Circuit Intuitions

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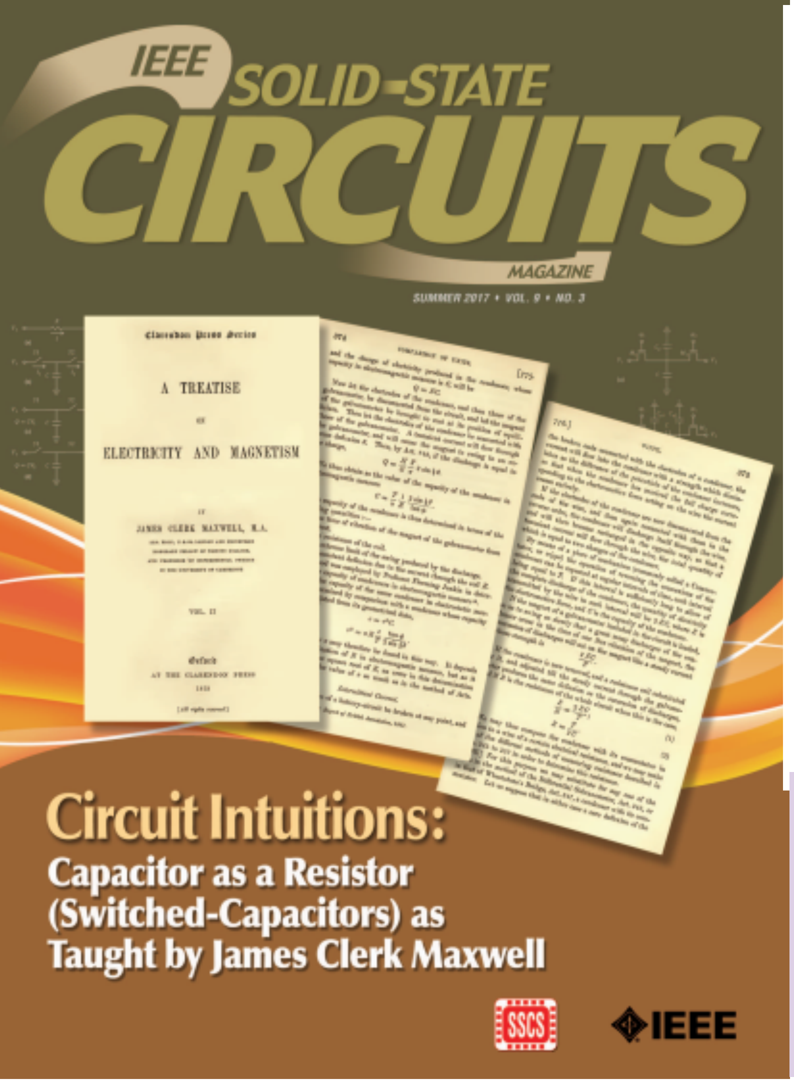
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A TREATISE
ON
ELECTRICITY AND MAGNETISM
Circuit Intuitions:
Capacitor as a Resistor
(Switched-Capacitors) as
Taught by James Clerk Maxwell
SSCS
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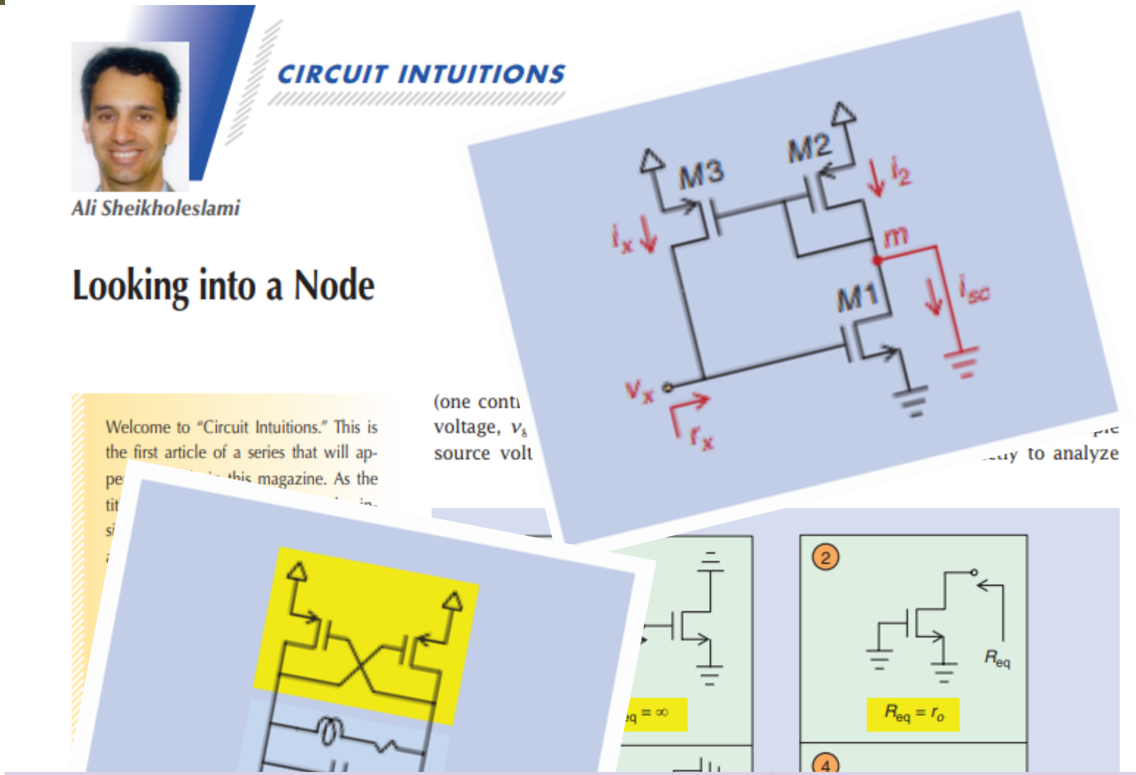
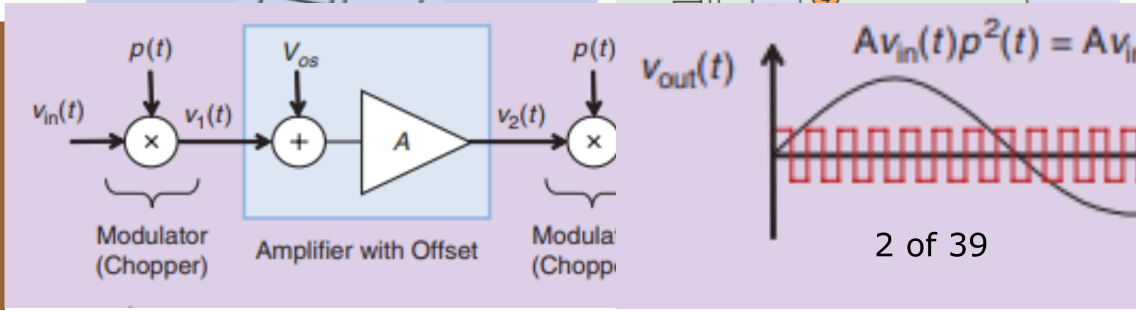

CIRCUIT INTUITIONS
Ali Sheikholeslami
Looking into a Node
Welcome to "Circuit Intuitions." This is
the first article of a series that will ap-
this magazine. As the
(one conti
voltage,
source volt
to analyze
M3
M2
M1
m
i_x
i_2
i_{sc}
v_x
r_x
2
4
R_{eq}
$R_{eq} = r_o$
$_{eq} = \infty$


$p(t)$
V_{os}
$p(t)$
$v_{in}(t)$
$v_1(t)$
$v_2(t)$
A
Modulator
(Chopper)
Amplifier with Offset
$v_{out}(t)$
$Av_{in}(t)p^2(t) = Av_{in}$

Outline

- Why Circuit Intuitions?
- Overview of Articles Series
- Looking into a Node
  - Use of Thevenin and Norton Equivalent Circuits

Why Circuit Intuitions?

- Turning circuit design/analysis into a fun game!
- Gain intimate understanding of how circuits behave
- Making things simple, obvious!
- A gateway to innovation!

Circuit Intuitions Series

- Methods of Analysis
  - Looking into a Node
  - Source Degeneration; Bandwidth Extension
  - Miller’s Theorem; Miller’s Approximation
- Fundamental Concepts
  - Process Variation and Pelgrom’s Law
  - Why Sinusoids? Reinventing the Wheels, Random Walk
  - Capacitor Analogy (3 articles)
  - Norton and Thevenin Equivalent Circuits (3 articles)
- Special Circuits
  - Negative Cap.; Chopper Amp.; Capacitor as a Resistor

Focus of this Talk

- Methods of Analysis
  - Looking into a Node
  - Source Degeneration; Bandwidth Extension
  - Miller's Theorem; Miller's Approximation
- Fundamental Concepts
  - Process Variation and Pelgrom's Law
  - Why Sinusoids? Reinventing the Wheels
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  - Norton and Thevenin Equivalent Circuits (3 articles)
- Special Circuits
  - Negative Cap.; Chopper Amp.; Capacitor as a Resistor

Linear Time Invariant (LTI) Circuits

- Output is a linear combination of inputs!
  - Simple Case: If there are no storage elements (C or L)

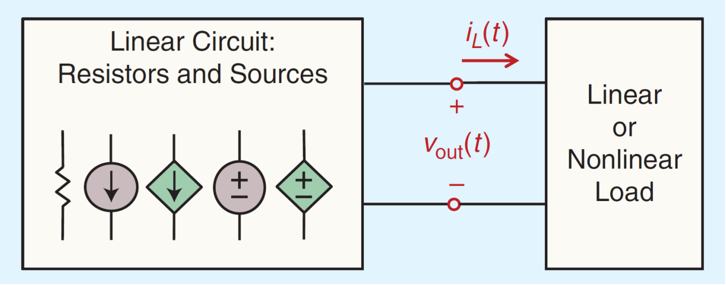
$$y = h_1 x_1 + h_2 x_2 + h_3 x_3 + \cdots$$

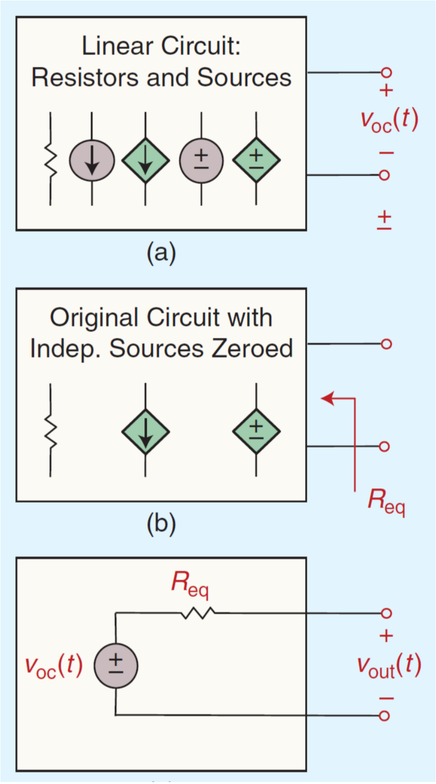
- Superposition holds!

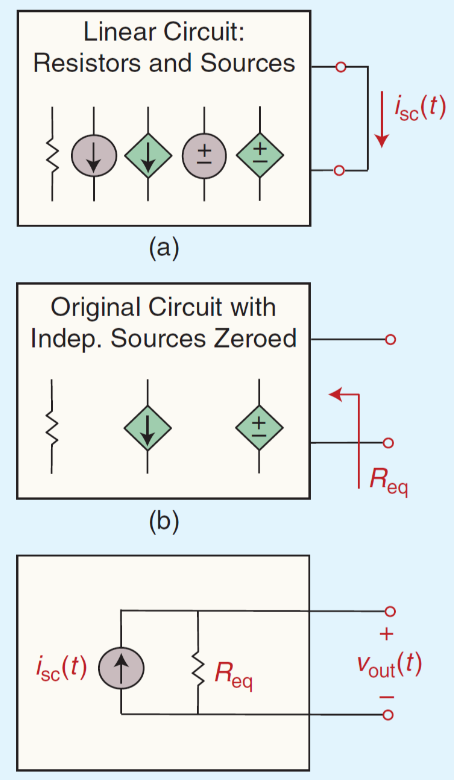
$$y = y_1 + y_2 + y_3 + \cdots$$

- Output is sum of contributions from individual inputs!
- Thevenin/Norton Theorem: consequence of superposition

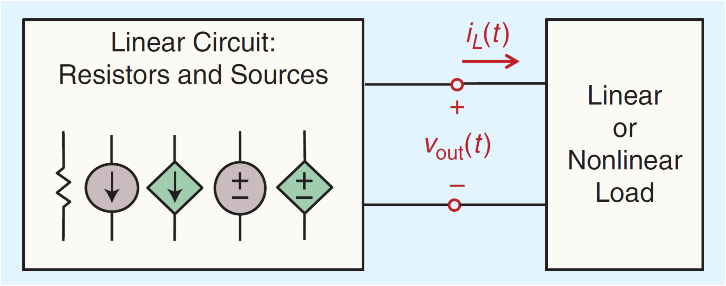
Thevenin/Norton Theorems

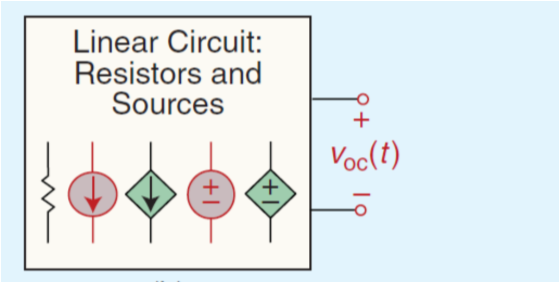


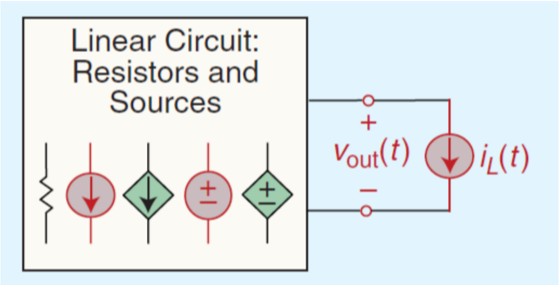


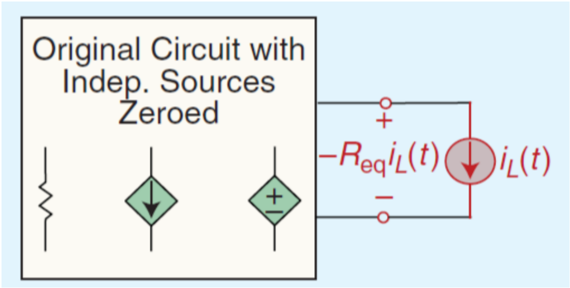


An Intuitive Proof of Thevenin



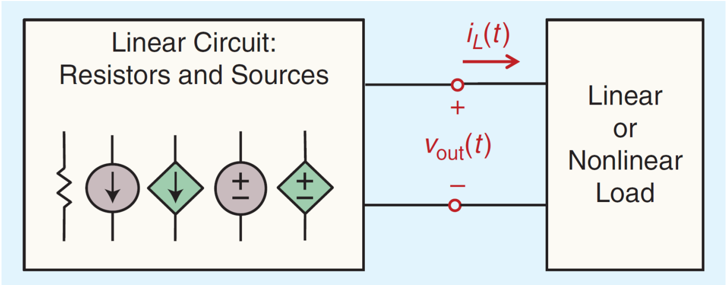


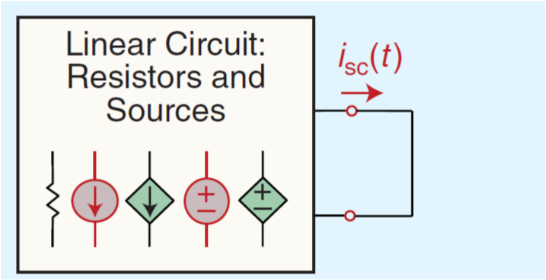


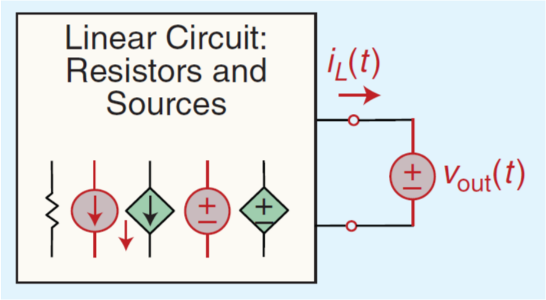


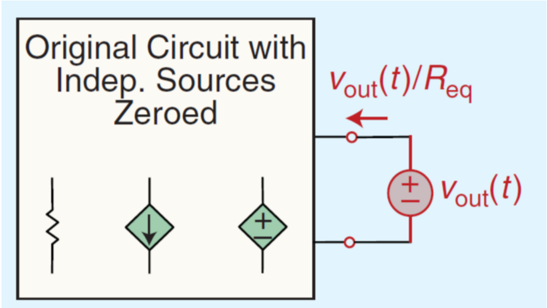
$$v_{out}(t) = v_{oc}(t) - R_{eq} i_L(t)$$

An Intuitive Proof of Norton



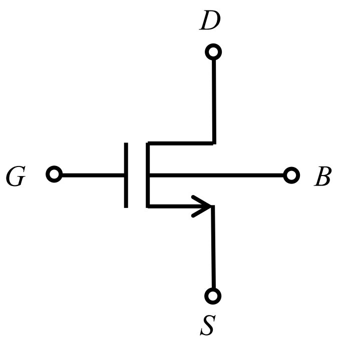


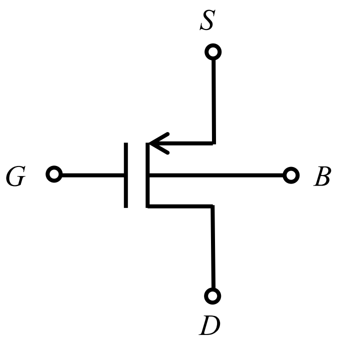


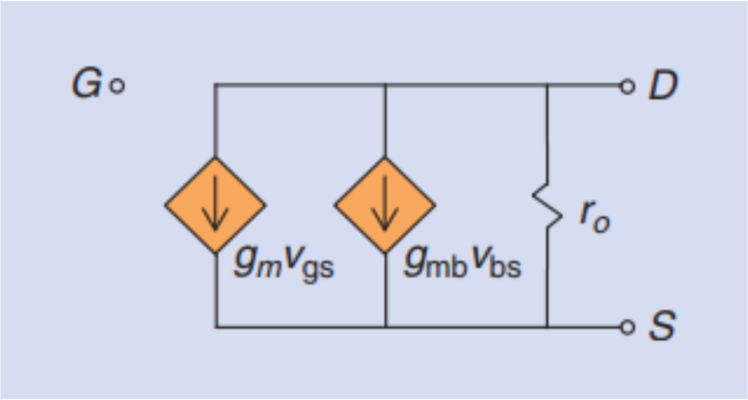


$$i_L(t) = i_{sc}(t) - v_{out}(t)/R_{eq}$$

Small-Signal Model of NMOS/PMOS



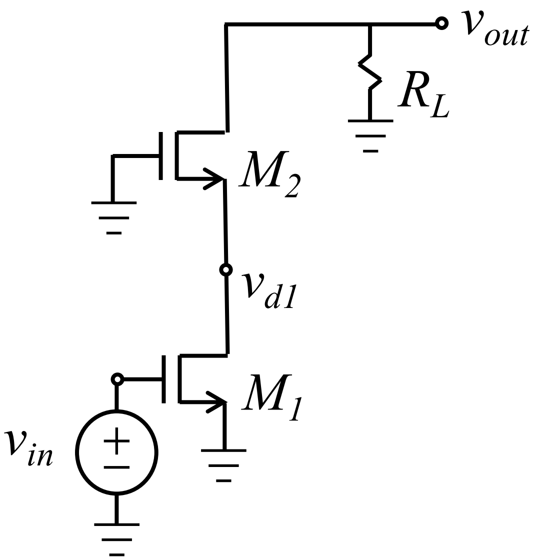


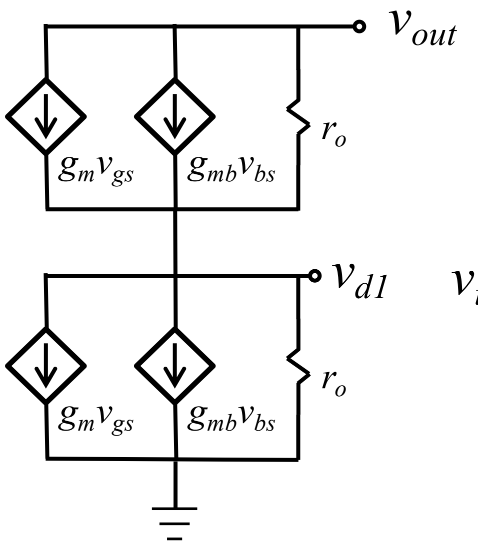


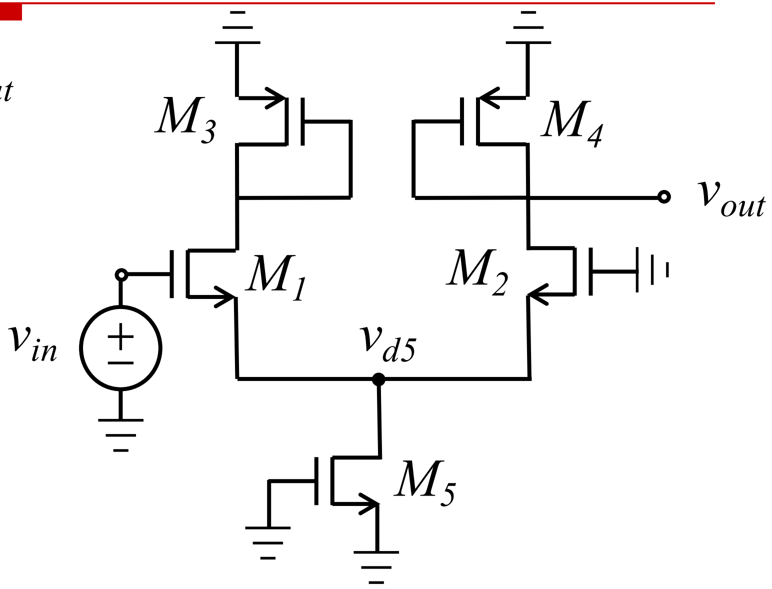
Assumption: All transistors are in Saturation Region!

- g_m is the short-circuit transconductance of the transistor
- g_{mb} is additional g_m due to non-zero v_{bs}
- r_o is the transistor output resistance

Circuits with More Transistors







- Analysis becomes too cumbersome very quickly
- Practice of KVL/KCL, leaves no room for intuition

Basic Premise

- Assume low-frequency analysis in this talk!
- That is, capacitors do not show up!
- In small-signal, transistors behave like LTI systems
- Superposition holds!

- Every circuit has a Thevenin/Norton equivalent circuit
  - Open-circuit voltage source (v_{oc}) in series with R_{eq}
  - Short-circuit current source (i_{sc}) in parallel with R_{eq}
  - When there is no input signal, they are just resistors!

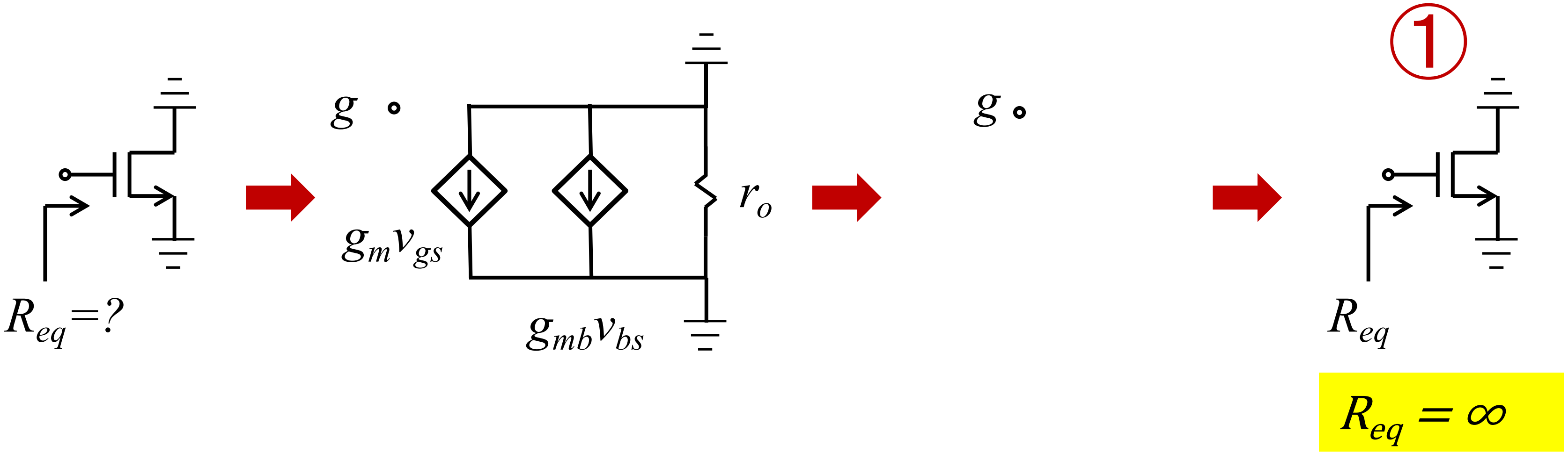
- We rely on Thevenin/Norton equivalent circuits ONLY!

“Library Elements”

- We call these “library elements”
- Commonly-Used Configurations
  - Looking into the gate and the drain
  - Looking into the source
  - Diode-Connected transistor
  - Looking into the drain with source degeneration
  - Looking into the source with load at the drain
  - Thevenin/Norton Equivalent looking into the drain
  - Thevenin/Norton Equivalent looking into the source

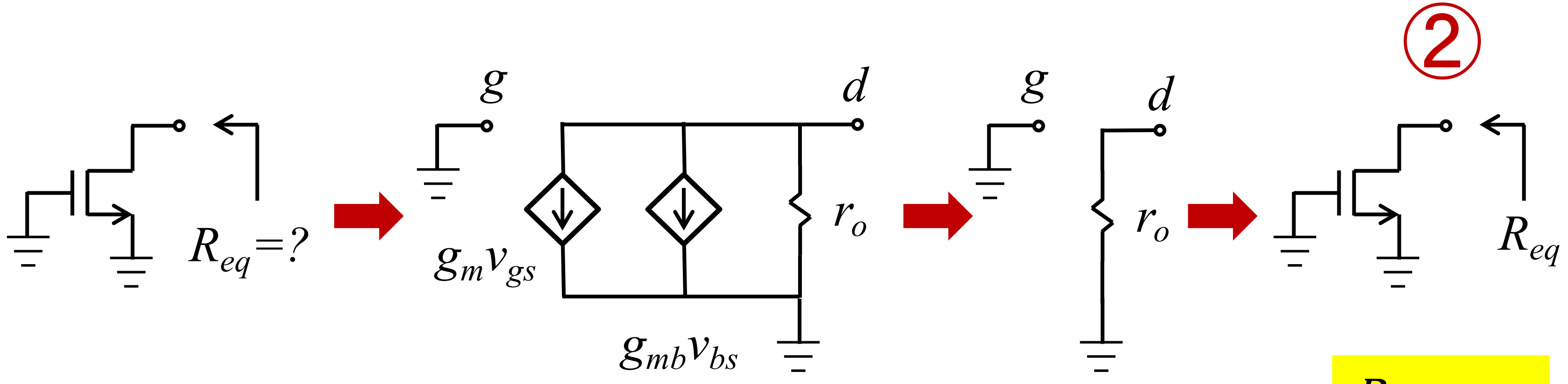
- Let us build this library; one element at a time

Looking into the Gate



- Looking into the gate, we see infinite resistance

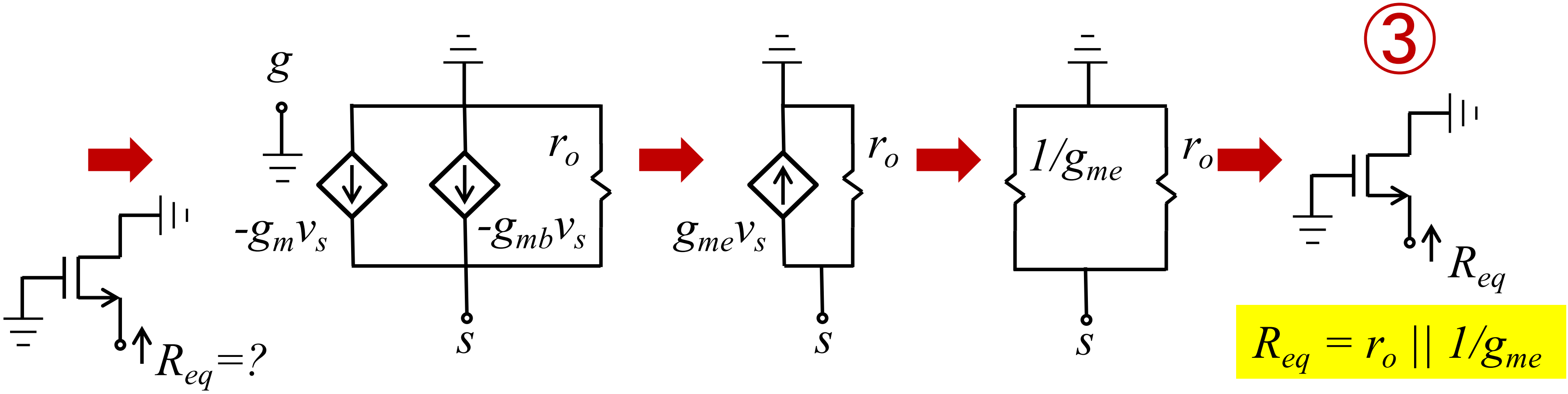
Looking into the Drain



$R_{eq} = r_o$

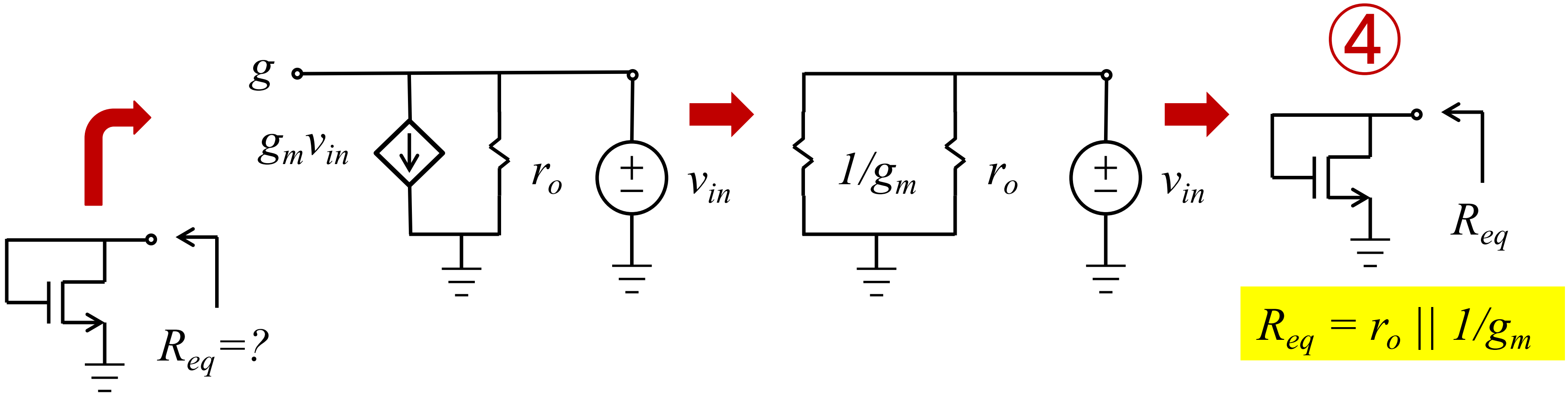
- Looking into the drain, while gate and source are grounded, we see r_o!

Looking into the Source



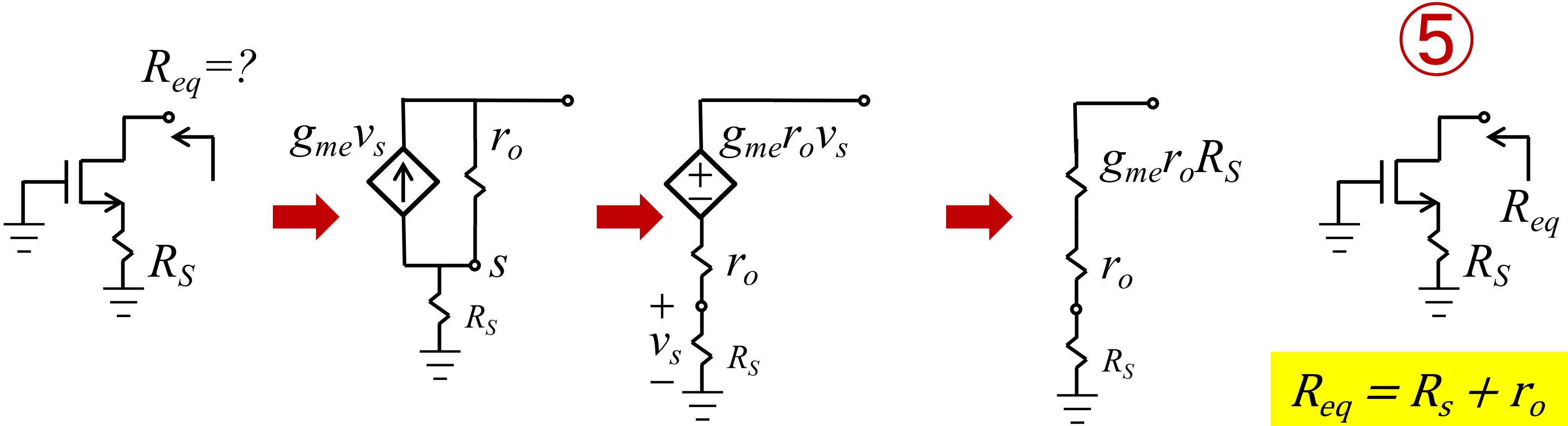
- $g_{me} = g_m + g_{mb} = 1.1\text{-}1.2\ g_m$
- Looking into the source, while gate and drain are grounded, we see a resistor whose value is $R_{eq} = r_o || 1/g_{me} \approx 1/g_{me}$

Diode-Connected Transistor



- A diode-connected transistor is a resistor whose value is $R_{eq} = r_o || 1/g_m \approx 1/g_m$

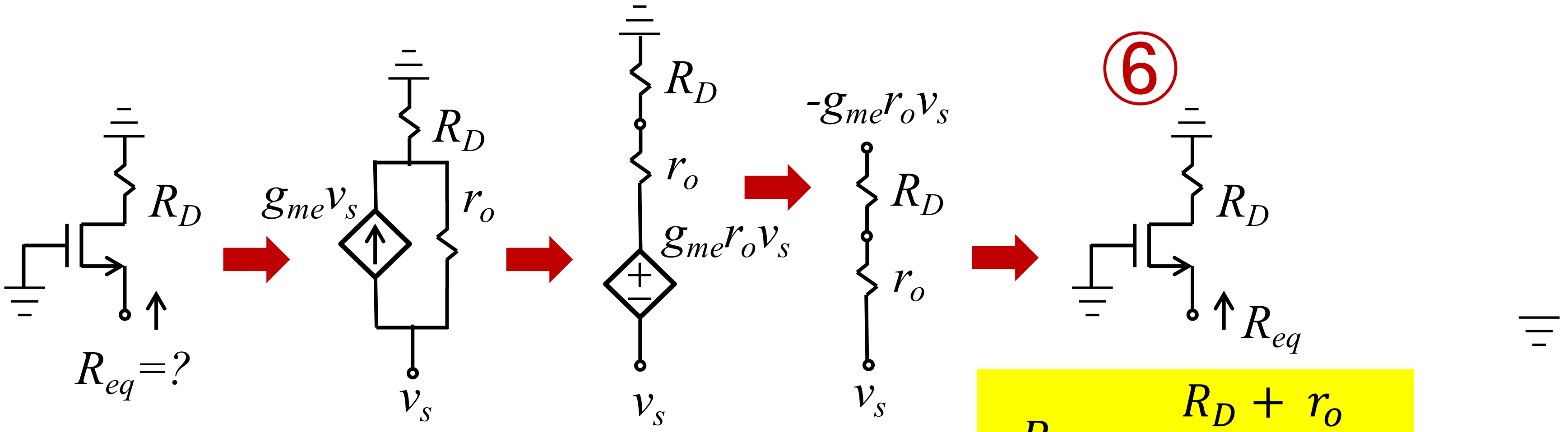
Looking into the Drain with R_S



$$R_{eq} = R_s + r_o + g_{me}\, r_o R_s$$

- Also known as Source Degeneration
- Effective way to increase the output impedance
- Multiplying the source resistance by $g_{me}r_o$

Looking into the Source with R_D



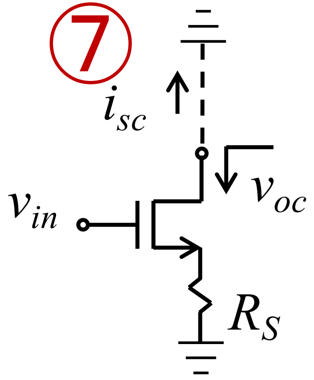
$$R_{eq} = \frac{R_D + r_o}{1 + g_{me}r_o}$$

- If $R_D << r_o$, then $R_{eq} \approx 1/g_{me}$
- R_D is divided by $(1+g_{me}r_o)$ and appears at the source

Looking into the Drain with Input

Two Methods:

- Find the Thevenin Equivalent Circuit:
  - open-circuit voltage v_{oc} in series with R_{eq}
- Find Norton Equivalent Circuit:
  - short-circuit current i_{sc} in parallel with R_{eq}
- Note that $v_{oc}=i_{sc} \times R_{eq}$



In this circuit, easier to find v_{oc} first

→ $v_s=0$, no body effect, $v_{oc}=-g_m r_o v_{in}$

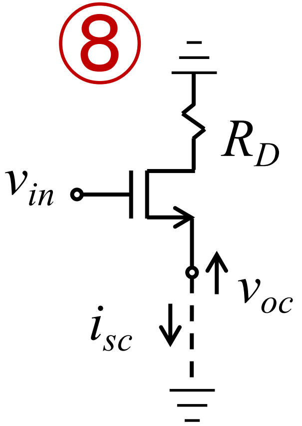
→ $i_{sc}=v_{oc}/R_{eq}$

Norton more intuitive due to high R_{eq}

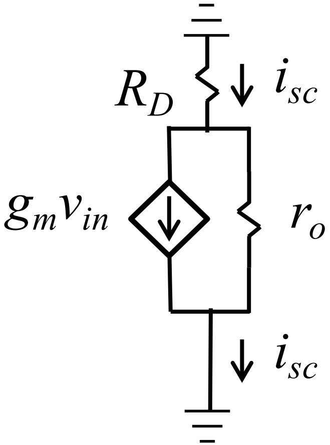
$$v_{oc} = -g_m r_o v_{in}$$
$$i_{sc} = v_{oc}/R_{eq}$$

Looking into the Source with Input

- Find Norton Equivalent Circuit first:
  - short-circuit current i_{sc} in parallel with R_{eq}



- Shorting output to ground → $v_s=0$



→ Use current division

→ $i_{sc} = g_m v_{in} \frac{r_o}{r_o + R_D}$

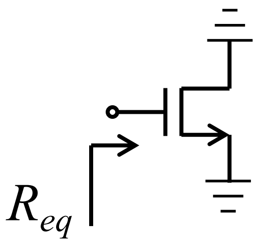
→ Given small R_{eq}, Thevenin is more intuitive

$$i_{sc} = \frac{g_m v_{in} r_o}{r_o + R_D}$$

$$v_{oc} = i_{sc} R_{eq}$$

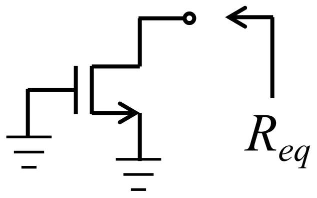
Putting It All Together (1 to 4)

①



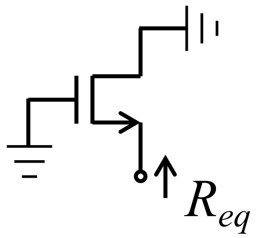
$R_{eq} = \infty$

②



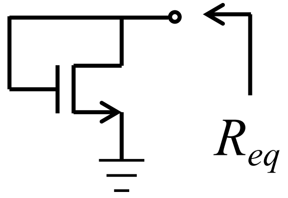
$R_{eq} = r_o$

③



$R_{eq} = r_o \,||\, 1/g_{me}$

④

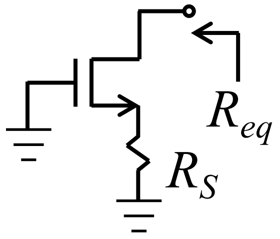


$R_{eq} = r_o \,||\, 1/g_m$

- Looking into one terminal while the other two grounded
- Looking into the drain of a diode-connected transistor
- No additional resistor in the circuit

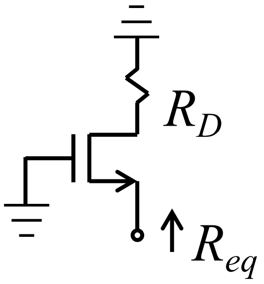
Putting It All Together (5 to 8)

⑤



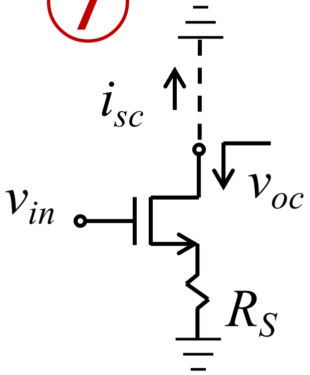
$$R_{eq} = R_s + r_o + g_{me}\, r_o R_s$$

⑥



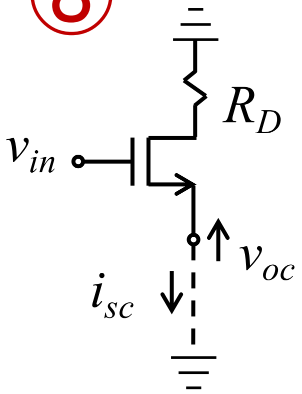
$$R_{eq} = \frac{R_D + r_o}{1 + g_{me} r_o}$$

⑦



$$v_{oc} = -g_m r_o v_{in}$$
$$i_{sc} = v_{oc}/R_{eq}$$

⑧



$$i_{sc} = \frac{g_m v_{in} r_o}{r_o + R_D}$$
$$v_{oc} = i_{sc} R_{eq}$$

- Adding resistors and input to the circuit

Finding Voltage of "any" Node

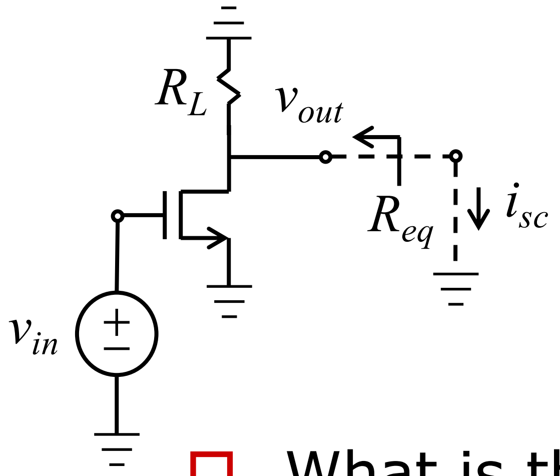
Method 1: Use Norton equivalent circuit

- Short the node to ground; find i_{sc}
- Find R_{eq} (using library elements)
- Multiply the two: $v_{out} = i_{sc} \times R_{eq}$

Method 2: Use Thevenin equivalent circuit

- Open the load; find v_{oc}
- Find R_{eq} (same as in Method 1)
- Use voltage divider rule to find v_{out}

Example: Common-Source Amplifier



Finding v_{out}: Use i_{sc} x R_{eq}:

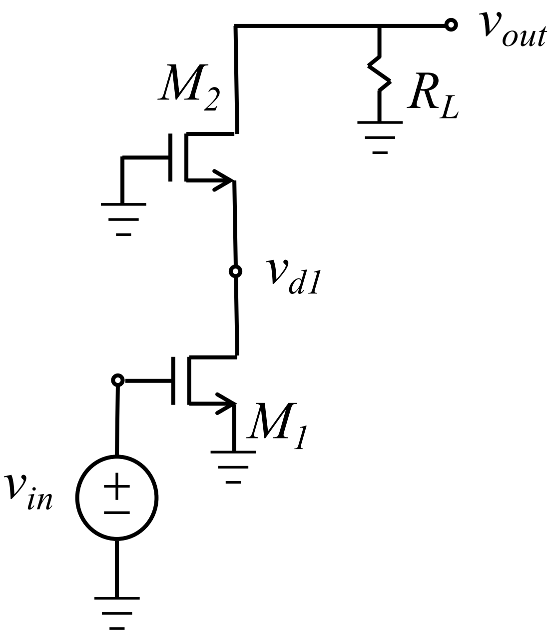
→ $i_{sc} = -g_m v_{in}$

→ $R_{eq} = R_L \,||\, r_o$

→ $v_{out} = -g_m (R_L \,||\, r_o)\, v_{in}$

- What is the intuition here?
  - How to increase the gain? How to increase v_{out} ?
  - Output voltage comes from i_{sc} x R_{eq}
  - → either increase i_{sc} or R_{eq}
- How can we increase these two?

Example: Cascode Circuit

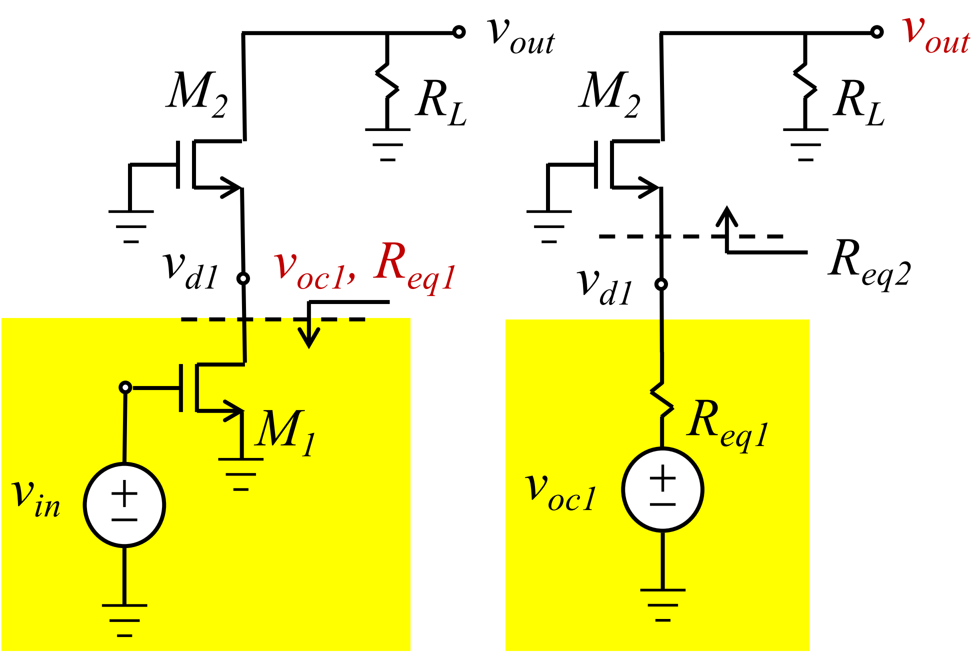


To find the output voltage

- Use i_{sc} x R_{eq}:
- Find out how i_{sc} and R_{eq} change compared to those for CS amplifier
- Gain intuition on why the gain changes from one to the other.
- Also, find intermediate voltages (such as vd1) for added insight

Cascode Circuit: Find any Voltage (1)

Method 1:



Finding v_{d1}: Use Thevenin:

② → $R_{eq1} = r_{o1}$

⑦ → $v_{oc1} = -g_m r_{o1} v_{in}$

⑥ → $R_{eq2} = (r_{o2} + R_L)/(1 + g_{me2}\, r_{o2})$

→ $v_{d1} = -g_m r_{o1} v_{in}\,(r_{o1} \,||\, R_{eq2})$

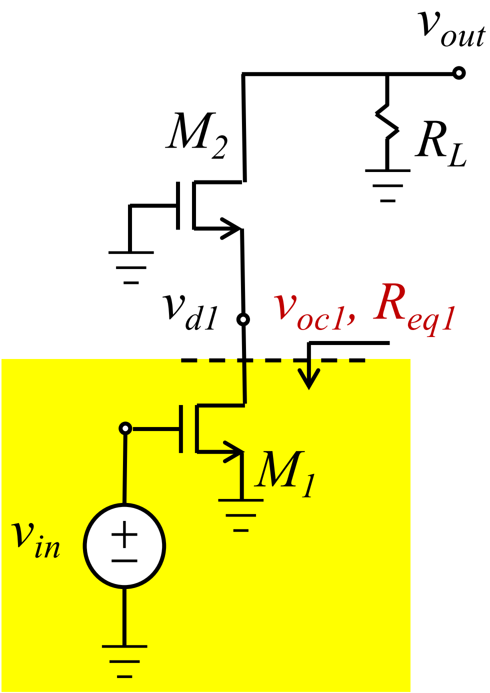
Finding v_{out}: Use R_{eq2} to find the load current first *:*

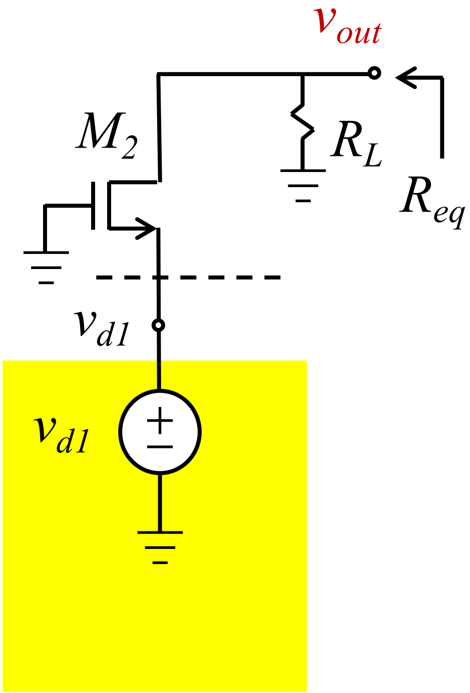
→ $i_L = v_{d1}/R_{eq2}$

→ $v_{out} = i_L R_L = v_{d1}\, R_L / R_{eq2}$

Cascode Circuit: Find any Voltage (2)

Method 2:





Finding v_{d1}:

(as in previous slide)

→ $v_{d1} = -g_m r_{o1} v_{in} (r_{o1} \,||\, R_{eq2})$

Treat v_{d1} as ideal source:

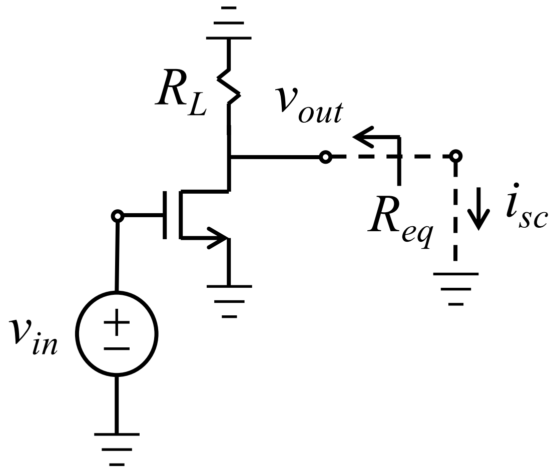
Find i_{sc} and R_{eq} at the output:

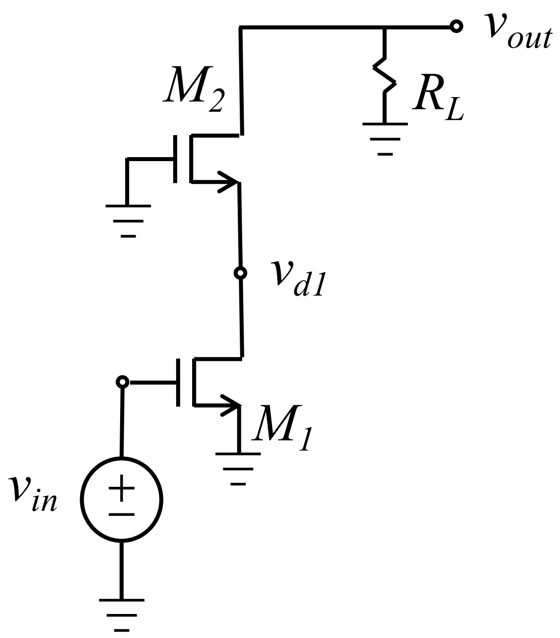
→ $i_{sc} = v_{d1}/(1/g_{me}||r_o)$

→ $R_{eq} = r_o||R_L$ *(not R_{eq} of original cct)*

→ $v_{out} = i_{sc}R_{eq}$

What Is the Intuition here?

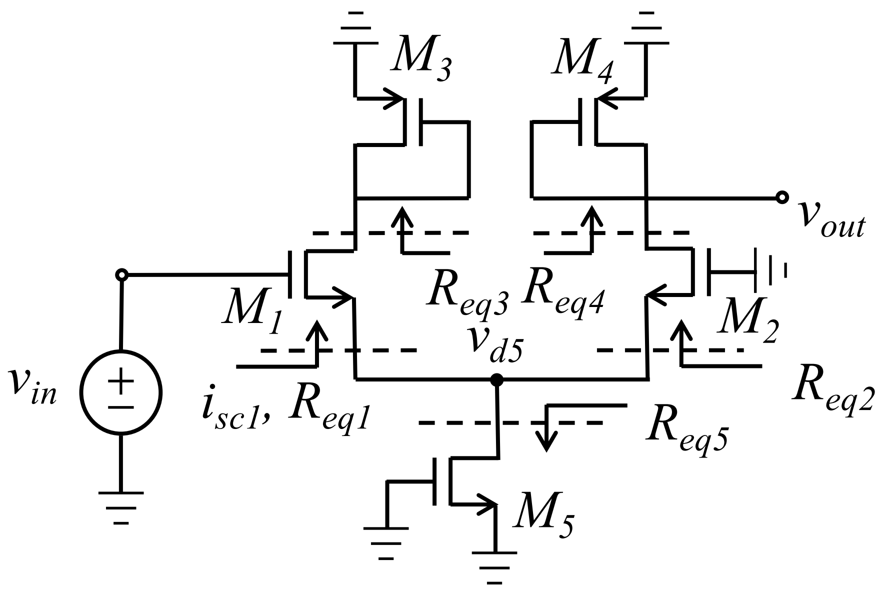




- i_{sc} remained the same
- R_{eq} increased by $g_m r_o$
- Voltage gain increased by $g_m r_o$

Example: Differential Pair (1)

Method 1:



Finding $\boldsymbol{v_{d5}}$: Use the Norton Equivalent at *the source of* M_1:

④ → $R_{eq3} = r_{o3} \,||\, 1/g_{m3}$

⑥ → $R_{eq1} = (R_{eq3}+r_{o1})/(1+g_{me1}r_{o1})$

⑧ → $i_{sc1} = g_{m1}\, v_{in}\, r_{o1}/(r_{o1}+R_{eq3})$

② → $R_{eq5} = r_{o5}$

④ → $R_{eq4} = r_{o4} \,||\, 1/g_{m4}$

⑥ → $R_{eq2} = (R_{eq4}+r_{o2})/(1+g_{me2}r_{o2})$

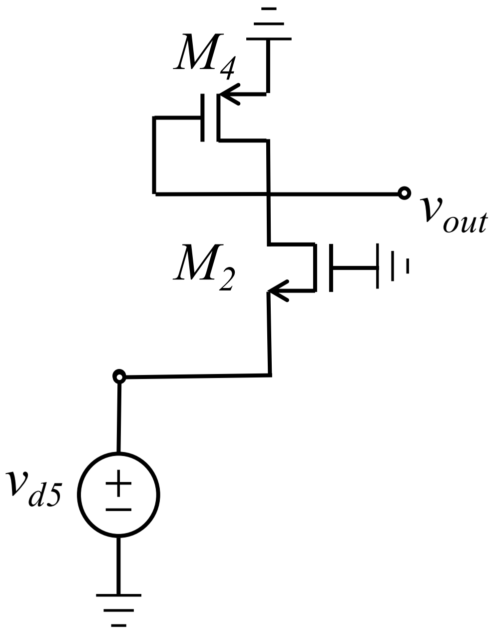
→ $v_{d5} = i_{sc1}\,(R_{eq1} \,||\, R_{eq5} \,||\, R_{eq2})$

Finding $\boldsymbol{v_{out}}$:

→ $v_{out} = (v_{d5}/R_{eq2})\, R_{eq4}$

Example: Differential Pair (2)

Method 2:

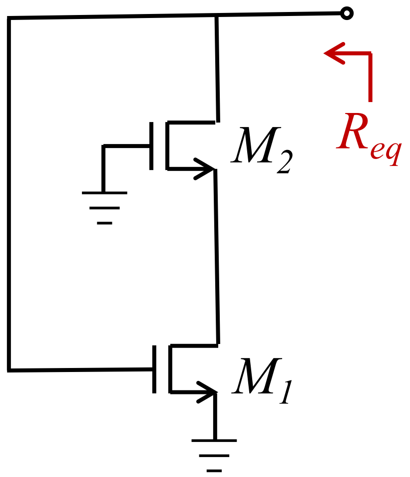


Treat v_{d5} as ideal source:
(from previous slide)
→ $v_{d5} = i_{sc1}\,(R_{eq1}\,||\,R_{eq5}\,||\,R_{eq2})$

Finding v_{out}:
Find i_{sc} and R_{eq} at the output:

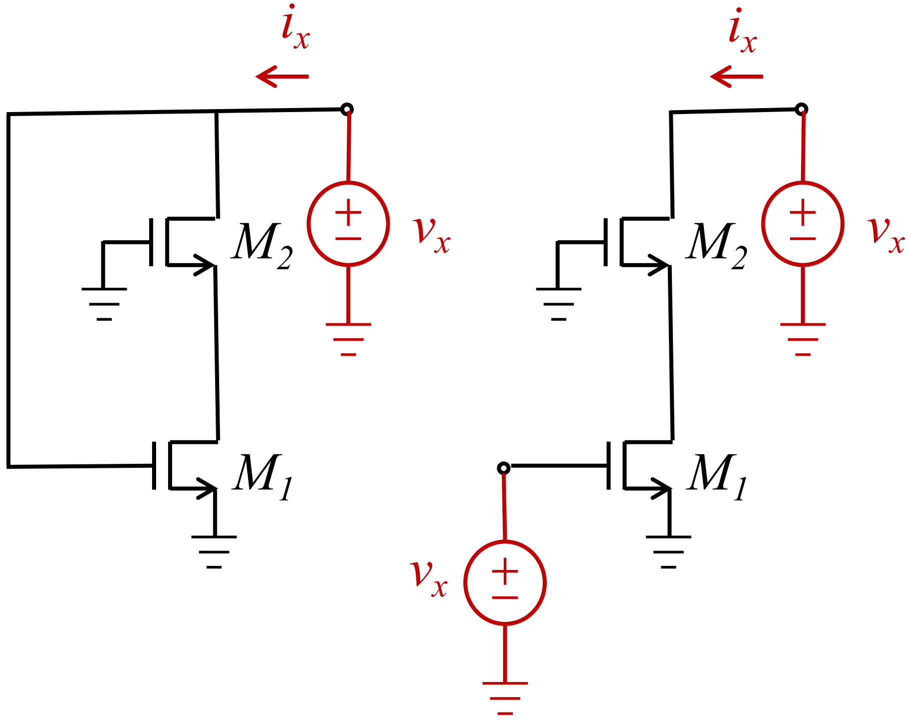
→ $i_{sc} = v_{d5}/(r_{o2}||1/g_{me2})$
→ $R_{eq} = r_{o2}||r_{o4}||1/g_{m4}$
Note: not equal to R_{eq} of original circuit
→ $v_{out} = i_{sc}\,R_{eq}$

Example: Diode-Connected Cascode



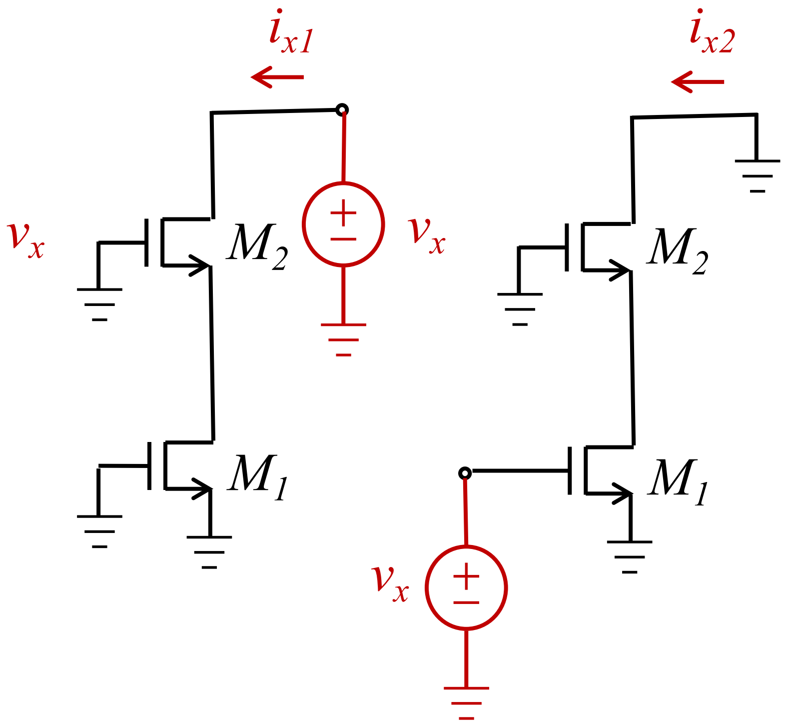
- Not one of the standard library elements
- Apply a voltage, measure current
- Use Superposition to find current
- Ratio of test voltage to current is Req
- Intuition?

Diode-Connected Cascode



- The two circuits are equivalent
- Treat two sources as independent
- Use Superposition

Diode-Connected Cascode



- $i_x = i_{x1} + i_{x2}$
- $R_{eq} = R_{eq1} || R_{eq2}$
- $R_{eq1} \cong g_{me2} r_{o1} r_{o2}$
- $R_{eq2} \cong 1/g_{m1}$
- $R_{eq} \cong 1/g_{m1}$

Summary

- In small-signal, all transistor circuits (assuming saturation region) can be treated as LTI circuit
- Transistor circuits without a signal source are resistors
  - This is for low-frequencies only
  - At high-frequencies, we should add capacitors
  - See other circuit intuition articles for high frequencies
- Transistor circuits with input signal are represented by:
  - Their Thevenin equivalent circuits, or
  - Their Norton equivalent circuits
- There are different ways to arrive at the solution
- Use the method that adds intuition!

References (1 of 2)

All in Solid-State Circuit Magazine

- [A20] Circuit Intuitions: Transfer Resistor, Winter 2019.
- [A19] Circuit Intuitions: Thevenin and Norton Equivalent Circuits, Part 3, Fall 2018.
- [A18] Circuit Intuitions: Thevenin and Norton Equivalent Circuits, Part 2, Su. 2018.
- [A17] Circuit Intuitions: Thevenin and Norton Equivalent Circuits, Spring 2018.
- [A16] Circuit Intuitions: Random Walk in a Ring, Winter 2018.
- [A15] Circuit Intuitions: Reinventing the Wheel, Fall 2017.
- [A14] Circuit Intuitions: Capacitor as a Resistor, Summer 2017.
- [A13] Circuit Intuitions: Why Sinusoids?, Spring 2017.
- [A12] Circuit Intuitions: A Capacitor Analogy, Part 3, Winter 2017.
- [A11] Circuit Intuitions: A Capacitor Analogy, Part 2, Fall 2016.
- [A10] Circuit Intuitions: A Capacitor Analogy, Part 1, Summer 2016.
- [A9] Circuit Intuitions: Chopper Amplifier, Spring 2016.

References (2 of 2)

All in Solid-State Circuit Magazine

- [A8] Circuit Intuitions: Offset Cancellation, Winter 2016.
- [A7] Circuit Intuitions: Miller's Approximation, Fall 2015.
- [A6] Circuit Intuitions: Miller's Theorem, Summer 2015.
- [A5] Circuit Intuitions: Bandwidth Extension, Spring 2015.
- [A4] Circuit Intuitions: Process Variation and Pelgrom's Law, Winter 2015.
- [A3] Circuit Intuitions: Negative Resistance, Fall 2014.
- [A2] Circuit Intuitions: Source Degeneration, Summer 2014.
- [A1] Circuit Intuitions: Looking into a Node, Spring 2014.
  - See also Correction to Looking Into a Node, Summer 2014.

Acknowledgement

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