MIMO Radar
SIMO Equivalence
And the Resulting Waveform Constraints

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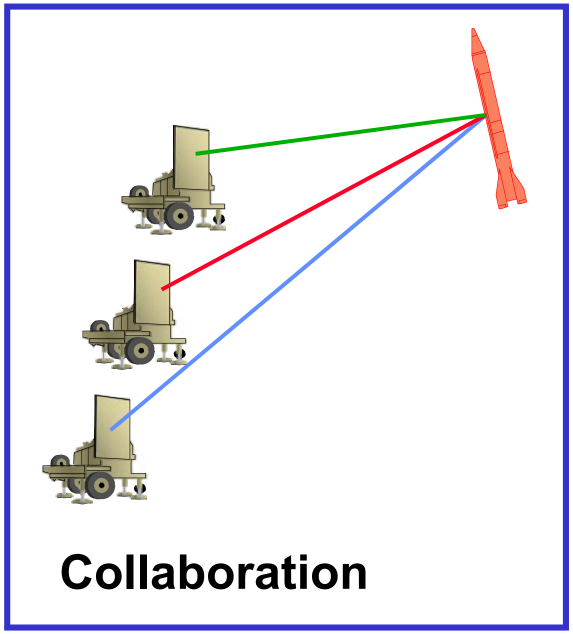
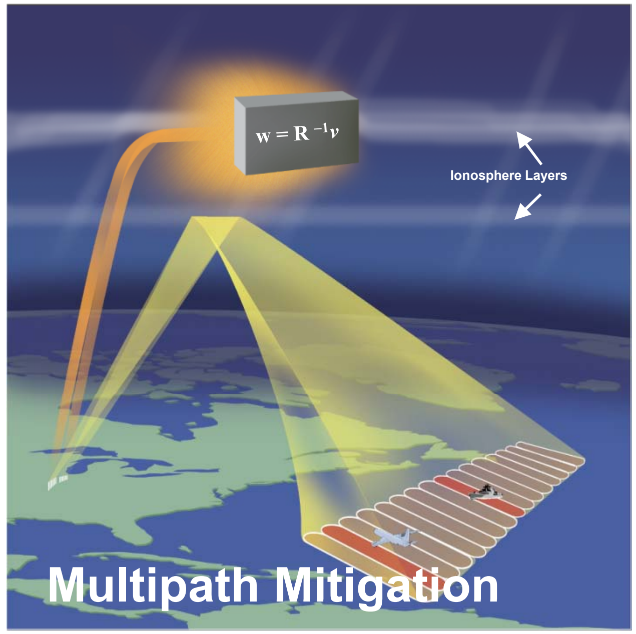
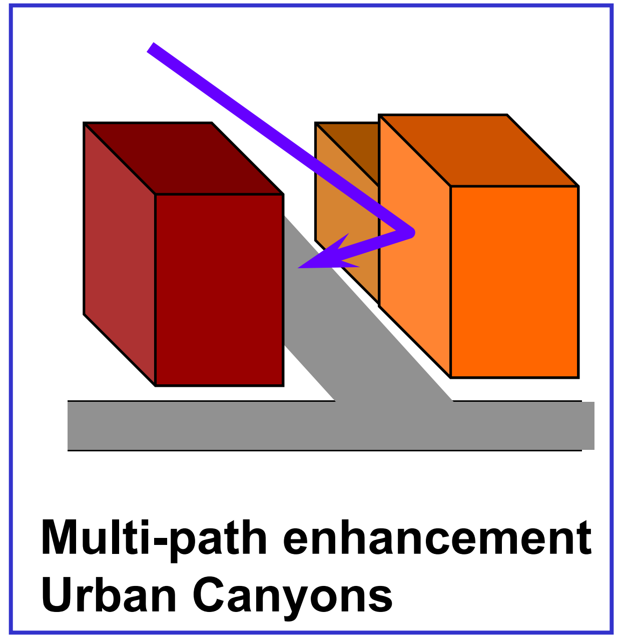
Coherent MIMO Application







- Coherent MIMO utilizes control system *observability*
  - Understand and observe propagation of signals in environment
- Application of multi-variate control to radar

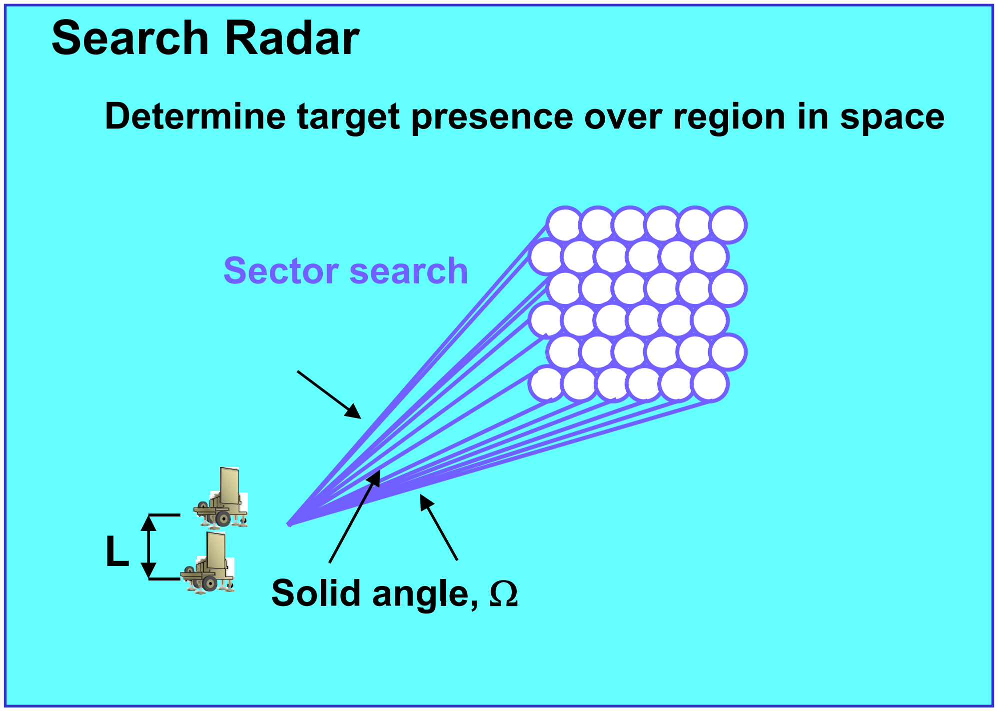
Outline

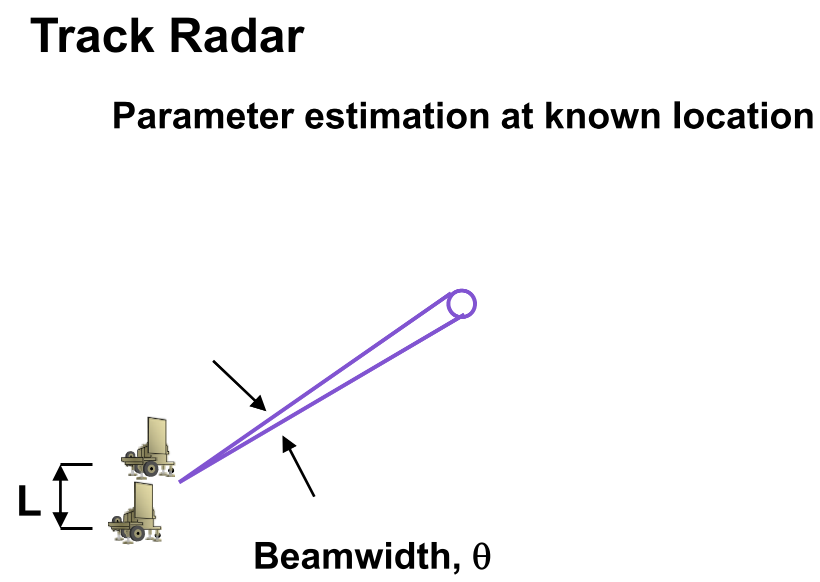
- Introduction
- Coherent MIMO Surveillance Radar Range Equation
- Comparison and Assumptions
- Conclusion

Briefing Purpose

- MIMO has observability advantages over SIMO radar
  - Ability to understand signal propagation in environment
  - Observability is key to controlling environment impact on radar
- What is impact on surveillance SNR?
- What assumptions are made?
  - How can this SNR be realized?

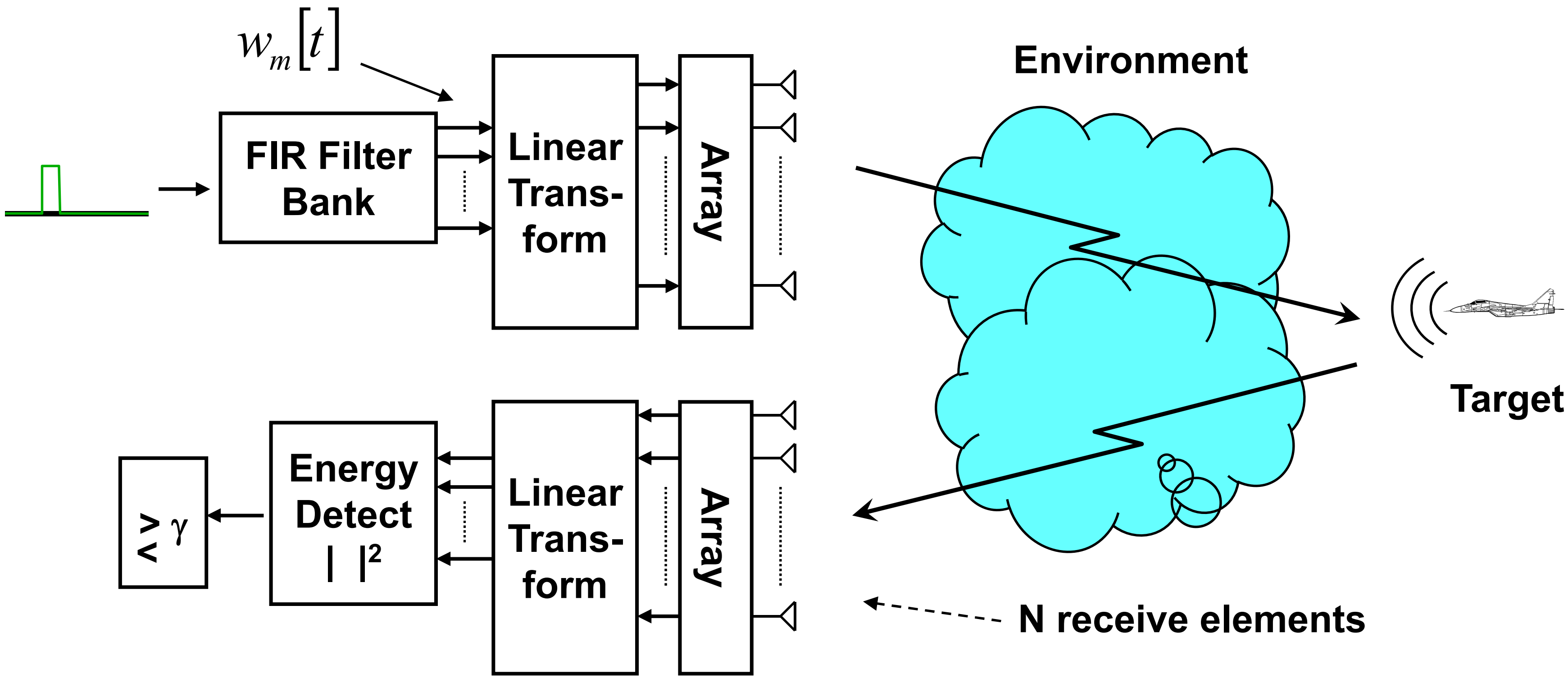
Pseudo-Monostatic MIMO Radar





- Detect and locate targets in desired surveillance sector Ω
- Sensor spacing and range are such that angle differences from each aperture are negligible

Coherent MIMO Radar



- Single impulse synchronizes system
- M linearly-independent/orthogonal waveforms

$$\sum_{i=1}^{I} w_m[i] w_n^*[i+j] = |a_m|^2 \delta[m-n]\delta[j]$$

- FIR filter bank model represents the three methods of achieving orthogonality: code division, frequency division, time division

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MIMO Radar Range Equation (1)

Energy density from radar transmitting spherical wave into solid angle Ω

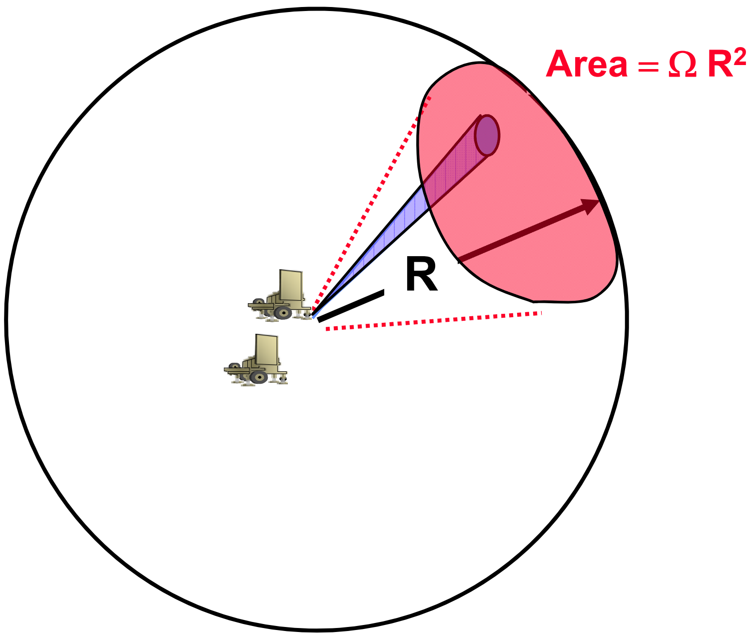
$$\frac{P\,T}{\Omega\, R^2}$$

P = average transmitter power

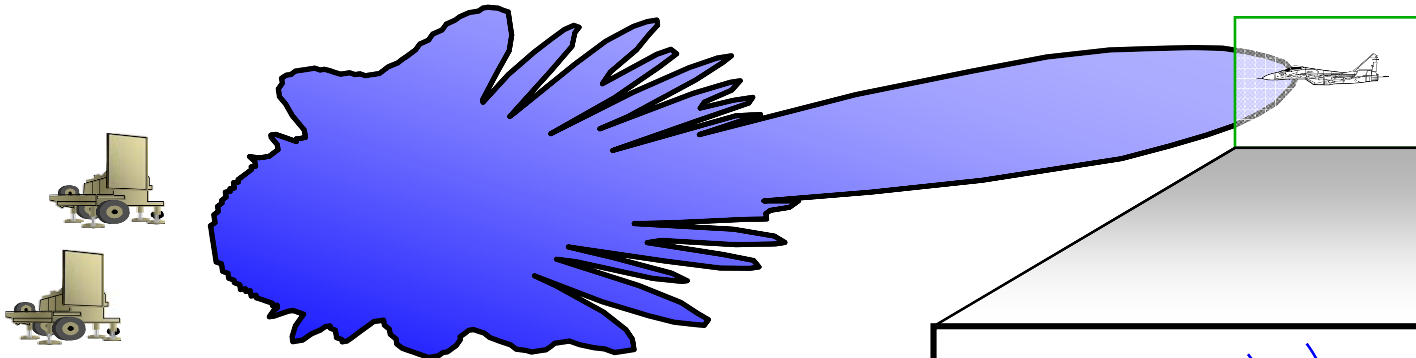
$$\sum_{m=1}^{M} |a_m|^2 = P\,T$$

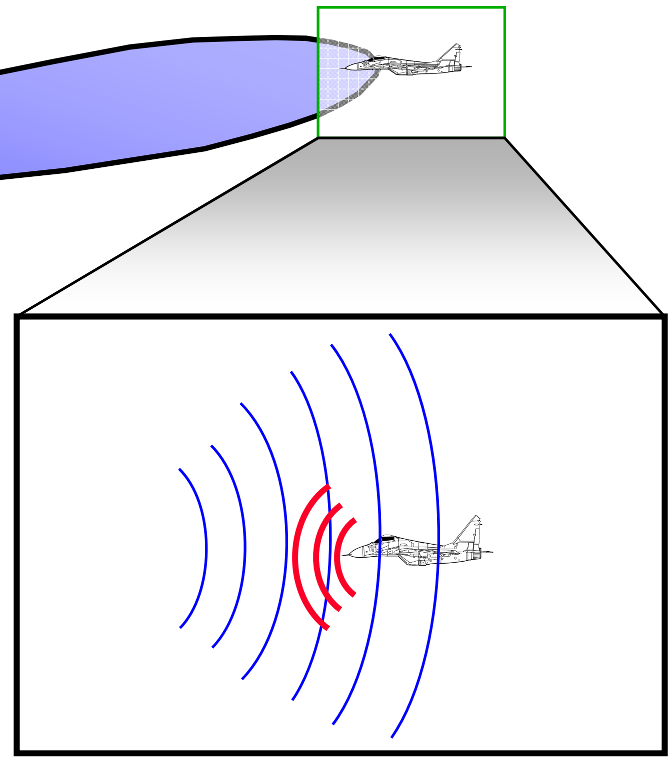
T = time to search solid angle Ω

R = distance from radar



Pseudo-Monostatic Radar Cross Section (RCS)





Measured in m^2 or dBsm

RCS

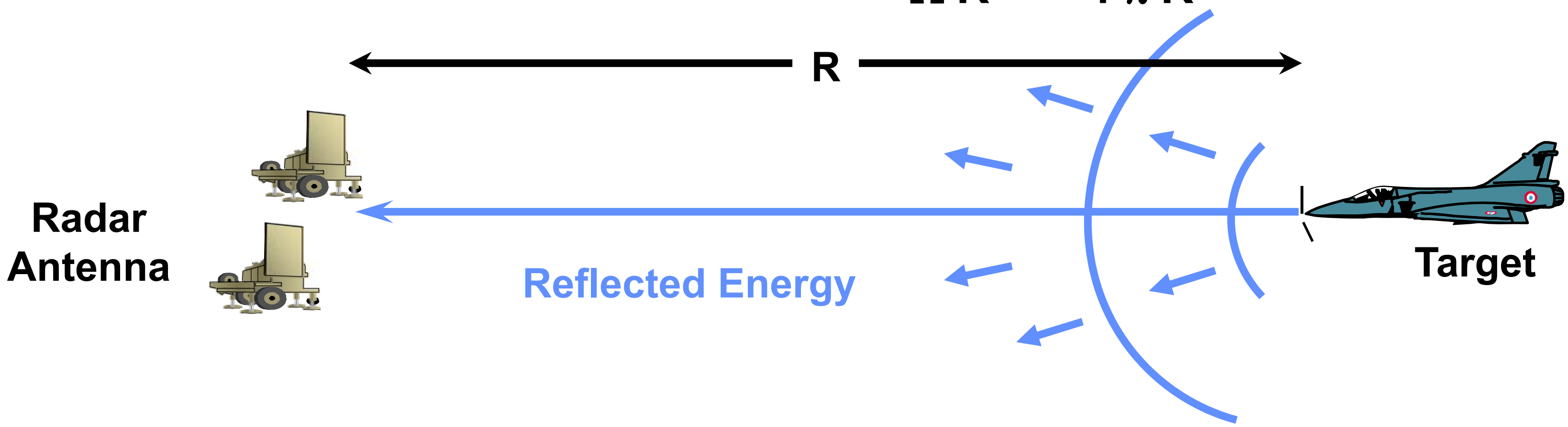
Incident Energy Density (Joules/m^2) x σ (m^2) = **Reflected Energy** (Joules)

- Radar Cross Section (RCS, or σ) magnitude is identical for all apertures (the effective cross-sectional area of the target as seen by the radar)

Search Radar Range Equation (2)

Energy density of reflected signal at radar receiver

$$\frac{P\,T}{\Omega\,R^2}\;\frac{\sigma}{4\,\pi\,R^2}$$



Energy of reflected signal from target and received by radar

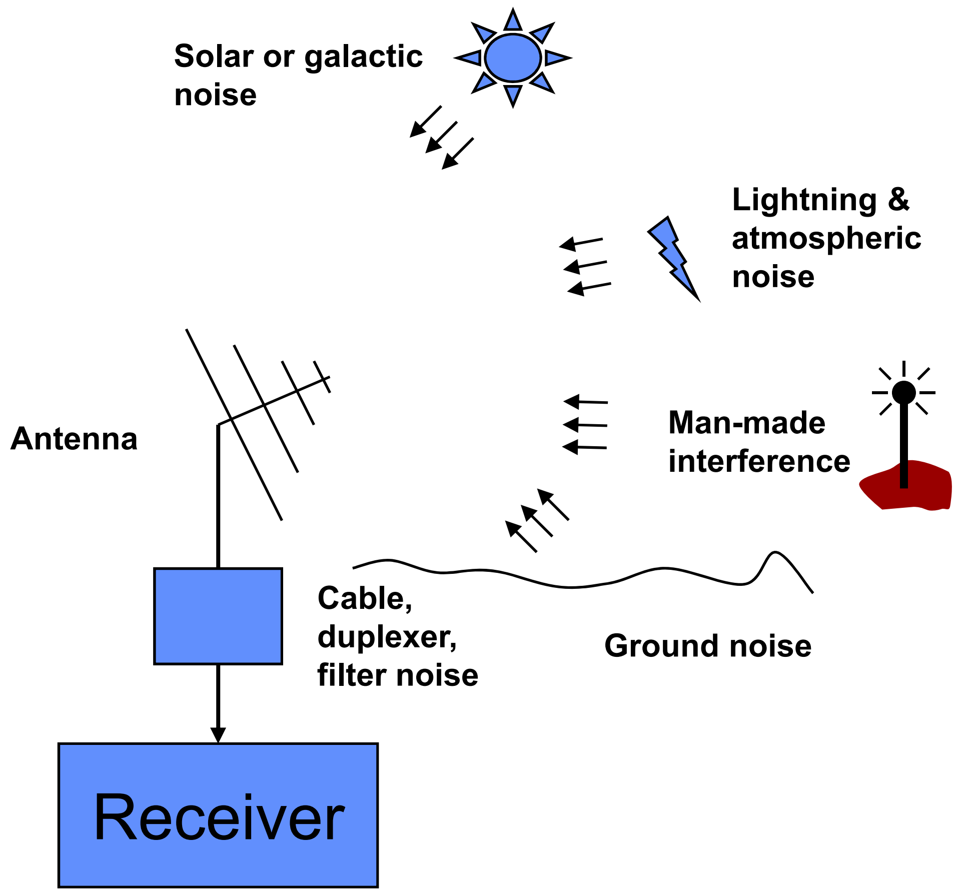
$$E_r = \frac{P\,T}{\Omega\,R^2}\;\frac{\sigma\,A_e\,N}{4\,\pi\,R^2}$$

E_r = Energy received

A_e = Effective area of each receiving antenna

N = Number of receive aperture

Received Noise



Total effect represented by a single noise source at the antenna output terminal.

The noise energy at the receiver is given by:

$$N_r = k\,T_s$$

k = Boltzmann's constant
= 1.38 x 10^{-23} joules/Kelvin
T_s= System Noise Temperature

Search Radar Range Equation (3)

Signal Energy reflected from target and received by radar $E_r = \frac{P\,T}{\Omega R^2} \frac{\sigma A_e N}{4 \pi R^2}$

Average Noise Energy $N_r = k\,T_s$

Signal to Noise Ratio $SNR = E_r / N_r L$

L = Loss Factor

$$SNR = \frac{P\,T\,A_e\,N\,\sigma}{4 \pi\ \Omega R^4\ k\,T_s\,L}$$

Receive Signal Processing

- **Matrix of receive data (N by P)**

$$\mathbf{Y} = \mathbf{d}(\varphi_{\mathrm{r}})\,\mathbf{s}^{H}(\varphi_{\mathrm{t}}) + \mathbf{N}$$

Each element of N, N(0,σ^2), total noise energy N_r

  – **Define orthonormal transformations**

$$\mathbf{A} = \left[\mathbf{d}(\varphi_{\mathrm{r}}) \vdots \mathbf{D}_{\perp}\right]$$

$$\mathbf{B} = \left[\tilde{\mathbf{s}}(\varphi_{\mathrm{t}}) \vdots \mathbf{S}_{\perp}\right]$$

  – **After transformation,**

$$\tilde{\mathbf{Y}} = \begin{bmatrix} \hat{a} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} + \tilde{\mathbf{N}}$$

Each element of $\tilde{\mathrm{N}}$, N(0,σ^2)

  – **Signal appears in upper left corner with total signal energy E_r**

No different for SIMO or MIMO: SNR is equal

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Target and Environment Assumptions

- **Target and environment must be constant for significantly longer time period**

$$\left|E\left[\sigma(t)\sigma^*(t+\tau)\right]\right| = \sigma^2 \forall t, t+\tau \in \left(0, \mathrm{T}\right)$$

  - **Single beam dwell area**

    Antenna Gain = 4 π $A_{t\lambda}$ **$A_{t\lambda}$: area in wavelengths**

  - **Dwell time increased by the number of beam positions to scan region:**

    k = Number of beams = Ω / 4 π $A_{t\lambda}$

- **It is difficult to meet coherence assumption in many applications**
  - **Alternative processing approaches likely can mitigate this**

- **Dwell time is often determined by required Doppler resolution rather than SNR**

Waveform Assumptions

- **Compact Uniform Spatial Spectrum**
  - Energy in region of interest is

$$\max \int_{\Omega} \left|S(k)\right|^2 dk$$

  - Or conversely,

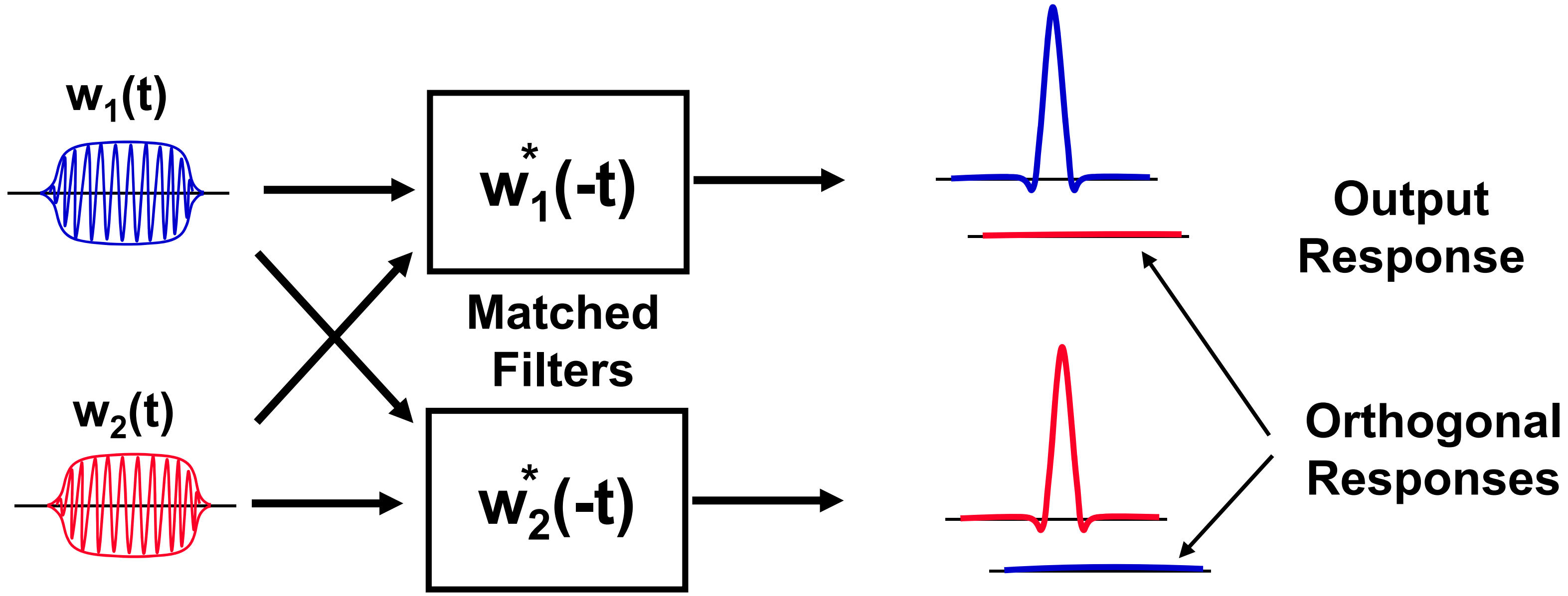
$$\min \int_{k \notin \Omega} \left|S(k)\right|^2 dk$$

  - Uniformity of illumination criteria is:

$$\min \max_{\Omega} (ave(\left|S(k)\right|^2) - \left|S(k)\right|^2)$$

  - SIMO radar precisely generates and scans a single beam
  - MIMO attempts to "uniformly" illuminate a larger area
- **Waveform Spectral Response**
  - Waveforms are not disjoint frequency for coherent operation
  - MIMO observability lost if disjoint
  - SIMO has no such restriction

Orthogonal Waveforms



- Orthogonal waveforms are the key enabler to MIMO
  - Limit mutual interference
  - Enable cooperative operation
  - Provide visibility into paths between transmitter and receivers
- Three methods of achieving orthogonality
  - Time division, frequency division, code division
  - Orthogonality achievable determined by time-bandwidth product

Waveform Assumptions (2)
Correlation Properties

- **Autocorrelation function**
  - **Autocorrelation function critical for both MIMO and SIMO**

$$\chi(\tau, f) = \int_{-\infty}^{\infty} w_m(t)\, w_m^{*}(t-\tau)e^{-j2\pi ft}\,dt$$
$$= \delta(\tau)\cdot\delta(f)$$

- **Cross correlation**

$$\int_{-\infty}^{\infty} w_A(t)\, w_B^{*}(t+\tau)\,d\tau = 0 \quad \forall\, t,\, A \neq B$$

- **Waveform packing of Abramovich and Fraser**
  - **Constraint on clear area of waveform ambiguity function**
  - **MIMO more difficult by factor of M for the same time-bandwidth**
  - **But time-bandwidth is likely higher by factor of M**

Additional Waveform Assumptions

- **Constant amplitude at array face**

$$|s(t)| = c \;\forall t$$

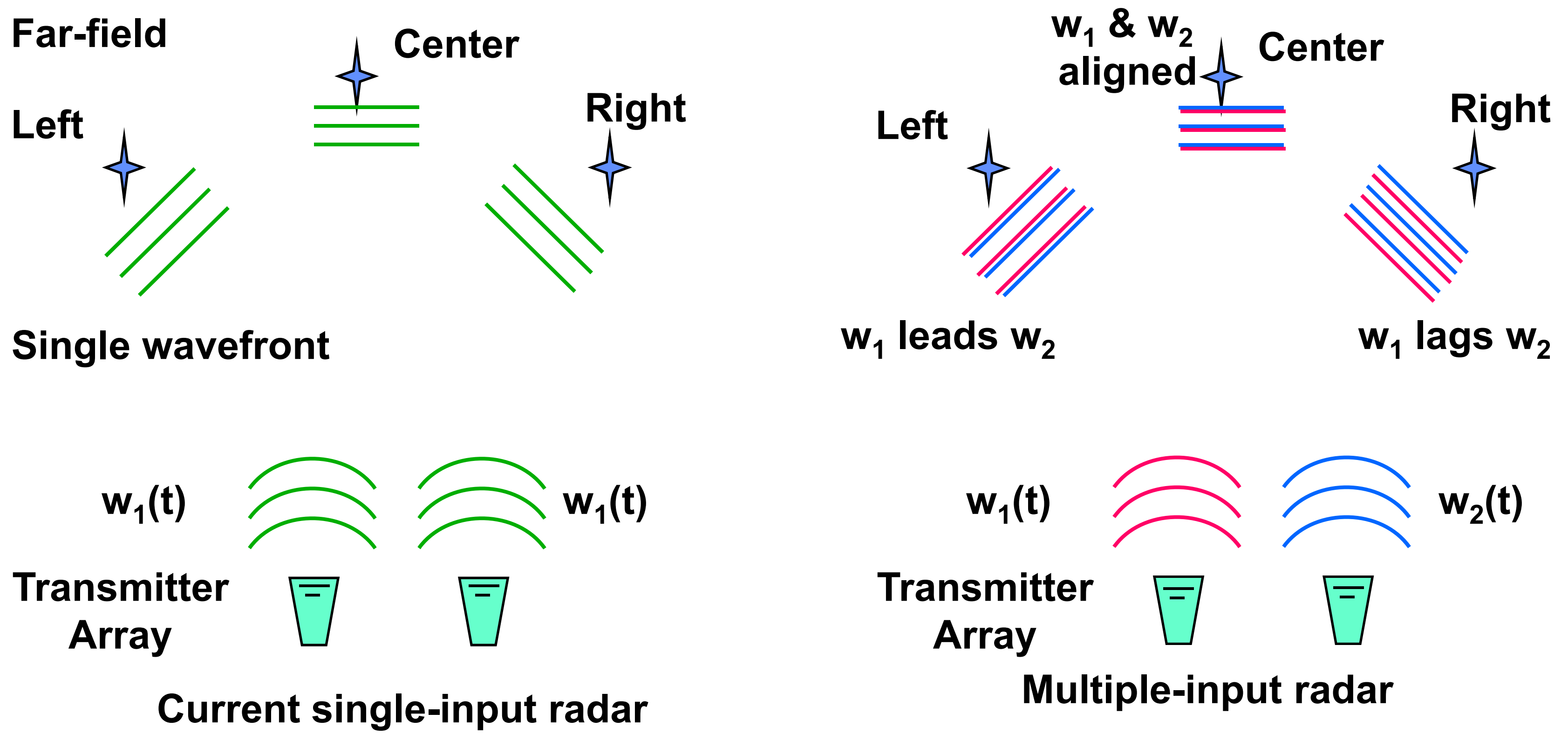
  - **Required for efficient power amplifier performance**
  - **Not met with linear transformation in generic MIMO model**
  - **Can be met in straightforward manner with SIMO**
- **Frequency spectrum to meet NTIA requirements**
  - **Maximize in-band energy**

$$\max \frac{\int_{f_L}^{f_H} |S(\omega)|^2 \, d\omega}{\int_0^{\infty} |S(\omega)|^2 \, d\omega}$$

  - **And minimize the maximum out-of-band spectral sidelobes**

$$\min \max_{\omega < f_L, \omega > f_H} \left(|S(\omega)|^2 \right)$$

MIMO Radar Observability

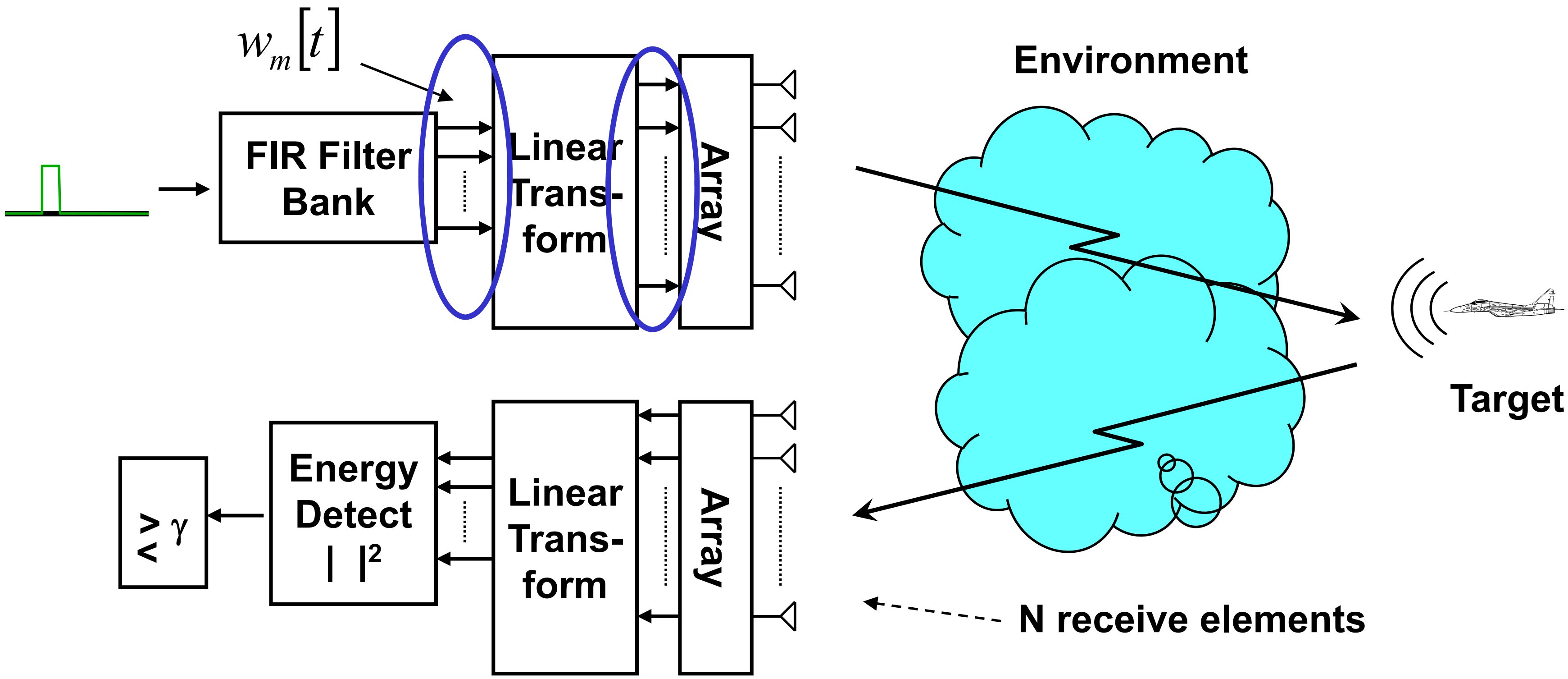


- Rank of illumination waveform must be >1 for observability
  - Does not have to be element-space waveforms
- Observability determined by transmit array/waveform manifold curvature

Maximizing MIMO Radar Observability

- **Small waveform rank is required to meet correlation limits**
- **Array/waveforms will interact to determine needed rank**
- **Array manifold arc-length and curvature will determine resolution and observability**
- **Equations need to be developed**
- **This is a work in progress**
  - **Cramer-Rao bounds for receive-only arrays apply directly for the virtual array or sum co-array**
  - **This should not be the same as maximizing observability**
  - **This should not be the same as maximizing ultimate SNR**

Waveform Assumptions



- Waveform assumptions were located at two points
- A linear transform model is correct given previous work
- Optimizing directly at the array face would appear productive

Waveform Assumption Summary

- General for MIMO or SIMO
  - Autocorrelation
  - Low out-of-band energy to meet NTIA requirements
  - Efficient (spectral shaping in-band)
  - Constant modulus
- MIMO specific assumptions
  - Compact uniform spatial spectrum (corresponds to low VSWR)
  - Provides MIMO observability (curvature of array manifold)
  - Cross-correlation
- Other practical assumptions
  - Resistant to small transmitter non-linearities
  - Ease in processing
  - Exploitation

Summary

- Coherent MIMO radar range equation developed
- MIMO and SIMO have equivalent SNR, but with significant assumptions
- All assumptions are unlikely to be met for MIMO radar
- The most difficult assumptions to meet are related to target characteristics and waveforms