## Hybrid PLL Architectures and Implementations

Presenter: Daniel Friedman September 26, 2019

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## Outline

- Introduction and motivation: hybrid architectures
- Initial implementation: proof-of-concept
- Flexible synthesizer: frac-N w/ noise cancellation
- Flexible synthesizer: coarse band elimination
- Flexible synthesizer: flicker noise suppression exploration
- Summary/conclusion

## Motivation: Application challenges

#### Context/relevant applications:

- High speed serial links
- Clock synthesis for mm-wave wireless



**Opportunities for hybrid analog/digital PLLs:** 

- Reduce dependence on special technology elements
- Reduce PLL area
- Introduce new functionality and improve testability

#### <u>Challenges</u>:

• must continue to meet stringent phase noise, tuning range, and robustness requirements set by application



- Requires large, low leakage capacitor (unfriendly to digital process)
- Requires wide dynamic range charge pumps
- Nonlinearity of digital components leads to spurs in the output spectrum
- PLL can slew in response to small phase errors
- Transfer function of all digital PLL is noisedependent

## Hybrid PLL: Core Concept



- Proportional path: pulse width modulated signal as in analog PLL
- Integral path: create control word using DSP as in digital PLL

#### Opportunity to achieve best-of-both worlds properties

- Linear proportional path with simplified charge pump versus analog PLL
- Compact digital integral path with extensive digital domain sensing and actuation enablement

## **Example Hybrid PLL Architecture**



- Proportional path drives a scaled pulsewidth modulated signal into VCO control node
- Integral path drives a digital control word to the IDAC/CDAC path
- Oscillator is current-controlled ring IBM Research - IEEE SSCS DL



Vision: Replace multiple custom synthesizers with a single, flexible, dynamically reconfigurable PLL for wireless and wireline high performance applications IBM Research - IEEE SSCS DL

## Preview: Flexible Synthesizer Features Achieved Using Hybrid PLL Approach

Characteristic	Objectives	Implementation Approach	
Tuning range 2-26GHz	Continuous tuning range to maximum frequency	Wide-range dual linearized transconductance VCO complex	
Loop transfer function control	High programmability over wide range through digital control	Leverage highly digital architecture of dual-path HPLL	
Supported reference rate	Flexible without sacrificing performance	Fractional-N synthesis	
Fractional-N quantization noise	Active noise cancellation scheme that does not limit bandwidth	Proportional & integral path highly digital cancellation architecture	
Area	Less than 1 mm <sup>2</sup> total	Single PLL with wide-range dual VCO complex	
Fast frequency hopping	Fast settling time based on digital memory	Leverage digital memory/parameter replay	
Spurious content	Minimize to enable broader use model	Reduced by utilization of digital/predictive techniques	
Extended range in main loop	Eliminate need for coarse bands in supporting wide tuning range	Enhance and extend digital integral path approach to include full VCO range	

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## High Performance Hybrid PLL

- Objective: demonstrate hybrid PLL in high performance (LC-VCObased) context while maintaining tuning range of analog implementation
- <u>Approach</u>
  - Proportional path varactor driven by simplified PWM path
  - Integral path controls fixed and  $\Sigma\Delta$  modulated switched capacitance in D/VCO (implicit DAC)
  - Coarse band architecture to enable wide tuning range without sacrificing phase noise performance
- Key challenges
  - Integral path design
  - D/VCO integration



- PLL utilizes a mixture of analog (proportional path) and digital (integral path) techniques
- Analog component simplified  $\rightarrow$  easy to implement in low voltage CMOS

Ferriss et al, VLSI 2013 IBM Research - IEEE SSCS DL



- Requires phase detector with analog (linear) and digital (bang-bang) outputs
- Do we need two phase detectors?



## Hybrid PLL Approach: Single PFD with two sets of outputs



- Offset of input latches is common to both paths
- Digital discriminator takes time to resolve similar to delay path in tri-state PFD

## Analog proportional path



# Dual path architecture enables proportional path simplification



- Proportional/Integral control path operates over large voltage range → requires charge pump to operate across wide voltage range
- Proportional path stays close to common-mode (once PLL is locked) → relaxes charge pump performance requirements



- Switched resistor scheme has lower flicker noise than conventional CP
- Common mode control greatly simplified

## PLL integral path control



### Integral path overview



- Digital accumulator step size, K<sub>1</sub>, programmable using serial interface
- VCO frequency control split into two paths
  - A 12-bit digitally switched capacitor bank
  - A  $\Sigma\Delta$ -controlled analog varactor

#### Composite hybrid PLL control path Analog Proportional Path



## VCO conceptual diagram



- Control of VCO comprised of a mixture of digitally switched capacitors and varactors
- Implementation strategy for integral path digital cap bank: *redundant numbering* IBM Research IEEE SSCS DL
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## The VCO capacitor bank: options



- Very large number of control lines
- Complex interconnect inductance, difficult to extract and simulate



- Stringent matching requirements
- Major code transition problem
   01111 → 10000

#### Is there a better way?

## Chosen approach: redundant numbering



- Fewer control lines than thermometer weighted
- Matching requirements easier than binary
- Many capacitor configurations can result in the same total capacitance: scheme is redundant
- Major code transitions can be avoided by utilizing redundancy

## Integral path digital control options

Coding Scheme	Redundant	Thermo	Binary
Steps	78	-meter 78	64-128
Control wires	12	78	6-7
Matching requirements	moderate	trivial	extreme
Simultaneous switching events per inc/dec	2	1	<b>Up to 7</b> 1000000→0111111

Redundant numbering based capacitor bank

- $\rightarrow$  Fewer switching events than binary
- $\rightarrow$  Fewer control lines than thermometer

#### Switching sequence example Example Control Sequence



We can increment or decrement with at most 2 capacitor changes → mitigates major code transition problem

... But transition problem remains—better solutions described later in talk
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#### Demonstration of independent control of proportional and integral path gains Sweeping Proportional Path Gain **Sweeping Integral Path Gain** -60 -60 VCO @ 28GHz VCO @ 28GHz -70 -70 -80 -80 -90 -100 -100 dBc/Hz -110 -110 Increasing Increasing -120 -120 **Proportional Path Integral Path** Gain Gain -130 -130 -140 Half Rate Freq:14.000GHz, Reference:194.4MHz, -140 Half Rate Freq:14.000GHz, Reference:194.4MHz, Noise@10MHz: -113.4dBc,-114.7dBc,-107.9dBc, Noise@10MHz: -113.4dBc,-113.0dBc,-113.8dBc, -150-Noise@1MHz: -90.5dBc,-84.7dBc,-98.4dBc -150-Noise@1MHz: -90.5dBc,-94.8dBc,-90.2dBc. 100kHz 1MHz 10MHz 100MHz 10kHz 1MHz 10MHz 100MHz 10kHz 100kHz Frequency Offset (Hz) Frequency Offset (Hz)

Hybrid PLL responds to changes in proportional/integral path gains in a similar fashion as a dual path analog PLL

## Measured effects of RC filter on $\Sigma\Delta$ output



### Conclusion: Initial High Performance Hybrid PLL Implementation

#### Key results

- A 28GHz, low noise hybrid PLL drawing 31mW from 1V demonstrated in 32nm CMOS
- Compact, digital friendly implementation comparable to that of digital PLL achieved
- Linear phase response and spurious spectral content similar to that of all analog PLL achieved

#### **Challenges**

- Integral path redundant numbering implementation still problematic
- Does not really exploit opportunities presented by hybrid architecture

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## Fractional-N Hybrid PLL

- Objective: demonstrate frac-N synthesizer with integrated noise cancellation compatible with hybrid PLL architecture, and demonstrate new features enabled by the architecture
- Approach
  - Introduce proportional path digital-to-time conversion infrastructure
  - Introduce integral path scale/ignore infrastructure
  - Explore use of replay to support fast hopping
- Key challenges
  - Introduction of new elements
  - Support background calibration of digital-to-time converter

Hybrid PLL Architecture: Frac-N



## Classic $\Delta\Sigma$ Problem



- Divider's  $\Delta\Sigma$  dithers PLL's feedback (FB) clock
- $\Delta\Sigma$  noise contributes to PLL's phase noise
## **Time-based Cancellation Concept**



- Digital-to-time based delay scheme used on analog path
  - Utilizes loop filter's low pass response to remove delay path quantization noise → do not need fine quantization step. IBM Research - IEEE SSCS DL

## **Delay Controller Architecture**



- (1) background calibrates DTC gain (2) shapes  $\Delta T$  quantization error
- (3) scrambles mismatch error

## **Delay Controller Operation Simulation**





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## **Integral Path Prediction Circuit**



- Integral path operates from single BB phase detector
  - Works in parallel with the analog proportional path
- Ignores BB results that match prediction



### Improving the Integral Path: Introduction to Varactor Folding

Objective: address disturbance that occurs in previous implementation's redundant numbering control scheme during digital switching



Similar concept used in P. Wang, et al., JSSC 2009 IBM Research - IEEE SSCS DL

### **VCO Linearization Challenge**



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### Linearization in the Varactor Folding Design



Concept: Introduce two parallel paths with offset control vectors that are combined in the D/VCO; the resulting transfer characteristic is a combination of the offset banks yielding linearization IBM Research - IEEE SSCS DL 45

### Varactor Folding and Linearization in the Full PLL Context



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### **Die Photo with Layout**



**Dual D/VCO Tuning Characteristic** 



## Measured Phase Noise and Output Spectrum



M. Ferriss et al, ISSCC15

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## **Demonstration of DTC Background Cal**



Background calibration engine finds the optimum setting M. Ferriss et al, ISSCC15 IBM Research - IEEE SSCS DL

### Trading off High Offset (>1MHz) Noise with Low Offset Noise: PLL Enables Flexible Bandwidth Control



Noise cancellation scheme demonstrated independent of bandwidth (<u>BW range does</u> <u>not affect dynamic range requirements on cancellation path</u>)

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### Fast Hopping with Hybrid PLL: Infrastructure



- Memory space to hold digital state information associated with a second frequency
- Fast switching state machine to support transition between active and stored frequency state words
- Within state machine, configurable delays to enable enhanced management of frequency transition

### Fast Hopping with Hybrid PLL: Use Model



- Two frequency case (simplest, illustrated on next chart):
  - Initialization: Lock to f1, store PLL state, then lock to f2
  - Hopping: on each transition, load stored state for new frequency and store current state for future use
- General case
  - Initialization: Lock to f1, . . . , fn, store PLL states in external memory using serial interface
  - Hopping: (1) load future target state from serial interface prior to hop request; (2) execute hop as in 2frequency case; (3) store updated state information for last frequency used in external memory using serial interface



previous charts (blue)

## Conclusion: Frac-N, Fast-Hopping Hybrid PLL Implementation

#### Key results

- A 13.1-28 GHz, frac-N, low noise hybrid PLL drawing 31mW from 1V demonstrated in 32nm CMOS
- Design features analog proportional path, and (mostly) digital  $\Delta\Sigma$  cancellation and integral paths
- Supports reference frequencies from 15-400MHz and includes infrastructure for fast hopping

### Challenges and opportunities

- Further improve integral path
- Can we do something about banded nature of design?

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# Fractional-N Hybrid PLL with Coarse Band Elimination

- Objective: retain benefits and capabilities of hybrid frac-N design, and remove need for VCO frequency banding without sacrificing noise performance
- <u>Approach</u>
  - Extend varactor folding concept to include coarse band capacitance
  - Enhance integral path design to improve performance
- Key challenges
  - Switched capacitor element choice
  - Introduction of enhanced control scheme

## Motivation

- Objective: Remove coarse bands of CMOS oscillators
- Impact: extends PLL lock range to entire VCO tuning range, allowing, for example, improvement in lock maintenance in wireline applications and enabling fast wide-range chirp generation for wireless/radar Typical VCO



### Varactor Folding in the Full PLL Context: Reminder



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### **Coarse Band Elimination in VCO Design Context**



## Digital control of a varactor



Trade off between  $\Sigma\Delta$  noise and gain/tuning range IBM Research - IEEE SSCS DL

## Extending the range: varactor folding (review)



## Comparing typical tuning mechanisms





Discrete tuning: MOM + Switch



Can we use MOM+switch for continuous tuning?

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## Varactors vs MOM+switch (Cap)



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## Varactors vs MOM+switch (Gain)



## Varactors vs MOM+switch (Q)



# **Q** degradation



# One band



 $Va_1$ 

.....

# Two bands with offset



# Two bands and two thresholds



# Varactor gain versus frequency



## VCO: Coarse Band Elimination



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# **PLL architecture**



M. Ferriss et al, ISSCC16

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# 32nm SOI CMOS prototype



# Measured tuning range and gain





**Measured transient frequency** 



An 80GHz oscilloscope directly samples PLLs output.

The 8GHz chirp was measured in segments due to scope memory limitations.

M. Ferriss et al, ISSCC16 IBM Research - IEEE SSCS DL



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# **Conclusion:** Frac-N Hybrid PLL with Coarse Band Elimination

Key results

- Can support wide-range designs within integral path loop
- Linearization enhancement demonstrated
- Features of previous versions maintained

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- Flexible synthesizer: initial thoughts on flicker noise suppression
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### Hybrid PLL and upconversion noise suppression

Recent Advances in CMOS oscillator flicker noise suppression

Strategy: Add phase shift to VCO loop [Pepe, et al., JSSC 2013]

 $\checkmark$  Simple implementation.

× Magnitude of phase shift depends on absolute component values.

Strategy: Add tank resonance modes at second / third harmonics. [Shahmohammadi, et al., JSSC 2016]

✓ Dramatic improvement in phase noise.

× Depends on component matching / difficult to precisely model impedances at very high harmonic frequencies

Opportunity: suppression of up-conversion mechanisms in a closed loop

An optimizing scheme is proposed that:

- Suppresses VCO upconverted noise
- Does not rely on absolute values of components
- Does not require accurate control of higher order resonance modes

M. Ferriss et al, RFIC 2018

#### Noise and bias up-conversion gain



Same up-conversion mechanism for V<sub>cm</sub> and V<sub>bias</sub> Minimizing bias-to-frequency gain minimizes noise up conversion



Hybrid PLL's bias optimizing state machine Monitors digital control word for frequency Actuates on bias point of transconducting transistors in VCO

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#### Behavioral sim shows the optimizer converging



Disturbance is added to bias voltage and detected on oscillator control word As bias approaches optimum, frequency disturbance disappears

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Fast frequency hopping	Fast settling time based on digital memory	Leverage digital memory/parameter replay
Spurious content	Minimize to enable broader use model	Reduced by utilization of digital/predictive techniques
Extended range in main loop	Eliminate need for coarse bands in supporting wide tuning range	Enhance and extend digital integral path approach to include full VCO range
Flicker noise suppression	Reduce low offset noise in CMOS PLLs	Closed-loop algorithm acting on noise proxy; <i>more research required!</i>

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