Circuit Intuitions

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Looking into a Node

Welcome to "Circuit Intuitions." This is the first article of a series that will appear periodically in this magazine. As the series progresses, we will explore various aspects of circuit analysis. In this installment, we focus on the analysis of a node in a circuit.
Outline

- Why Circuit Intuitions?
- Overview of Articles Series
- Looking into a Node
  - Use of Thevenin and Norton Equivalent Circuits
Why Circuit Intuitions?

- Turning circuit design/analysis into a fun game!
- Gain intimate understanding of how circuits behave
- Making things simple, obvious!
- A gateway to innovation!
Circuit Intuitions Series

- Methods of Analysis
  - Looking into a Node
  - Source Degeneration; Bandwidth Extension
  - Miller’s Theorem; Miller’s Approximation

- Fundamental Concepts
  - Process Variation and Pelgrom’s Law
  - Why Sinusoids? Reinventing the Wheels, Random Walk
  - Capacitor Analogy (3 articles)
  - Norton and Thevenin Equivalent Circuits (3 articles)

- Special Circuits
  - Negative Cap.; Chopper Amp.; Capacitor as a Resistor
Focus of this Talk

- **Methods of Analysis**
  - Looking into a Node
  - Source Degeneration; Bandwidth Extension
  - Miller’s Theorem; Miller’s Approximation

- **Fundamental Concepts**
  - Process Variation and Pelgrom’s Law
  - Why Sinusoids? Reinventing the Wheels
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- **Special Circuits**
  - Negative Cap.; Chopper Amp.; Capacitor as a Resistor
Linear Time Invariant (LTI) Circuits

- Output is a linear combination of inputs!
  - Simple Case: If there are no storage elements (C or L)
    \[ y = h_1 x_1 + h_2 x_2 + h_3 x_3 + \cdots \]

- Superposition holds!
  \[ y = y_1 + y_2 + y_3 + \cdots \]

- Output is sum of contributions from individual inputs!
- Thevenin/Norton Theorem: consequence of superposition
Thevenin/Norton Theorems
An Intuitive Proof of Thevenin

\[ v_{out}(t) = v_{oc}(t) - R_{eq} i_L(t) \]
An Intuitive Proof of Norton

\[ i_L(t) = i_{sc}(t) - \frac{v_{out}(t)}{R_{eq}} \]
Assumption: All transistors are in Saturation Region!

- $g_m$ is the short-circuit transconductance of the transistor
- $g_{mb}$ is additional $g_m$ due to non-zero $v_{bs}$
- $r_o$ is the transistor output resistance
Circuits with More Transistors

- Analysis becomes too cumbersome very quickly
- Practice of KVL/KCL, leaves no room for intuition
Basic Premise

- Assume low-frequency analysis in this talk!
- That is, capacitors do not show up!
- In small-signal, transistors behave like LTI systems
- Superposition holds!

- Every circuit has a Thevenin/Norton equivalent circuit
  - Open-circuit voltage source ($v_{oc}$) in series with $R_{eq}$
  - Short-circuit current source ($i_{sc}$) in parallel with $R_{eq}$
  - When there is no input signal, they are just resistors!

- We rely on Thevenin/Norton equivalent circuits ONLY!
We call these “library elements”

Commonly-Used Configurations
- Looking into the gate and the drain
- Looking into the source
- Diode-Connected transistor
- Looking into the drain with source degeneration
- Looking into the source with load at the drain
- Thevenin/Norton Equivalent looking into the drain
- Thevenin/Norton Equivalent looking into the source

Let us build this library; one element at a time
Looking into the Gate

Looking into the gate, we see infinite resistance

\[ R_{eq} = \infty \]
Looking into the drain, while gate and source are grounded, we see $r_o$!
Looking into the source, while gate and drain are grounded, we see a resistor whose value is \( R_{eq} = r_o \parallel 1/g_{me} \approx 1/g_{me} \)

- \( g_{me} = g_m + g_{mb} = 1.1-1.2 \ g_m \)
- \( R_{eq} = r_o \parallel 1/g_{me} \approx 1/g_{me} \)
A diode-connected transistor is a resistor whose value is $R_{eq} = r_o \parallel \frac{1}{g_m} \approx \frac{1}{g_m}$.
Looking into the Drain with $R_S$

- Also known as Source Degeneration
- Effective way to increase the output impedance
- Multiplying the source resistance by $g_{me}r_o$

$$R_{eq} = R_s + r_o + g_{me}r_oR_s$$
Looking into the Source with $R_D$

If $R_D << r_o$, then $R_{eq} \approx 1/g_{me}$

$R_D$ is divided by $(1+g_{me}r_o)$ and appears at the source.
Looking into the Drain with Input

Two Methods:

- Find the Thevenin Equivalent Circuit:
  - open-circuit voltage $v_{oc}$ in series with $R_{eq}$
- Find Norton Equivalent Circuit:
  - short-circuit current $i_{sc}$ in parallel with $R_{eq}$
- Note that $v_{oc} = i_{sc} \times R_{eq}$

In this circuit, easier to find $v_{oc}$ first

$\Rightarrow v_s = 0$, no body effect, $v_{oc} = -g_m r_o v_{in}$
$\Rightarrow i_{sc} = v_{oc} / R_{eq}$

Norton more intuitive due to high $R_{eq}$

$$v_{oc} = -g_m r_o v_{in}$$
$$i_{sc} = v_{oc} / R_{eq}$$
Looking into the Source with Input

Find Norton Equivalent Circuit first:
- short-circuit current \( i_{sc} \) in parallel with \( R_{eq} \)

Shorting output to ground \( \rightarrow v_s = 0 \)

\[ i_{sc} = g_m v_{in} \frac{r_o}{r_o + R_D} \]

\( \Rightarrow \) Given small \( R_{eq} \), Thevenin is more intuitive

\[ v_{oc} = i_{sc} R_{eq} \]
Putting It All Together (1 to 4)

1. Looking into one terminal while the other two grounded
2. Looking into the drain of a diode-connected transistor
3. No additional resistor in the circuit

\[ R_{eq} = \infty \]
\[ R_{eq} = r_o \]
\[ R_{eq} = r_o \parallel 1/g_{me} \]
\[ R_{eq} = r_o \parallel 1/g_m \]
Putting It All Together (5 to 8)

5. \[ R_{eq} = R_s + r_o + g_{me} r_o R_s \]

6. \[ R_{eq} = \frac{R_D + r_o}{1 + g_{me} r_o} \]

7. \[ V_{oc} = -g_m r_o V_{in} \quad i_{sc} = V_{oc} / R_{eq} \]

8. \[ i_{sc} = \frac{g_m V_{in} r_o}{r_o + R_D} \quad V_{oc} = i_{sc} R_{eq} \]

Adding resistors and input to the circuit
Finding Voltage of “any” Node

Method 1: Use Norton equivalent circuit
- Short the node to ground; find $i_{sc}$
- Find $R_{eq}$ (using library elements)
- Multiply the two: $v_{out} = i_{sc} \times R_{eq}$

Method 2: Use Thevenin equivalent circuit
- Open the load; find $v_{oc}$
- Find $R_{eq}$ (same as in Method 1)
- Use voltage divider rule to find $v_{out}$
Example: Common-Source Amplifier

Finding $v_{out}$: Use $i_{sc} \times R_{eq}$:

$\Rightarrow i_{sc} = -g_m v_{in}$
$\Rightarrow R_{eq} = R_L \parallel r_o$
$\Rightarrow v_{out} = -g_m (R_L \parallel r_o) v_{in}$

- What is the intuition here?
  - How to increase the gain? How to increase $v_{out}$?
  - Output voltage comes from $i_{sc} \times R_{eq}$
  - $\Rightarrow$ either increase $i_{sc}$ or $R_{eq}$

- How can we increase these two?
Example: Cascode Circuit

To find the output voltage
- Use $i_{sc} \times R_{eq}$:
- Find out how $i_{sc}$ and $R_{eq}$ change compared to those for CS amplifier
- Gain intuition on why the gain changes from one to the other.
- Also, find intermediate voltages (such as $v_{d1}$) for added insight
Finding $v_{d1}$: Use Thevenin:

1. $R_{eq1} = r_{o1}$
2. $v_{oc1} = -g_m r_{o1} v_{in}$
3. $R_{eq2} = (r_{o2} + R_L) / (1 + g_{me2} r_{o2})$
4. $v_{d1} = -g_m r_{o1} v_{in} (r_{o1} || R_{eq2})$

Finding $v_{out}$: Use $R_{eq2}$ to find the load current first:

1. $i_L = v_{d1}/R_{eq2}$
2. $v_{out} = i_L R_L = v_{d1} R_L / R_{eq2}$
Finding $v_{d1}$:
(as in previous slide)

$\rightarrow v_{d1} = -g_m r_{o1} v_{in} (r_{o1} \parallel R_{eq2})$

Treat $v_{d1}$ as ideal source:
Find $i_{sc}$ and $R_{eq}$ at the output:

$\rightarrow i_{sc} = v_{d1} / (1/g_{me} \parallel r_o)$

$\rightarrow R_{eq} = r_o \parallel R_L \text{ (not } R_{eq} \text{ of original cct)}$

$\rightarrow v_{out} = i_{sc} R_{eq}$
What Is the Intuition here?

- $i_{sc}$ remained the same
- $R_{eq}$ increased by $g_m r_o$
- Voltage gain increased by $g_m r_o$
Finding $v_{d5}$: Use the Norton Equivalent at the source of $M_1$:

1. $R_{eq3} = r_{o3} \parallel 1/g_{m3}$
2. $R_{eq1} = (R_{eq3}+r_{o1})/(1+g_{me1}r_{o1})$
3. $i_{sc1} = g_{m1}v_{in}r_{o1}/(r_{o1}+R_{eq3})$
4. $R_{eq5} = r_{o5}$
5. $R_{eq4} = r_{o4} \parallel 1/g_{m4}$
6. $R_{eq2} = (R_{eq4}+r_{o2})/(1+g_{me2}r_{o2})$

$\Rightarrow v_{d5} = i_{sc1} \left(R_{eq1} \parallel R_{eq5} \parallel R_{eq2}\right)$

Finding $v_{out}$:

$\Rightarrow v_{out} = (v_{d5}/R_{eq2})R_{eq4}$
Example: Differential Pair (2)

Method 2:

Treat $v_{d5}$ as ideal source:
(from previous slide)

$\Rightarrow v_{d5} = i_{sc1} (R_{eq1} \parallel R_{eq5} \parallel R_{eq2})$

Finding $v_{out}$:
Find $i_{sc}$ and $R_{eq}$ at the output:

$\Rightarrow i_{sc} = v_{d5} / (r_{o2} \parallel 1/g_{me2})$

$\Rightarrow R_{eq} = r_{o2} \parallel r_{o4} \parallel 1/g_{m4}$

Note: not equal to $R_{eq}$ of original circuit

$\Rightarrow v_{out} = i_{sc} R_{eq}$
Example: Diode-Connected Cascode

- Not one of the standard library elements
- Apply a voltage, measure current
- Use Superposition to find current
- Ratio of test voltage to current is $R_{eq}$
- Intuition?
Diode-Connected Cascode

The two circuits are equivalent

Treat two sources as independent

Use Superposition
Diode-Connected Cascode

\[ i_x = i_{x1} + i_{x2} \]

\[ R_{eq} = R_{eq1} || R_{eq2} \]

\[ R_{eq1} \approx g_{me2}r_{o1}r_{o2} \]

\[ R_{eq2} \approx 1/g_{m1} \]

\[ R_{eq} \approx 1/g_{m1} \]
Summary

- In small-signal, all transistor circuits (assuming saturation region) can be treated as LTI circuit.
- Transistor circuits without a signal source are resistors:
  - This is for low-frequencies only.
  - At high-frequencies, we should add capacitors.
  - See other circuit intuition articles for high frequencies.
- Transistor circuits with input signal are represented by:
  - Their Thevenin equivalent circuits, or
  - Their Norton equivalent circuits.
- There are different ways to arrive at the solution.
- Use the method that adds intuition!
References (1 of 2)

All in Solid-State Circuit Magazine

References (2 of 2)

All in Solid-State Circuit Magazine

  - See also Correction to Looking Into a Node, Summer 2014.
Acknowledgement

I would like to thank my undergraduate students at the University of Toronto, who have been the inspirations behind writing these articles.