Hybrid Random-Structured Coding

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December 28, 2013

Outline

1. **Random vs structured codes: Point-to-point systems**

2. **Random vs. structured coding: Multiterminal systems**
   - Slepian-Wolf coding
   - Körner & Marton's binary two-help-one problem

3. **The Gaussian two-help-one problem**
   - Berger-Tung (BT) random coding
   - Krithivasan-Pradhan (KP) structured coding
   - Hybrid random-structured coding
     - Partial sum-rate tightness
     - Gap to optimal sum-rate

4. **Conclusions**

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What are they?

Codes with randomness

- Invented by Shannon in 1948

Key theoretical tool

- In proving Shannon's classic source and channel coding theorems

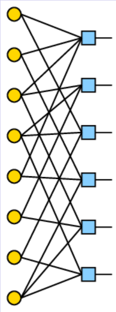
Recent invention/rediscovery of codes based on random graphs

- Turbo/LDPC codes
- Fulfilled Shannon's prophecy (after 50 years)

What do we learn from Shannon?

- Random codes are optimal for point-to-point systems

Random graphs



What are they?

Codes with structure

- Linear/lattice/trellis codes

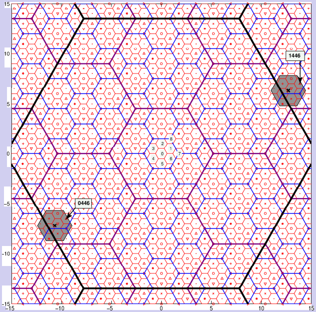
Widely used in communication systems

- For their low complexity
- Potentially limit-approaching (as promised by info theory)
  - Achievable capacity $\frac{1}{2}\log(\text{SNR}) \rightarrow \frac{1}{2}\log(1+\text{SNR})$ for AWGN channels
  - Optimal for at least some point-to-point systems as well
  - Structure "comes for free"

Can structured codes exceed the info-theoretic limits?

- No, at least for memoryless point-to-point systems

Nested lattice codes



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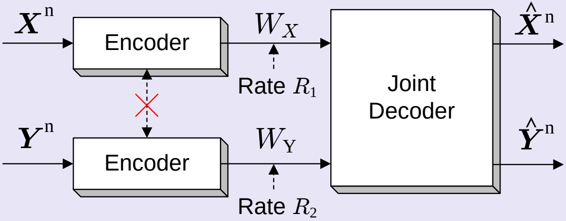
Slepian-Wolf coding'73

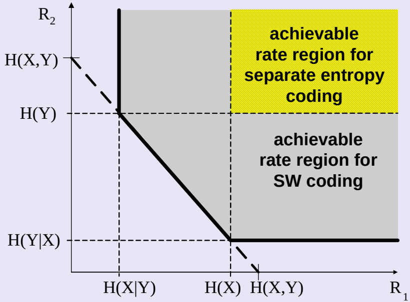
- X, Y: two correlated sources with finite alphabet
- Separate encoding and joint decoding
- Near-lossless decoding, i.e., $\lim_{n\to\infty} P_e\Big((X^n, Y^n) \neq (\hat{X}^n, \hat{Y}^n)\Big) = 0$



Slepian-Wolf theorem

- Slepian-Wolf rate region (proved by using random coding/bining)

$$\begin{aligned} R_1 &\geq H(X|Y) \\ R_2 &\geq H(Y|X) \\ R_1 + R_2 &\geq H(X,Y) \end{aligned}$$



- No rate loss compared to joint encoding of X and Y

Slepian-Wolf coding example

Assumptions:

- $X, Y \in \{0,1\}^3 \rightarrow$ binary triplets
- $H(X) = H(Y) = 3$ bits $\rightarrow$ equiprobable triplets
- Correlation between X and Y
  - x and y differ at most in one position
  - Hamming distance $d_H(\text{x,y}) \leq 1$
  - $H(X|Y) = 2$ bits

Question:

- If y is perfectly known at the decoder but not at the encoder, is it possible
  - to send 2 bits instead of 3 for x and
  - reconstruct x without loss?

Slepian-Wolf coding example

Solution:

- Form 4 cosets and send 2-bit index of the coset x belongs to:
  - coset Z_{00}: $\{000,111\}$ ← codewords of rate-$\frac{1}{3}$ repetition code
  - coset Z_{01}: $\{001,110\}$
  - coset Z_{10}: $\{010,101\}$
  - coset Z_{11}: $\{011,100\}$
- In each set: 2 members at $d_H = 3$
- Joint decoder: in the set indexed by Z:
  - Using y, pick $\hat{x}$ s.t. $d_H(\hat{x},y) \leq 1$
  - This guarantees correct/lossless decoding
    Example: y=[000], index 00 from encoder, $\hat{x}$ =[000]
    index 01 from encoder, $\hat{x}$ =[001]
    index 10 from encoder, $\hat{x}$ =[010]
    index 11 from encoder, $\hat{x}$ =[100]
- Separate encoding as efficient as joint encoding!

Equivalent way of viewing last example from a syndrome concept

- Form parity-check matrix of rate-$\frac{1}{3}$ repetition code

$$\boldsymbol{H} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

- Syndrome=coset/bin index
  - Coset/bin 0: $\{000,111\}$ has syndrome $Z = 00$
  - Coset/bin 1: $\{001,110\}$ has syndrome $Z = 01$
  - Coset/bin 2: $\{010,101\}$ has syndrome $Z = 10$
  - Coset/bin 3: $\{011,100\}$ has syndrome $Z = 11$
- All 4 cosets preserve the distance properties of the repetition code
- Encoding corresponds to matrix multiplication $\boldsymbol{H}$x
  - Compression 3:2
- Separate encoding as efficient as joint encoding!

1. Random coding optimal for this symmetric Slepian-Wolf coding example

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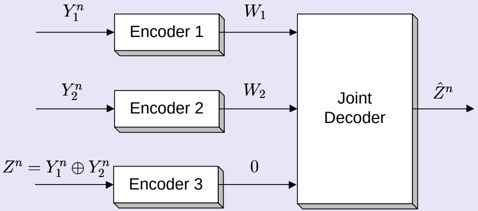
The binary two-help-one problem

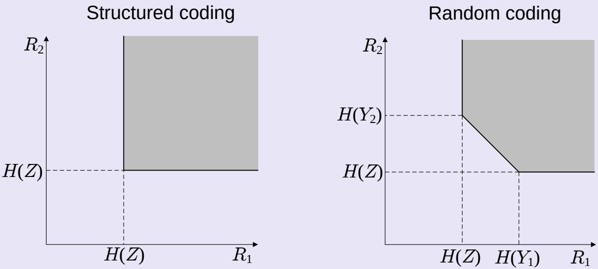
How to encode the mod-2 sum of binary sources? Introduced by Körner & Marton in 1979



- Y_1 and Y_2 are doubly symmetric binary sources
- Only the first two encoders are allowed to transmit, the decoder reconstructs $Z = Y_1 \oplus Y_2$ losslessly
  - This is combined compression and inference (for big data)
  - Goes beyond Slepian-Wolf coding

The rate region of Körner-Marton coding

- Slepian-Wolf coding before forming $Z = Y_1 \oplus Y_2$ certainly works
- Structured coding strictly improves random (Slepian-Wolf) coding!



- Using the same linear code, encoder i transmits $\boldsymbol{H}\mathrm{y}_i$
  - So both encoders use the same rate $\boldsymbol{H}(Z)$
- The decoder forms $\boldsymbol{H}\mathrm{y}_1 \oplus \boldsymbol{H}\mathrm{y}_2 = \boldsymbol{H}(\mathrm{y}_1 \oplus \mathrm{y}_2) = \boldsymbol{H}\mathrm{z}$ before recovering z
  - by picking the element with $d_H \leq 1$ in the coset indexed by z

Körner-Marton coding example

Recall 4 cosets at each encoder (from the Slepian-Wolf coding example)

- coset Z_{00}: {000,111} ; coset Z_{01}: {001,110}
- coset Z_{10}: {010,101} ; coset Z_{11}: {011,100}

Suppose y_1=[101], encoder 1 transmits index 10

- y_2=[101], encoder 2 transmits 10, decoder forms 00 before reconstructing z as [000]
- y_2=[001], encoder 2 transmits 01, decoder forms 11 before reconstructing z as [100]
- y_2=[111], encoder 2 transmits 00, decoder forms 10 before reconstructing z as [010]
- y_2=[100], encoder 2 transmits 11, decoder forms 01 before reconstructing z as [001]
- First instance of linear coding beating the best-known random coding

2. Linear coding optimal for this symmetric Körner-Marton coding example

Structured vs. random codes

It was long held that random codes are optimal for many comm. systems

- Körner and Marton's work showed advantage of structured code for some multiterminal systems
- Partially responsible for their recent back-to-back Shannon awards
- Structured coding for multiterminal systems currently a very active area of research

Reprise

- Linear coding optimal for symmetric Körner-Marton coding
  - Symmetry means $H(Y_1|Y_2) = H(Y_2|Y_1)$ or $P(Y_1 = 0|Y_2 = 1) = P(Y_1 = 1|Y_2 = 0)$

What about the general asymmetric case?

- Y_1 and Y_2 are binary sources with joint PMF ($p_{01} \neq p_{10}$ in general)

$Y_1 \backslash Y_2$	0	1
0	p_{00}	p_{01}
1	p_{10}	p_{11}

- Q: Which coding scheme is better? A: Neither

How to encode the mod-2 sum of binary sources?

Ahlswede-Han coding

- Introduced by Ahlswede & Han in 1983
- Random coding: First quantize Y_1^n and Y_2^n as U_1^n and U_2^n
- Structured coding: Then apply Körner-Marton coding on Y_1^n and Y_2^n with U_1^n and U_2^n as decoder side info
- Achievable rate pairs

$$\begin{aligned} R_1 &\geq I(Y_1;U_1|U_2) &+ H(Z|U_1,U_2) \\ R_2 &\geq I(Y_2;U_2|U_1) &+ H(Z|U_1,U_2) \\ R_1+R_2 &\geq I(Y_1,Y_2;U_1,U_2) &+ 2H(Z|U_1,U_2) \end{aligned}$$

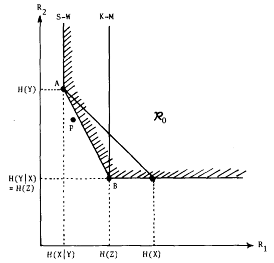
- Performs no worse than Slepian-Wolf and Körner-Marton coding for general correlation

How to encode the mod-2 sum of binary sources?

Ahlswede-Han coding example

- Expanded rate region when

$Y_1 \backslash Y_2$	0	1
0	0.00392	0.97608
1	0.01992	0.00008



- Due to asymmetry, time sharing between Slepian-Wolf and Körner-Marton always exists
- Ahlswede-Han coding achieves points (e.g., P) outside the time-sharing region
  - Optimal coding scheme not known

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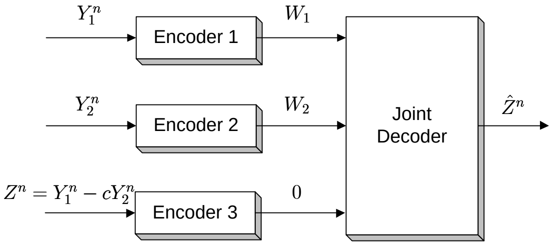
Definition

- Separate compression and joint decompression of a linear combination $Z = Y_1 - cY_2$ of jointly Gaussian sources Y_1 and Y_2 subject to an MSE distortion constraint on Z
- Problem characterized by the linear coefficient c, the source correlation coefficient ρ, and the MSE distortion constraint D on Z



Motivation

- Arises in many practical video surveillance applications, e.g., reconstructing the motion difference between two video sequences

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Berger-Tung's generic random coding scheme 1977

- Independent quantization of
  - Y_1 to U_1 s.t. $U_1 = Y_1 + Q_1$ with $Q_1 \sim \mathcal{N}(0, q_1)$
  - Y_2 to U_2 s.t. $U_2 = Y_2 + Q_2$ with $Q_2 \sim \mathcal{N}(0, q_2)$
- Followed by Slepian-Wolf compression (or binning) of U_1 and U_2 leads to rate region

$$\mathcal{R}^{BT}(q_1, q_2) = \left\{ (R_1, R_2) : \begin{array}{cc} R_1 & \geq H\left(U_1 | U_2\right) \\ R_2 & \geq H\left(U_2 | U_2\right) \\ R_1 + R_2 & \geq H\left(U_1, U_2\right) \end{array} \right\}$$

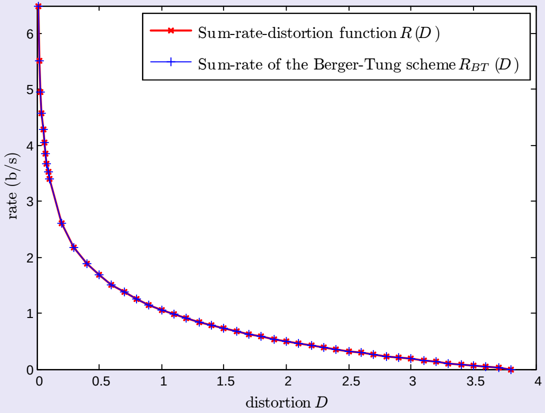
- Berger-Tung (BT) achievable rate region

$$\mathcal{R}_{BT}(D) = \text{conv}\Big(\bigcup_{(U_1, U_2) \in \mathcal{U}(Y_1, Y_2)} \{(R_1, R_2) : R_1 \geq I(Y_1; U_1 | U_2),\ R_2 \geq I(Y_2; U_2 | U_1),$$

$$R_1 + R_2 \geq I(Y_1, Y_2; U_1, U_2)\} \Big)$$

When $c \cdot \rho \leq 0$, e.g., $\rho > 0, c = -1$

- Referred to as the μ-sum problem (e.g., with $Z = Y_1 + Y_2$)
- Considered by Wagner et al. in 2005
- Berger-Tung (or random QB) coding is optimal



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When $c \cdot \rho > 0$, e.g., $\rho > 0, c = 1$

- Referred to as the μ-difference problem (e.g., with $Z = Y_1 - Y_2$)
- Considered by Krithivasan-Pradhan in 2009 using structured/lattice codes
- Two lattices (Λ_1, Λ_2) for SC; one lattice Λ_C with $\Lambda_C \subseteq \Lambda_i$ for CC
  - Independent lattice quantizers (Λ_1, Λ_2) on Y_1 and cY_2
  - Encoders send quantized versions modulo the same lattice Λ_C
  - Transmission rates

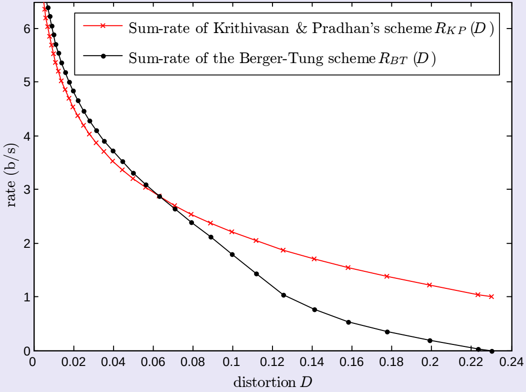
$$R_i = \log_2 \frac{\sigma^2(\Lambda_C)}{\sigma^2(\Lambda_i)}$$

- Krithivasan-Pradhan (KP) achievable rate region

$$\mathcal{R}_{KP}(D) = \left\{ (R_1, R_2) : 2^{-2R_1} + 2^{-2R_2} \leq \frac{D}{1 + c^2 - 2c\rho} \right\}$$

When $c \cdot \rho > 0$, e.g., $\rho > 0, c = 1$

- Referred to as the μ-difference problem (e.g., with $Z = Y_1 - Y_2$)
- Considered by Krithivasan-Pradhan in 2009 using structured/lattice codes
- Smaller sum-rate than the random Berger-Tung scheme for certain (ρ, c) pairs
  - Structured codes beat random codes!



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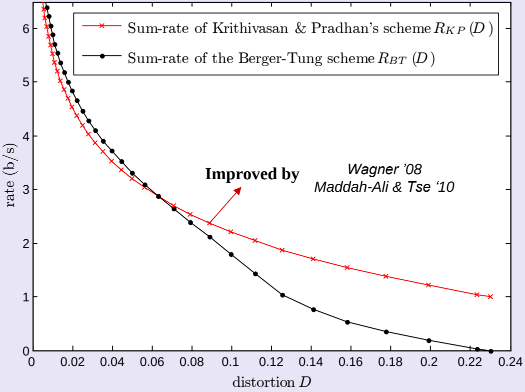

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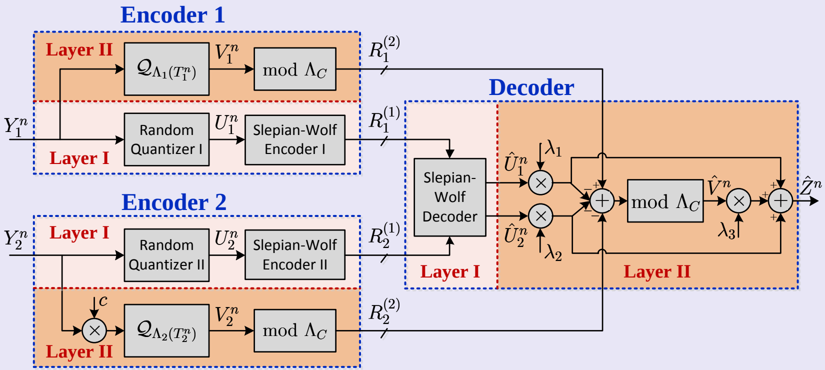
When $c \cdot \rho > 0$, e.g., $\rho > 0, c = 1$

- For the μ-difference problem, can we do better?
- New hybrid random-structured coding scheme
    - Inspired by Ahlswede & Han's for encoding the mod-2 sum of binary sources
    - Hybrid random BT coding (layer I) and structured KP coding (layer II)



When $c \cdot \rho > 0$, e.g., $\rho > 0, c = 1$

- Theorem 1: Achievable rate region of hybrid random-structured coding

$$\mathcal{R}_{new}(D) = \text{conv}\Big(\bigcup_{(U_1,U_2)\in\mathcal{U}(c,D)} \{(R_1,R_2) :$$

$$R_1 \geq I(Y_1;U_1|U_2) + I(V_1 - V_2; Y_1, V_2|U_1,U_2),$$

$$R_2 \geq I(Y_2;U_2|U_1) + I(V_1 - V_2; V_1, Y_2|U_1,U_2),$$

$$R_1 + R_2 \geq I(Y_1,Y_2;U_1,U_2) + I(V_1 - V_2; Y_1, V_2|U_1,U_2)$$

$$+ I(V_1 - V_2; V_1, Y_2|U_1,U_2)\}\Big)$$

$$\mathcal{U}(c,D) \triangleq \Big\{(U_1,U_2,V_1,V_2) : U_i = Y_i + P_i, V_1 = Y_1 + Q_1, V_2 = cY_2 + Q_2,$$

$$P_i \sim \mathcal{N}(0,p_i), Q_i \sim \mathcal{N}(0,q_i), i = 1,2, \text{ indep. of each other and}$$

$$(Y_1,Y_2), \text{such that } E\big[(Z - E(Z|U_1,U_2,V_1 - V_2))^2\big] \leq D\Big\}$$

When $c \cdot \rho > 0$, e.g., $\rho > 0, c = 1$

- Comparison between $\mathcal{R}_{new}(D)$, $\mathcal{R}_{KP}(D)$ and $\mathcal{R}_{BT}(D)$
  - Hybrid coding always subsumes structured KP coding

$$\mathcal{R}_{new}(D) \supseteq \mathcal{R}_{KP}(D)$$

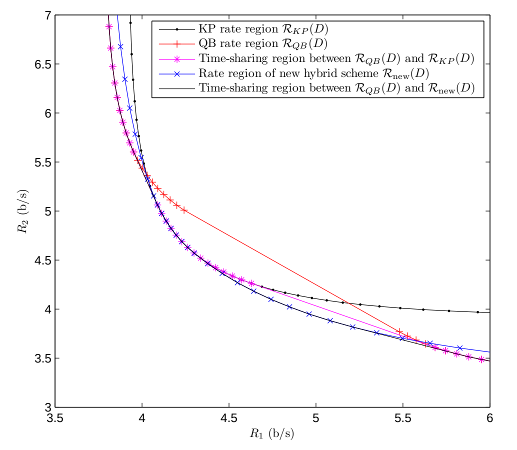
  - It becomes a QB random coding with additional rate of $\frac{1}{2}\log_2\frac{1}{\alpha}$ and $\frac{1}{2}\log_2\frac{1}{\beta}$ (with $\alpha + \beta = 1$) at the two encoders
  - Lemma 1:

$$\mathcal{R}_{\text{new}}(D) \supseteq \mathcal{R}_{KP}(D) \cup \left[\mathcal{R}_{BT}(D) \uplus \{(\frac{1}{2}\log_2\frac{1}{\alpha}, \frac{1}{2}\log_2\frac{1}{\beta}) : \alpha + \beta = 1\}\right]$$

New rate region $\mathcal{R}_{\text{new}}(D)$ strictly improves the time-sharing region between $\mathcal{R}_{KP}(D)$ and $\mathcal{R}_{BT}(D)$

When $c \cdot \rho > 0$, e.g., $\rho > 0, c = 1$

- Comparison between $\mathcal{R}_{new}(D)$, $\mathcal{R}_{KP}(D)$ and $\mathcal{R}_{BT}(D)$



When $c \cdot \rho > 0$, e.g., $\rho > 0, c = 1$

- We look deeper at the minimum achievable sum-rate

$$R_{\text{new}}(D) \triangleq \min\left\{R_1 + R_2 : (R_1, R_2) \in \mathcal{R}_{\text{new}}(D)\right\}$$

- Theorem 2: Achievable sum-rate of hybrid random-structured coding

$$R_{\text{new}}(D) = \begin{cases} \frac{1}{2}\log_2 \frac{16c^2(1-\rho^2)(1-c\rho)^2}{D^2}, & \text{if } c \le \frac{1}{\rho+\sqrt{1-\rho^2}} \ \& \ D \le 2c^2(1-\rho^2) \\ \min\left(\log_2 \frac{2\sigma_Z^2}{D}, \frac{1}{2}\log_2 \frac{16c((1-\rho^2)c-\rho D)}{D^2}\right), & \text{if } c > \frac{1}{\rho+\sqrt{1-\rho^2}} \ \& \ D \le \frac{2c^2(1-\rho^2)}{1+c\rho} \\ \min\left(\log_2^+ \frac{2\sigma_Z^2}{D}, \frac{1}{2}\log_2^+ \frac{4(1-c\rho)^2}{D-c^2(1-\rho^2)}\right), & \text{otherwise} \end{cases}$$

When $c \cdot \rho > 0$, e.g., $\rho > 0, c = 1$

- Comparison between $R_{new}(D)$, $R_{KP}(D)$ and $R_{BT}(D)$
  - Corollary to Lemma 1:

$$R_{\text{new}}(D) \leq \min\left(R_{KP}(D), R_{BT}(D) + 1\right)$$

  - When can $R_{new}(D)$ strictly improve both $R_{KP}(D)$ and $R_{BT}(D)$?
  - Lemma 2:

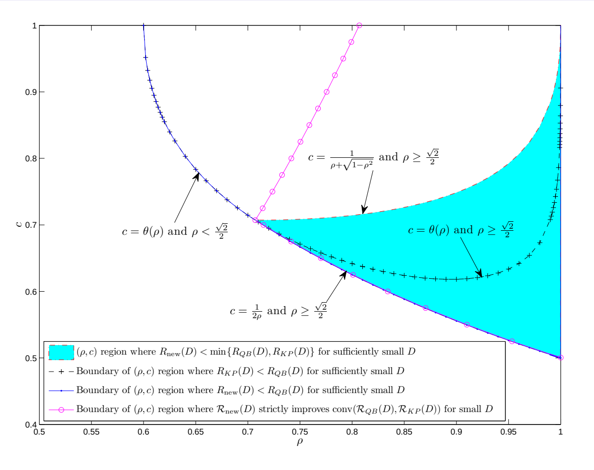
$$R_{\text{new}}(D) < \min\left(R_{KP}(D), R_{BT}(D)\right)$$

if either $\frac{1}{2\rho} < c < \min\left(\frac{\sqrt{3}}{2\rho}, \frac{1}{\rho+\sqrt{1-\rho^2}}\right)$ & $D < \frac{c(1-\rho^2)(3-2c\rho)(2c\rho-1)}{\rho}$

or $\frac{\sqrt{3}}{2\rho} < c < \frac{1}{\rho+\sqrt{1-\rho^2}}$ & $D < 4(2-\sqrt{3})c^2(1-\rho^2)$

When $c \cdot \rho > 0$, e.g., $\rho > 0, c = 1$

- Comparison between $R_{new}(D)$, $R_{KP}(D)$ and $R_{BT}(D)$



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Partial sum-rate tightness

- Sum-rate lower bound needed –– to compared with achievable sum-rate upper bound from hybrid coding
  - Consider a new and more general problem of Gaussian two-terminal SC problem with covariance matrix $\Sigma_Y = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}$ and covariance matrix distortion constraint $\mathcal{D} = \begin{bmatrix} k_1^2 & \theta k_1 k_2 \\ \theta k_1 k_2 & k_2^2 \end{bmatrix}$
  - Fact: If the minimum sum-rate for the above problem is $R(\mathcal{D})$, then

$$R(D) = \min_{\mathcal{D} \in \Upsilon(\rho, c, D)} R(\mathcal{D}),$$

where $\Upsilon(\rho, c, D)$ contains all real 2×2 symmetric matrices $\mathcal{D}$ such that $\mathbf{0} \leq \mathcal{D} \leq \Sigma_Y$ and $[1 \ \ -c]\mathcal{D}[1 \ \ -c]^T \leq D$

Partial sum-rate tightness

- Lemma 3: A new sum-rate lower bound $R(\mathcal{D}) \geq \max\left(\underline{R}^{\dagger}(\mathcal{D}), \underline{R}^{\ddagger}(\mathcal{D})\right)$, where

$$\underline{R}^{\dagger}(\mathcal{D}) \triangleq \begin{cases} \frac{1}{2}\log_2\left[\frac{1-\rho^2+2\rho k_1 k_2(1+\theta)}{(1+\theta)^2 k_1^2 k_2^2}\right], & \theta \leq \theta^{\star} \\ \frac{1}{2}\log_2\left[\frac{(1-\rho^2)^2}{(1-\theta)^2 k_1^2 k_2^2(1-\rho^2+2\rho k_1 k_2(1+\theta))}\right], & \theta > \theta^{\star} \end{cases}$$

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with $k_i = \min(k_1, k_2)$, $k_j = \max(k_1, k_2)$, and

$$\theta^{\star} \triangleq \frac{1}{2\rho k_1 k_2}\left(\sqrt{(1-\rho^2)^2 + 4\rho^2 k_1^2 k_2^2} - (1-\rho^2)\right)$$

$$\theta^{\ddagger} \triangleq \frac{1}{2\rho k_1 k_2}\left(\sqrt{(1-\rho^2)^2 + 4\rho^2 k_1^2 k_2^2 - 8k_i^2\rho^2(1-\rho^2)} + (1-\rho^2)\right)$$

Partial sum-rate tightness

- Lemma 3: A new sum-rate lower bound $R(\mathcal{D}) \geq \max\left(\underline{R}^{\dagger}(\mathcal{D}), \underline{R}^{\ddagger}(\mathcal{D})\right)$, where

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- The 1st lower bound $\underline{R}^{\dagger}(\mathcal{D})$ proved in Xiong'13 using the estimation-theoretic approach of Wang'10

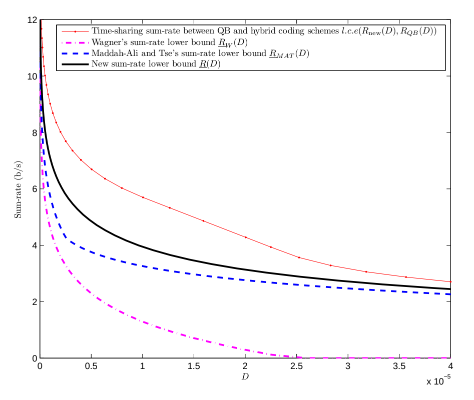
- The 2nd lower bound $\underline{R}^{\ddagger}(\mathcal{D})$ newly obtained by combining the approach of Wang'10 and the technique in Wagner'11, which exploits stochastic degradedness of the channel $Y_1 \to Y_2$ with respect to $Y_1 \to Z$

Partial sum-rate tightness

- Theorem 3: A new sum-rate lower bound

$$R(D) \geq \underline{R}(D) \triangleq \min_{\boldsymbol{\mathcal{D}} \in \boldsymbol{\Upsilon}(\rho,c,D)} \max\left(\underline{R}^{\dagger}(\boldsymbol{\mathcal{D}}), \underline{R}^{\ddagger}(\boldsymbol{\mathcal{D}})\right)$$

- Comparison among sum-rate lower and upper bounds

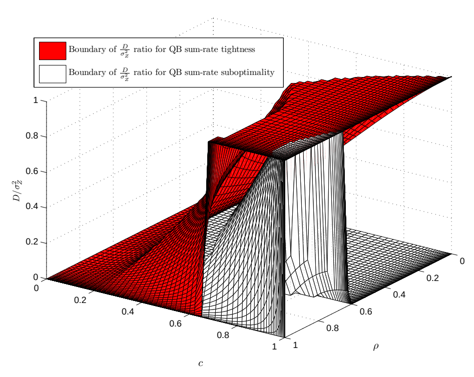


Partial sum-rate tightness

- Theorem 4: First partial sum-rate tightness result

$$R_{QB}(D) = R(D) = \underline{R}(D)$$

if $\rho \in (0,1)$, $0 \le c \le \frac{1}{1+2\rho}$, $D \ge \frac{2c^2(1-\rho^2)(1-2c\rho)}{1-3c\rho}$ or $0 \le c \le \frac{2\rho}{1+2\rho^2}$, $D \ge \frac{c(1-\rho^2)(1-2c^2\rho^2)}{\rho(2-3c\rho)}$



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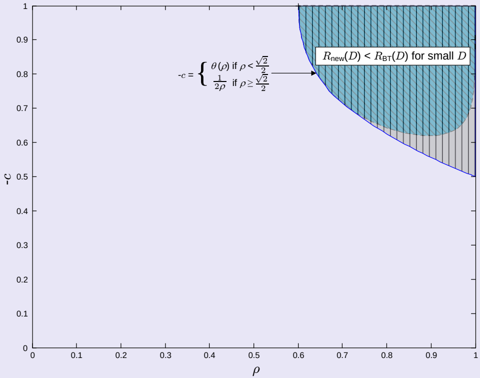

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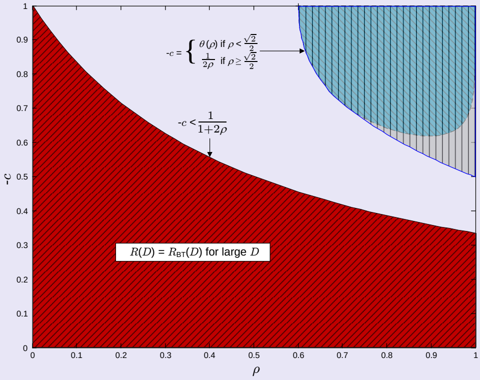


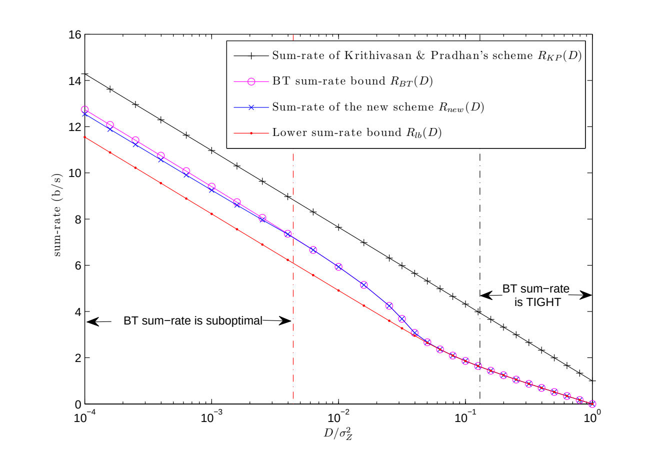
Outline

1. Random vs structured codes: Point-to-point systems
2. Random vs. structured coding: Multiterminal systems
   - Slepian-Wolf coding
   - Körner & Marton's binary two-help-one problem
3. **The Gaussian two-help-one problem**
   - Berger-Tung (BT) random coding
   - Krithivasan-Pradhan (KP) structured coding
   - Hybrid random-structured coding
     - Partial sum-rate tightness
     - Gap to optimal sum-rate
4. Conclusions

Gap to optimal sum-rate

- Theorem 5: For any (ρ, c, D) triple, it holds that

$$R_{\text{new}}(D) - R(D) \leq R_{\text{new}}(D) - \underline{R}(D) \leq 2$$



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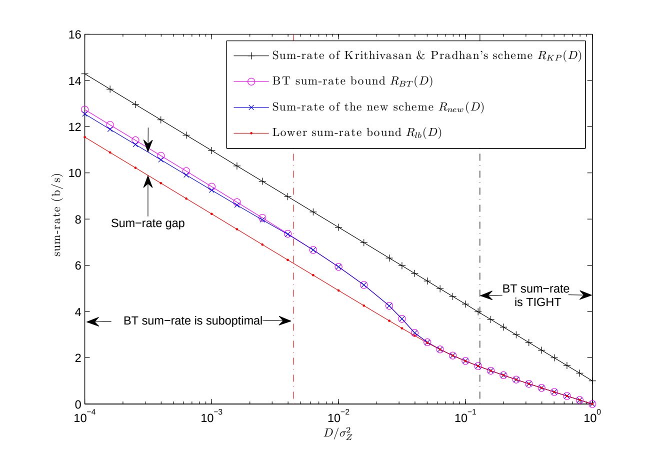
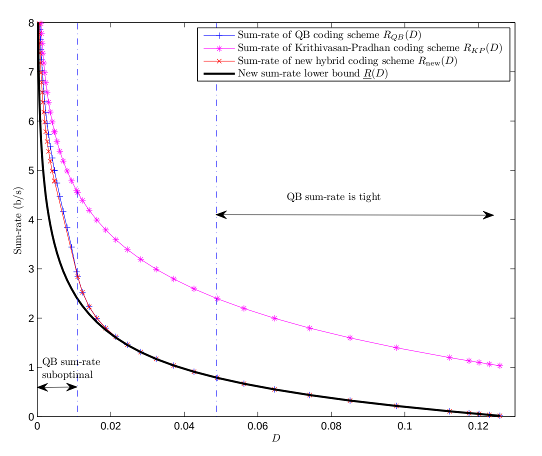


Gap to optimal sum-rate

- Lemma 4: If $c = 1$ or $c = \rho$, it holds that

$$R_{\text{new}}(D) - R(D) \leq R_{\text{new}}(D) - \underline{R}(D) \leq 1$$



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- Philosophically the right approach (with better performance)

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  - Combined SC and inference for big data
  - The more general many-help-one problem
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  - Many open issues (e.g., optimality of hybrid coding)

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Broader impact:

- Hybrid approach applicable to many other network comm. scenarios
  - Cooperative networks: The two-way relay channel
  - The interference channels

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