What's Next After OFDM ?

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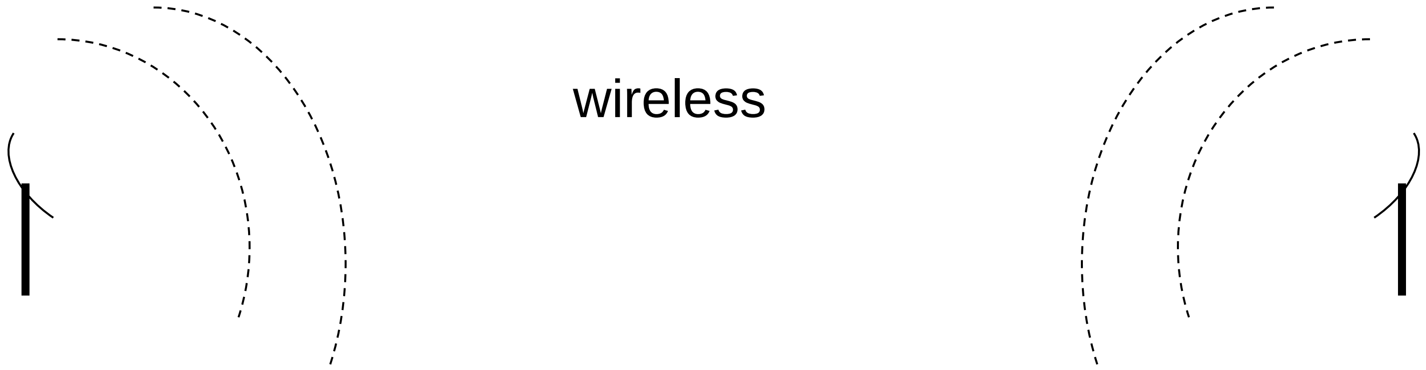
Outline

- Background
  - Current Standards
- OFDM Overview
- Vector OFDM
- VOFDM with Matrix Modulation
- Linear Receivers
- CDD VOFDM for Multiple Antennas
- Conclusion and Future Research

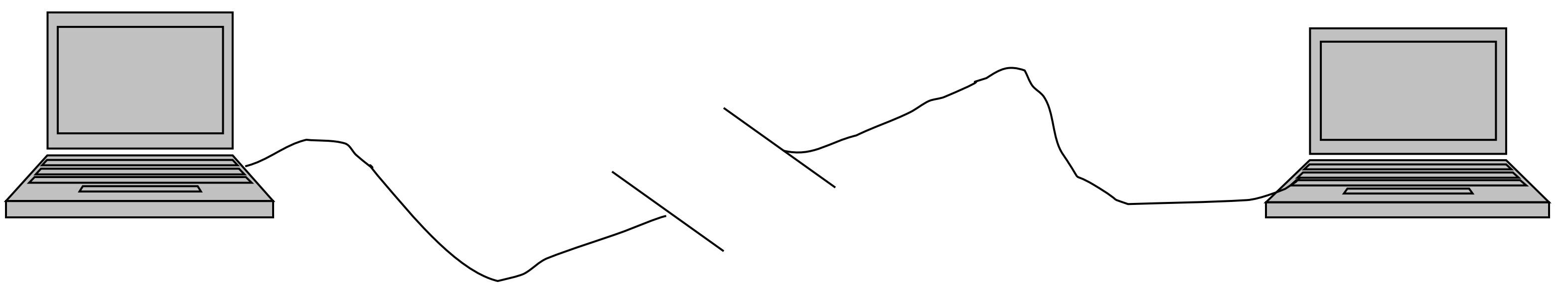
Communications



wireless

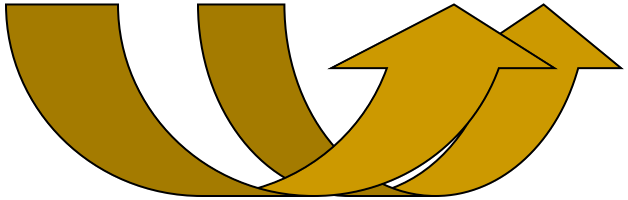
transmission through EM wave propagation

wired



Signal:

A(t) cos(p(t)) carry information

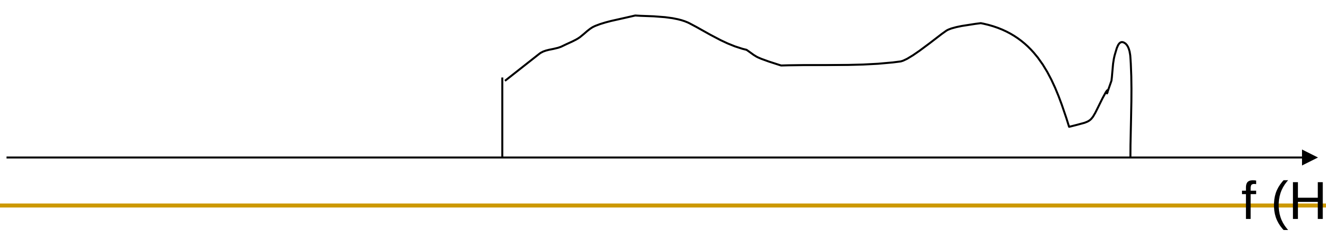


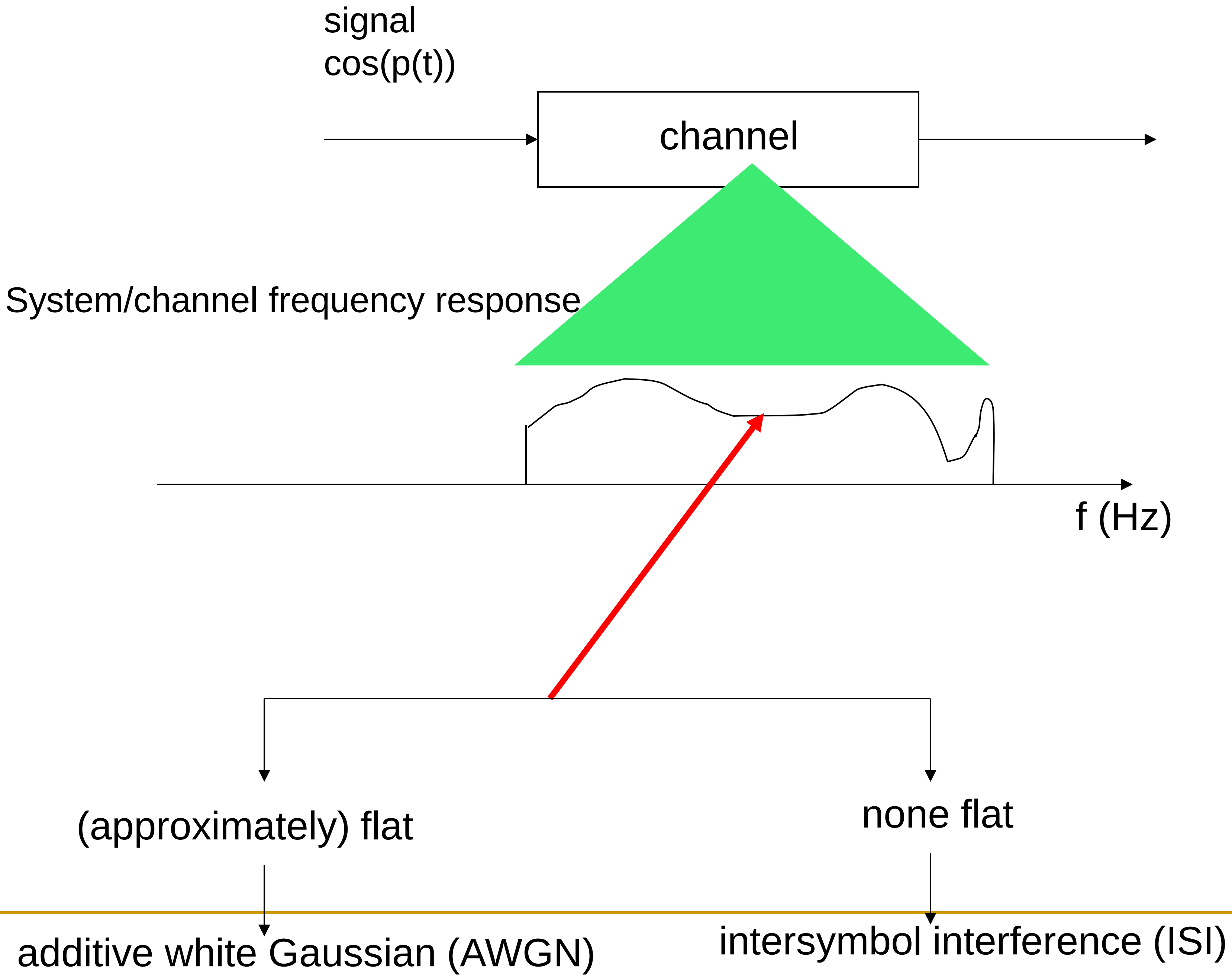
Channel:

a media that wave carrying information propagates through

→ Approximately a linear system

system frequency response

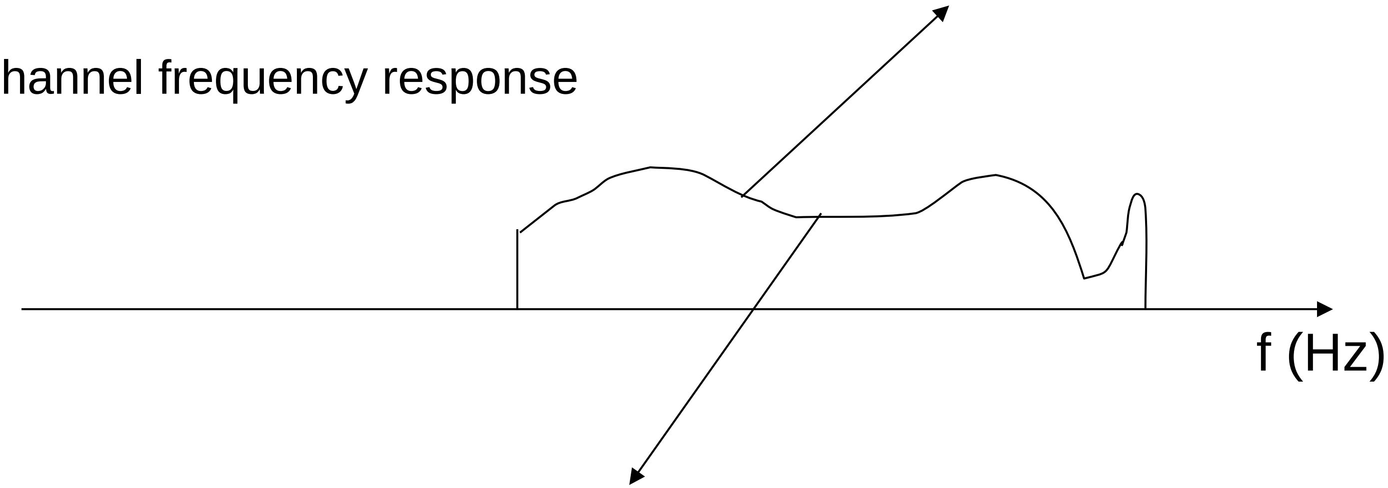



signal
cos(p(t))
channel
System/channel frequency response
f (Hz)
(approximately) flat
none flat
additive white Gaussian (AWGN)
intersymbol interference (ISI)

In broadband systems

this is wide

channel frequency response



It is not possible to be flat → ISI occurs

Wireless: Multipath

Wired: None flat ISI channel
None ideal

Wired (modem): Channel is fixed and has high SNR

< 9.6 kbs/s	equalization	(Lucky 60s)
9.6 kbs/s	TCM +equalization (DFE)	
14.4 kbs/s 28.8 kbs/s 56 kbs/s	TCM + equalization	

Squeeze more bits to a symbol

Asymmetric Digital Subscriber Line (ADSL)

6 Mbs/s — **orthogonal frequency division multiplexing (OFDM)**
or called discrete multi-tone (DMT)

Use more bandwidth

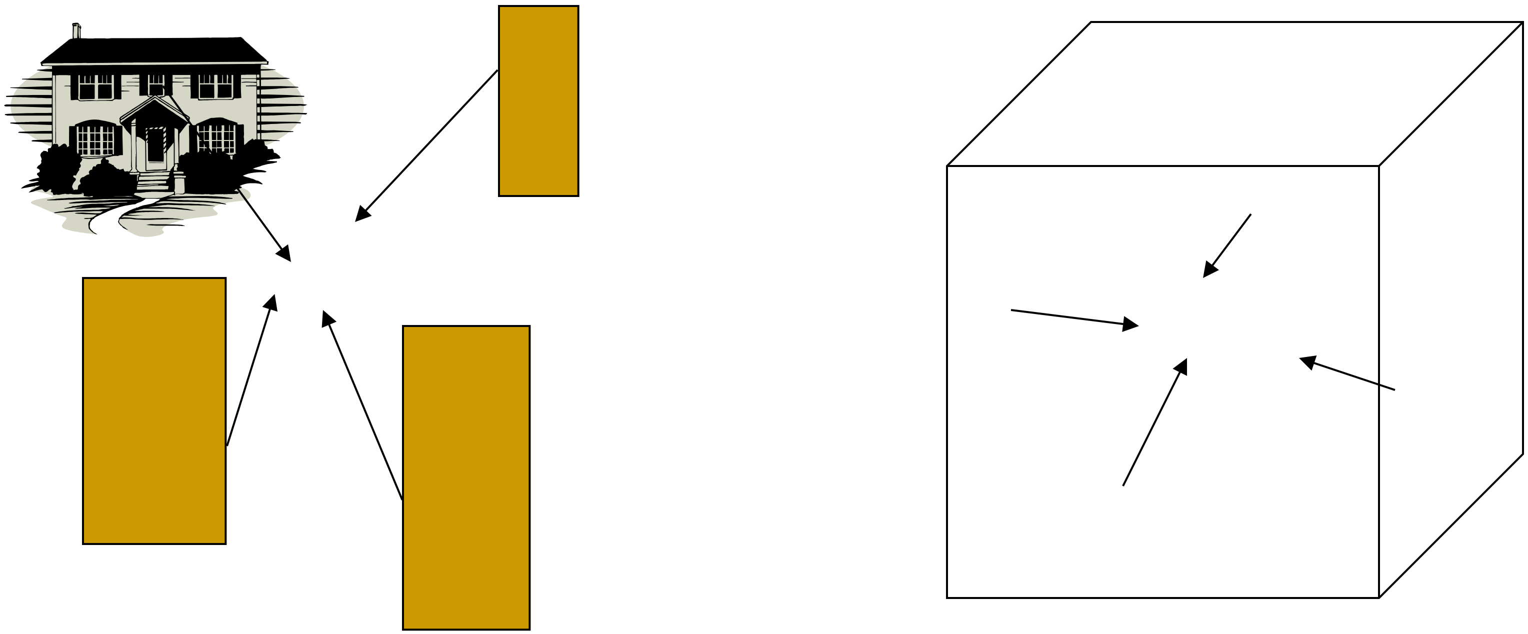
Data Rate	Wire Size	Distance
1.5 or 2 Mbps	0.5 mm	5.5 Km
1.5 or 2 Mbps	0.4 mm	4.6 Km
6.1 Mbps	0.5 mm	3.7 Km
6.1 Mbps	0.4 mm	2.7 Km

Wireless Systems: Channel varies/fades and not high SNR

outdoor

indoor

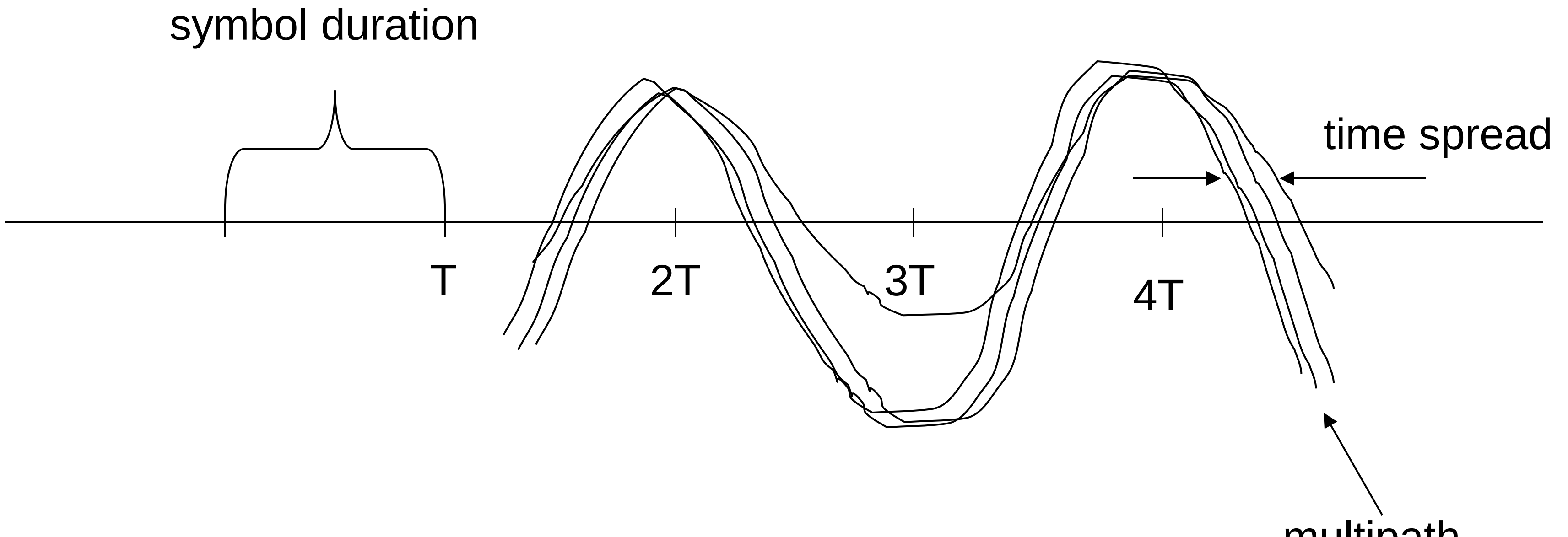
multipaths



multiple reflections

Multipath

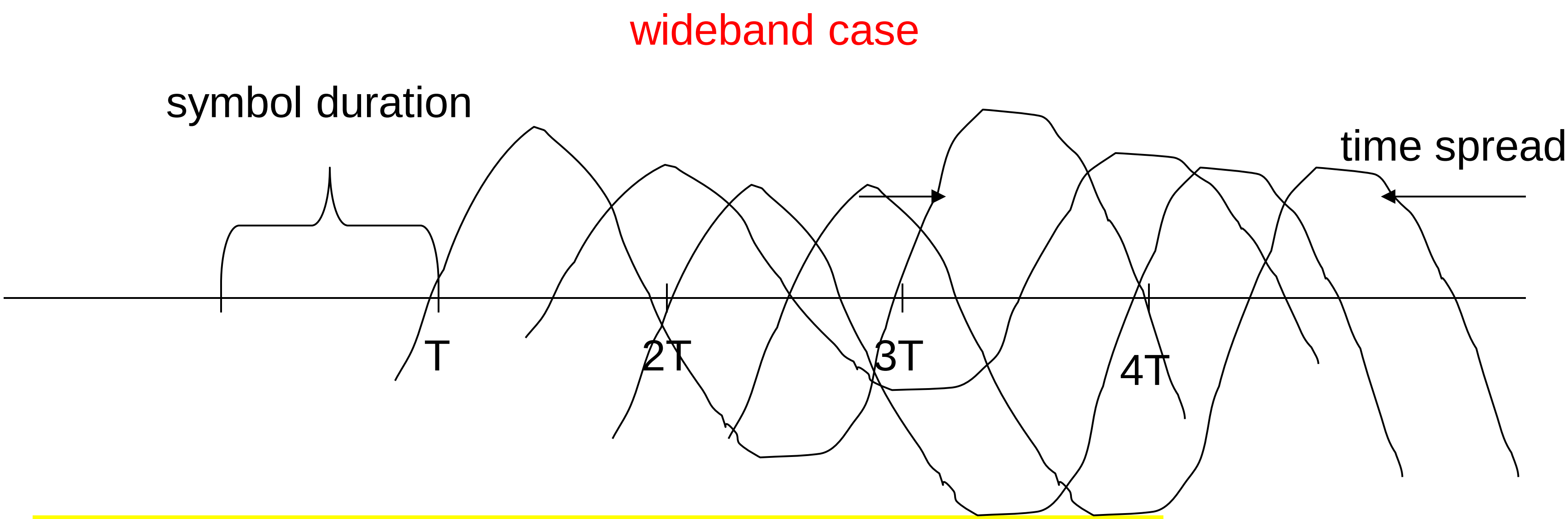
narrowband case



1/T bandwidth

No intersymbol interferences

When the bandwidth is too wide or T is too small, the time spread may be across over multiple symbols. In this case, intersymbol interference (ISI) occurs.



x(t): transmitted signal; r(t): received signal

$$r(t) = \sum_{n=0}^{L} h(n)x(t-nT) + W(t)$$

ISI AWGN

Number of Multipath vs. Modulation Methods in Wireless Applications

2G (IS-95)	1.23 MHz	Almost optimal for single path
3G (WCDMA CDMA2000)	< 11 MHz	6--8 multipath almost the break point to use CDMA
IEEE 802.11b (LAN)	similar to 3G	
IEEE 802.11a (LAN)	20 MHz	16 multipath OFDM
IEEE 802.11n (LAN)	20 & 40MHz	40MHz doubles everything in 20MHz OFDM
3GPP/4G LTE	20 MHz	16 multipath OFDM and SC-FDE
XG	100 MHz	??

Digital Wireless Standards vs. Bandwidth (#of Multipaths)

- A standard is determined by a bandwidth (so far)
- 2G: 1.23MHz, almost the highest for non-ISI
  - Both TDMA and CDMA (DS spread spectrum) work well
- 3G: 10 MHz, a few multipath
  - Due to the ISI and wireless varying channels, time domain equalization may not work well, TDMA is not used, but CDMA (DS spread spectrum) is used in all standards since it is good to resist a few chip level time delays (RAKE receiver)
- 4G: 20 MHz, more multipath
  - **Even CDMA RAKE receiver may not work well**
  - **OFDM is adopted (down link)**
    - Due to wireless channel varying, the number of subcarriers, N=64, is used, 25% data overhead for the cyclic prefix (CP) to deal with the multipath

5G: Bandwidth >>20 MHz (?)

- Can OFDM Still Work?
  - Much more multipath exist

    → much large CP length to deal with multipath

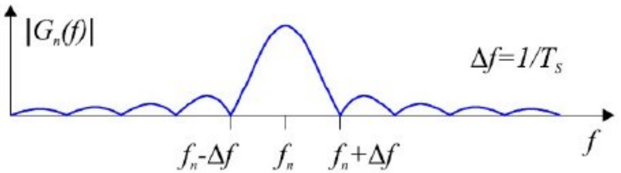
    → much large number *N* of subcarriers/IFFT_size

    → may lead to break down OFDM??
      - High PAPR (?)
      - Time varying channels (?)
- Is multiband OFDM bandwidth efficient?
  - Five 20 MHz bandwidth OFDM systems to form 100 MHz band
- Is 20 MHz a breakpoint for OFDM in **cellular systems**? How large will a bandwidth go?
  - Can we make it work with a **fixed *N*** while it still can deal with the increased # of multipath?
    - **We next think about single antenna VOFDM** [Xia, TCOM, August, 2001]

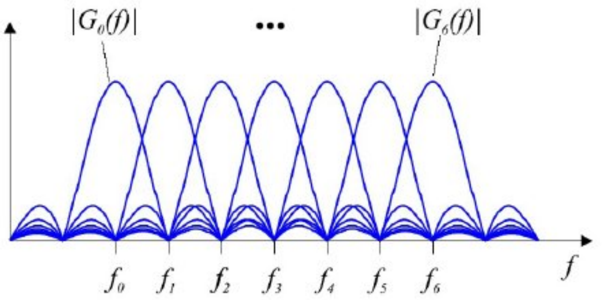
OFDM and VOFDM

- OFDM: orthogonal frequency division multiplexing

1 subcarrier

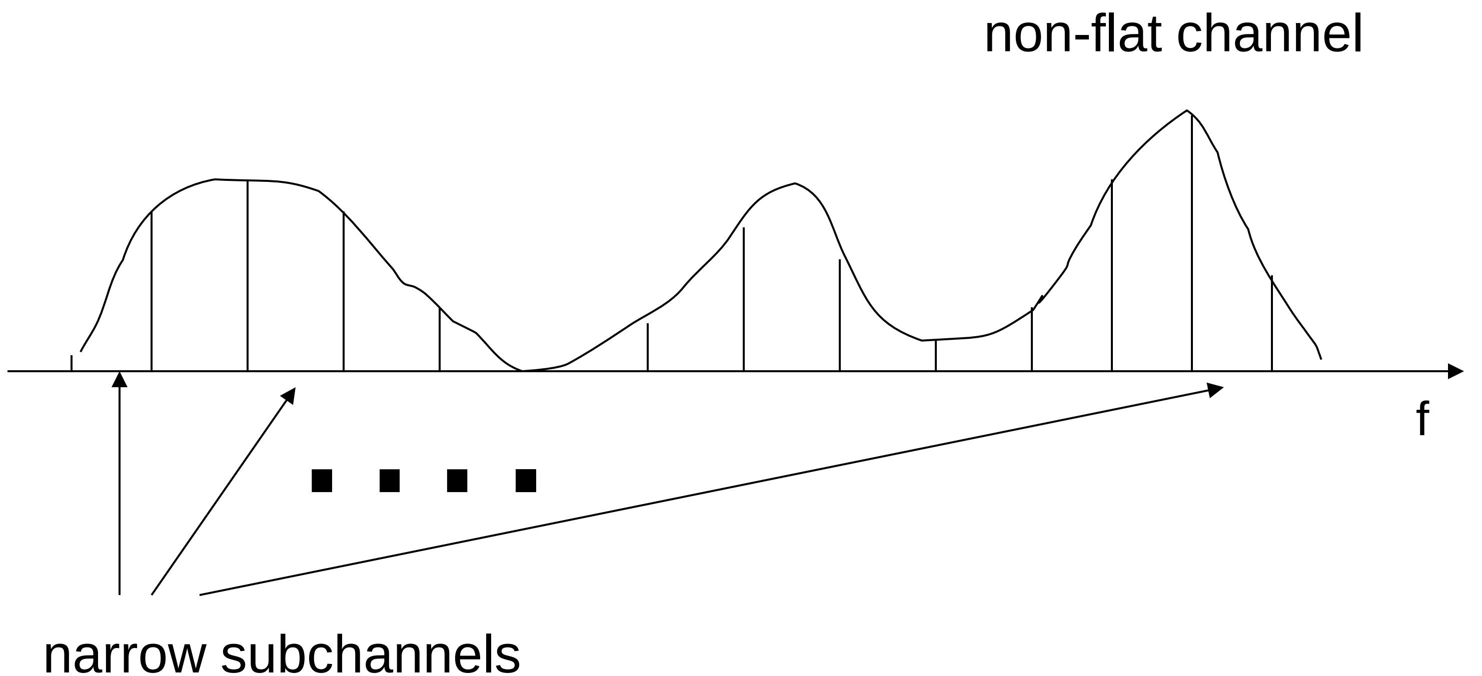


6 subcarriers



- VOFDM: vector OFDM
  - It is nature for multiple antenna systems, when every antenna employs OFDM
    - Cisco's VOFDM for MIMO systems (MIMO-OFDM)
  - It is not trivial for single transmit antenna systems
    - **Today's focus**

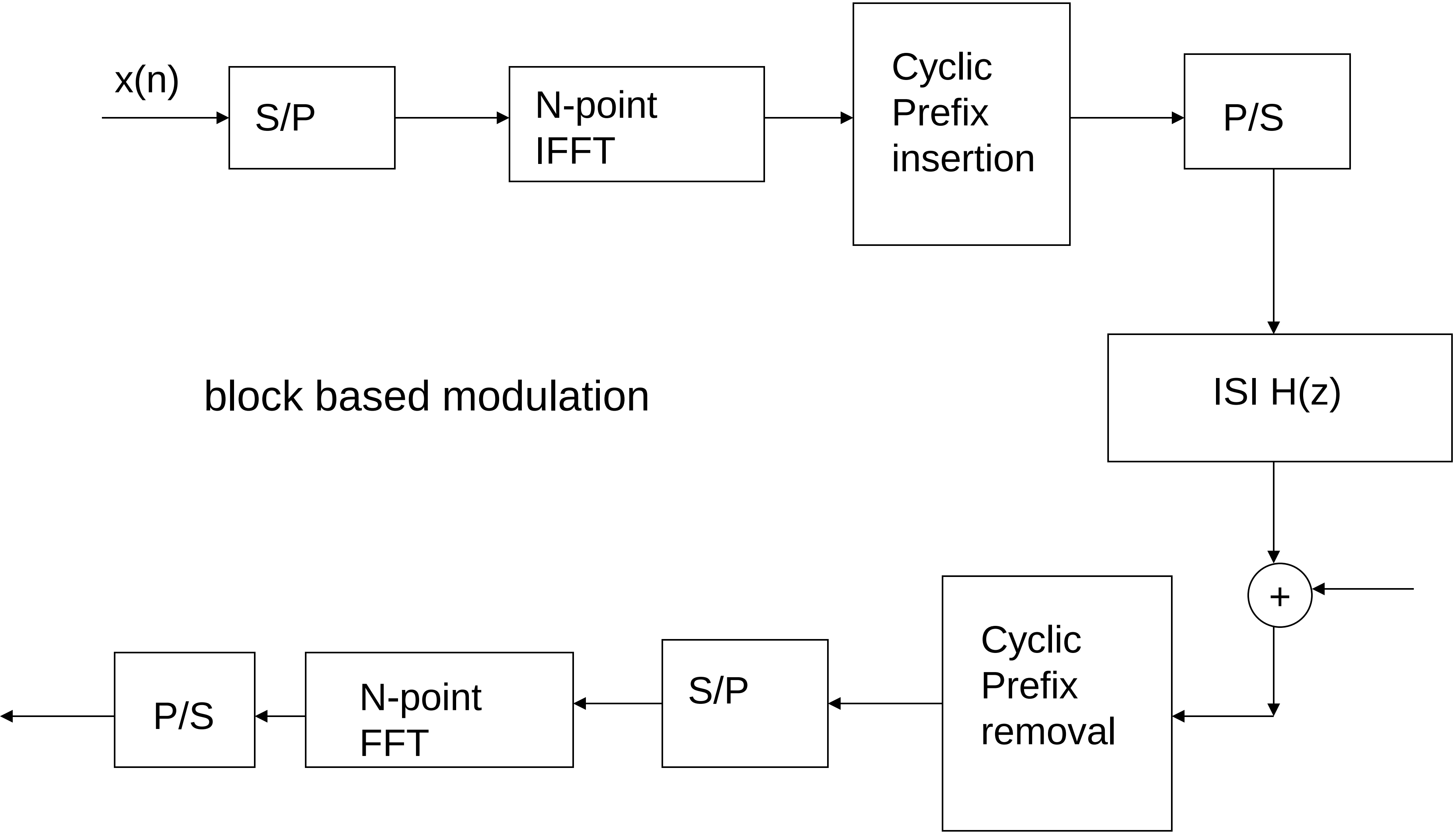
Why OFDM ? -------- Rough Idea



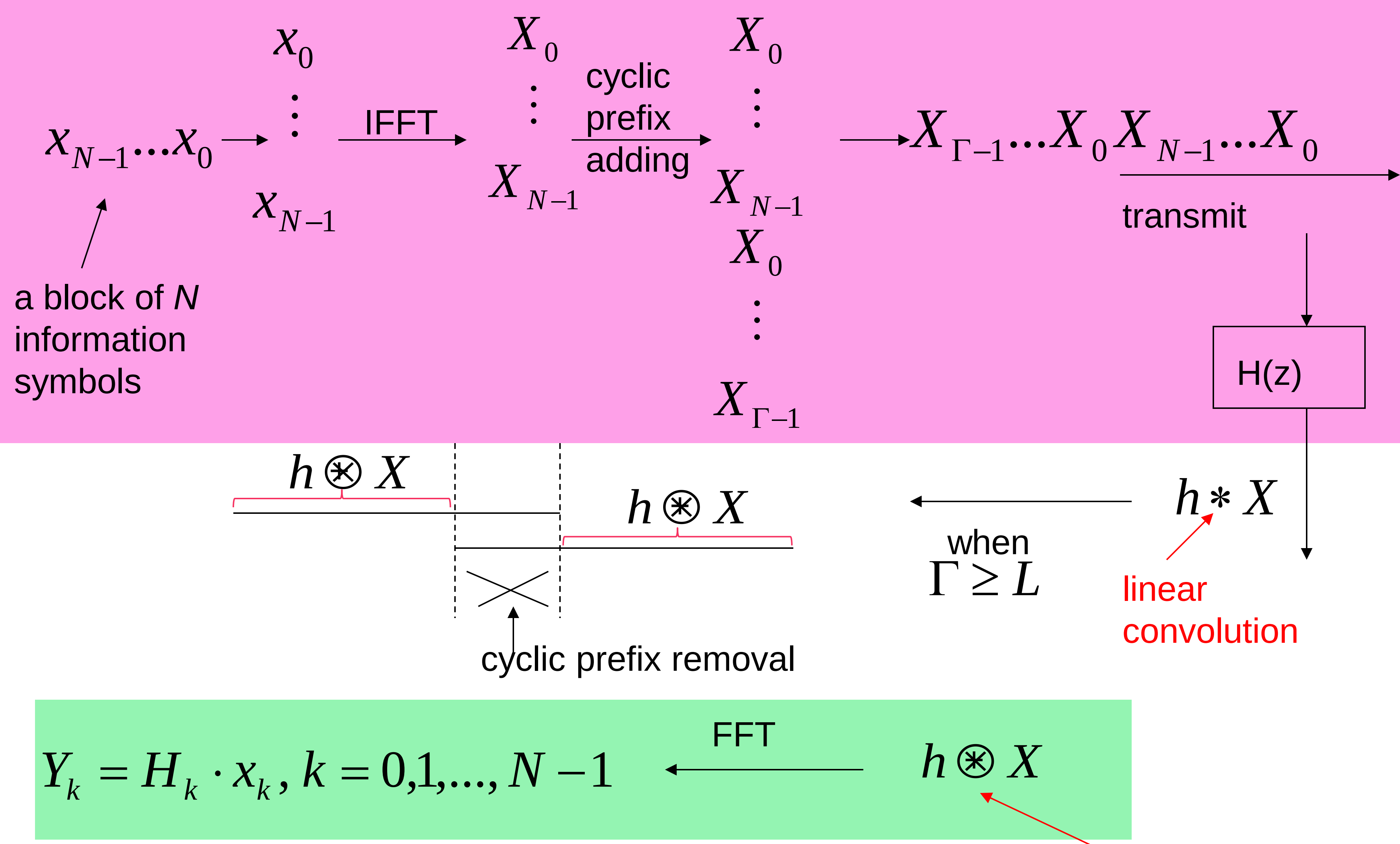
Each subchannel is narrow and therefore more flat;

does not have ISI

OFDM



block based modulation



$$H_k = H(e^{j2\pi \frac{k}{N}}) = \sum_{n=0}^{L} h(n) e^{-j\frac{2\pi nk}{N}}$$

ISI channel

$$y(k) = \sum_{n=0}^{L} h(n)x(k-n) + w(k)$$

adding cyclic prefix as an additional data rate overhead

N ISI-free subchannels

$$Y_k = H_k \cdot x_k + W_k, \quad k = 0,1,\ldots,N-1$$

Each subchannel corresponds to a DFT component H_k of the ISI channel.
If the frequency component H_k is small (bad), then this subchannel is bad.

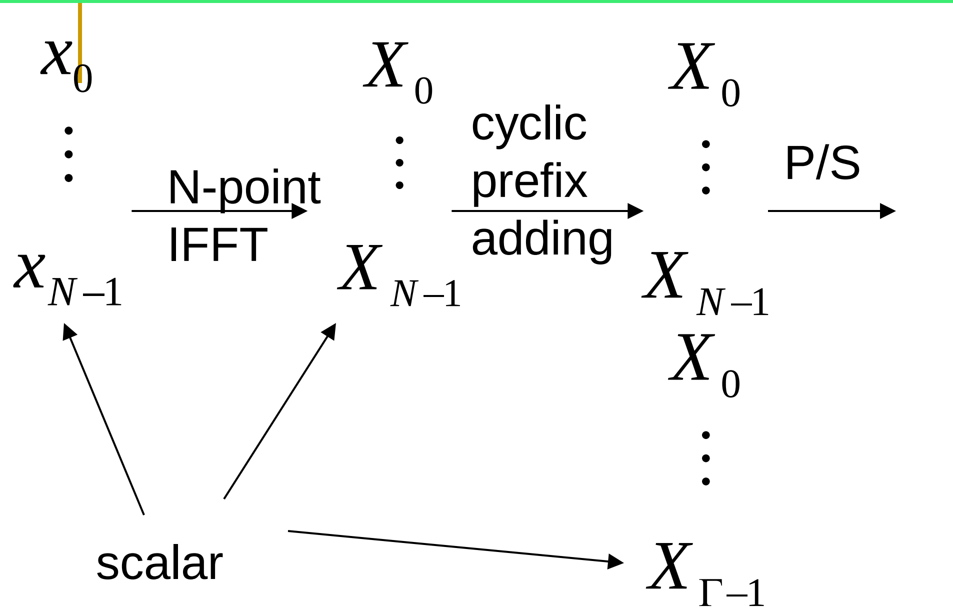
For 20 MHz Channel, OFDM

L <= 16

N=64

Γ=L=16, 25% data rate overhead

Basic Idea for Vector OFDM Single Transmit Antenna System



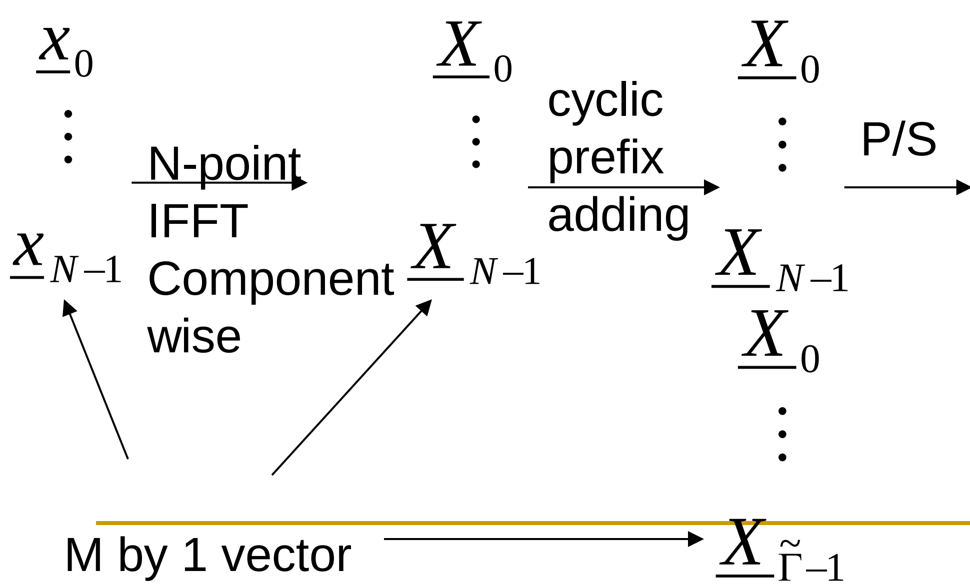
OFDM, when

data rate overhead $\Gamma \geq L$

Receiver:

$$Y_k = H_k x_k + W_k$$

N scalar channels/equations



VOFDM, when

data rate overhead $\tilde{\Gamma} \geq \tilde{L} \approx \frac{L}{M}$

Receiver:

$$\underline{Y}_k = \underline{H}_k \underline{x}_k + \underline{W}_k$$

N M by 1 vector channels/equations

VOFDM: Vectorized Channel

- The ISI channel *H(z)* is converted into N vector channels

M symbols in each vector are in ISI

$$\underline{Y}_k = \underline{H}_k \underline{x}_k + \underline{W}_k, \quad k=0,1,\ldots,N-1$$

where $\underline{H}_k$ is the M by M blocked version of the original frequency responses of the ISI *H(z)*:

$$\underline{H}_k = \underline{H}(e^{j2\pi k/N}),$$

$$\underline{H}(z) = \begin{bmatrix} H_0(z) & z^{-1}H_{M-1}(z) & \cdots & z^{-1}H_1(z) \\ H_1(z) & H_0(z) & \cdots & z^{-1}H_2(z) \\ \vdots & \vdots & \vdots & \vdots \\ H_{M-1}(z) & H_{M-2}(z) & \cdots & H_0(z) \end{bmatrix}$$

$$H_m(z) = \sum_{l=0}^{\tilde{L}} h(Ml+m)z^{-l}, \quad 0 \le m \le M-1.$$

*m*th polyphase component of *H(z)*

$$\tilde{L} = \left\lceil \frac{L}{M} \right\rceil$$

Vectorized Channel Example

If $H(z) = 1 + 0.9z^{-1} - 0.8z^{-2} + 0.6z^{-3} + 0.5z^{-4} - 0.4z^{-5}$, vector size M=2,

then, its polyphase components are

$$H_0(z) = 1 - 0.8z^{-1} + 0.5z^{-2}, \quad H_1(z) = 0.9 + 0.6z^{-1} - 0.4z^{-2}$$

and the vector channel coefficient matrices are

$$L = 5$$

$$\tilde{L} = \left\lceil \frac{L}{M} \right\rceil = \left\lceil \frac{5}{2} \right\rceil = 3$$

$$\underline{H}(z) = \begin{bmatrix} H_0(z) & z^{-1}H_1(z) \\ H_1(z) & H_0(z) \end{bmatrix}$$

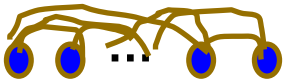
$$= \begin{bmatrix} 1 & 0 \\ 0.9 & 1 \end{bmatrix} + \begin{bmatrix} -0.8 & 0.9 \\ 0.6 & -0.8 \end{bmatrix} z^{-1} + \begin{bmatrix} 0.5 & 0.6 \\ -0.4 & 0.5 \end{bmatrix} z^{-2} + \begin{bmatrix} 0 & -0.4 \\ 0 & 0 \end{bmatrix} z^{-3}$$

VOFDM, OFDM, SC-FDE

- When *M*=1, VOFDM=OFDM
- When *M*=*N* and the FFT size is 1, VOFDM=SC-FDE:
  - at the transmitter, no IFFT is implemented (so the PAPR is not changed) but just CP of the information symbols is inserted; low PAPR
  - at the receiver, both FFT and IFFT, and frequency domain equalizer are implemented
- VOFDM is a bridge between OFDM and SC-FDE
  - Its ML receiver complexity is also in the middle

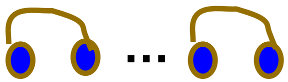
Time domain single carrier vs. equalization

Maximum # symbols in ISI



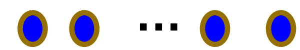
VOFDM

No, or 2, or 3, …, or Maximum # (**you choose**) symbols in ISI



Frequency domain OFDM

No ISI



VOFDM: Advantages

- Cyclic prefix data rate overhead reduction when the FFT/IFFT size is fixed
  - For OFDM, it is $\frac{L}{N}$
  - For VOFDM, it is $\frac{L}{MN}$
- For fixed cyclic data rate overhead, the FFT/IFFT size can be reduced by M times
  - The IFFT size reduction reduces the peak-to-average power ratio (PAPR), which is important in cellular communications.

VOFDM: Advantages

- VOFDM can be combined with matrix modulation: at the receiver $\underline{Y}_k = \underline{H}_k \underline{x}_k + \underline{W}_k$ where $\underline{x}_k$ are vectors of information bits or symbols. By grouping two vectors of size 2 together considering BPSK for each information symbol, these vectors become 16 matrices

$$\left\{ \begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix} : x_{ij} \in \{1, -1\} \right\}$$

- These 16 matrices are not good in terms of matrix modulation due to the channel matrices have random components (fading).
  - These 16 matrices can be replaced by the ones with the best known diversity product.

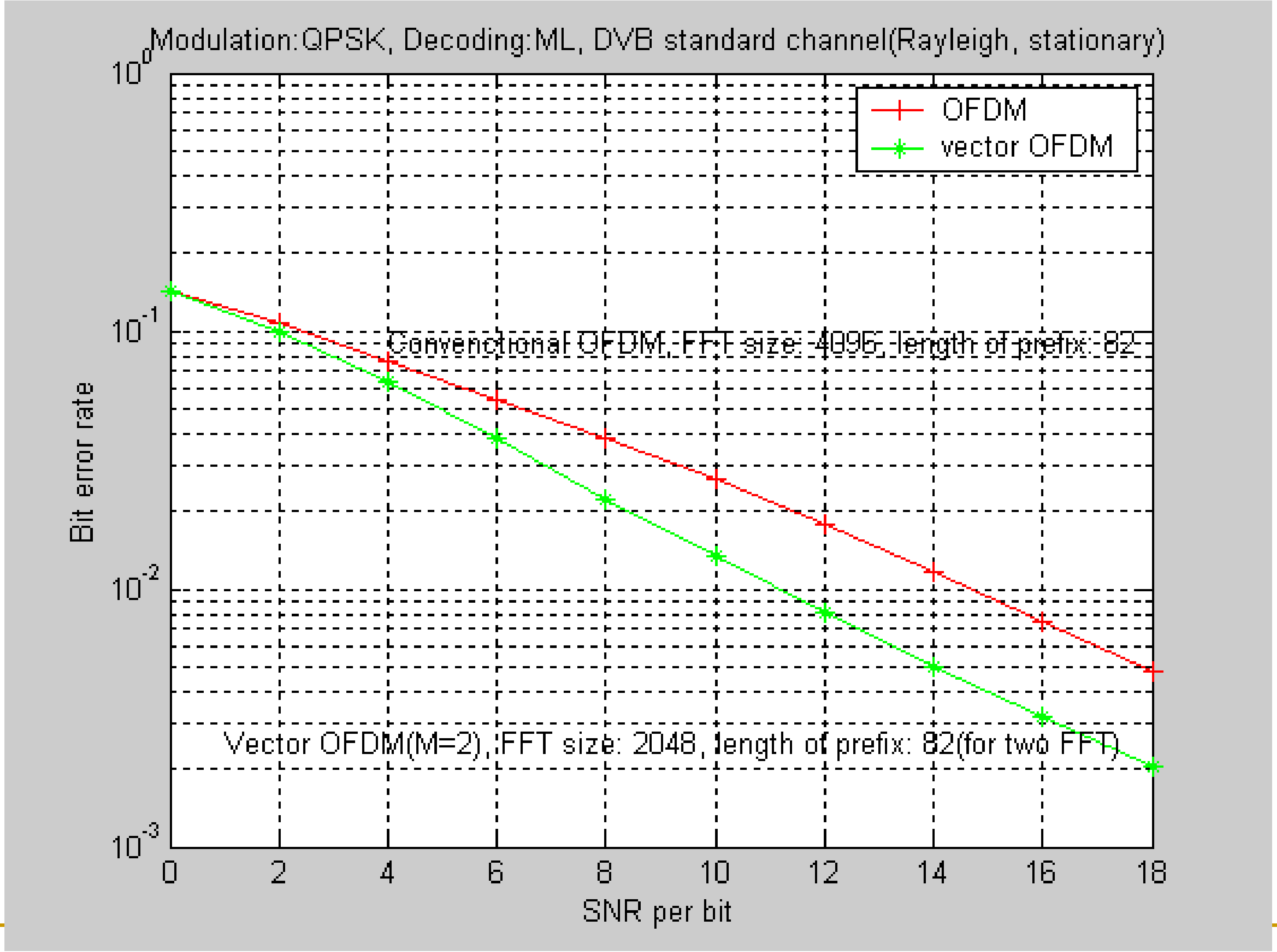
Unitary Matrix Modulation

- 16 best known 2 by 2 unitary matrices in the literature (Liang-Xia, IEEE Trans. Information Theory, Aug. 2002) with the best known diversity product (*the minimum absolute value of all the determinants of difference matrices of any two distinct matrices*):

$$\left\{ \begin{bmatrix} e^{jl\pi/8} & 0 \\ 0 & e^{jl3\pi/8} \end{bmatrix} \begin{bmatrix} \cos\frac{l\pi}{2} & \sin\frac{l\pi}{2} \\ -\sin\frac{l\pi}{2} & \cos\frac{l\pi}{2} \end{bmatrix} \begin{bmatrix} e^{jl\pi/4} & 0 \\ 0 & e^{-jl\pi/4} \end{bmatrix} : 0 \le l \le 15 \right\}$$

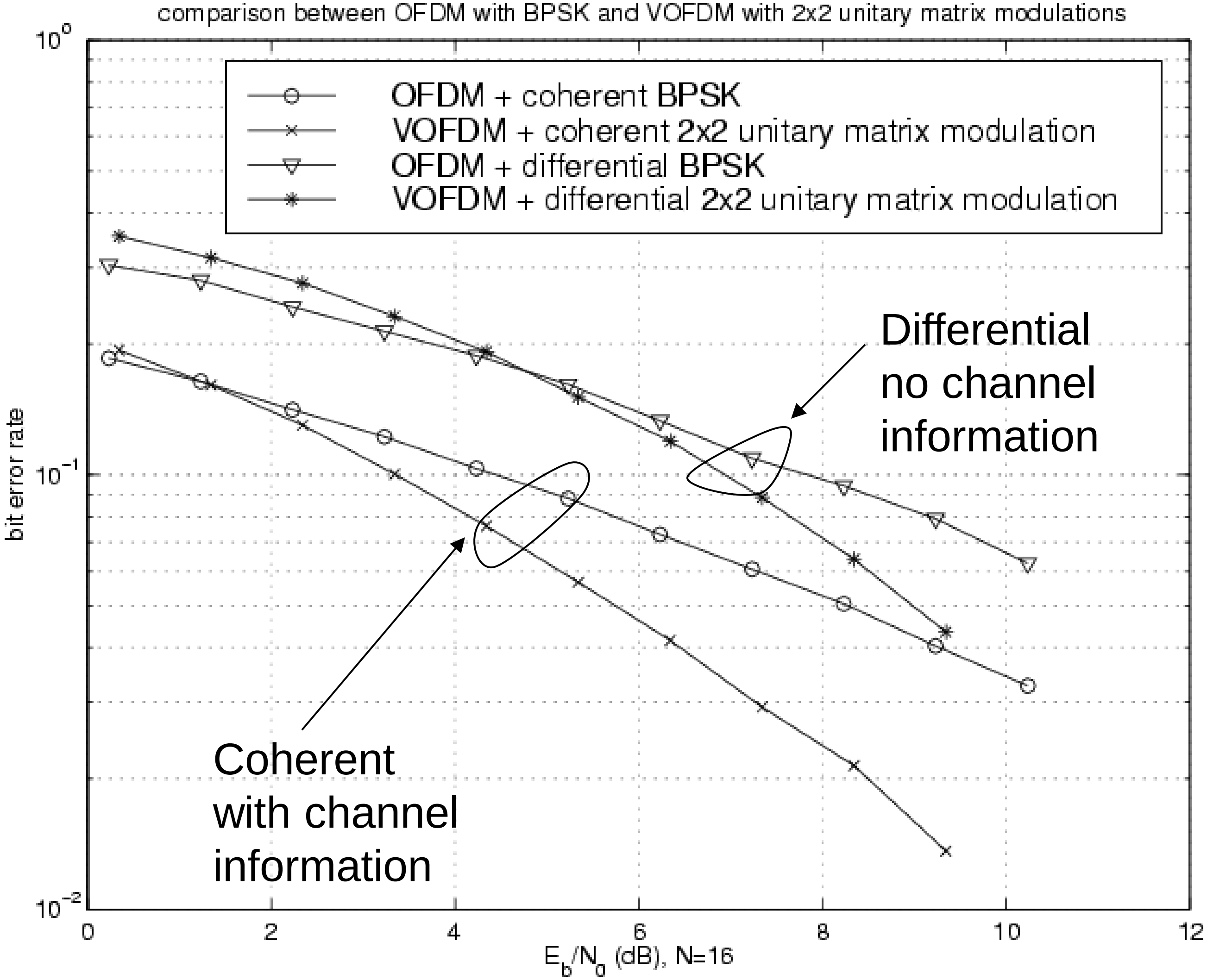
Simulations

DVB



CP data rate overhead is the same for the two curves, matrix modulation is not used.

ML receivers



Linear Receivers for Single Antenna VOFDM (Yabo Li, Ngebani, Xia and Host-Madsen, *IEEE Trans. on Signal Processing*, Oct. 2012)

- Zero-Forcing (ZF) receiver

- Minimum mean square error (MMSE) receiver

Detection SNR Gap Between ZF and MMSE Receivers

Theorem *1: Denote the m-th column of* $\mathbf{H}_l$ *as* $\mathbf{h}_{l,m}$ *and the matrix of* $\mathbf{H}_l$ *after deleting the m-th column as* $\mathbf{H}_{l,m}$*, which is an* $M \times (M-1)$ *matrix. When* $\rho \to \infty$*, the gap between the detection SNRs of the ZF-V-OFDM and the MMSE-V-OFDM can be written as*

$$\lim_{\rho \to \infty} \left(\rho_l^{MMSE} - \rho_l^{ZF} \right) = \left\| \mathbf{h}_l^H \mathbf{H}_{l,m} \left(\mathbf{H}_{l,m}^H \mathbf{H}_{l,m} \right)^{-1} \right\|^2, \tag{12}$$

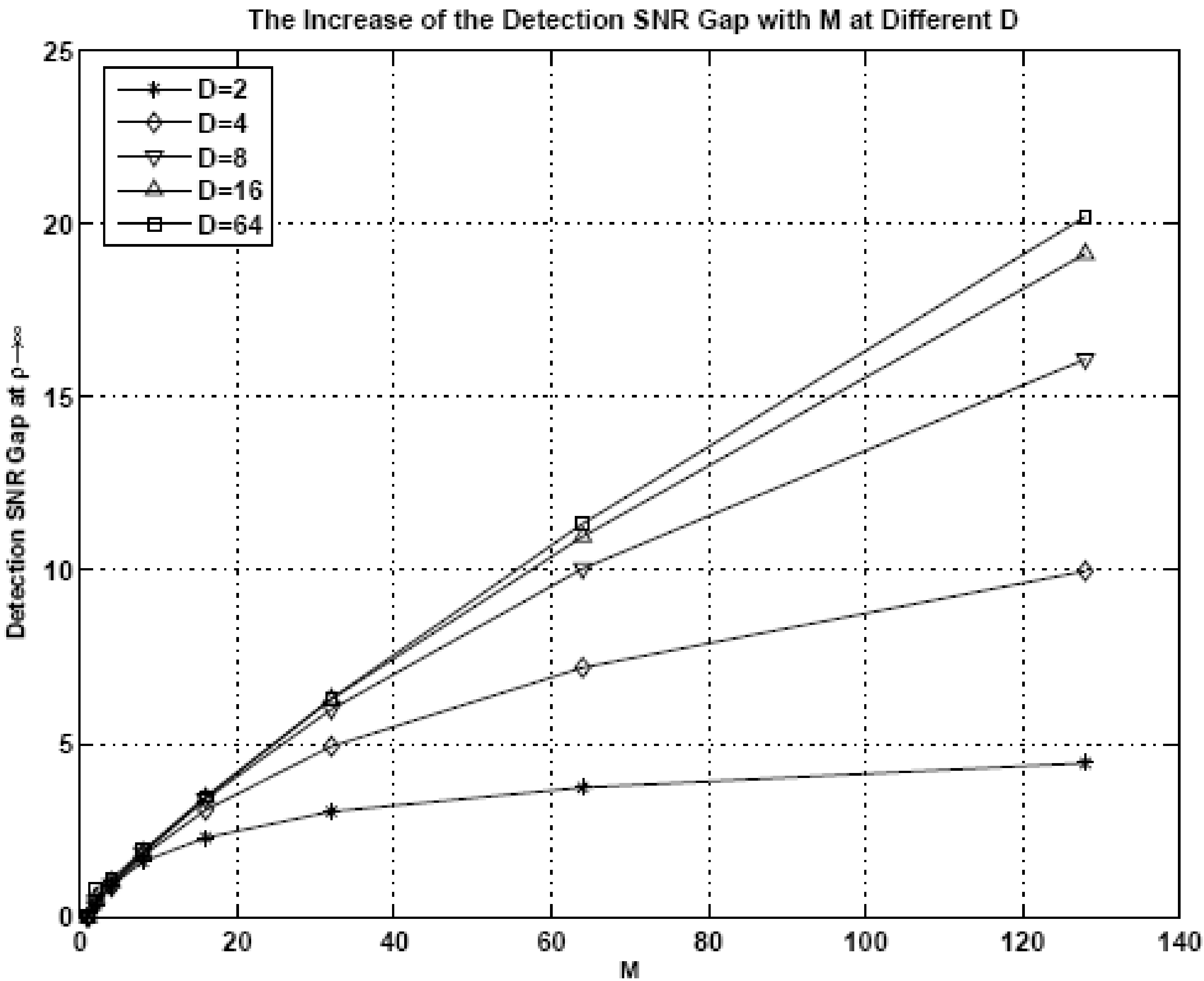
which is independent of m. As $\rho \to \infty$*, the ratio between* ρ_l^{MMSE} *and* ρ_l^{ZF} *approaches* 1*, i.e.,*

$$\lim_{\rho \to \infty} \frac{\rho_l^{MMSE}}{\rho_l^{ZF}} = 1.$$

Detection SNR Gap Between ZF and MMSE Receivers

- Conventionally, MMSE detection is thought of as equivalent to ZF detection when SNR approaches infinity.
- However, for V-OFDM, the SNR gap between ZF and MMSE detection doesn't approach zero as SNR approaches infinity
- On average, the performance gap increases with the vector block (VB) block size *M* and the maximum delay *L (*or *D* used as below*)* of the channel

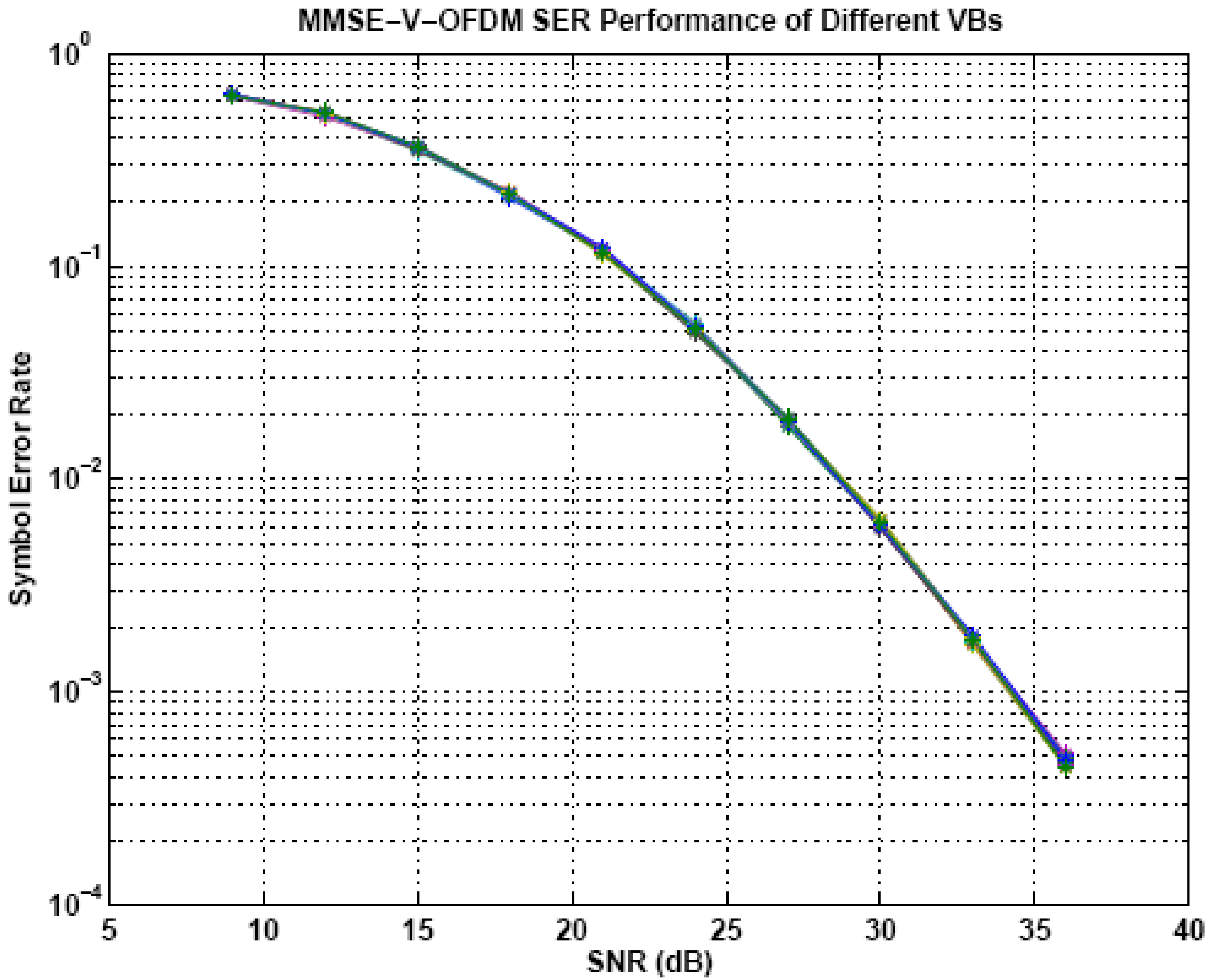
Detection SNR Gap Between ZF and MMSE Receivers



The Performance Independence of Vector Block Index

- **Theorem 2**: For ZF-VOFDM and MMSE-VOFDM, after averaging over all the channel, the *NM* transmitted symbols have the same error rate performance.
  - For VOFDM with ML receiver (i.e., ML-VOFDM), different VBs may have different performances (See Han *et al* 2010 and Cheng *et al.* 2011).
  - However, for VOFDM with ZF and MMSE receivers, all the VBs have the same performance.

The Performance Independence of Vector Block Index



Diversity Order of MMSE and ZF Receivers

- Definition of the diversity order

$$d\left(R,M,D,N\right) = -\lim_{\rho\to\infty}\frac{\log P_{ser}\left(R,M,D,N\right)}{\log\rho}.$$

  - *R* is the spectrum efficiency defined as bits/sec/Hz

- **Theorem** *5: For MMSE-V-OFDM, the diversity* $d^{MMSE}\left(R,M,D,N\right)$ *equals*

$$d^{MMSE}\left(R,M,D,N\right) = \min\left\{\lfloor M2^{-R}\rfloor, D\right\} + 1.$$

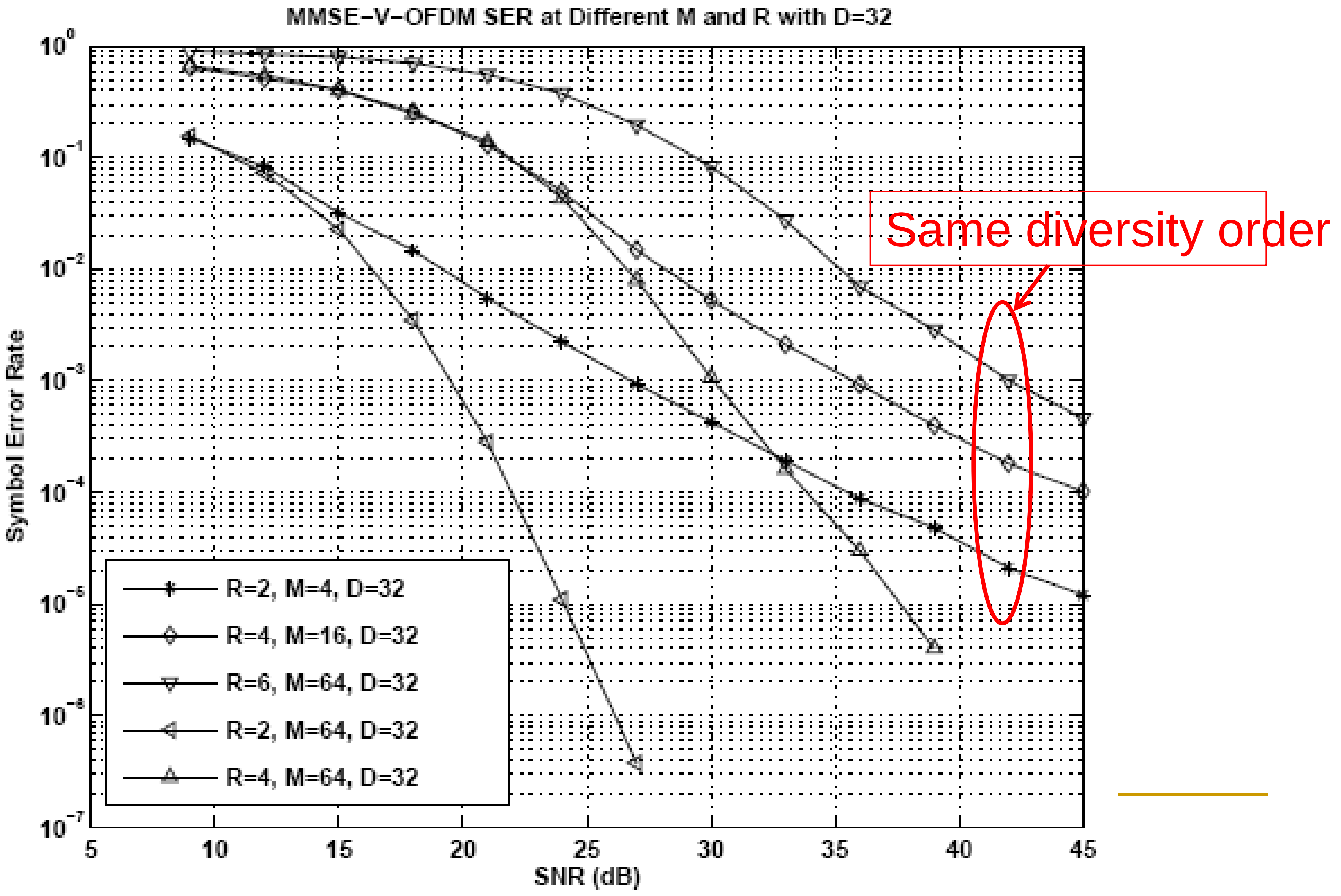
- **Theorem** *6: For ZF-V-OFDM, the diversity order* $d^{ZF}\left(R,M,D,N\right) = 1.$

Diversity Order of MMSE and ZF Receivers

- Both ZF and MMSE detections are scalar detections, they have the similar complexities.
- However, the MMSE detection can exploit the diversity inside the VOFDM, while ZF detection cannot.
- The only required extra information for MMSE detection is the channel SNR, which can be obtained at the receiver.

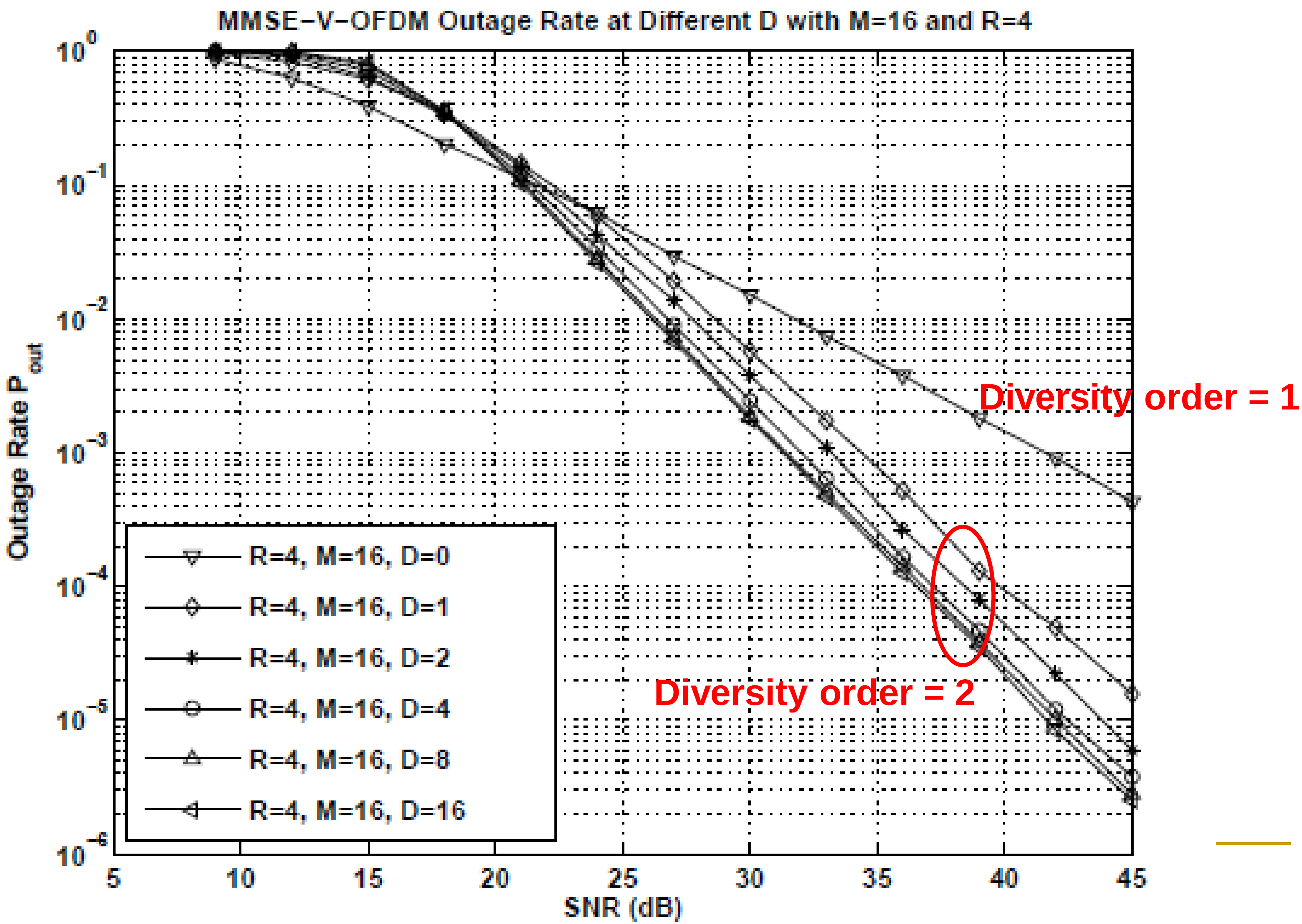
Diversity Order of MMSE Receiver

$$\text{diversity order} = \min\left\{\left\lfloor M\,2^{-R}\right\rfloor, D\right\} + 1$$



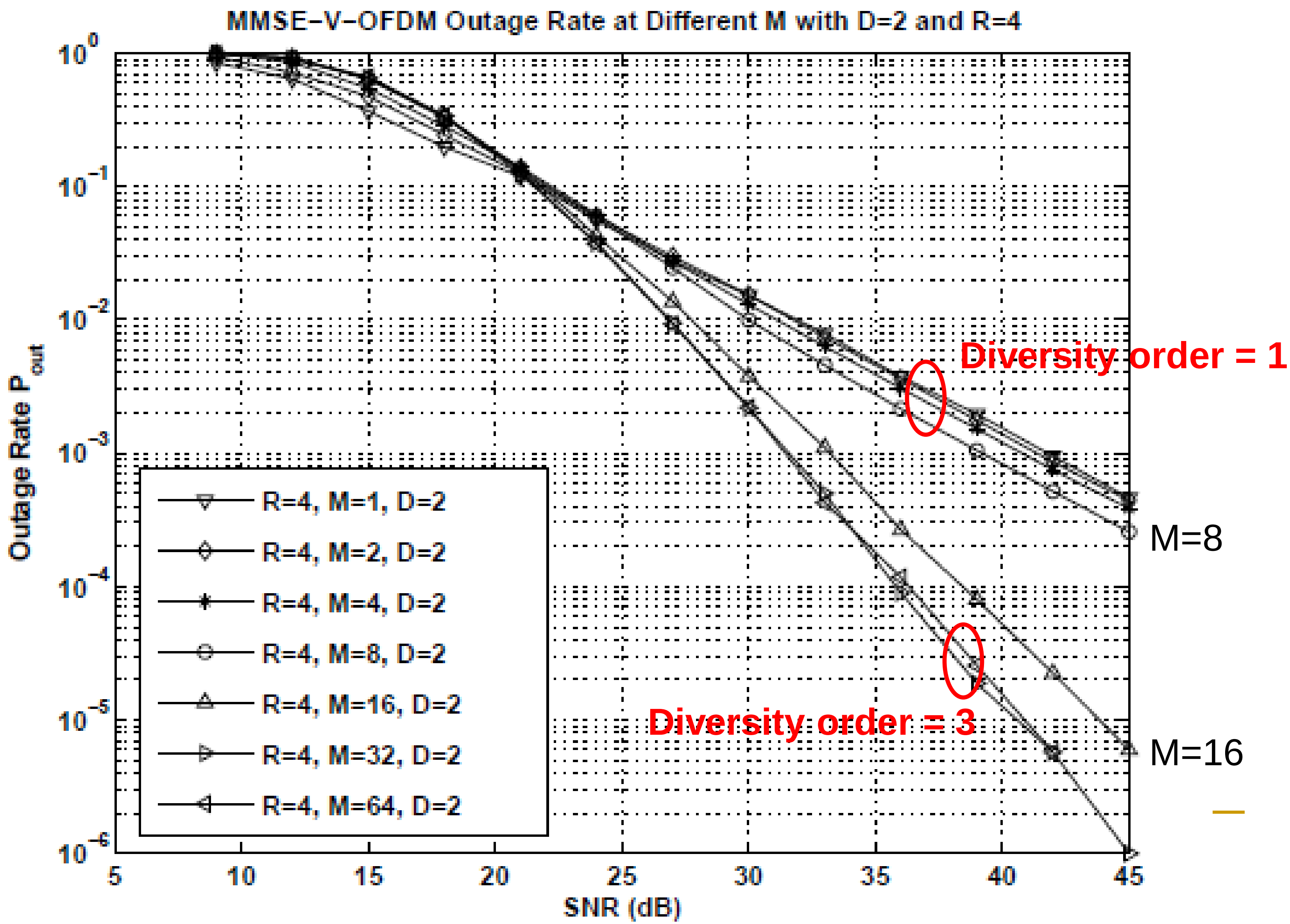
Diversity Order of MMSE Receiver

$$\text{diversity order} = \min\left\{\left\lfloor M\,2^{-R}\right\rfloor, D\right\} + 1$$



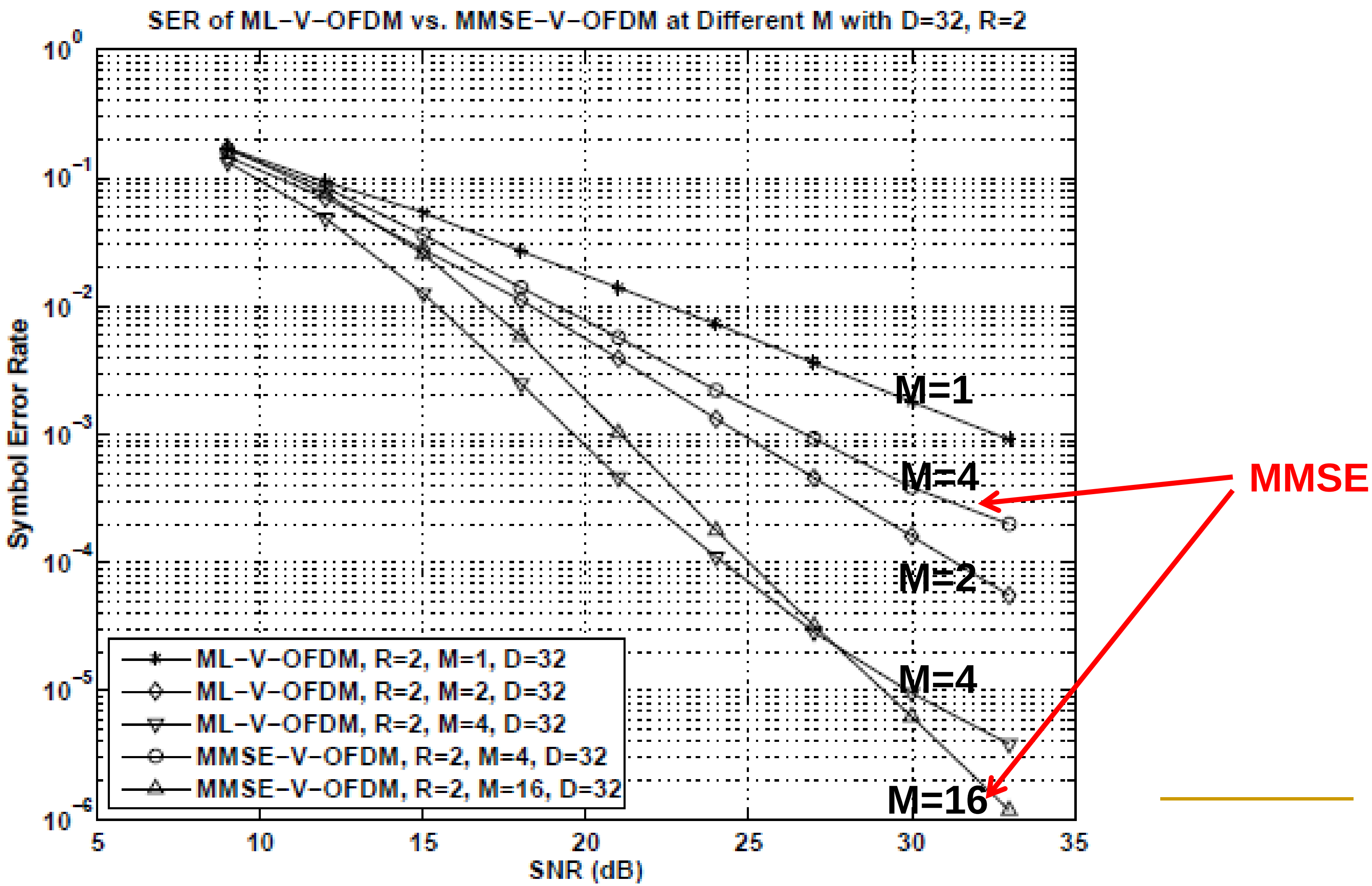
Diversity Order of MMSE Receiver

$$\text{diversity order} = \min\{\lfloor M\,2^{-R}\rfloor, D\}+1$$

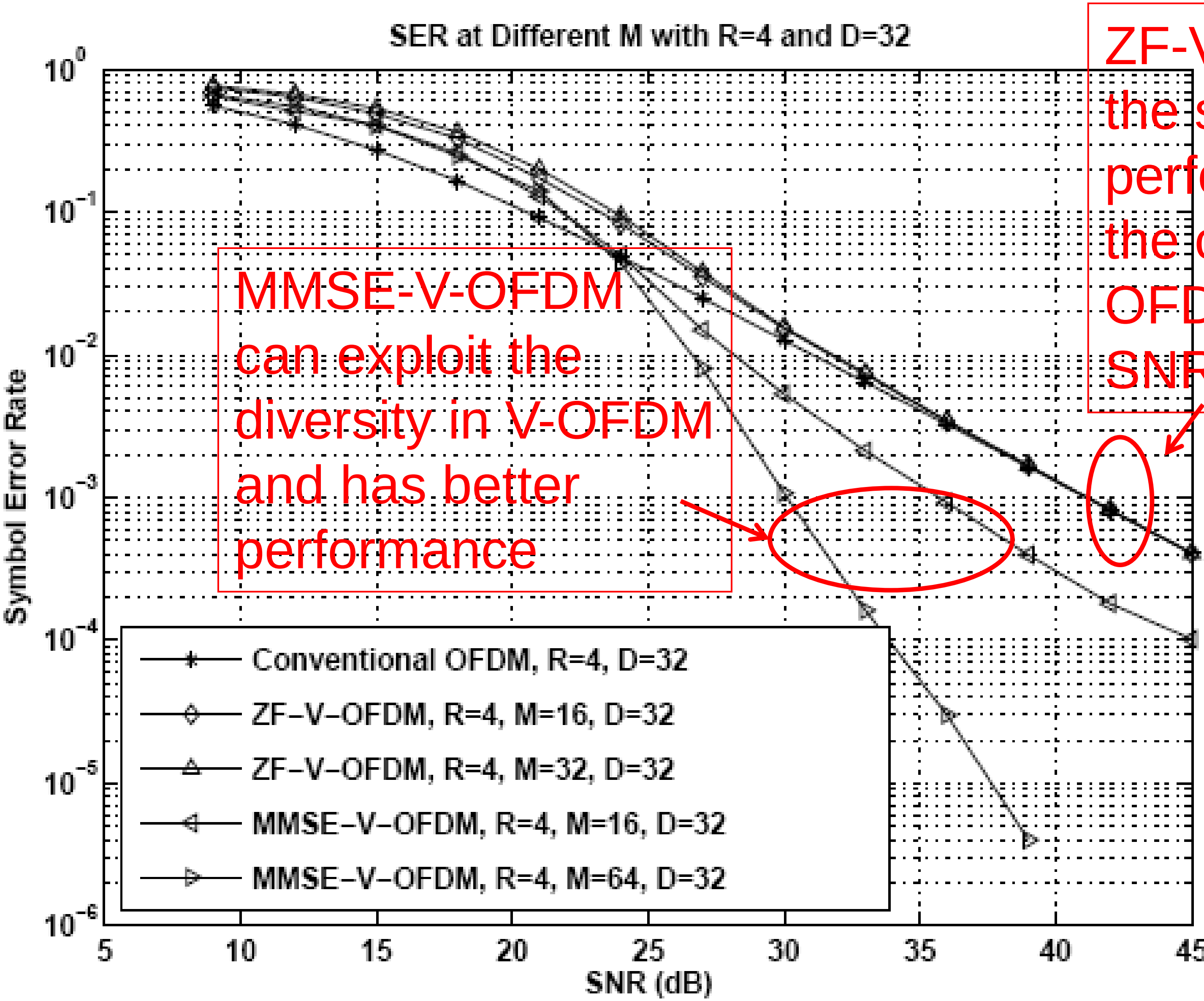


Performances for ML and MMSE Receivers

D=32, R=2



Diversity Order of ZF Receiver



MMSE-V-OFDM can exploit the diversity in V-OFDM and has better performance

ZF-V-OFDM has the same performance of the conventional OFDM at high SNR

Multiple Antenna VOFDM Using Cyclic Delay Diversity (CDD)

- CDD can be used to collect both spatial and multipath diversities in a MIMO-OFDM systems

$$h_{11}, h_{12}, \cdots, h_{1L}$$

$$\vdots$$

$$h_{n_t 1}, h_{n_t 2}, \cdots, h_{n_t L}$$

After CDD

$$h_{11}, \cdots, h_{1L}, \cdots\cdots, h_{n_t 1}, \cdots, h_{n_t, L}$$

It is equivalent to

if $N \geq n_t L$

When the bandwidth is larger, the number *L* of multipaths will be larger too. Then, CDD in this case may not be able to collect full spatial and multipath diversities anymore.

Multiple Antenna VOFDM Using Cyclic Delay Diversity (CDD)

- CDD VOFDM can collect both spatial and multipath diversities despite of a large bandwidth

$$\underrightarrow{H_{11}, H_{12}, \cdots, H_{1\frac{L}{M}}}$$

$$\vdots$$

$$\underrightarrow{H_{n_t 1}, H_{n_t 2}, \cdots, H_{n_t \frac{L}{M}}}$$

After CDD $\longrightarrow$

$$\underrightarrow{H_{11}, \cdots, H_{1\frac{L}{M}}, \cdots\cdots, H_{n_t 1}, \cdots, H_{n_t \frac{L}{M}}}$$

It is equivalent to

if $N \geq n_t \dfrac{L}{M}$

The number of multipaths is equivalently reduced by *M* times for VOFDM with a vector size *M*

Conclusion and Future Research

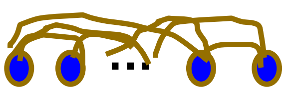
- Single antenna VOFDM can be used either to reduce the PAPR by reducing the IFFT size while fixed the CP data rate overhead; or reduce the CP data rate overhead while fixed the IFFT size
- Matrix modulation designs optimally fitting to the VOFDM
- Optimal combinations with forward error correction coding: fast soft decoding and iterative decoding
- CDD VOFDM for multi-antennas can collect both spatial and multipath diversities, where CDD OFDM is not be able to do so in a large bandwidth system
- Applications in future generations of 1) broadband cellular systems; 2) digital video broadcasting systems; 3) underwater acoustics communications; 4) power line communications, …

Recall Physical Layer Communications Developments in Recent Decades for Both Wireless and Wired Systems

- It has been always on dealing with ISI

Time domain single carrier vs. equalization

Maximum # symbols in ISI

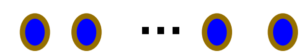


VOFDM

No, or 2, or 3, …, or Maximum # (**you choose**) symbols in ISI

Frequency domain OFDM

No ISI



Is this VOFDM something to think about after OFDM?

Or what's next???

References

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Thank you!