuracy								



Applications		
	Application 3: Online Manipulator Control	
Manipulato	or dynamics:	
	$\mathbf{M}(q) \mathbf{q} + Q(q, \mathbf{q}) + \tau_d = \tau = \begin{bmatrix} \tau_1 & \tau_2 \end{bmatrix}^T$	
where $\tau_{1} = [(I_{1} + m_{1}l_{c1}^{2} + (I_{e} + m_{e}l_{c2}^{2}) + (I_{e} + m_{e}l_{c2}^{2}) + (I_{e}e^{l_{1}}l_{ce}\cos\delta_{e}) + (I_{e}e^{l_{1}}l_{ce}\cos\delta_{e}) + \tau_{d1}$	$+ I_{e} + m_{e} l_{ce}^{2} + m_{e} l_{1}^{2}) + 2(m_{e} l_{1} l_{ce} \cos \delta_{e}) \cos q_{2} + 2(m_{e} l_{1} l_{ce} \sin \delta_{e}) \sin q_{2}] \dot{q}_{1}$ $(m_{e} l_{1} l_{ce} \cos \delta_{e}) \cos q_{2} + (m_{e} l_{1} l_{ce} \sin \delta_{e}) \sin q_{2}] \dot{q}_{2}$ $i \sin q_{2} - (m_{e} l_{1} l_{ce} \sin \delta_{e}) \cos q_{2}] \dot{q}_{1} \dot{q}_{2}$ $i \sin q_{2} - (m_{e} l_{1} l_{ce} \sin \delta_{e}) \cos q_{2}] \dot{q}_{1} \dot{q}_{2}$	
$\tau_2 = [(I_e + m_e l_{ce}^2 + [(m_e l_1 l_{ce} \cos \delta_e) + \tau_{d2}])]$	$ + (m_{e}l_{1}l_{ce}\cos\delta_{e})\cos q_{2} + (m_{e}l_{1}l_{ce}\sin\delta_{e})\sin q_{2}]\dot{q}_{1} + (I_{e} + m_{e}l_{ce}^{2})\dot{q}_{2} + (m_{e}l_{1}l_{ce}\sin\delta_{e})\cos q_{2}]\dot{q}_{1}^{2} $	68



Applications								
Application 3: Online Manipulator Control								
Performance comparison of GD-FNN with PD and (adaptive fuzzy controller) AFC controllers:								
		Steady State Error (rad)	Overshoot (%)	Rise Time (sec)	Maximum Control Torque (Nm)			
PD	q ₁	0.050	9.3	0.287	312.7			
K _p =6000; K _ν =100	q ₂	0.018	7.1	0.254	109.9			
AFC	q ₁	0.022	0	0.237	425.9			
K _p =25; K _v =35	q ₂	0.025	0	0.218	255.6			
GD-FNN	\mathbf{q}_1	0.001	0	0.01	280.3			
K _p =25; K _v =7	q_2	0.001	0	0.01	110.0			
					70			











