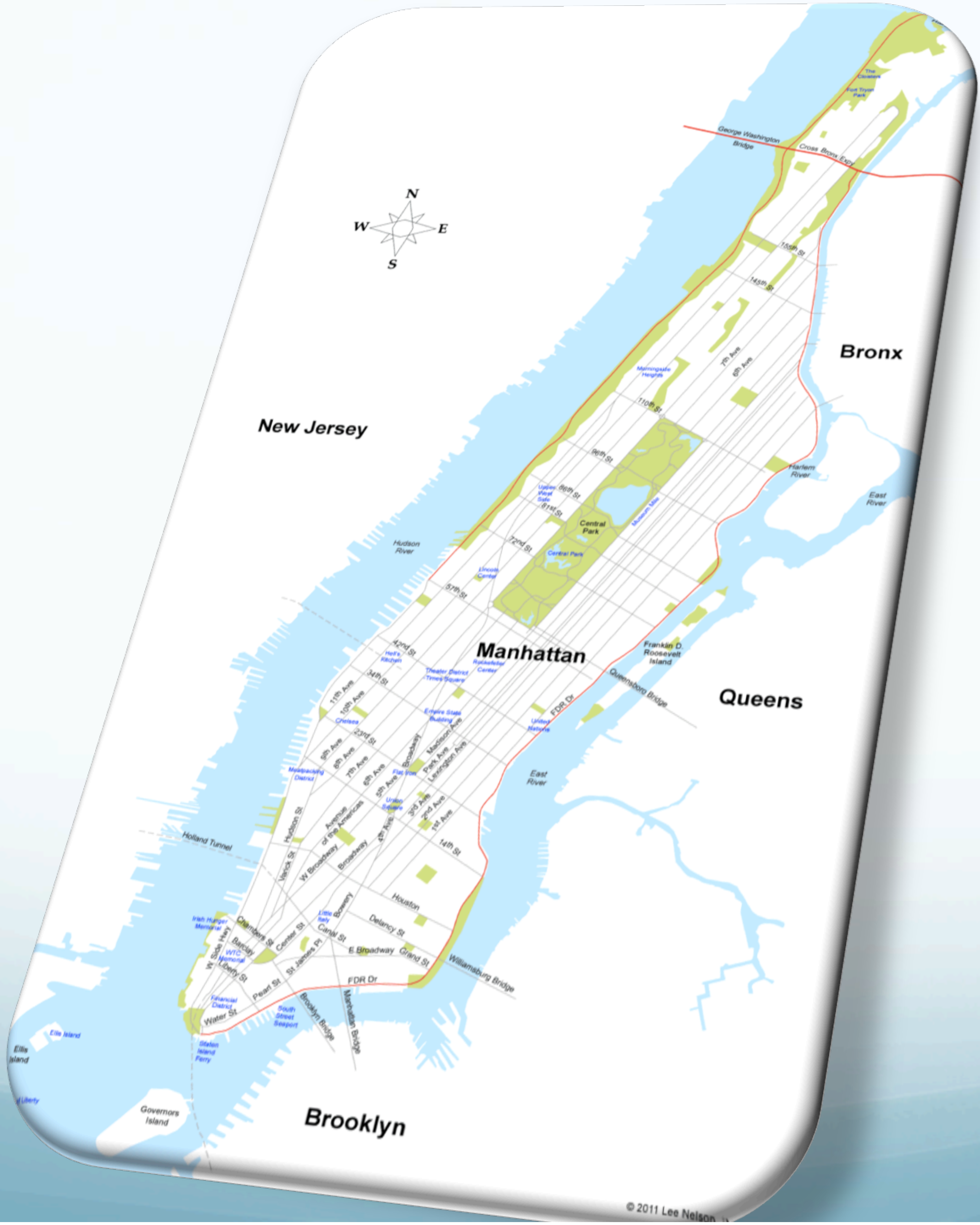
Columbia University

In the City of New York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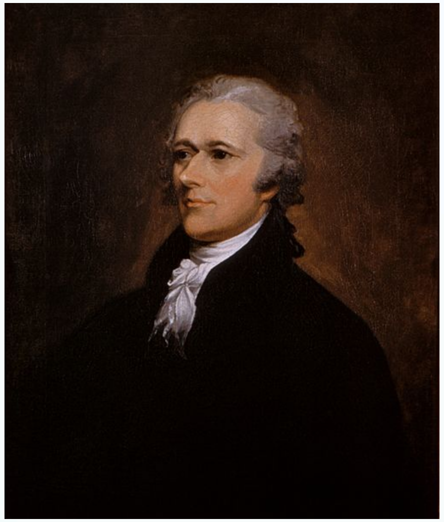
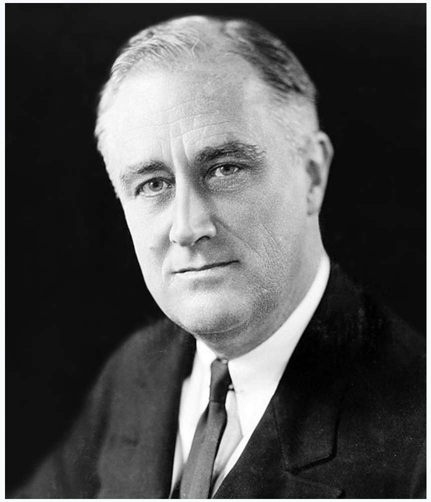
OF COLUMBIA UNIVERSITY

Notable Columbia Students






Columbia Nobel Laureates

- Columbia has most Nobel Laureates of any university in the world, with 98 affiliated prize-winners

Chinese Columbia Alumni

- 徐志摩
- 闻一多
- 陶行知
- 蒋梦麟
- 潘光旦
- 吴文藻
- 梁实秋
- 侯德榜
- ……
- 顾维钧
- 蒋廷黻
- 马寅初
- 宋子文
- 冯友兰
- 胡适
- 李政道
- 吴健雄
- ……

Group Decoding for Interference Management

Xiaodong Wang

Department of Electrical Engineering

Columbia University

joint work with

Ali Tajer   Narayan Prasad   Chen Gong

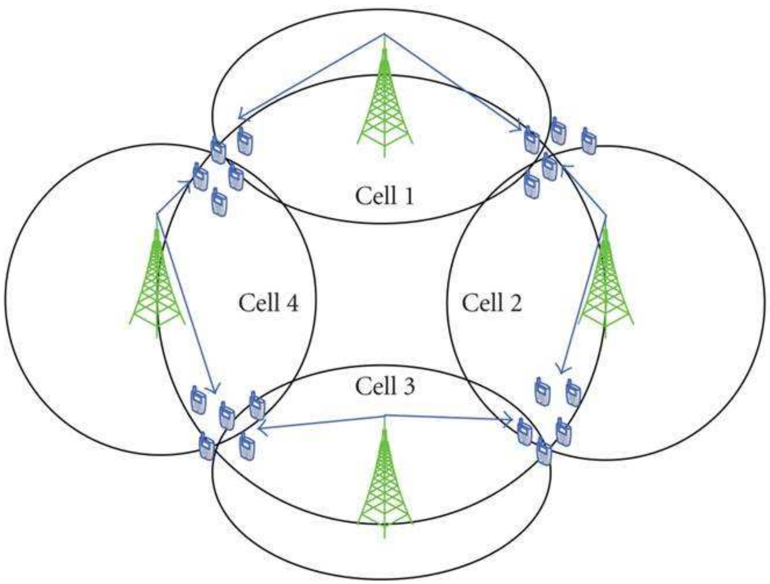
Multi-cell Networks

future networks are interference-limited

- shrinking cell sizes
- ambitious spectral efficiencies
- universal frequency reuse

recent developments

- MIMO networks
- game theory
- interference alignment



Multi-cell Downlink Systems

interference management: *decode* or *suppress*?

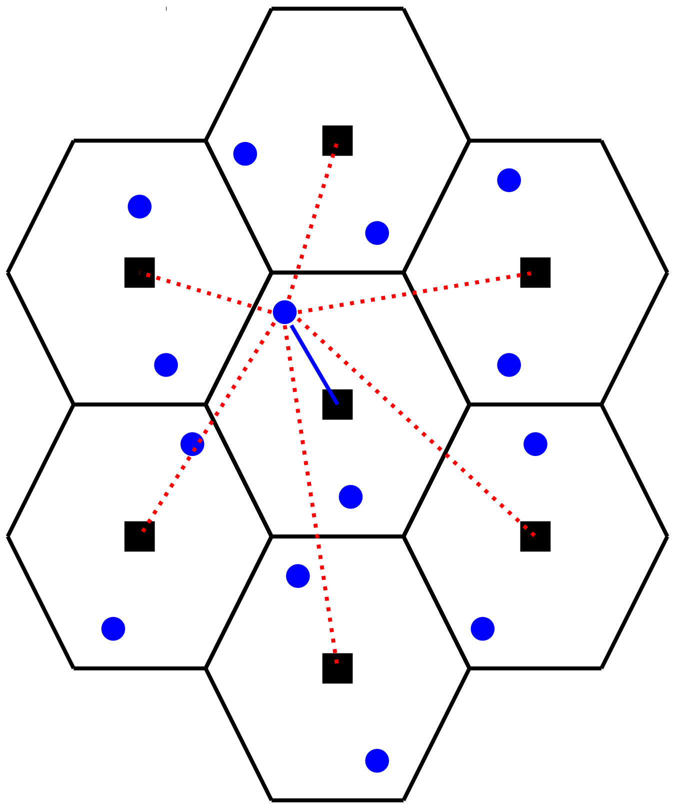
conventional way:

suppress interference

- precoding
- scheduling

new look: decode interference

- interference has structure
- decoding it *might* be helpful



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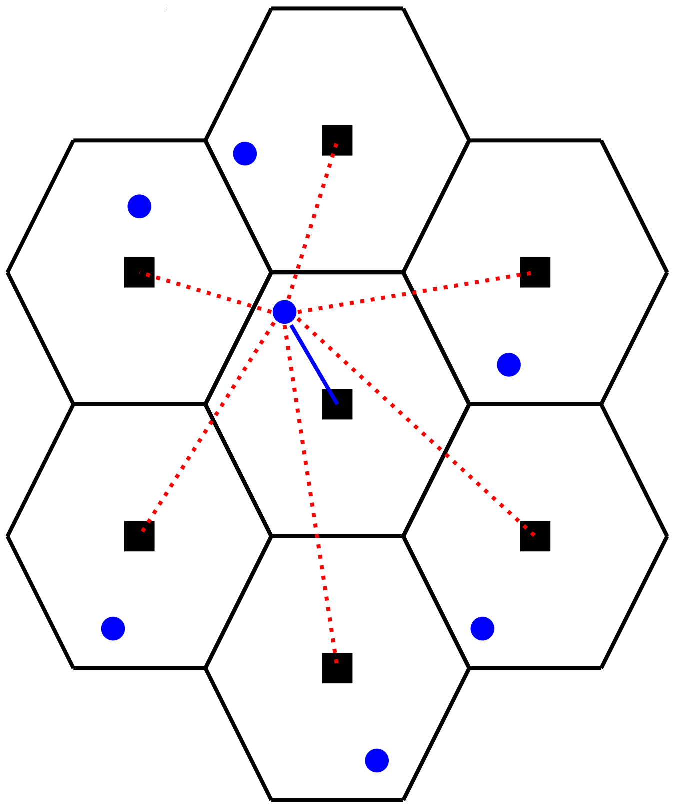


Decoding Interference

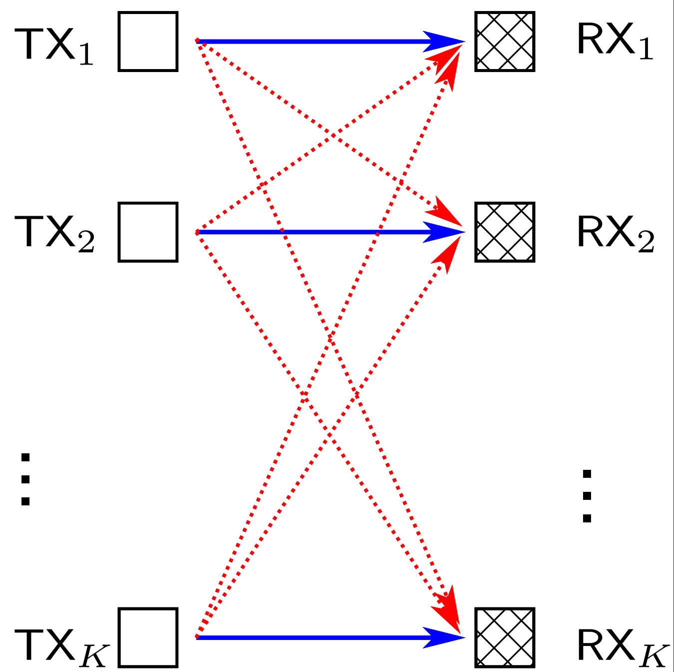
user RX_m must decode TX_m

1) which interferers should RX_m decode? (2^{K-1} options)

(group decoding)

2) what fraction of an interferer should be decoded?

(rate splitting)



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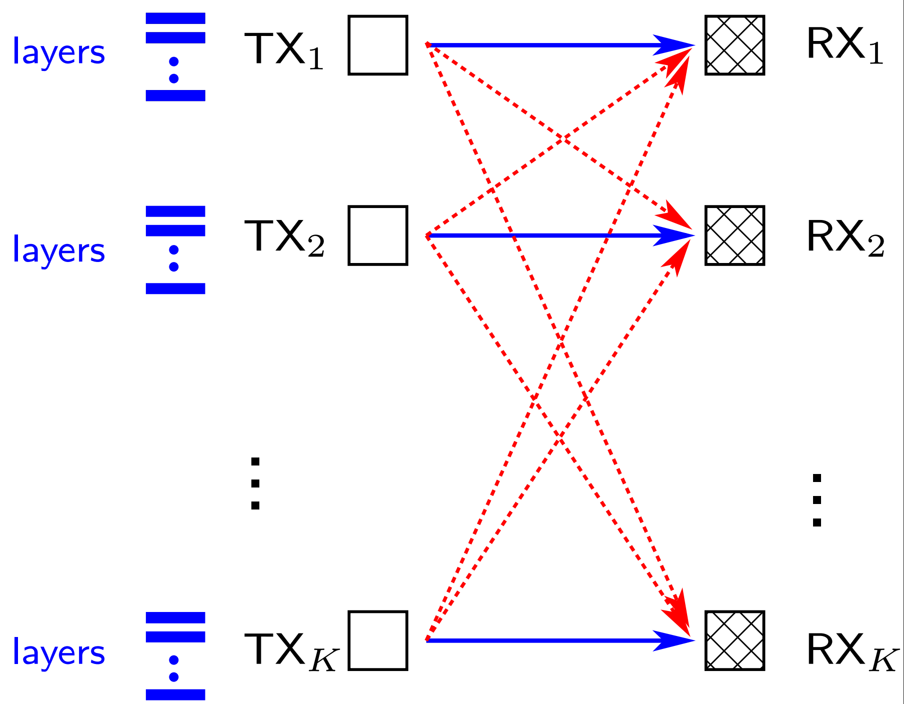
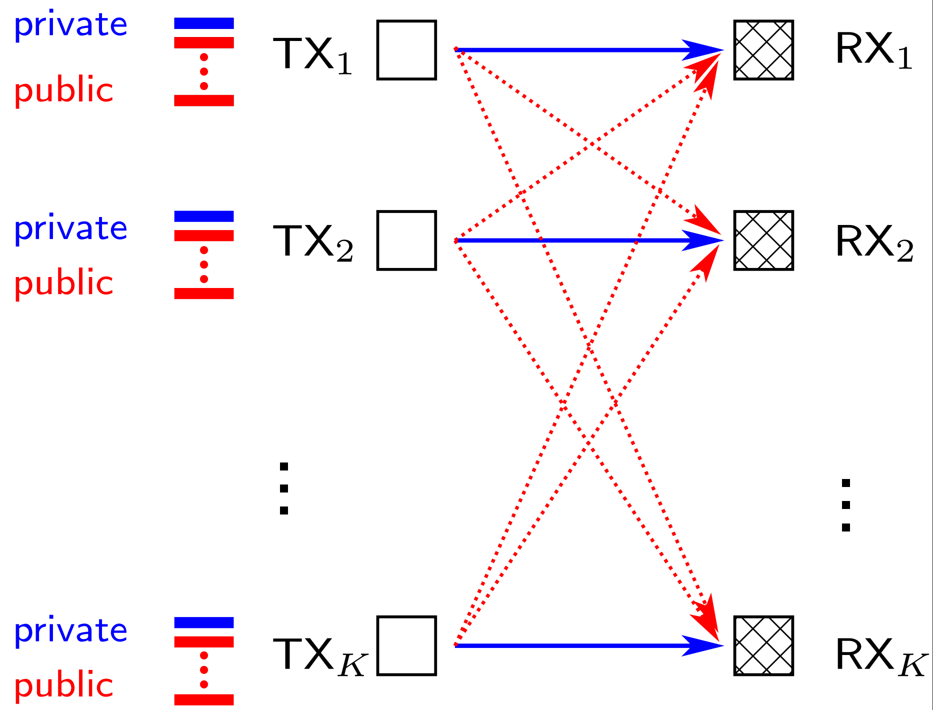


Rate Splitting

multiple codebooks per BS

all codebooks are independent



rate splitting:

- one private message
- multiple public messages

- private $\longrightarrow$ to intended user
- public $\longrightarrow$ to all users

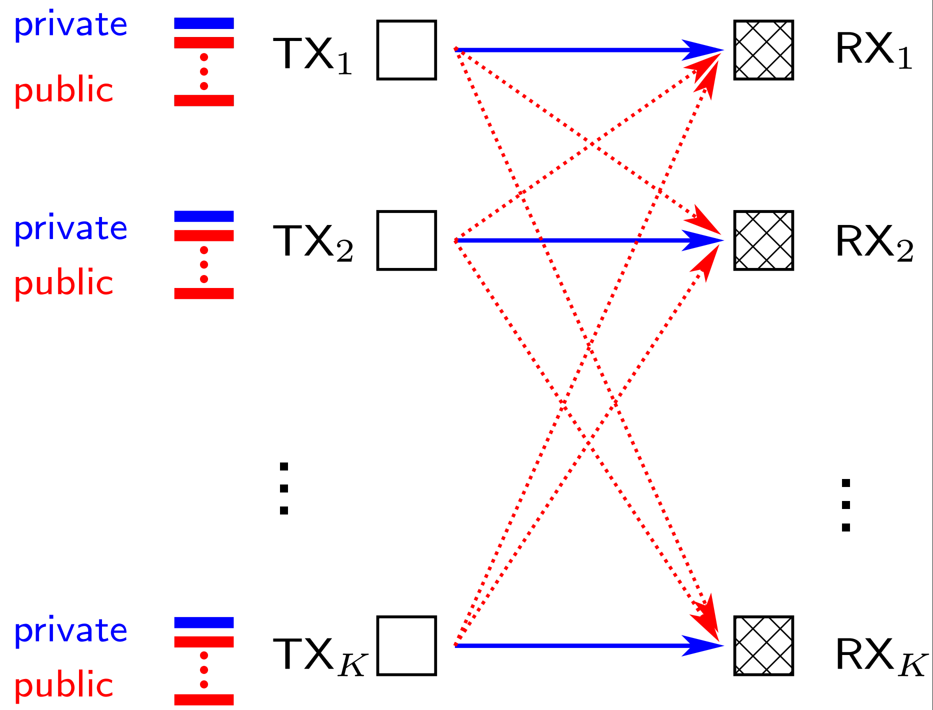
How Many Codebooks per BS?

private message:

- 1 codebook

public messages:

- 1 codebook for each arbitrary set of unintended receivers
- $2^{K-1} - 1$ codebooks



2^{K-1} codebooks per BS

$K = 2 \rightarrow$ Han-Kobayashi

Group Decoding

receiver U$_m$:

1. must decode all the 2^{K-1} segments of the message of BS$_m$
2. receives interference from $(K-1)$ interferers $\{\mathrm{BS}_n\}_{n \neq m}$
3. each interferer has a message consisting of 2^{K-1} segments
4. $M = (K-1)2^{K-1}$ is the total message segments interfering with U$_m$
5. decodes a subset of the interfering segments along with its intended message
6. there exist $2^M - 1$ choices for group decoding

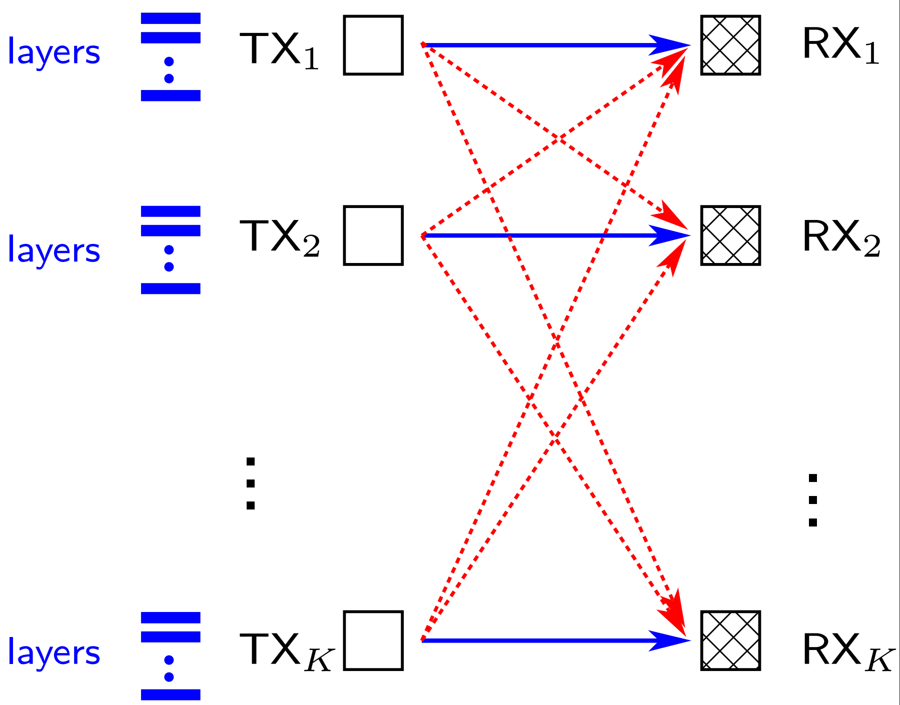
Group Decoding

advantages:

- freedom in managing the interference
- higher achievable rates

2^{K-1} codebooks per TX

$K = 2 \rightarrow$ Han-Kobayashi



challenges:

- complexity
- TX coordination

Group Decoding

simplification: one codebook per user

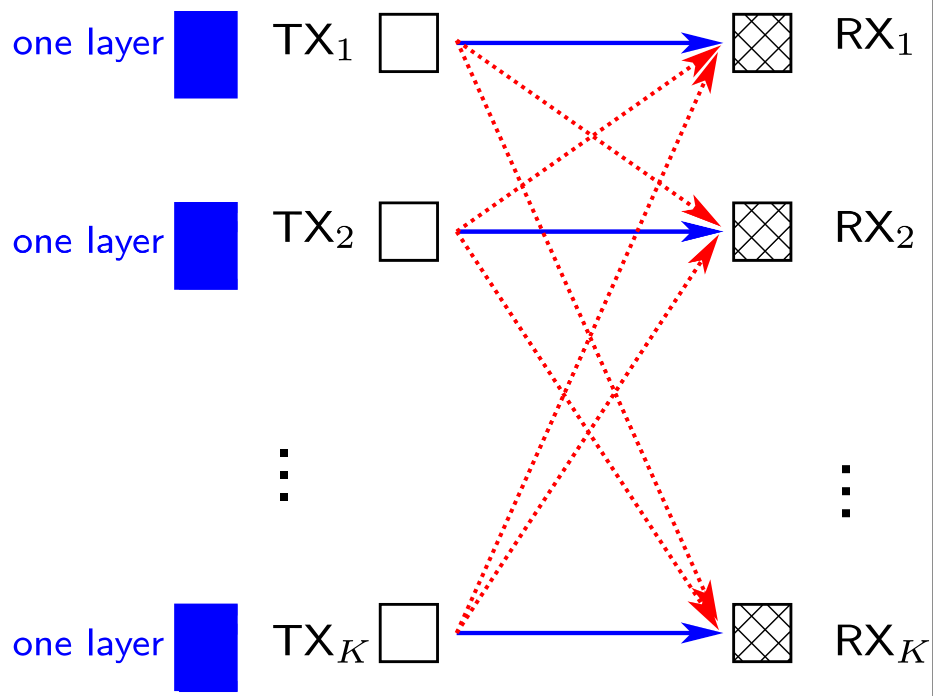
generalization: follows same ideas

assumptions:

- users share their codebooks

at each receiver:

- jointly decode a group of interferers along with the desired user
- treat the rest as Gaussian noise

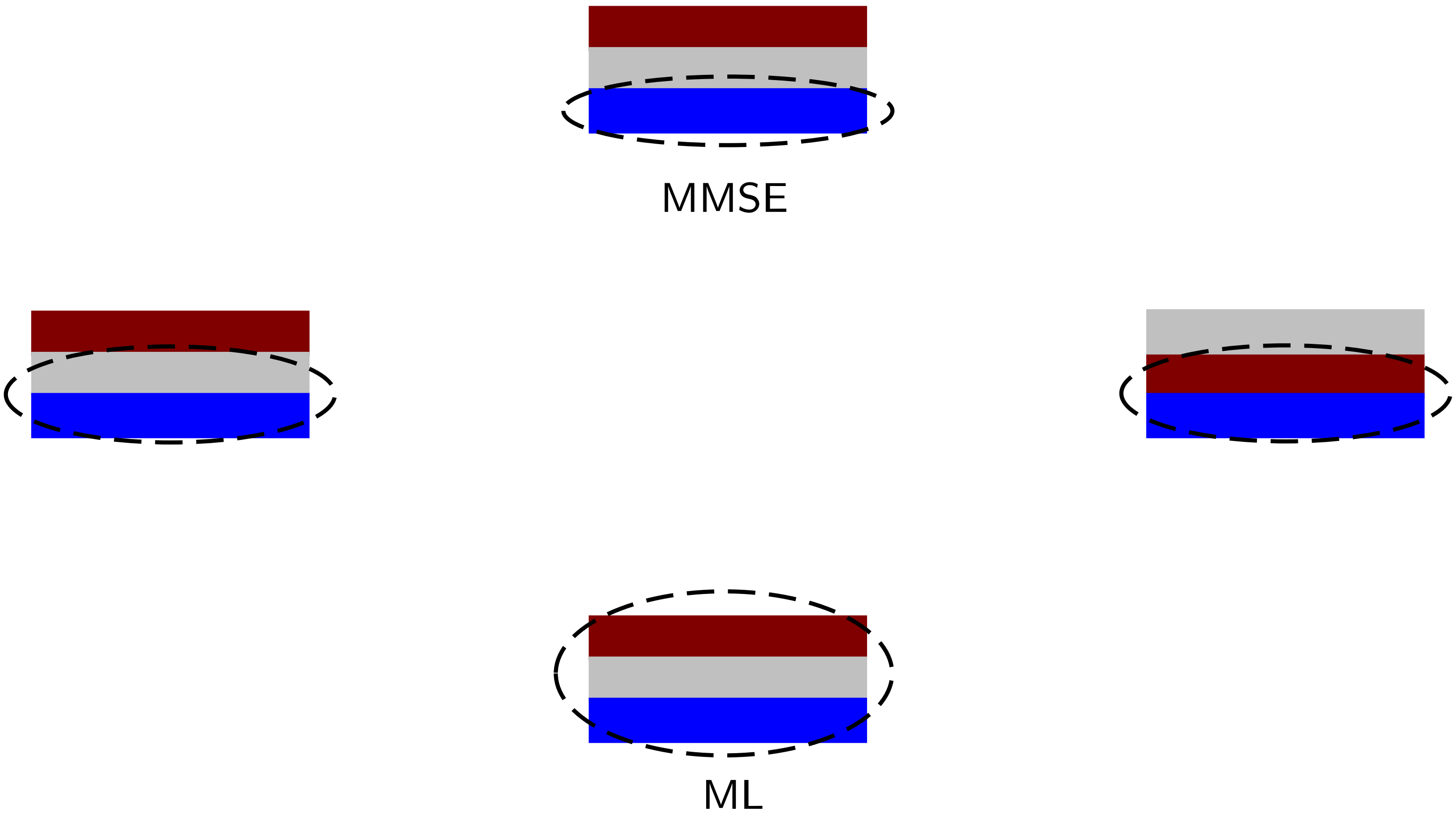


3-user Interference Channel

if deemed beneficial:

decode the desired user jointly with any other set of interferers

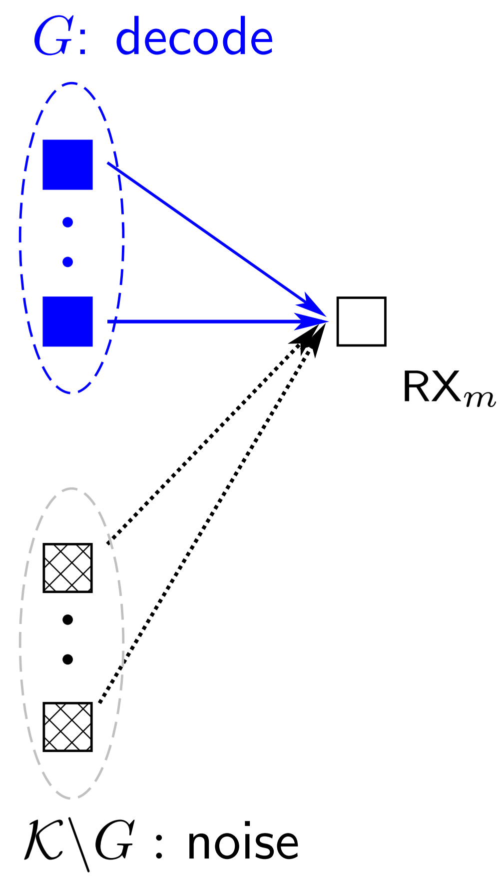
objective: decode the blue message



Rate Region

$$\{\text{TX}_1, \ldots, \text{TX}_K\} \to \text{RX}_m$$

- for partition $\mathcal{K} = \{G, \mathcal{K}\backslash G\}$, $m \in G$
  - RX_m jointly decodes users in G
  - RX_m suppresses users in $\mathcal{K}\backslash G$ as noise



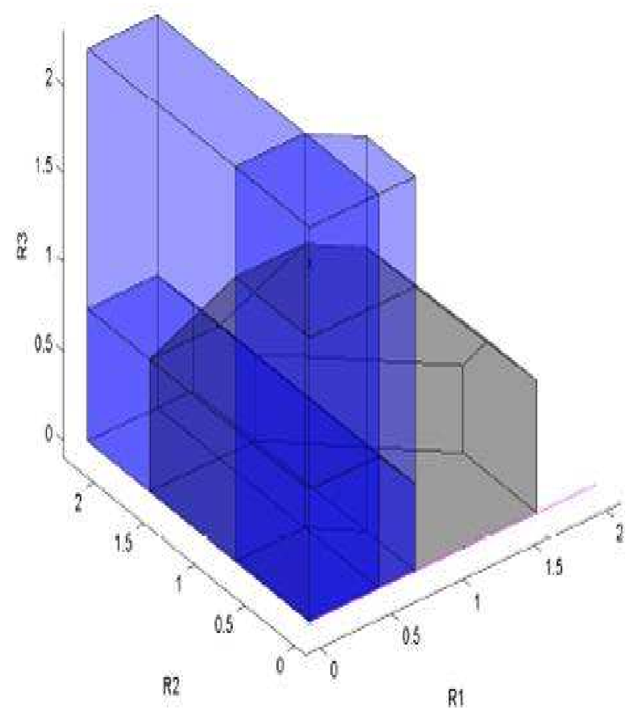
- each partition $\{G, \mathcal{K}\backslash G\}$ $\Rightarrow$ one MAC
- $\mathcal{R}_m^G \triangleq$ achievable rate region of this MAC
- achievable rate region for RX_m:

$$\mathcal{R}_m = \bigcup_{G \subseteq \mathcal{K}:\, m \in G} \mathcal{R}_m^G$$

Rate Region

- 3-user Interference Channel

- 3 rate regions
  - $\mathcal{R}_1^{\{1,2\}} = (R_1, R_2)$ (Blue)
  - $\mathcal{R}_1^{\{1,3\}} = (R_1, R_3)$ (Blue)
  - $\mathcal{R}_1^{\{1,2,3\}} = (R_1, R_2, R_3)$ (Gray)



- RX_1 can decode any rate-vector $\boldsymbol{R} \in \mathcal{R}_1 \triangleq \mathcal{R}_1^{\{1,2\}} \cup \mathcal{R}_1^{\{1,3\}} \cup \mathcal{R}_1^{\{1,2,3\}}$

- $\mathcal{R}_1$ is unbounded along R_2 and R_3

Applications of Group Decoders

1. K-user interference channel
   - generalized HK: an achievable rate region for any arbitrary K
   - network optimization with limited coordination (*almost distributed*)
   - fairness

2. multi-cell downlink systems
   - joint precoding + group decoding
   - rate allocation

3. joint channel coding + group decoding

Two Types of Problems

fixed-rate problems $\Rightarrow$ RX design

examples: outage minimization, error minimization ...

challenge: complexity

variable-rate problems $\Rightarrow$ joint TX & RX design

example: rate allocation, fairness, precoding + group decoding, ...

challenge: complexity + coordination

Complexity

Both types of problems need to identify decodable sets for each user

Each user observes 2^{K-1} MAC channels

A MAC channel of cardinality m involves $2^m - 1$ inequalities

$$\text{in total} \quad \sum_{m=1}^{K-1} \binom{K-1}{m} (2^m - 1) = 3^{K-1} - 2^{K-1} \quad \text{inequalities}$$

Each receiver has to check $(3^{K-1} - 2^{K-1})$ inequalities

decoding has exponential complexity in K

Unconstrained Group Decoding

How group decoders are useful?

we develop a class of group decoders with the following features:

1. **controlled** complexity in implementation

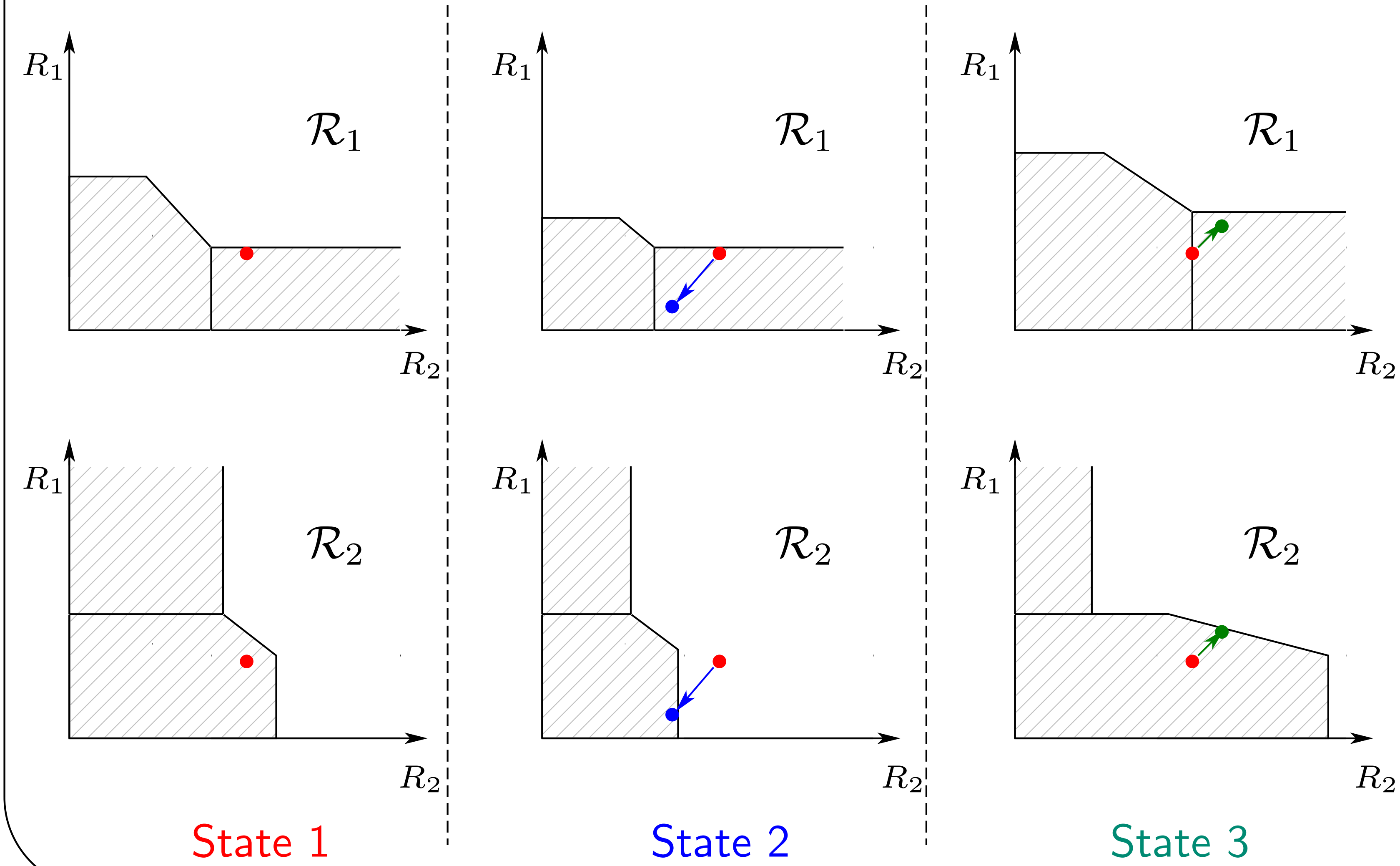
2. **limited** coordination among the TXs

K-user Interference Channels

Fair Rate Adaptation

Fair Rate Adaptation

adapt the rates to channel variations with some fairness constraints



Symmetric Fairness

state S, rate $\boldsymbol{R}$ $\Rightarrow$ state S', rate $\boldsymbol{R}'$

$$\text{RAF} = \begin{cases} \max & x \\ \text{s.t.} & \boldsymbol{R}' = \boldsymbol{R} + x \cdot \boldsymbol{t} \quad \text{remains decodable} \end{cases}$$

- $\boldsymbol{t} = [1, \ldots, 1]$ $\Rightarrow$ identical rate increment/decrement
- $\boldsymbol{t} = \boldsymbol{R}$ $\Rightarrow$ identical rate scaling

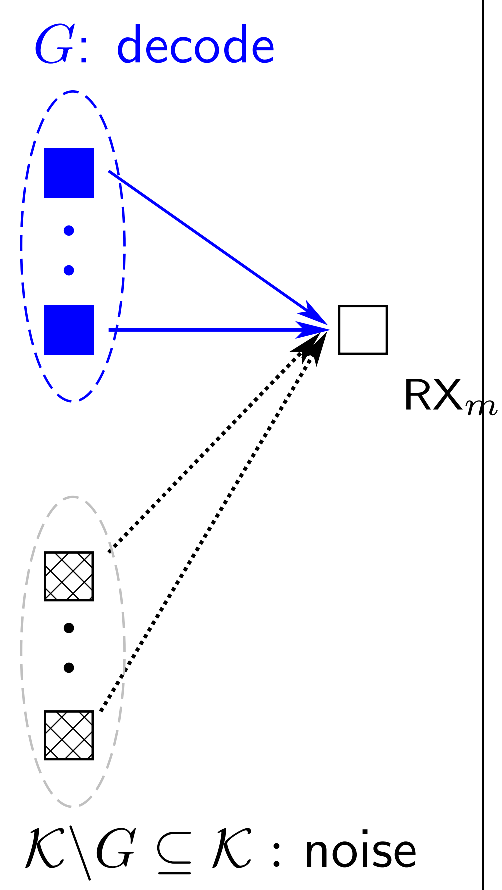
RAF≥ 0 $\Rightarrow$ $\boldsymbol{R}$ $\in$ the capacity region of S' $\Rightarrow$ rate **increment**

RAF< 0 $\Rightarrow$ $\boldsymbol{R}$ $\notin$ the capacity region of S' $\Rightarrow$ rate **decrement**

Symmetric Fair Rate Adaptation

1) **local rate adaptation for a given partition:** complexity

$$\mathsf{RAF}_m(G) = \begin{cases} \max & x \\ \text{s.t.} & \boldsymbol{R}'_G = \boldsymbol{R}_G + x \cdot \boldsymbol{t}_G \ \text{ is decodable} \end{cases}$$



2) **local rate adaptation:** complexity

$$\mathsf{RAF}_m = \max_{G \subseteq \mathcal{K}:\ m \in G} \mathsf{RAF}_m(G)$$

3) **global rate adaptation:** coordination

$$\mathsf{RAF} = f(\mathsf{RAF}_1, \ldots, \mathsf{RAF}_K)$$

Step 1

local rate adaptation for a given partition

Theorem 1 *The solution of*

$$\mathsf{RAF}_m(G) = \begin{cases} \max & x \\ \textit{s.t.} & \boldsymbol{R}'_G = \boldsymbol{R}_G + x \cdot \boldsymbol{t}_G \ \textit{ is decodable} \end{cases}$$

is given by

$$\mathsf{RAF}_m(G) = \min_{D \neq \emptyset, D \subseteq G} \frac{f(D,G)}{\sum_{j \in D} t_j} \ .$$

where

$$f(D,G) = I(x_D; y_m \mid x_{\mathcal{K} \setminus G}) - \sum_{i \in D} R_i$$

$f(D,G)$ is a **submodular** function

$\Rightarrow$ polynomial complexity in $|G|$

Step 2 - Local Rate Adaptation

Unconstrained Group Decoder

$$\mathsf{RAF}_m = \max_{G \subseteq \mathcal{K}:\ m \in G} \mathsf{RAF}_m(G)$$

1. we offer a successive decoding procedure

2. include all users to be jointly decoded: $G = \mathcal{K}$

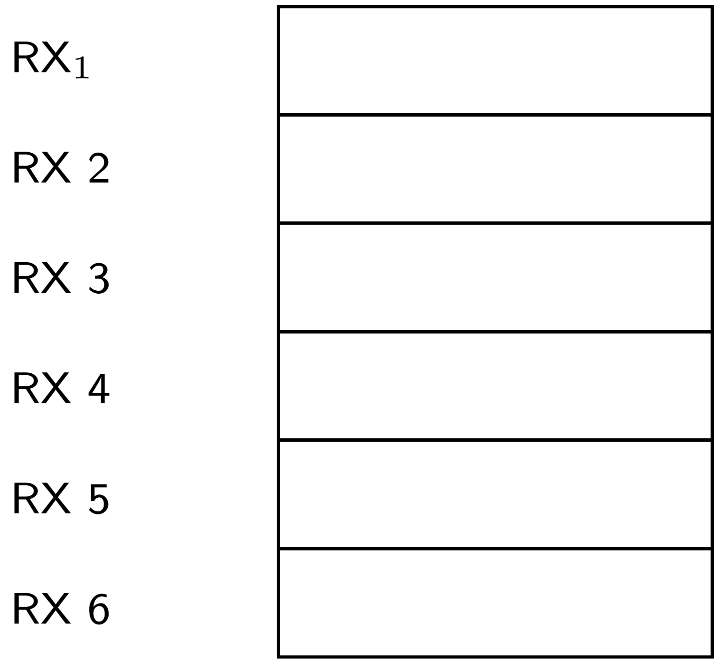
3. at each iteration identify the bottleneck users (V)

4. is m a bottleneck user?

   - No: discard users in V, i.e., $G \leftarrow G \backslash V$; repeat step 3
   - Yes: users in G should be jointly decoded

Local Rate Adaptation for RX_1

Objective: Determine RAF_1 and the partition $\{G_1^*, \mathcal{K} \backslash G_1^*\}$



$$V = \underset{D \neq \emptyset, D \subseteq G}{\arg\min} \frac{f(D,G)}{\sum_{j \in D} t_j}$$

$G = \{1, 2, 3, 4, 5, 6\}$ and $\mathcal{K} \backslash G = \{\}$

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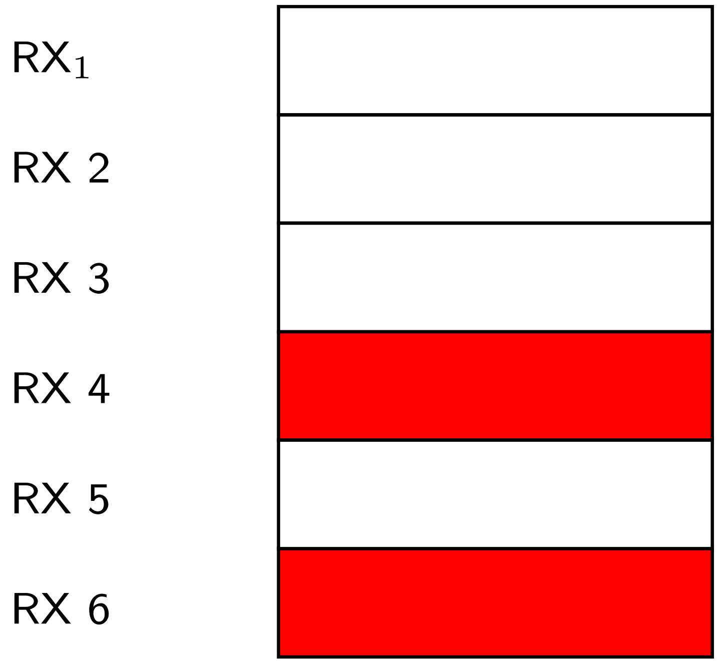

$V = \{4, 6\}$

$G = \{1, 2, 3, 4, 5, 6\}$ and $\mathcal{K} \backslash G = \{\}$

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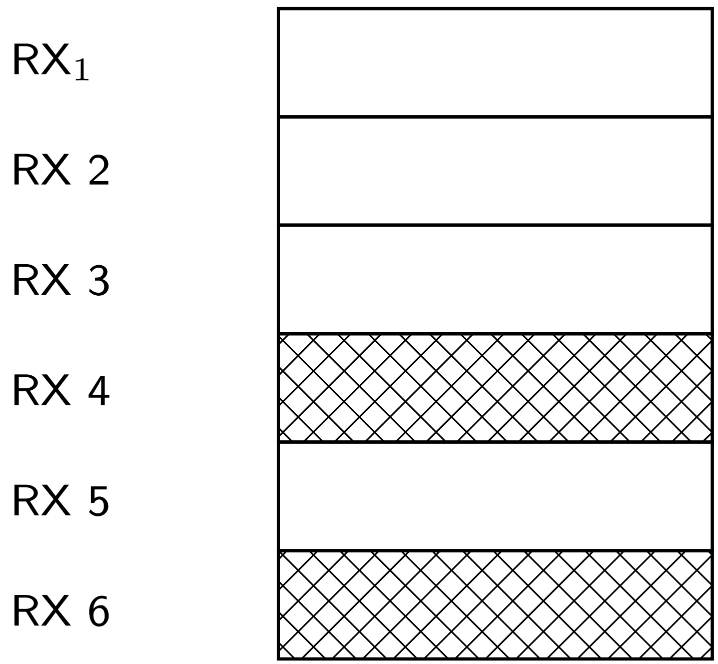

$$V = \arg\min_{D \neq \emptyset, D \subseteq G} \frac{f(D,G)}{\sum_{j \in D} t_j}$$

$$G = \{1, 2, 3, 5\} \text{ and } \mathcal{K}\backslash G = \{4, 6\}$$

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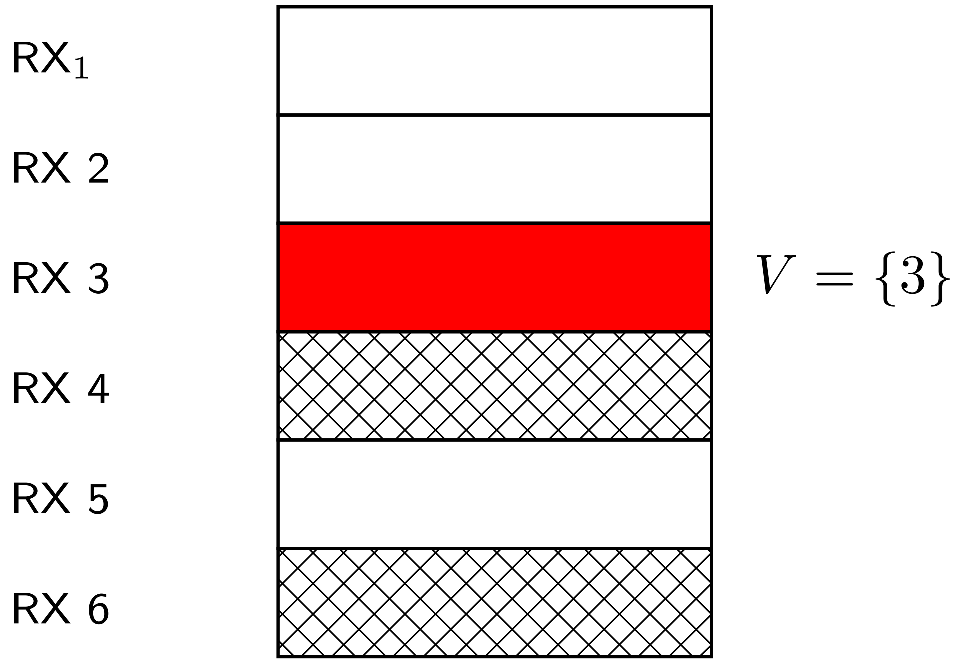

$G = \{1, 2, 3, 5\}$ and $\mathcal{K} \backslash G = \{4, 6\}$

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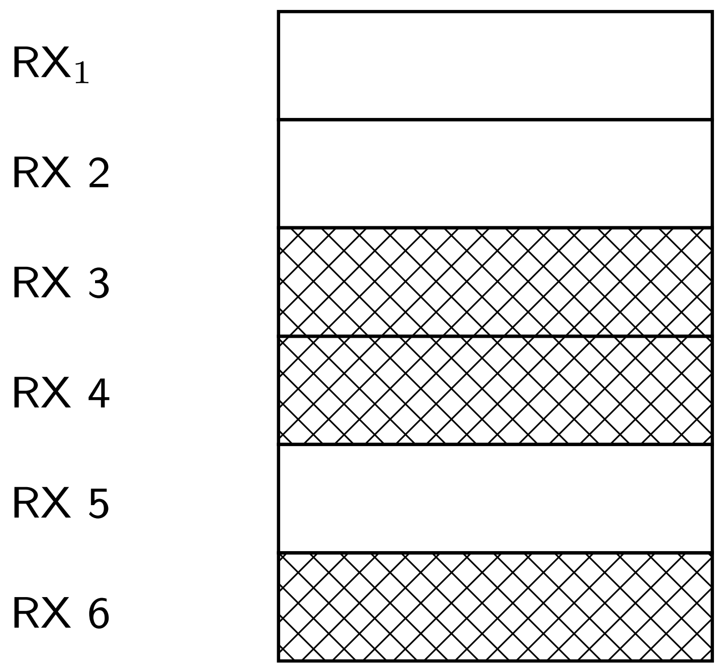

$$V = \arg\min_{D \neq \emptyset, D \subseteq G} \frac{f(D,G)}{\sum_{j \in D} t_j}$$

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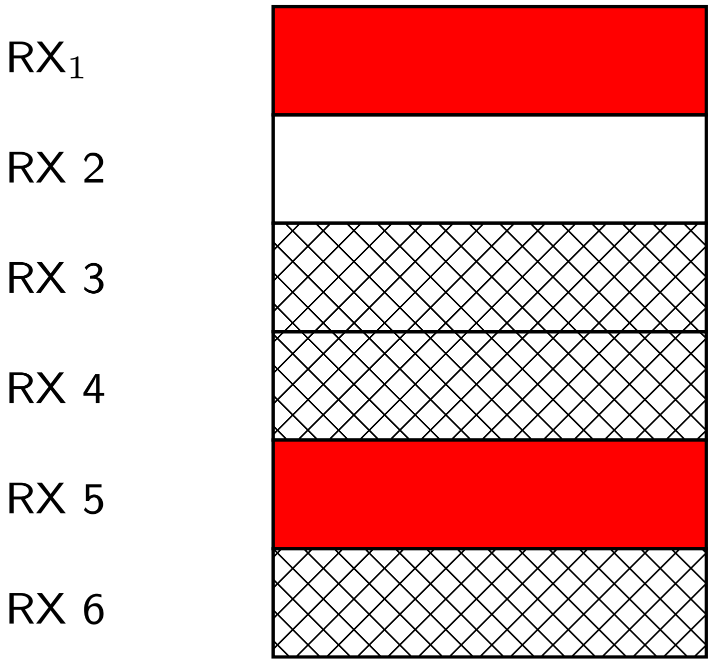


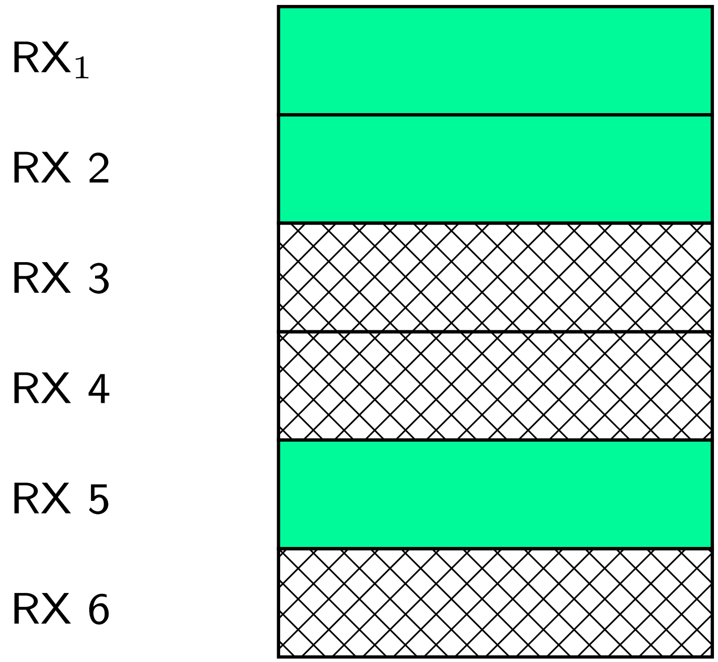
$V = \{1, 5\}$

$1 \in V \Rightarrow G_1^* = G$

$G = \{1, 2, 5\}$ and $\mathcal{K}\backslash G = \{3, 4, 6\}$

Local Rate Adaptation for RX_1

Objective: Determine RAF_1 and the partition $\{G_1^*, \mathcal{K}\backslash G_1^*\}$



$$G_1^* = \{1, 2, 5\}$$

$$\text{RAF}_1 = \frac{f_k(V, G_1^*) - \sum_{i \in V} R_i}{\sum_{j \in V} t_j}$$

$G = \{1, 2, 5\}$ and $\mathcal{K}\backslash G = \{3, 4, 6\}$

Local Rate Adaptation for RX_m

Optimality

Theorem 2 *The partitioning* $\{G_m^*, \mathcal{K} \backslash G_m^*\}$ *yielded by the Unconstrained Group Decoder maximizes* $\mathsf{RAF}_m(G)$ *over all valid* G*, i.e.,*

$$\mathsf{RAF}_m = \max_{G \subseteq \mathcal{K}:\ m \in G} \mathsf{RAF}_m(G) = \mathsf{RAF}_m(G_m^*)$$

where RAF_m *is the maximum rate adaptation factor sustained by user* m*.*

- at most K iterations
- each iteration polynomial in at most K

polynomial complexity in K

Step 3 - Global Rate Adaptation

obtain RAF as a function of $\{$ RAF$_1$,. . . ,RAF$_K\}$

- Each user computes a rate increment factor RAF$_m$ independently
- The optimal rate increment factor is RAF $= \min_m\{\text{RAF}_m\}$

Input $\boldsymbol{R}$
for $m = 1, \ldots, K$ **do**
  Determine RAF$_m$ and G^*_m
end for
Update $\boldsymbol{R}' \leftarrow \boldsymbol{R} + \min_{1\le m\le K}\{\text{RAF}_m\} \cdot \boldsymbol{t}$
Output $\boldsymbol{R}'$ and $\{\mathcal{G}^*{}_m\}_{m=1}^K$

---

distributed

Step 3 - Global Rate Adaptation

Optimality

Theorem 3 *The rate vector yielded satisfies*

$$\boldsymbol{R} \succeq \tilde{\boldsymbol{R}},$$

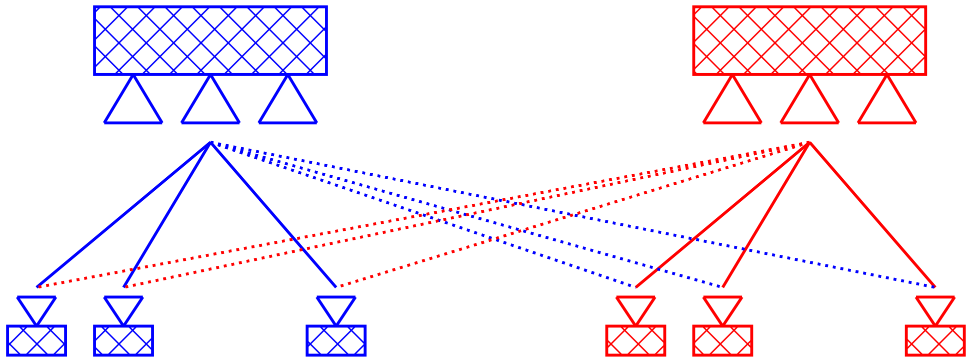
where $\tilde{\boldsymbol{R}}$ is any decodable rate-vector such that $\tilde{\boldsymbol{R}} = \boldsymbol{R} + x \cdot \boldsymbol{t}$ for some $x \geq 0$.

The overall algorithm is distributed with polynomial complexity

MIMO Networks

Network Optimization

Downlink in MIMO Networks

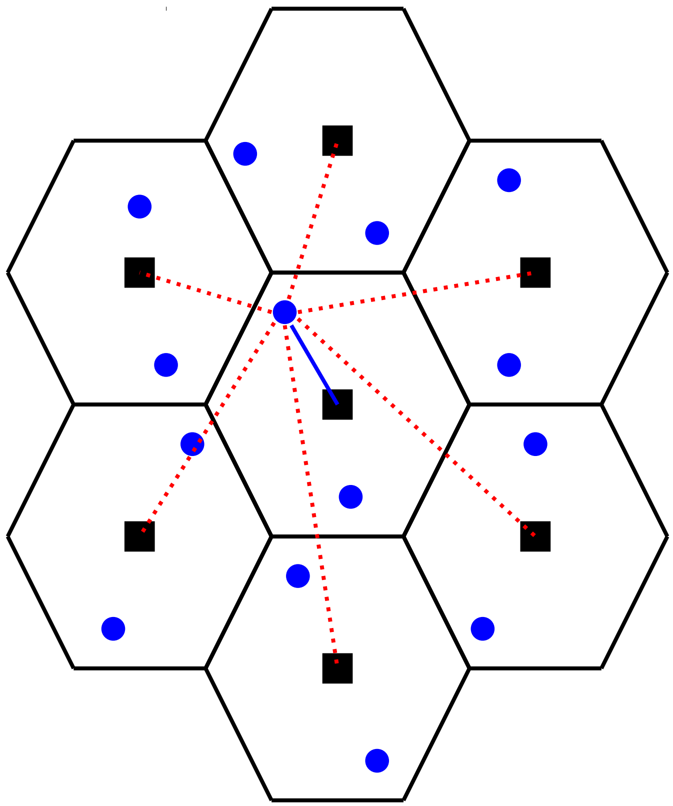


- multi-cell network
- broadcast transmission (MISO)
- collaborative (among BSs) linear precoding to harness
  - inter-cell interference
  - intra-cell interference

Downlink in MIMO Networks

interference management: *decode* or *suppress*?

- three-layer transmission
  1. multi-antenna pre-coding
  2. rate allocation
  3. UGD



jointly solve these two problems

- design the precoders
- design the optimal group decoders

Network Optimization

Max-Min Rate Optimization

Maximize worst-case weighted rate subject to power constraint

$$\mathcal{R}(P_0) = \begin{cases} \max_{\{\boldsymbol{w}_i\}} & \min_i \frac{R_i}{\rho_i} \\ \text{s.t.} & \sum_{i=1}^{M} \alpha_i \|\boldsymbol{w}_i\|^2 \leq P_0 \\ & \boldsymbol{R} \text{ is decodable} \end{cases}$$

Power Optimization

Minimize weighted sum-power subject to QoS guarantees

$$\mathcal{P}(\boldsymbol{\rho}) = \begin{cases} \min_{\{\boldsymbol{w}_i\}} & \sum_{i=1}^{M} \alpha_i \|\boldsymbol{w}_i\|^2 \\ \text{s.t.} & \text{Rate } \boldsymbol{\rho} \text{ is decodable} \end{cases}$$

Rate Optimization for UGD

Solving $\mathcal{R}(P_0)$ can be facilitated by solving $\mathcal{P}(\boldsymbol{\rho})$

Theorem 4 *The problems $\mathcal{R}(P_0)$ and $\mathcal{P}(\boldsymbol{\rho})$ are related as*

$$\text{if } \mathcal{P}(\boldsymbol{\rho}) \text{ feasible, then } \mathcal{R}\Big(\mathcal{P}(\boldsymbol{\rho})\Big) = 1$$

$$\text{and } \mathcal{P}\Big(\mathcal{R}(P_0) \cdot \boldsymbol{\rho}\Big) \leq P_0,$$

with the equality only if the weighted sum-power constraint of $\mathcal{R}(P_0)$ holds with equality.

We formulate and treat the power optimization problem $\mathcal{P}(\boldsymbol{\rho})$

Power Optimization for UGD

Theorem 5 *For identical rate weights* $\rho_i = \rho$ *the power optimization problem with UGD is given by*

$$\mathcal{P}(\boldsymbol{\rho}) = \begin{cases} \min_{\{\boldsymbol{w}_i\}} & \sum_{i=1}^{M} \alpha_i \|\boldsymbol{w}_i\|^2 \\ \textit{s.t.} & \textit{Rate } \boldsymbol{\rho} \textit{ is decodable} \end{cases}$$

the constraint $\boldsymbol{\rho}$ *being decodable is a non-linear non-convex one*

- $\mathcal{P}(\boldsymbol{\rho})$ is a non-linear non-convex problem
- not guaranteed to have a solution even when solved in a centralized setup

Remedy: A two-stage distributed suboptimal approach

Two-stage *distributed* Rate Optimization

1. Beamforming design
   - use single-user (MMSE) decoders (treat interference as noise)
   - offer distributed algorithms for solving $\mathcal{P}(\boldsymbol{\rho})$ and $\mathcal{R}(P)$
2. Excess Rate Allocation
   - BSs exchange their codebooks
   - For the given set of beamformers, BSs deploy UGD
   - UGD allows BSs support rates higher than those yielded by MMSE decoders

Distributed Beamforing Design

For MMSE receivers the problem simplifies to

$$\mathcal{P}(\boldsymbol{\rho}) = \begin{cases} \min_{\{\tilde{\boldsymbol{w}}_i\}} & \sum_{i=1}^{M} \|\tilde{\boldsymbol{w}}_i\|^2 \\ \text{s.t.} & \frac{|\tilde{\boldsymbol{h}}_{i,i}\tilde{\boldsymbol{w}}_i|^2}{\sum_{j \neq i} |\tilde{\boldsymbol{h}}_{i,j}\tilde{\boldsymbol{w}}_j^s|^2 + \sigma_i^s} \geq 1 \end{cases}$$

Lemma 1 *Problem $\mathcal{P}(\boldsymbol{\rho})$ is feasible only if the channel realizations are such that*

$$\text{rank}(\boldsymbol{Q}) \geq \frac{M}{2}.$$

where the matrix $\boldsymbol{Q}$ is determined by the channel coefficients and M is the number of transmit antennas.

Distributed Beamforing Design

Lemma 2 *The problem $\mathcal{P}(\boldsymbol{\rho})$ and its Lagrangian dual exhibit a zero duality gap.*

The design involves the following optimization methods

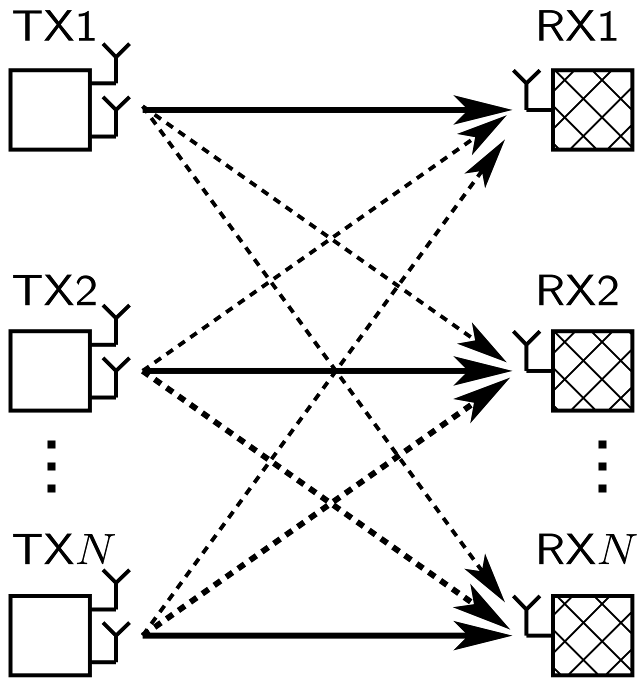
1. Partial Lagrangian: Obtained by dualizing interference margin constraints
2. Subgradient: Distributed algorithm for minimizing non-differentiable convex functions

Theorem 6 *The problems $\mathcal{R}(P_0)$ and $\mathcal{P}(\boldsymbol{\rho})$ can be solved optimally in a distributed way.*

Excess Rate Allocation

Channels + beamformers: MU Interference Channel

- $\boldsymbol{R}_{\min}$ is the rate achieved by using MMSE receivers
- BSs share their codebooks (very limited information exchange)
- RXs use UGD
- RXs can boost their rates beyond $\boldsymbol{R}_{\min}$



Excess Rate Allocation

- UGD: A compromise between rate increments of different RXs
- There should exist coordination among RXs for incrementing rates
- Coordinations should be carried out in a distributed way
- Computationally efficient algorithms
- Rate increments should satisfy max-min fairness (the original problem was max-min rate optimization)

Max-Min Rate Allocation

MMSE receiver, rate $\boldsymbol{R}_{\min}$ $\Rightarrow$ UGD receiver, rate $\boldsymbol{R}$

$$\mathsf{RAF}_{\mathsf{mm}} = \begin{cases} \max & \min_k \frac{r_k}{\rho_k} \\ \text{s.t.} & \boldsymbol{R} = \boldsymbol{R}_{\min} + \boldsymbol{r} \quad \text{remains decocable} \end{cases}$$

$\mathsf{RAF}_{\mathsf{mm}}$: the max-min rate adaptation factor

UGD is superior to MMSE $\Rightarrow$ $\mathsf{RAF}_{\mathsf{mm}} \geq 0$

Solving Max-Min Rate Allocation

connections with the rate adaptation problem in interference channels

<u>similarities:</u>

- similar steps:
  1. solving RAF_{mm} for any given user and partition
  2. locally solving RAF_{mm} for each user (optimizing over partitions)
  3. finding the globally optimal partitions
- similar successive decoding procedure
- same complexity (polynomial)

Solving Max-Min Rate Allocation

connections with the rate adaptation problem in interference channels

<u>differences:</u>

- amount of information exchange (higher)
- the relationship between the global and local adaptation factors
- each receiver suggests a set of rate changes for all BSs
- therefore, each BS receives multiple suggestions
- each BS obtains its adaptation factor by picking the smallest suggested change by all users

Max-Min Fair Rate Adaptation

Initialize $\boldsymbol{R}^{(0)} = \boldsymbol{R}$ and $q = 0$
repeat
  for $k = 1, \ldots, K$ **do**
    Find $\{r_k^i\}_{i=1}^K$
  end for
  Update $q \leftarrow q + 1$ and $R_k^{(q)} = R_k + \min_{1 \le i \le K}\{r_i^k\}$ and $\boldsymbol{R} \leftarrow \boldsymbol{R}^{(q)}$
until $\boldsymbol{R}^{(q)}$ converges

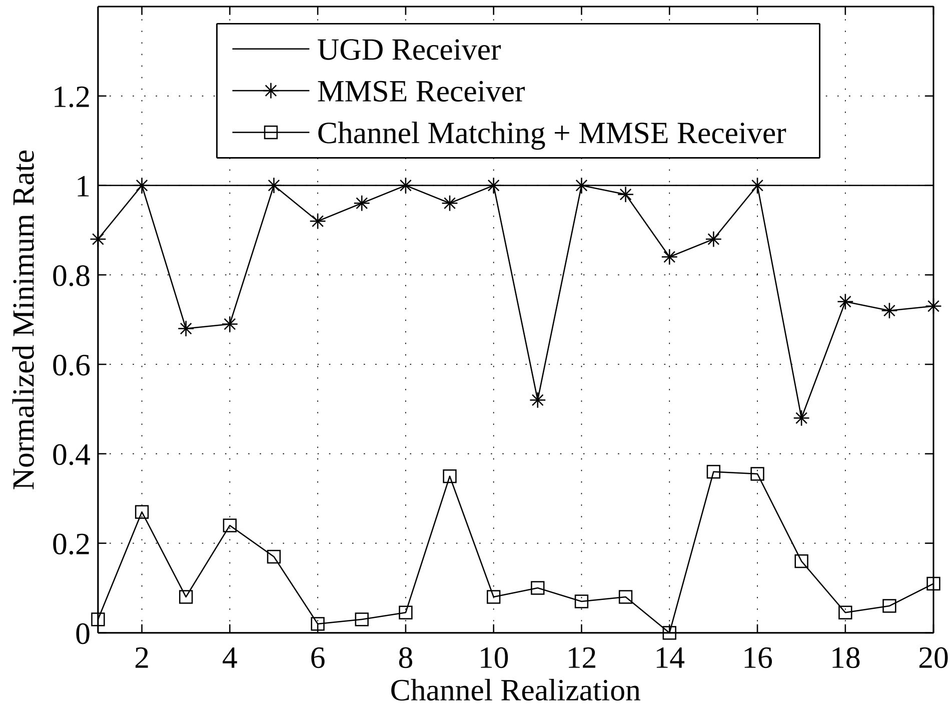
Theorem 7 *The algorithm*

1. *is monotonic in the sense that* $\boldsymbol{R}^{(q+1)} \succeq \boldsymbol{R}^{(q)}$ *and is convergent.*
2. *at each iteration the vector* $\boldsymbol{R}^{(q)}$ *is max-min optimal, i.e., for any other arbitrary decodable rate vector* $\tilde{\boldsymbol{R}} \succeq \boldsymbol{R}^{\min}$ *we have*

$$\min_{k \in \mathcal{K}} \frac{R_k^{(q)} - R_k^{(0)}}{\rho_k} \ge \min_{k \in \mathcal{K}} \frac{\tilde{R}_k - R_k^{(0)}}{\rho_k}.$$

3. *provides pareto-optimal rate adaptaion*

Max-Min Rate for Different Channel Realizations



- three cells, each with 3 users
- BSs: 4 transmit antennas, users: 1 receive antenna
- rates normalized with those achievable with UGD

Comment on Decoding Complexity

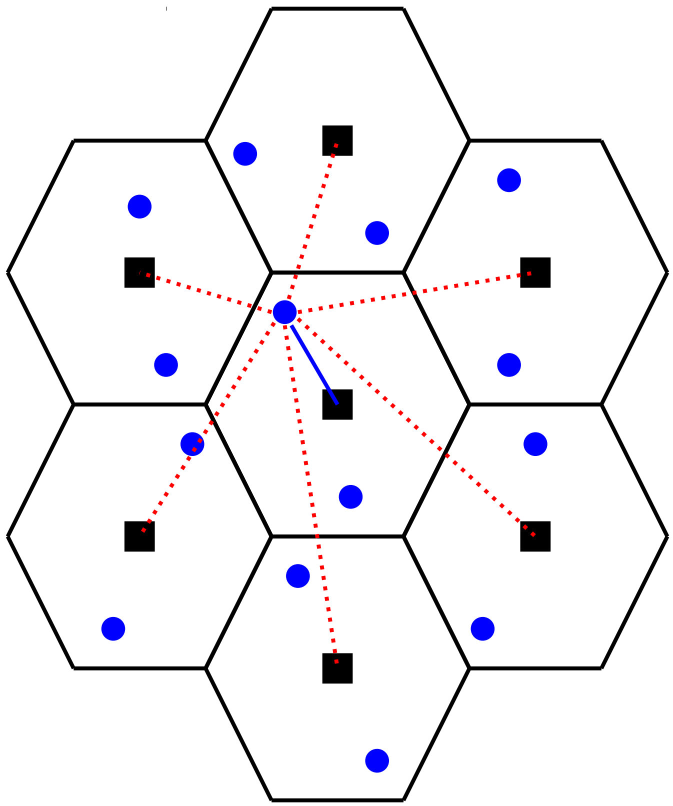
- each receiver partitions the interferers into two sets:
  - decodable set: a group to be decoded jointly via ML decoding
  - noise set: a group to be treated as Gaussian noise
- the size of the decodable set is between 1 and K
- for large decodable sets ML is computationally prohibitive
- to harness such ML decoding complexity
  - partition the decodable set to smaller sets of cardinality $\leq \mu$
  - devise a successive decoding procedure and decoding these partitions successively
  - all the optimality claims still hold true

Joint Channel Coding and Group Decoding

Multi-cell Downlink Systems

interference management: *decode* or *suppress*?

- adding another layer:
  1. multi-antenna pre-coding
  2. rate allocation
  3. UGD
  4. channel coding



Practical Challenges

- the rates suggested by group decoders might *not* be practical rates
  1. impractical signal constellation
  2. impractical code rates
- real codes yield degraded rates compared to Gaussian codeboks
- allow each receiver to decoded the interferers only *partially*
- decoding complexity: the size of decodable set must be $\leq \mu$

Strategies

1. assign multiple codebooks to each transmitter
   - allows decoding it by the non-intended receivers only partially
2. quantize the rates yielded by the algorithm
   - quantization according to a given set of implementable rates
   - quantization level are chosen to minimize the rate distortion
3. use rateless codes to implement discrete rates
   - easy to adjust to channel (and rate) fluctuations
4. there is a gap between the practical rates and the Shannon rate
   - optimize the code profile to minimize the gap

Multiple Codebooks for Each Transmitter

- The message of TX j transmitter split into L_j smaller layers
- $x_{j,k}$: the k^{th} layer of TX j drawn from an independent codebook
- codebook superposition at TX j

$$x_j = \sum_{k=1}^{L_j} x_{j,k}$$

- Equal power allocation for all layers, i.e., $\mathbb{E}(|x_{j,k}|^2) = \frac{P}{L_j}$.
- The transmitters collectively have $\sum_{j=1}^{K} L_j$ codebooks

Constrained Partial Group Decoder (CPGD)

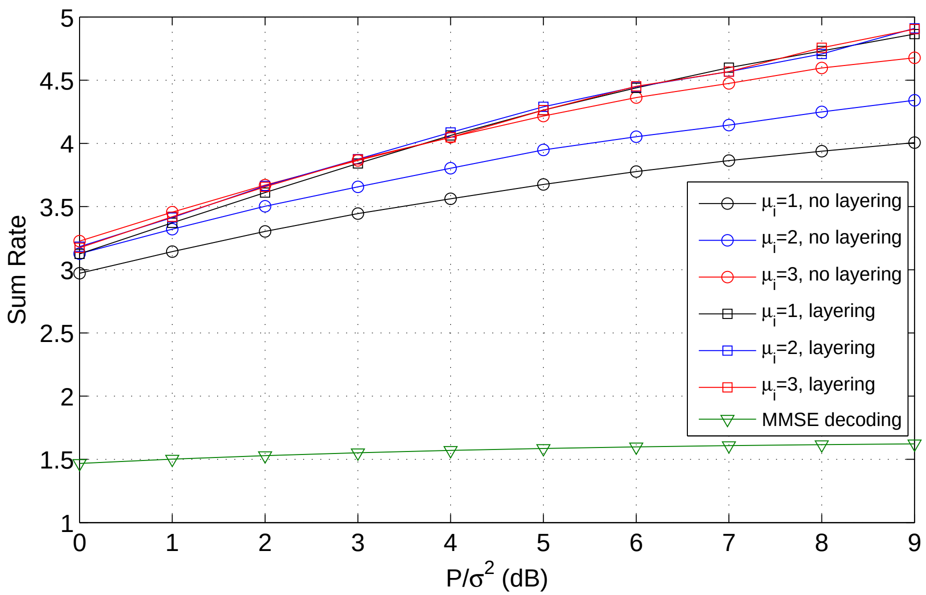
- The i^{th} receiver partitions *the set of all* codebooks to $\underline{\mathcal{Q}}^i$ and $\underline{\tilde{\mathcal{Q}}}^i$
  - codebooks in $\underline{\mathcal{Q}}^i$ to be decoded
  - codebooks in $\underline{\tilde{\mathcal{Q}}}^i$ to be treated as noise
- $\underline{\mathcal{Q}}^i$ is also partitioned to $\underline{\mathcal{Q}}^i \triangleq \{\mathcal{Q}^i_1, \ldots, \mathcal{Q}^i_{p_i}\}$:
  - $|\mathcal{Q}^i_m| \leq \mu$ for $m \in \{1, \ldots, p_i\}$
- constrained partial group decoder:
  - p_i-stage successive decoding;
  - during the m^{th} stage, decode $\mathcal{Q}^i_m$ while treating the following codebooks as noise

$$\left(\cup_{\ell > m} \mathcal{Q}^i_\ell\right) \bigcup \underline{\tilde{\mathcal{Q}}}^i$$

Sum-rate Maximization

no practical constraints on rate selection and channel coding yet

- sum-rate by CPGD
- ideal Gaussian codebooks
- ideal infinite-length codes
- 6 pairs of transceivers
- 5 codebooks per TX



Practical Rate Selection

a two-step rate selection procedure

1. coarse tuning: suggests rates based on the long-term statistical knowledge of the channels

2. fine tuning: further improve the rates based on the instantaneous channel states

Practical Rate Selection (2)

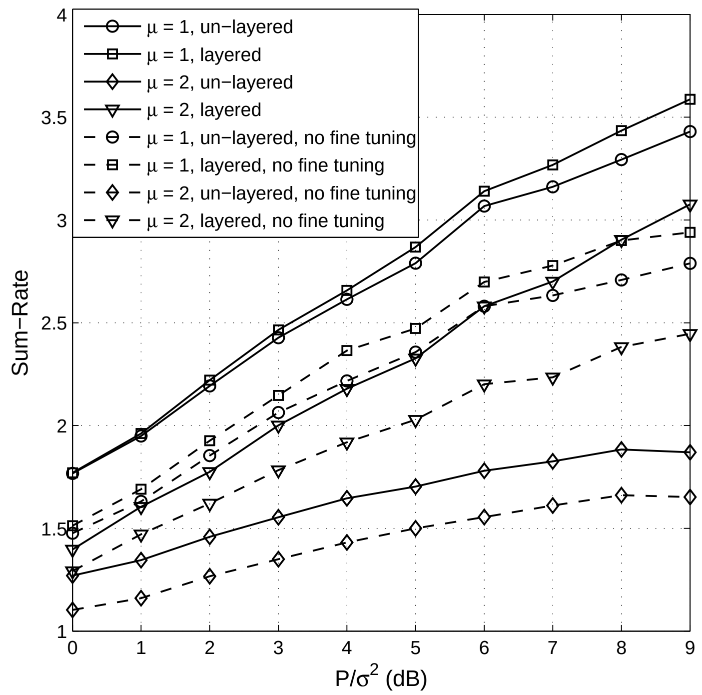
- coarse tuning: Quantize the rates yielded by the CPGD according to the quantization vector $\boldsymbol{d} = [d_0, d_1, \ldots, d_T]$ where $d_0 = 0$

$$\text{if} \quad R_i \in (d_k, d_{k+1}] \quad \Rightarrow \quad \tilde{R}_i = d_k$$

- $\boldsymbol{d}$ is designed
  - a pre-designed channel code for N channel uses
  - a pre-designed modulation scheme
  - and such that the *average* rate distortions due to rate quantization is minimized
- fine tuning: scale up the rates from $\tilde{R}_i$ to $\eta^* \cdot \tilde{R}_i$
  - $\eta^* = \max \eta$ such that $\{\eta \cdot \tilde{R}_i\}$ still decodable
  - achieve $\eta^* \cdot \tilde{R}_i$ via $\frac{N}{\eta^*}$ channel uses

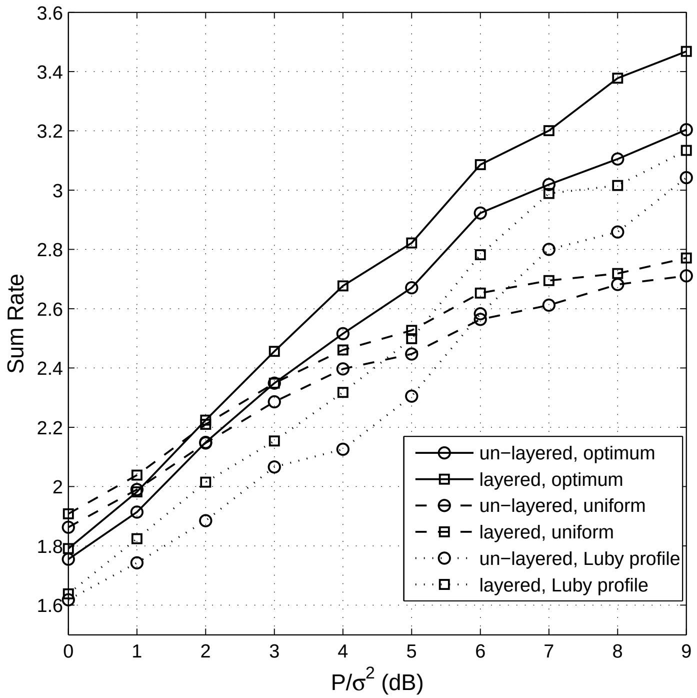
Sum-rate vs. SNR

- $\{R_i\}$ yielded by CPGD
- $\{\tilde{R}_i\}$ by coarse tuning
- $\{\eta^* \tilde{R}_i\}$ by fine tuning
- 6 pairs of transceivers
- 5 codebooks per TX
- Layered outperforms unlayered
- Fine tuning offers 20% rate enhancement



Code Profile Optimization

- Reducing the gap to Gaussian rates
- Performance gain from
  - Layered scheme
  - Optimized $\boldsymbol{d}$
  - Profile optimization



Conclusions

- we have proposed group decoders with constrained and unconstrained sizes
- we characterize new achievable rate regions for K-user interference channels
- group decoders can be used in conjunction with any other interference management techniques, e.g., linear precoding
- certain advantages in some rate allocation problems
  - fair rate adaptation in interference channels
  - max-min rate optimization in MIMO networks
- implementing group decoders is practically feasible
  - polynomial complexity with the network size
  - distributed with very limited information exchange

References

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