# Single User or Multiple User ?

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Single- and Multi-User Communication

2 Superposition Coded Modulation



- Block Markov Superposition Transmission

#### 5 Conclusions

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#### 1 Single- and Multi-User Communication

2 Superposition Coded Modulation

#### 3 Kite Codes

- 4 Block Markov Superposition Transmission
- 5 Conclusions

# Single-User Communication System

C. E. Shannon, "A mathematical theory of communication," *Bell Sys. Tech. Journal*, 27, 379-423, 623-656, 1948.

Digital Communication Framework



Figure: Block diagram of communication system.

- A *source* is nothing more than and nothing less than an arbitrary random process;
- The task of the *encoder* is to transform the output from the source into signals that matched to the channel, which can be split into two parts:
  - Source encoder: Everything is binary!
  - Channel encoder: Key techniques in the physical layer.
- A *channel* transforms an input to an output in a random manner dominated by a probability transition law.

Shannon showed that a channel can be characterized by a parameter, C, called the *channel capacity*, which is a measure of how much information the channel can convey.

#### The Channel Coding Theorem

- codes exist that provide "reliable" communication provided that the code rate satisfies *R* < *C*;
- conversely, if R > C, there exists no code that provides reliable communication.

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## Capacity of the Ideal AWGN Channel

• Consider the discrete-time channel model,

$$Y_t = X_t + Z_t,$$

where  $E[X_t^2] < P$  and  $Z_t \sim (0, \sigma^2)$ .

• The capacity of the channel is given by

$$C = \frac{1}{2}\log_2(1 + \frac{P}{\sigma^2})$$
 bits/channel symbol.

• Frequently, one is interested in a channel capacity in units of bits per second rather than bits per channel symbol,

$$C = W \log_2(1 + \frac{P}{\sigma^2})$$
 bits/second.

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# Single-User Communication System



Figure: Capacity versus SNR curves for selected modulation schemes.

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# Power Efficiency

## Traditional Codes

- Hamming codes, Golay codes, Reed-Muller codes
- Bose-Chaudhuri-Hocquenghem codes, Reed-Solomon codes
- Convolutional Codes

## Capacity-Approaching Codes

- Turbo codes
- Low-density parity-check (LDPC) codes
- Repeat-accumulate codes
- Accumulate-repeat-accumulate codes
- Concatenated zigzag codes, concatenated tree codes
- Precoded concatenated zigzag codes
- Convolutional LDPC codes
- Polar codes

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## Existing Coded Modulation Schemes

- Trellis-coded modulation (TCM): proposed by Ungerboeck in 1982;
- Bit-interleaved coded modulation (BICM): proposed by Zehavi in 1992 for coding for fading channels;
- Multilevel codes (MLC): first proposed by H. Imai in 1977.

## Capacity-Approaching Coded Modulation Schemes

- Turbo-TCM schemes: two (or multiple) TCM codes are concatenated in the same fashion as binary turbo codes
- BICM with iterative decoding
  - The output stream of a binary (turbo or LDPC) encoder is bit-interleaved and then mapped to an M-ary constellation
  - The de-mapper is viewed as a APP decoder
- MLC with iterative multistage decoding
- Superimposed binary codes
- Coded modulation using non-binary LDPC codes

# Typical Multi-User Channels

#### Multiple-Access Channel



- two (or more) senders send information to a common receiver;
- senders must contend not only with the receiver noise but with interference from each other as well.

#### Broadcast Channel



- one sender send information to two or more receivers;
- the basic problem is to find the set of simultaneously achievable rates for communication.

# Typical Multi-User Channels

## Relay Channel



- one sender and one receiver with a number of relays;
- relays help the communication from the sender to the receiver.

#### Interference Channel



- two senders and two receivers;
- Sender 1 wishes to send information to receiver 1 without caring what receiver 2 receives or understands;
- Similarly with sender 2 and receiver 2.

# Typical Multi-User Channels

## Two-Way Channel



- The two-way channel is very similar to the interference channel;
- Sender 1 is attached to receiver 2 and sender 2 is attached to receiver 1;
- This channel introduces another fundamental aspect of network information theory: namely, feedback;
- Feedback enables the senders to use the partial information that each has about the other's message to cooperate with each other.

#### Question

Can we apply the strategies in the multi-user communication system to the single-user communication system?

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#### Single- and Multi-User Communication

#### 2 Superposition Coded Modulation

#### 3 Kite Codes

4 Block Markov Superposition Transmission

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# Superposition Coded Modulation

## Gaussian Multiple-Access Channel

• Two senders,  $X_1$  and  $X_2$ , communicate to the single receiver, Y. The received signal at time t is

$$Y_t = X_{1t} + X_{2t} + Z_t$$

where  $Z_t \sim (0, N)$ ;

• We assume that there is a power constraint  $P_j$  on sender j.

## Interpretation of the Corner Points

- Point A: the maximum rate achievable C(<sup>P<sub>1</sub></sup>/<sub>N</sub>) from sender 1 to the receiver when sender 2 is not sending any information.
- Point B: decoding as a two-stage process
  - The receiver decodes the second sender, considering the first sender as part of the noise. This decoding will have low probability of error if  $R_2 < C(\frac{P_2}{P_{1+N}})$ ;
  - After the second sender has been decoded successfully, it can be subtracted out and the first sender can be decoded correctly if R<sub>1</sub> < C(<sup>P<sub>1</sub></sup>/<sub>N</sub>).
- Points C and D correspond to B and A, respectively, with the roles of the senders reversed.



# Superposition Coded Modulation

## Main Ideas

- Key technology: successive cancellation decoding (consider the other as noise; subtract its effect);
- Higher spectral efficiency is achieved for larger number of users.

We proposed a coded modulation using superimposed binary codes.

## The Multilevel Coding/Sigma-Mapping Scheme

- Partition the information sequence into many subsequences;
- Each subsequence is encoded by a component code;
- Each coded sequence is then randomly-interleaved;
- All the random-interleaved versions are then mapped to a signal sequence by a sigma-mapper

[MA04]X. Ma and Li Ping, "Coded modulation using superimposed binary codes", IEEE Trans. Inform. Theory, 2004

# Multilevel Coding/Sigma-Mapping Scheme

## Multilevel Coding/Sigma-Mapping Scheme



• Component codes: turbo-like codes or LDPC codes;

- Power-allocation strategy:
  - A simulation-based recursive search algorithm;
  - Gaussian approximation power allocation.

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# Multilevel Coding/Sigma-Mapping Scheme

## Normal Graph



Figure: A normal graph for the multilevel coding/sigma-mapping scheme.

#### Decoding

- three kinds of nodes: C,  $\prod$  and  $\sum$ ;
- messages are processed and exchanged over the normal graph.

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## Simulation Result

#### Using ten Brink's doped code of length 500000 as component codes.



Figure: Performance of the four-level coding/sigma-mapping system with coding rate of 2 bits/dim.

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- The multilevel coding/sigma-mapping scheme is an instance of MLC scheme. For the conventional MLC with lattice-based constellations and set-partitioning-based bit mappings, different levels are usually protected by codes with different rates.
- In contrast, by choosing appropriate amplitudes α<sub>i</sub> in the multilevel coding/sigma-mapping systems, the component codes at different levels can be the same.
- The multilevel coding/sigma-mapping system can be treated as a multiuser system by viewing one level as one user. So it is not surprising that most important methods and results for the multiuser system are applicable here.
- Since the cooperation among different "users" is perfect, we are able to play more at both the transmitter and the receiver.

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# Time Varying Channels

#### Gaussian Broadcast Channels

- A sender of power P and two distant receivers;
- $Y_1 = X + Z_1$  and  $Y_2 = X + Z_2$ , where  $Z_1$  and  $Z_2$  are arbitrarily correlated Gaussian random variables with variances  $N_1$  and  $N_2$ , respectively.
- Assume that  $N_1 < N_2$ . That is, the receiver  $Y_1$  sees a better channel. The message consists of "common" information for both users and private information for  $Y_1$ .
- The results of the broadcast channel can be applied to the case of a single-user channel with an unknown distribution.
- The objective is to get at least the minimum information through when the channel is bad and to get some extra information through when the channel is good.

#### Time Varying Channels

- different channel states at different time;
- adaptive coding scheme: rateless coding.

A coding method that can generate potentially infinite parity bits for any given fixed-length sequence.

## Existing Rateless Codes

- LT-codes;
- Raptor codes.
- ...

## Motivations

- Raptor codes are optimized by degree distribution for erasure channels;
- No universal degree distributions exist for AWGN channels.
- How to construct good codes for AWGN channels with arbitrarily designated coding rate?

We will propose a class of rateless codes for AWGN channels.

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An ensemble of Kite codes with dimension k, denoted by  $\mathcal{K}[\infty, k; \underline{p}]$ , is specified by a real sequence p, called *p*-sequence.

#### Encoder of Kite Codes

$$\underline{c} = (v_0, \cdots, v_{k-1}, w_0, w_1, \cdots, w_t, \cdots)$$



- Initially, load information sequence of length k into a buffer.
- At time t ≥ 0, randomly choose, with success probability pt, several bits from the buffer.
- Calculate the XOR of these chosen bits and use it to drive the accumulator to generate a parity bit  $w_t$ .

Image: A math the second se

For any given  $n \ge k$ , the prefix code with length n of a Kite code  $\mathcal{K}[\infty, k; \underline{p}]$  is denoted by  $\mathcal{K}[n, k]$  and also called Kite code.

From the encoding process of Kite codes, we can see that the parity-check matrix of Kite codes has the following form.



By choosing  $p_t \ll 0.5$ , we can construct Kite codes as LDPC codes. If so, the receiver can perform the iterative sum-product decoding algorithm.

A specific Kite code (a realization of Kite code) is a kind of LDPC code, which is closely related to generalized IRA code.

A specific Kite code can also be considered as a partially serially concatenated code with a systematic LDGM code as outer code and an accumulator as an inner code.



However, as an ensemble, Kite codes are new.

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A binary linear code *ensemble* is a probability space (C, Q)-a sample space C and a probability assignment  $Q(\mathbf{C})$  to each  $\mathbf{C} \in C$ . Each sample  $\mathbf{C} \in C$  is a binary linear code, and the probability Q(C) is usually implicitly determined by a random construction method.

- Code ensemble  $C_g$ : random generator matrix G of size  $k \times n$ .
- **Q** Code ensemble  $C_h$ : random parity-check matrix H of size  $(n k) \times n$ .
- Solution Code ensemble  $C_s$ : random parity-check matrix  $H_s = [P, I_{n-k}]$ .



An ensemble of Kite codes (of length n) with  $p_t < 1/2$  has the same sample space as that of  $C_s$  but different probability assignment to each code.

An ensemble of general LDPC code is specified by the a pair of degree distributions

$$\lambda(x) = \sum \lambda_i x^i \text{ and } \rho(x) = \sum \rho_i x^i,$$

where  $\lambda_i$  and  $\rho_i$  are fractions. Given  $\lambda_i > 0$ , there must exist nodes of degree *i*.

These fractions are fixed.

An ensemble of Kite code is specified by the p-sequence. Its degree distributions are

$$\lambda(x) = \sum \lambda_i x^i \text{ and } \rho(x) = \sum \rho_i x^i,$$

where  $\lambda_i$  and  $\rho_i$  are probabilities. Even if  $\lambda_i > 0$ , it is possible for a specific Kite code to have no nodes of degree *i*.

# Design of Kite Code

#### **Original Problem**

Evidently, the performance of Kite codes is determined by the p-sequence. The whole p-sequence should be optimized jointly such that all the prefix codes of Kite codes are good enough, which is too complex to implement.



Too complex due to too many (may be infinite) variables involved in.

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# Design of Kite Code: A Simple Idea (layer by layer)

Partitioning the coding rate into 9 subintervals.



- Firstly, we choose  $q_9$  such that the prefix code  $\mathcal{K}[\lfloor k/0.9 \rfloor, k]$  is as good as possible.
- Secondly, we choose  $q_8$  with fixed  $q_9$  such that the prefix code  $\mathcal{K}[\lfloor k/0.8 \rfloor, k]$  is as good as possible.
- Thirdly, we choose  $q_7$  with fixed  $(q_9, q_8)$  such that the prefix code  $\mathcal{K}[\lfloor k/0.7 \rfloor, k]$  is as good as possible.
- we choose q<sub>1</sub> with fixed

   (q<sub>9</sub>, q<sub>8</sub>, ..., q<sub>2</sub>) such that the prefix code K[[k/0.1], k] is as good as possible.

At each step, it is a one-dimensional optimization problem and can be implemented with density evolution or simulations.

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Coding Group

...

## Numerical Results

With data length k = 1890 and rates from 0.1 to 0.9, we have the following curves.



Issue: Error floors.

[MA11]X. Ma *et al*, "Serial Concatenation of RS Codes with Kite Codes: Performance Analysis, Iterative Decoding and Design", http://arxiv.org/abs/1104.4927, 2011 [BAI11]B. Bai, B. Bai, X. Ma, "Semi-random Kite Codes over Fading Channels", AINA 2011

## Numerical Results (continued)

With k = 50000, we utilize RS code as outer codes to lower down the error floor.



Issue: Relative large gap between the performance and the Shannon limits.

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- Issue I: In the high-rate region, there exist error floors, which is caused by the existence of all-zero (or extremely-low-weight) columns in the randomly generated matrix H<sub>v</sub>.
- Issue II: In the low-rate region, there exists a relatively large gap between the performances of the Kite codes and the Shannon limits.
- **Solution** Ill: The optimized p-sequence depends on the data length k.

The objective of this work is to solve these issues in simple ways.

We partition the coding rates into 20 intervals.

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# Row-weight concentration algorithm: to lower down the error floor.

Given the parity-check matrix constructed layer by layer, we swap the "1"s and "0"s within each layer as follows.



# Accumulator randomization algorithm: to mitigate the performance loss.

To introduce more randomness in the dual-diagonal matrix.



## Improved design: Numerical result

Improved Kite codes with data length k = 1890 are constructed and the performances are shown as below.



#### Remark: lower error-floors, better performances.

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## Improved design: Numerical result

Improved Kite codes with data length k = 3780 are constructed and the performances are shown as below.



#### Remark: lower error-floors, better performances.

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# Improved design: Numerical result



- In high-rate region: the row weight concentration algorithm lowers down the error-floors.
- In low-rate region: the row weight concentration algorithm and the accumulator randomization algorithm gain about 0.9 dB.
- In moderate-rate region: no much gain.

To accelerate the design of improved Kite codes, we present the following empirical formula for the *p*-sequence.

$$q_{\ell} = \frac{1}{k} \left( \frac{1.65}{(1.5 - 0.05\ell)^6} + 2.0 \right)$$

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The procedure to construct an improved Kite code:

- Calculate the *p*-sequence according to the empirical formula;
- Sandomly generate the parity-check matrix according to the p-sequence;
- Onduct the Row weight concentration algorithm;
- Conduct the Accumulator randomization algorithm.

#### Improved design: Numerical result

A Kite code with data length k = 9450. The average decoding rates (at "zero" error probability) of this improved Kite code over AWGN channels is shown below.



The system model for adaptive coded modulation is shown below.



#### Rate-compatible codes and adaptive coded modulation

The average decoding spectral efficiency (at "zero" error probability) of the improved Kite code with data length k = 9450 over AWGN channels.



Given

- any code length k,
- any code rate r,

we construct a well-performed binary LDPC code.

#### Application

- Broadcasting common information;
- Adaptive coded modulation;
- Easily extended to group code[MA11b];
- Joint source-channel code[YANG12];
- Useful for research.

[MA11a]X. Ma *et al*, "Kite codes over groups", ITW 2011 [YANG12]Z. Yang, S. Zhao, X. Ma and B. Bai "A new joint source-channel coding scheme based on nested lattice codes", IEEE Communication Letters 2012

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Pair of rates: (0.48, 0.88).

Performance of Two Users with different K1 and K2 User KiteConv User Kite BER = 10<sup>-4</sup> -1 -2 --3 BER -1 -5 -6 0.85 0.48 0.8 0.46 0.44 0.75 0.42 0.7 0.4 code rate of User Kite code rate of User KiteConv

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A rateless transmission scheme for two-user Gaussian broadcast channels.



 $\underline{v}$  - lattice point sequence  $\underline{c}$  - coded lattice point sequence

Figure: Encoding structure of the two-way lattice-Kite code.



Figure: Gaussian broadcast channel.

 two receivers, R<sub>1</sub> and R<sub>2</sub> with sinal-to-noise ratios (SNR) SNR<sub>1</sub> and SNR<sub>2</sub>, respectively;

• we assume that  $\Delta SNR = SNR_1 - SNR_2 > 0.$ That is , receiver  $R_1$  sees a better channel.

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Figure: Bandwidth efficiency of the proposed rateless transmission scheme for TU-GBC.

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- Single- and Multi-User Communication
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#### Gaussian Relay Channel



- A sender X and an ultimate intended receiver Y;
- The Gaussian relay channel is given by

$$\begin{array}{rcl} Y_1 &=& X + Z_1 \\ Y &=& X + Z_1 + X_1 + Z_2, \end{array}$$

where  $Z_1$  and  $Z_2$  are independent zero-mean Gaussian random variables with variance  $N_1$  and  $N_2$ , respectively;

• The encoding allowed by the relay is the causal sequence

$$X_{1i} = f_i(Y_{11}, Y_{12}, \dots, Y_{1i-1});$$

• Sender X has power P and sender  $X_1$  has power  $P_1$ .

#### Capacity of the Gaussian Relay Channel

The capacity is

$$C = \max_{0 \le \alpha \le 1} \min \Big\{ C(\frac{P + P_1 + 2\sqrt{\alpha PP_1}}{N_1 + N_2}), C(\frac{\alpha P}{N_1}) \Big\},$$

where  $\overline{\alpha} = 1 - \alpha$ .

#### Basic Techniques for the Proof of Achievability

- Random coding;
- List codes;
- Slepian-Wolf partitioning;
- Coding for the cooperative multiple-access channel;
- Superposition coding;
- Block Markov encoding at the relay and transmitter.

#### Superposition Block Markov Encoding (SBME) in the Relay Channel

- $\bullet\,$  The data are equally grouped into B blocks;
- Initially, the source broadcasts a codeword that corresponds to the first data block;
- Then the source and the relay cooperatively transmit more information about the first data block;
- In the meanwhile, the source "superimposes" a codeword that corresponds to the second data block;
- Finally, the destination recovers the first data block from the two successive received blocks;
- After removing the effect of the first data block, the system returns to the initial state;
- This process iterates B + 1 times until all B blocks of data are sent successfully.

We apply a similar strategy (SBME) to the single-user communication system, resulting in the block Markov superposition transmission (BMST) scheme.

#### BMST scheme

- The data are equally grouped into  ${\cal B}$  blocks;
- Initially, the transmitter sends a codeword that corresponds to the first data block;
- Since the short code is weak, the receiver is unable to recover reliably the data from the current received block. Hence the transmitter transmits the codeword (in its interleaved version) one more time.
- In the meanwhile, a fresh codeword that corresponds to the second data block is superimposed on the second block transmission.
- Finally, the receiver recovers the first data block from the two successive received blocks.
- After removing the effect of the first data block, the system returns to the initial state;
- This process iterates B + 1 times until all B blocks of data are sent successfully.

[MA13]X. Ma *et al*, "Obtaining extra coding gain for short codes by block Markov superposition transmission", submitted to ISIT, 2013

#### Encoding



Figure: Encoding structure of BMST with memory m.

#### Recursive Encoding of BMST

- Initialization: For t < 0, set  $\mathbf{v}^{(t)} = \mathbf{0} \in \mathbb{F}_2^n$ .
- **2** *Recursion*: For  $t = 0, 1, \dots, L 1$ ,
  - Encode  $\mathbf{u}^{(t)}$  into  $\mathbf{v}^{(t)} \in \mathbb{F}_2^n$  by the encoding algorithm of the basic code  $\mathscr{C}$ ;
  - For  $1 \le i \le m$ , interleave  $\mathbf{v}^{(t-i)}$  by the *i*-th interleaver  $\Pi_i$  into  $\mathbf{w}^{(i)}$ ;
  - Compute  $\mathbf{c}^{(t)} = \mathbf{v}^{(t)} + \sum_{1 \le i \le m} \mathbf{w}^{(i)}$ , which is taken as the *t*-th block of transmission.

**Orrepresentation:** For  $t = L, L + 1, \dots, L + m - 1$ , set  $\mathbf{u}^{(t)} = \mathbf{0} \in \mathbb{F}_2^k$  and compute  $\mathbf{c}^{(t)}$  recursively.

#### Normal Graph



Figure: The normal graph of a code with L = 4 and m = 2.

#### Decoding

- an iterative sliding-window decoding algorithm is used;
- four types of nodes: C, =, +, and  $\prod$ ;
- messages are processed and passed through different decoding layers forward and backward over the normal graph;

# Coding Gain Analysis of the BMST

#### Genie-Aided Lower Bound on BER

• The performance of the BMST under MAP decoding is determined by

$$\Pr\{u_j^{(t)}|\mathbf{y}\} = \sum_{\mathbf{u}'} \Pr\{\mathbf{u}'|\mathbf{y}\} \Pr\{u_j^{(t)}|\mathbf{u}',\mathbf{y}\},$$

where the summation is over  $\mathbf{u}' = {\mathbf{u}^{(i)}, t - m \le i \le t + m, i \ne t};$ 

• the BER performance can be lower-bounded by

$$f_n(\gamma_b) \ge f_o(\gamma_b + 10 \log_{10}(m+1) - 10 \log_{10}(1+m/L));$$

• noticing that  $Pr\{u'|y\} \approx 1$  for the transmitted data block u' in the low error rate region, we can expect that

$$f_n(\gamma_b) \approx f_o(\gamma_b + 10 \log_{10}(m+1) - 10 \log_{10}(1+m/L))$$

as  $\gamma_b$  increases.

• the maximum coding gain can be  $10 \log_{10}(m+1) \text{ dB}$  for large L in the low error rate region.



Figure: Coding gain analysis of the BMST system.

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Figure: Coding gain analysis of the BMST system.

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Figure: The basic code  $\mathscr{C}$  is a concatenated code of dimension k = 10000 and length n = 20068, where the outer code is a 32-bit CRC code and the inner code is a terminated 4-state (2, 1, 2) convolutional code.

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Figure: The basic code  $\mathscr{C}$  is a concatenated code of dimension k = 10000 and length n = 20068, where the outer code is a 32-bit CRC code and the inner code is a terminated 4-state (2, 1, 2) convolutional code.

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Figure: The basic code  $\mathscr{C}$  is a concatenated code of dimension k = 11968 and length n = 21000, where the outer code is a 32-bit CRC code and the inner code is the Cartesian product of Hamming code [7, 4]<sup>3000</sup> Guangzhou, February 2013 67/80



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### Performance of the BMST System



Figure: The basic code  $\mathscr{C}$  is the Consultative Committee on Space Data System (CCSDS) standard code of dimension k = 1784 and length n = 4092, where the outer code is a [255, 223] Reed-Solomon (RS) code over  $\mathbb{F}_{256}$  and the inner code is a terminated 64-state (2, 1, 6) convolutional code.

#### Performance of the BMST System



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- Single- and Multi-User Communication
- 2 Superposition Coded Modulation
- 3 Kite Codes
- 4 Block Markov Superposition Transmission
- **5** Conclusions

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# Conclusions

### Superposition Coded Modulation

- We proposed a coded modulation system using superimposed binary codes;
- Using the unequal power-allocations and the Gaussian-approximation-based suboptimal demapping algorithm, coded modulation with high bandwidth efficiency can be implemented.

#### Kite Codes

- We proposed a kind of rateless codes for AWGN channels;
- A greedy optimization was presented to optimize Kite codes;
- Three methods were presented either to improve the performance of Kite codes, or to accelerate the design of Kite codes;
- Possible applications of Kite codes were investigated.

#### Block Markov Superposition Transmission

- We presented a new method for constructing long codes from short codes;
- The encoding process can be as fast as the short code, while the decoding has a fixed delay.

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### Thank You for Your Attention!

**A D > A A**