Imaging moving objects in the presence of a horizontal reflector

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Integrity ★ Service ★ Excellence

OVERVIEW

- Introduction to Microlocal Analysis
- Outline of Work
- Forward Problem
- Inverse Problem
- Numerical Results and Image Analysis
- Conclusion and Future Work

INTRODUCTION TO MICROLOCAL ANALYSIS

- An ***operator*** is a function between two vector spaces
- A ***pseudodifferential operator*** is an extension of the differential operator which is a differential function between two vectors spaces

Microlocal analysis suggests that there are a class of operators with properties that are useful to solve differential equations.

Consider a general 1-D homogeneous wave equation, a PDE:

$$\left(\frac{\partial}{\partial x^2}^2 + \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\psi(x,t) = 0$$

$$L(D)\psi(x,t) = 0$$

$$D = \text{ partial differential operator}$$

$$c = \text{ speed of light}$$

The operator L(D) that represents the wave equation operation is clearly a linear differential operator. In physical terms, it describes how the total wavefield ψ travels at the speed of light and propagates through a vacuum.

So how can we solve for ψ? Let's examine a non-rigorous visualization of how.

Apply a Fourier Transform

$$\Im(f(x,t)) = \hat{f}(\xi,t) = \int e^{-2\pi i \xi x} f(x,t)\,dx$$

to our wave equation

$$L(\xi,D)\hat{\psi}(\xi,t) = \left[(2\pi i \xi)^2 + \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right]\hat{\psi}(\xi,t) = 0$$

INTRODUCTION TO MICROLOCAL ANALYSIS

We now have an Ordinary Differential Equation (ODE) as long as

$L(\xi, D)$ is never zero (a symbol of compact support)

We solve for $\hat{\psi}$ using known ODE methods

Finally we use Fourier Inversion Theorem to solve for ψ.

This method is taught in elementary PDE coursework, however, important mathematical theory is often brushed under the rug.

We have used the following key properties to solve our 1-D wave equation problem:

1. Our operators and wave fields have clearly defined Fourier transforms
2. The symbol is related to the phase of the Fourier transform and is a function of compact support
3. The wave field is smooth
4. The manifolds we integrate over are smooth and non-degenerate

INTRODUCTION TO MICROLOCAL ANALYSIS

- some basic definitions

The ***support*** of a function is the set of points where the function is not zero. If this set of points forms a compact set, we say it is a ***compact support***. The ***singular support*** of a function is the set of points where a function is not smooth (not infinitely differentiable i.e. edges)

A ***Fourier integral operator*** is an integral operator with a generalized kernel that is a rapidly oscillating function or the integral of such a function.

$$\mathcal{F}[f](x) = \int e^{\mathrm{i}\varphi(x,y,\xi)} A(x,y,\xi) f(y) \mathrm{d}y \mathrm{d}\xi$$

INTRODUCTION TO MICROLOCAL ANALYSIS

$A(x,y,\xi)$ - symbol of compact support

$\phi(x,y,\xi)$ - phase function is real valued, smooth,

non-degenerate for $\xi \neq 0$, and infinitely differentiable

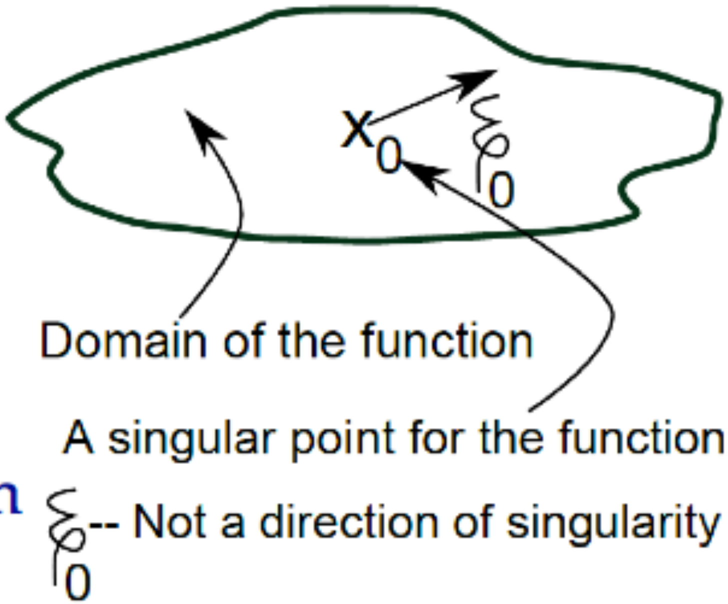
The Lagrangian manifold constructed by $\phi(x,y,\xi)$ coincides locally with a conic smooth Lagrangian manifold Λ.

In real words: for any point in Λ there is a $\phi(x,y,\xi)$ such that the Lagrangian manifold constructed with respect to $\phi(x,y,\xi)$ locally intersects Λ.

INTRODUCTION TO MICROLOCAL ANALYSIS

A ***wavefront set*** is the set of directions of singularity at the location of a singularity.

- x_0 a location of singularity for a function f and consider a direction ξ_0
- f is microlocally smooth near (x_0, ξ_0) if the localized Fourier transform decays rapidly
- The complement of the set of points (x_0, ξ_0) near which f is microlocally smooth is called the wavefront set of f
- Denoted $WF(f)$



Pseudodifferential Operators

- May be formed by composing an FIO with its adjoint
- Can solve a PDE
- Has information about local singularities via singular supports and wavefront sets

Distributions are a class of linear functionals that map a set of test functions (conventional and well-behaved functions) onto the set of real numbers.

Example: The set of test functions considered is D(R), which is the set of functions from R to R having two properties:

** The function is smooth (infinitely differentiable);*
** The function has compact support (is identically zero outside some interval).*

Then, a distribution d is a linear mapping from D(R) to R. A bump function in this set.

How FIO theory applies to Radar

Oscillatory Integral w/ phase

Singularities in Scene

Fourier Integral Operator: $d(t,s) = \mathcal{F}f(y) = \int e^{i\phi(y,x,\omega)} a(y,x,\omega) f(x) d\omega dx$

Singularities in Data

Amplitude

- ❑ Well-developed FIO Theory tells us about relationship between singularities.
- ❑ Easily Simulated in Matlab using FFT for many waveform types
- ❑ Data is just a transformed function of an actual scene

INTRODUCTION TO MICROLOCAL ANALYSIS

$$d(t,\boldsymbol{z},\boldsymbol{y}) = \mathcal{P}q_{\boldsymbol{v}}(\boldsymbol{x}) = \int \Upsilon(\boldsymbol{x},\boldsymbol{v},t,\boldsymbol{z},\boldsymbol{y})q_{\boldsymbol{v}}(\boldsymbol{x})d\boldsymbol{x}d\boldsymbol{y}$$

$$\begin{aligned} I(\boldsymbol{p},\boldsymbol{u}) &= \mathcal{P}^* d(\boldsymbol{y},\boldsymbol{z},t) \\ &= \int \sum_{\boldsymbol{z},\boldsymbol{y}} \Upsilon^*(\boldsymbol{p},\boldsymbol{u},t,\boldsymbol{z},\boldsymbol{y})\Upsilon(\boldsymbol{x},\boldsymbol{v},t,\boldsymbol{z},\boldsymbol{y})dtq_{\boldsymbol{v}}(\boldsymbol{x})d\boldsymbol{x}d\boldsymbol{v} \end{aligned}$$

Ideal

Image

Reconstruction

✧ Data in the form of FIO

- ✧ Ideal Reconstruction
- ✧ Cauchy-Schwartz Inequality implies PSF is maximal when p = x, u = v
- ✧ The Kernel for the image is approximate but if it represents the kernel of a pseudodifferential operator

INTRODUCTION TO MICROLOCAL ANALYSIS

- Microlocal Analysis →
  - If $\mathcal{K}\mathcal{F}$ is a pseudo-differential operator → preserves location and orientation of singularities
  - Analyze propagation of singularities /wavefront sets

$$WF(\mathcal{K}\mathcal{F}) \subset \mathcal{C}^{\mathcal{K}} \circ \mathcal{C}^{\mathcal{F}}$$

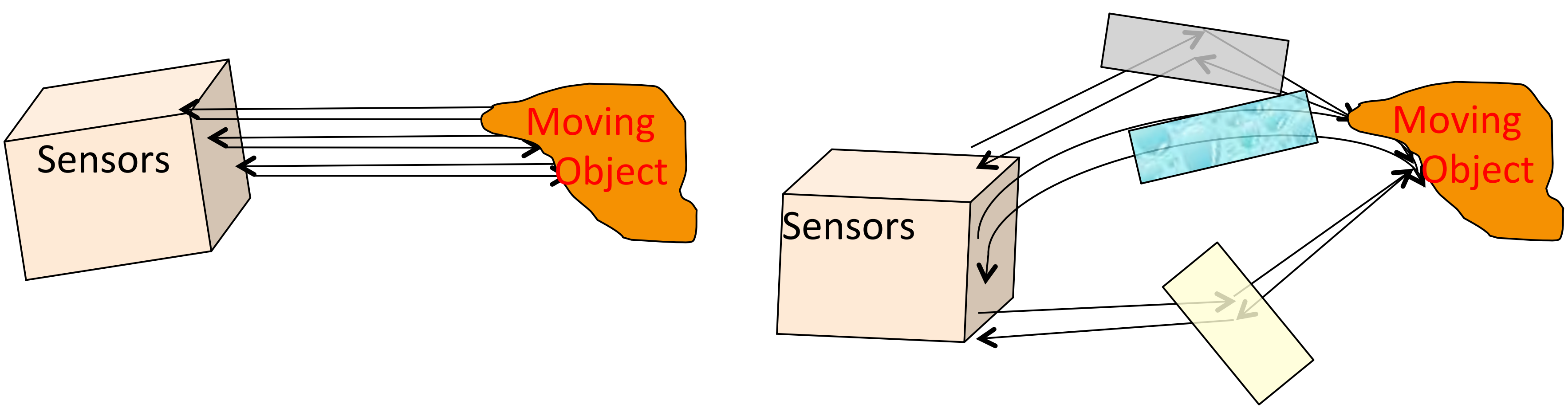
- $\mathcal{F}^*$ is the L^2-adjoint (backprojection operator).
- $\mathcal{F}^*\mathcal{F}$ is a PseudoDO. Reconstructs the singularities at the correct location and orientation.
- Will not reconstruct at the right strength. Design a filter to reconstruct singularities at the right strength.

OUTLINE OF WORK

Multipath Environment for Wave-based Imaging Systems

Direct Path

Indirect Path



MULTIPATH = Indirect Path + Direct Path

SOURCES OF MULTIPATH – BUILDINGS, TREES, ATMOSPHERE, WATER, ETC.

OUTLINE OF WORK

GOAL

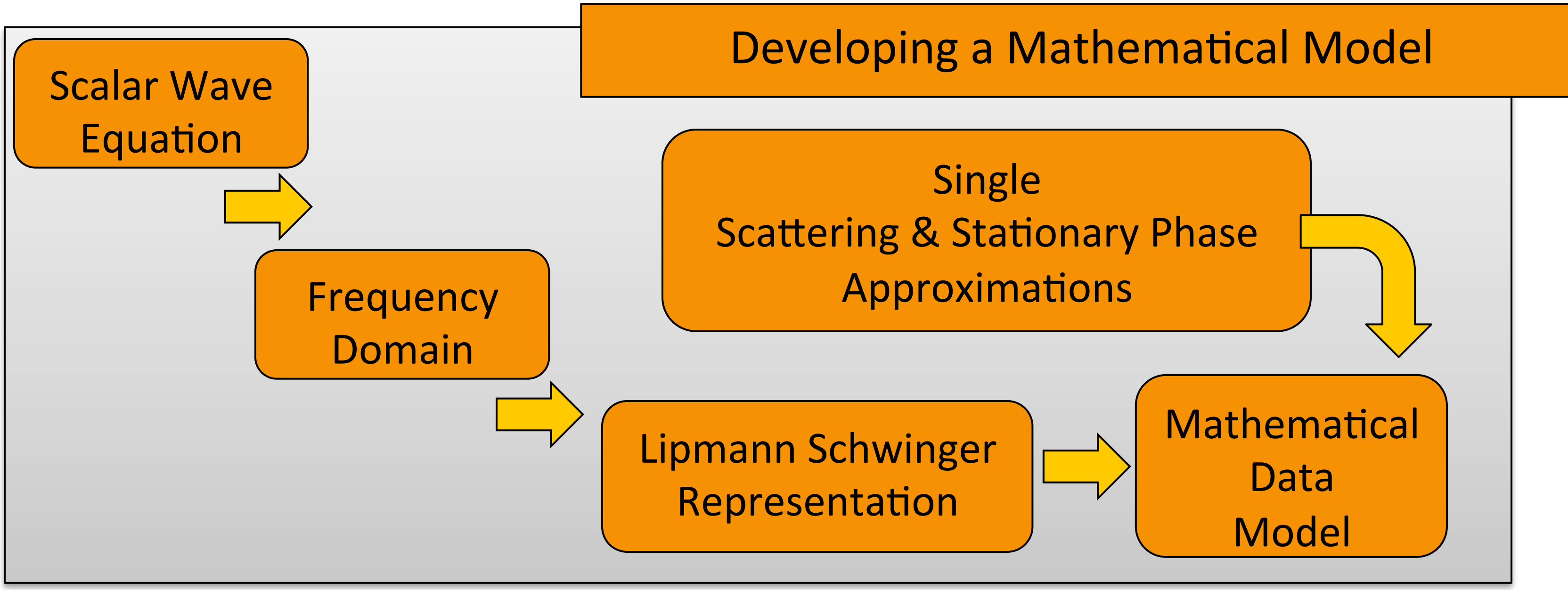
FORM AN IMAGE
THAT EXPLOITS MULTIPATH SCATTERING FROM
MULTIPLE SENSORS

STRATEGY

SIMULATE DATA VIA FORWARD OPERATOR

APPROXIMATE INVERSE OF FORWARD OPERATION VIA
ADJOINT

FORWARD PROBLEM



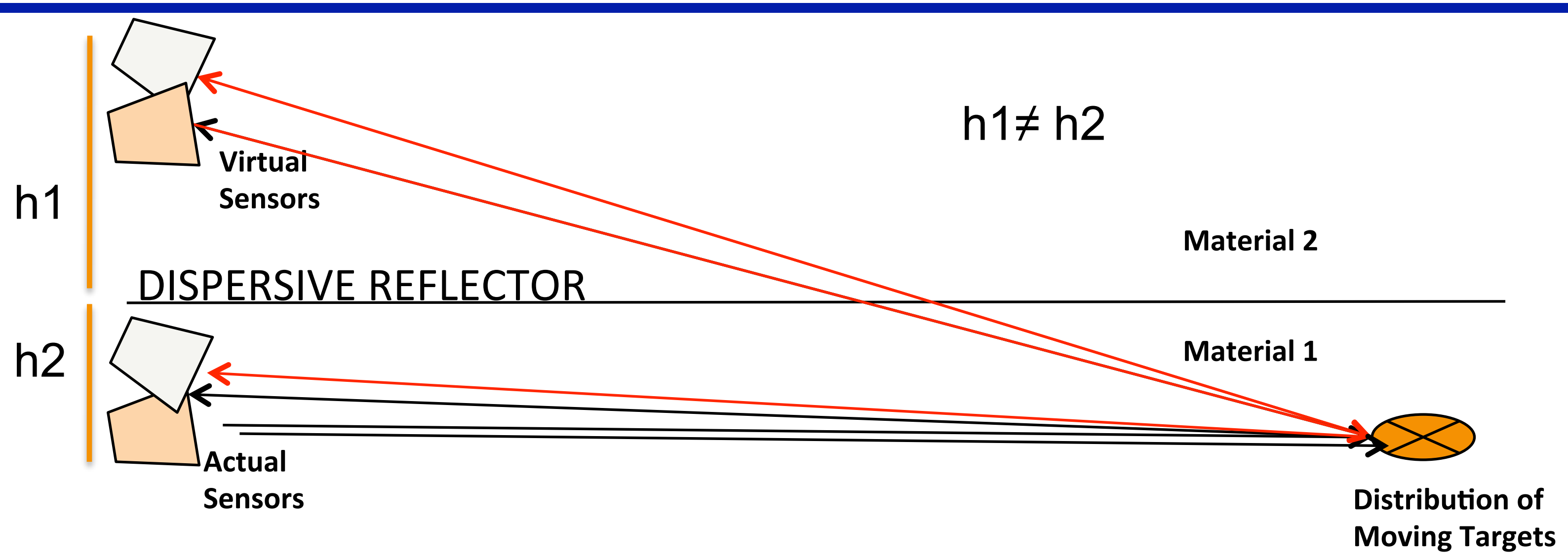
Incorporates

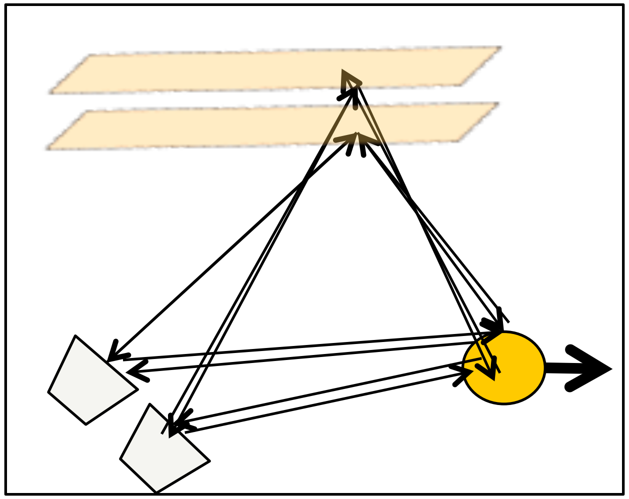
Temporal
Spatial
Spectral
} components

Includes automatically…

- Multiple Sensors
- Take-off angles
- Frequency of reflection
- Waveforms
- Scattering from a distribution of moving objects

FORWARD PROBLEM





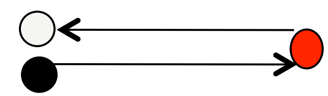
We arrive at our model using

- Method of Images
- Stationary Phase approximation

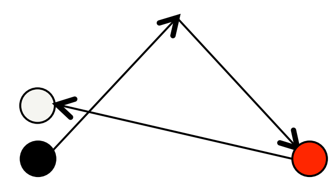
FORWARD PROBLEM

Our Data = Sum of Operators Representing Wave Front Paths

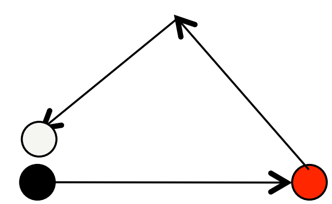
$$d(t,\boldsymbol{z},\boldsymbol{y}) = -\int \frac{\omega'^2 q_v(\boldsymbol{x}) s_y(\alpha_{\boldsymbol{\zeta},\boldsymbol{\zeta}',\boldsymbol{v}}\omega')}{4(2\pi)^6 F(\boldsymbol{\zeta}^{\mathbf{0}})F(\boldsymbol{\zeta}'^{\mathbf{0}})} \times e^{-i\omega'\left(t - t_y + \frac{\alpha_{\boldsymbol{\zeta}^{\mathbf{0}},\boldsymbol{\zeta}',\boldsymbol{v}}|\boldsymbol{z}(\alpha_{\boldsymbol{\zeta}^{\mathbf{0}},\boldsymbol{\zeta}',\boldsymbol{v}}\omega',\alpha_{\boldsymbol{\zeta}^{\mathbf{0}},\boldsymbol{\zeta}',\boldsymbol{v}}k'\hat{\boldsymbol{e}}_{\boldsymbol{\zeta}^{\mathbf{0}}})-\boldsymbol{x}|}{c} + \frac{|\boldsymbol{x}-\boldsymbol{y}(\omega',\hat{\boldsymbol{e}}_{\boldsymbol{\zeta}'^{\mathbf{0}}})|}{c}\right)} d\boldsymbol{x}\, d\omega'$$



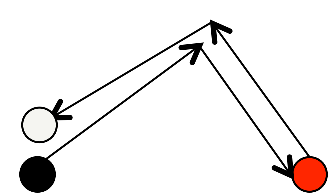
$$+\int \frac{\omega'^2 q_v(\boldsymbol{x}) s_{y'}(\alpha_{\boldsymbol{\zeta},\boldsymbol{\zeta}',\boldsymbol{v}}\omega')}{4(2\pi)^6 F(\boldsymbol{\zeta}^{\mathbf{0}})F(\boldsymbol{\zeta}'^{\mathbf{0}})} \times e^{-i\omega'\left(t - t_{y'} + \frac{\alpha_{\boldsymbol{\zeta}^{\mathbf{0}},\boldsymbol{\zeta}',\boldsymbol{v}}|\boldsymbol{z}(\alpha_{\boldsymbol{\zeta}^{\mathbf{0}},\boldsymbol{\zeta}',\boldsymbol{v}}\omega',\alpha_{\boldsymbol{\zeta}^{\mathbf{0}},\boldsymbol{\zeta}',\boldsymbol{v}}k'\hat{\boldsymbol{e}}_{\boldsymbol{\zeta}^{\mathbf{0}}})-\boldsymbol{x}|}{c} + \frac{|\boldsymbol{x}-\boldsymbol{y}'(\omega',\hat{\boldsymbol{e}}_{\boldsymbol{\zeta}'^{\mathbf{0}}})|}{c}\right)} d\boldsymbol{x}\, d\omega'$$



$$+\int \frac{\omega'^2 q_v(\boldsymbol{x}) s_y(\alpha_{\boldsymbol{\zeta},\boldsymbol{\zeta}',\boldsymbol{v}}\omega')}{4(2\pi)^6 F(\boldsymbol{\zeta}^{\mathbf{0}})F(\boldsymbol{\zeta}'^{\mathbf{0}})} \times e^{-i\omega'\left(t - t_y + \frac{\alpha_{\boldsymbol{\zeta}^{\mathbf{0}},\boldsymbol{\zeta}',\boldsymbol{v}}|\boldsymbol{z}'(\alpha_{\boldsymbol{\zeta}^{\mathbf{0}},\boldsymbol{\zeta}',\boldsymbol{v}}\omega',\alpha_{\boldsymbol{\zeta}^{\mathbf{0}},\boldsymbol{\zeta}',\boldsymbol{v}}k'\hat{\boldsymbol{e}}_{\boldsymbol{\zeta}^{\mathbf{0}}})-\boldsymbol{x}|}{c} + \frac{|\boldsymbol{x}-\boldsymbol{y}(\omega',\hat{\boldsymbol{e}}_{\boldsymbol{\zeta}'^{\mathbf{0}}})|}{c}\right)} d\boldsymbol{x}\, d\omega'$$



$$-\int \frac{\omega'^2 q_v(\boldsymbol{x}) s_y(\alpha_{\boldsymbol{\zeta},\boldsymbol{\zeta}',\boldsymbol{v}}\omega')}{4(2\pi)^6 F(\boldsymbol{\zeta}^{\mathbf{0}})F(\boldsymbol{\zeta}'^{\mathbf{0}})} \times e^{-i\omega'\left(t - t_y + \frac{\alpha_{\boldsymbol{\zeta}^{\mathbf{0}},\boldsymbol{\zeta}',\boldsymbol{v}}|\boldsymbol{z}'(\alpha_{\boldsymbol{\zeta}^{\mathbf{0}},\boldsymbol{\zeta}',\boldsymbol{v}}\omega',\alpha_{\boldsymbol{\zeta}^{\mathbf{0}},\boldsymbol{\zeta}',\boldsymbol{v}}k'\hat{\boldsymbol{e}}_{\boldsymbol{\zeta}^{\mathbf{0}}})-\boldsymbol{x}|}{c} + \frac{|\boldsymbol{x}-\boldsymbol{y}'(\omega',\hat{\boldsymbol{e}}_{\boldsymbol{\zeta}'^{\mathbf{0}}})|}{c}\right)} d\boldsymbol{x}\, d\omega'$$



Includes: Waveform, Object Position, Doppler Shift, Time delay, Sensor Positions, Reflection, take-off angle, frequency of reflection

INVERSE PROBLEM

$$I(\boldsymbol{p},\boldsymbol{u}) = \mathcal{P}^* d(\boldsymbol{y},\boldsymbol{z},t)$$

$$= \int \sum_{\boldsymbol{z},\boldsymbol{y}} \Upsilon^*(\boldsymbol{p},\boldsymbol{u},t,\boldsymbol{z},\boldsymbol{y})\Upsilon(\boldsymbol{x},\boldsymbol{v},t,\boldsymbol{z},\boldsymbol{y})dt q_{\boldsymbol{v}}(\boldsymbol{x})d\boldsymbol{x}d\boldsymbol{v}$$

$$d(t,\boldsymbol{z},\boldsymbol{y}) = \mathcal{P}q_{\boldsymbol{v}}(\boldsymbol{x}) = \int \Upsilon(\boldsymbol{x},\boldsymbol{v},t,\boldsymbol{z},\boldsymbol{y})q_{\boldsymbol{v}}(\boldsymbol{x})d\boldsymbol{x}d\boldsymbol{y}$$

Ideal Image Reconstruction

Multipath case has

$$\mathcal{P} = \sum_{j,k} \mathcal{P}^{jk}$$

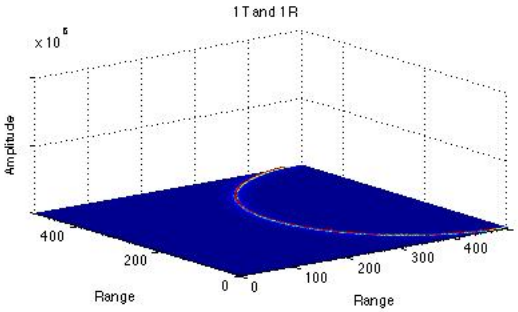
$$\Upsilon = \sum_{j,k} \Upsilon^{jk}$$

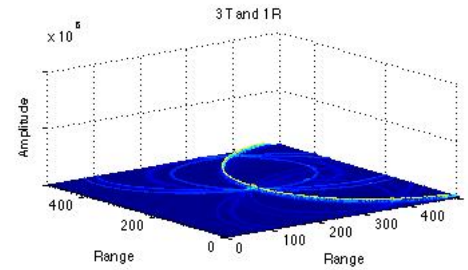
$$d(\boldsymbol{y},\boldsymbol{z},t) = \sum_{j,k} d^{jk}(\boldsymbol{y},\boldsymbol{z},t)$$

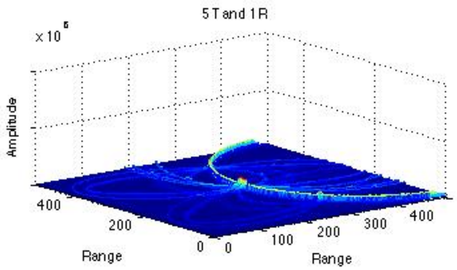
- ✧ If Data is Separable (Known Paths)
  - ✧ Nearly ideal reconstruction in dispersive case
  - ✧ See only diagonal terms but arise due to dispersion of reflector
- ✧ If Data is not Separable (Unknown Paths)
  - ✧ See non-diagonal terms as ambiguities in image

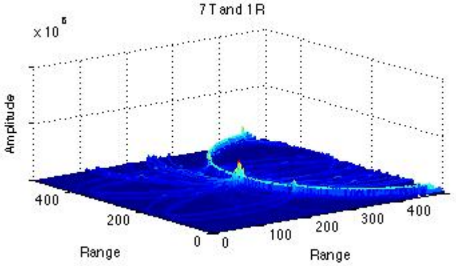
NUMERICAL RESULTS AND IMAGE ANALYSIS

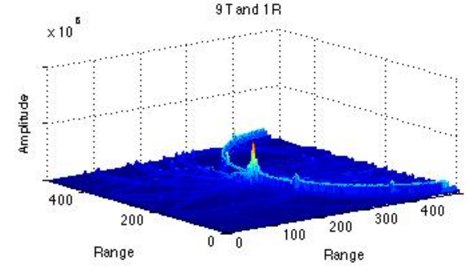
If we knew where all the paths where and where the reflector was located, then





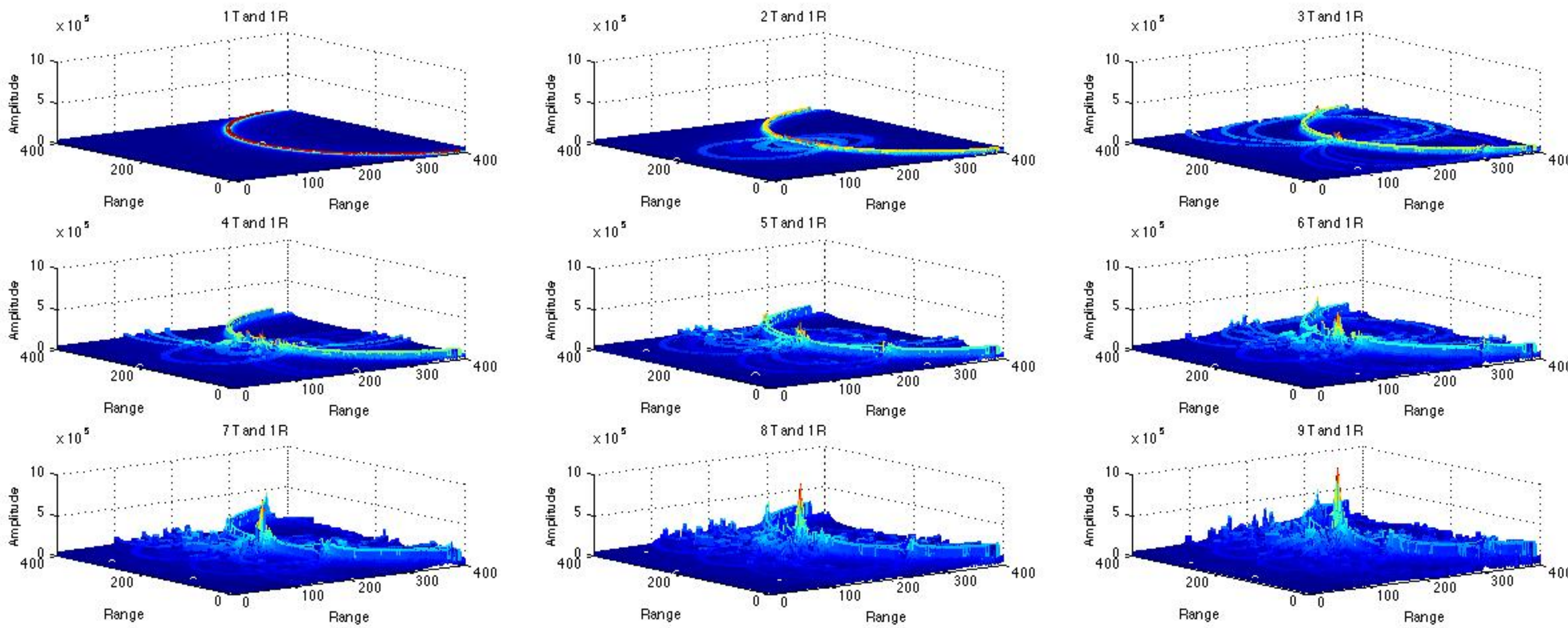






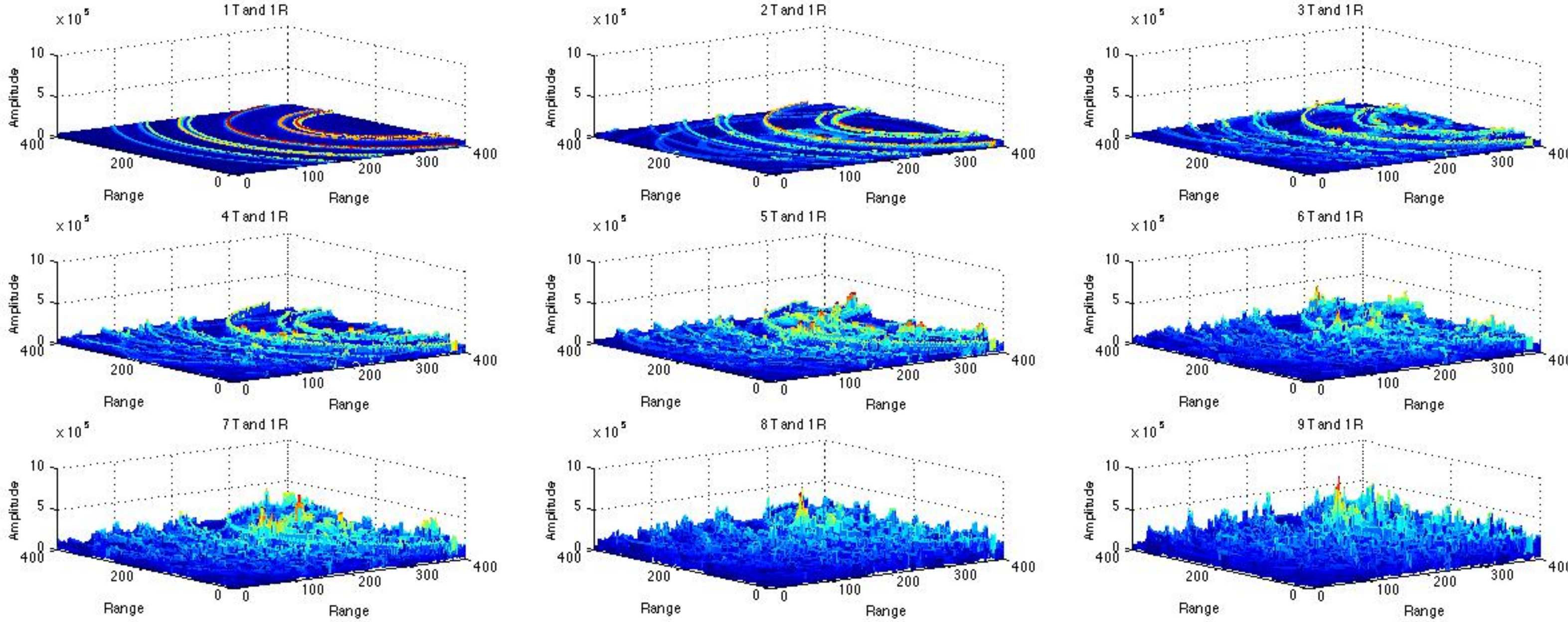
NUMERICAL RESULTS AND IMAGE ANALYSIS

If we knew where all the paths where but NOT where the reflector was located…



NUMERICAL RESULTS AND IMAGE ANALYSIS

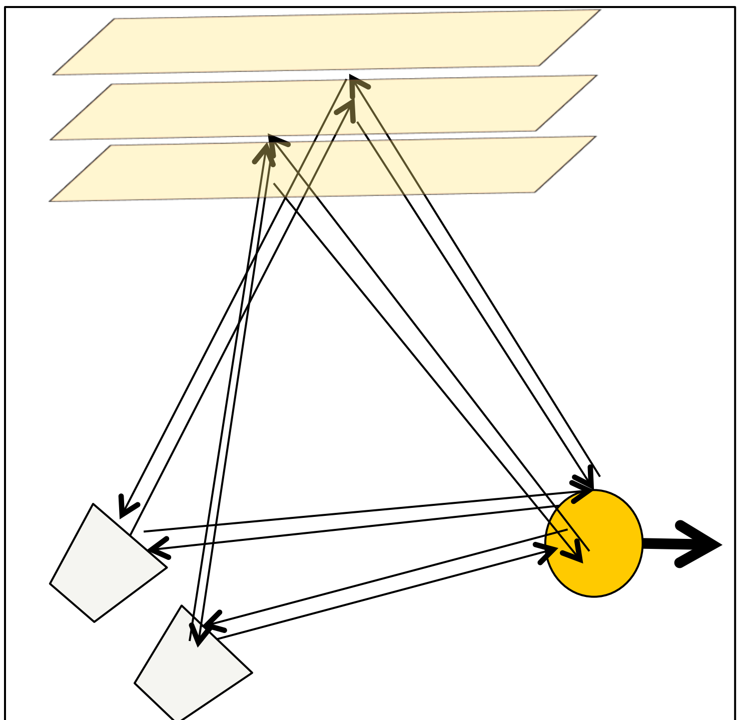
If we knew NOTHING about path or reflector locations…



CONCLUSION AND FUTURE WORK

CONCLUSION

- Developed model for scattering from stationary or moving objects embedded in a dispersive multipath environment
- Found that image resolution improved for a stationary target if data from multiple sensors is used even when no information is known about the dispersive reflector or path.



FUTURE SIMULATIONS

- Detect the velocity of moving objects
- Investigate the MIMO case
  - Sensors are Activated at Different Times with different waveforms

QUESTIONS

Questions?

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