Enhancing Image Fidelity through Spatio-Spectral Design for Color Image Acquisition, Reconstruction, and Display

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Joint work with Xiao-Li Meng (Harvard Statistics) and Patrick Wolfe (Harvard SEAS)

Outline

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Image Processing

natural scene statistics

digital camera & hardware

signal & image processing

display device, human vision

Image Processing

natural scene statistics

data generating model

digital camera & hardware

discretization, noise

signal & image processing

analysis, estimation, processing

display device, human vision

subjective analysis

Image Processing

natural scene statistics

digital camera & hardware

DATA LOST HERE!!

⇓

impose limits on DSP

signal & image processing

display device, human vision

DATA LOST HERE!!

⇓

impose limits on vision

Avenues for Improved Color Imaging

- Color Image Acquisition
  - quantitative analysis of the information loss
  - fundamental limitations to DSP imposed by current hardware
  - new hardware designs that minimize these losses and limitations
  - new hardware designs that enable fast algorithms
- Color Image Display
  - quantitative analysis of the visual information loss
  - fundamental limitations to vision imposed by current hardware
  - new hardware designs that minimize these losses and limitations
- Spatio-Spectro Sampling
  - the loss of data comes from hardware noise and from representing the image signal with discrete samples of pixels and colors.
  - we argue that there exist logical tradeoffs between spatial and spectral information.

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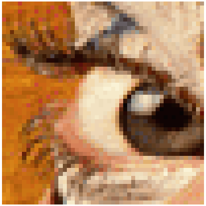
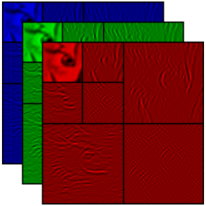
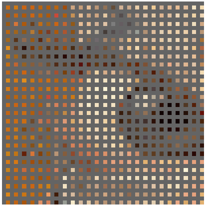
Wavelet Transform with Missing Data?



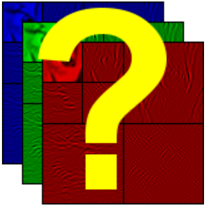
$\boldsymbol{f}$



$\boldsymbol{d} = \boldsymbol{W}\boldsymbol{f}$

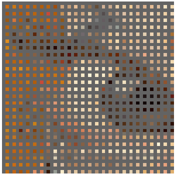


$\boldsymbol{f}_{\text{obs}}$

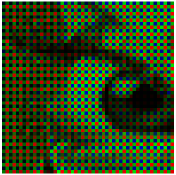


$\boldsymbol{d} = ?$

Types of Estimation Problems



$\boldsymbol{y}_{\mathrm{obs}} = \boldsymbol{f}_{\mathrm{obs}}$



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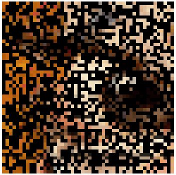

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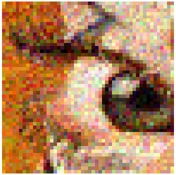


$\boldsymbol{y}_{\mathrm{obs}} = \boldsymbol{f} + \boldsymbol{e}$

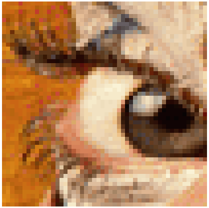
observed data

$\hat{\boldsymbol{f}} = E[\boldsymbol{f} | \boldsymbol{y}_{\mathrm{obs}}, \theta]$

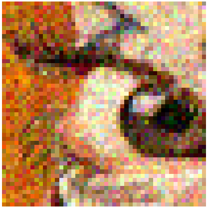
our estimate

where θ is an estimate of hyper-parameter and nuisance parameter.

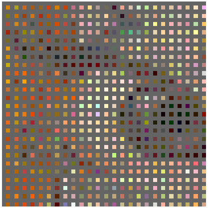
Interpolation + Denoising Problem



$\boldsymbol{f}$
desired image



$\boldsymbol{y}_{\text{com}} = \boldsymbol{f} + \boldsymbol{e}$
noisy image



$\boldsymbol{y}_{\text{obs}} = \boldsymbol{f}_{\text{obs}} + \boldsymbol{e}_{\text{obs}}$
observed data

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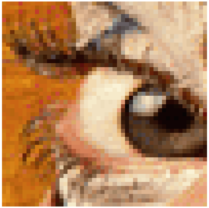
our estimate

- Attempt to preserve sharpness in image also amplifies noise.
- Noise patterns form false edge structures.
- Interpolation adds structure to noise.
- Denoising before interpolation results in blurry output images.

Interpolation + Denoising Problem



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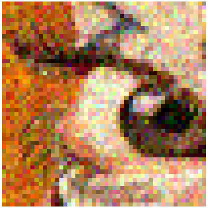

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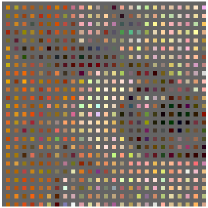

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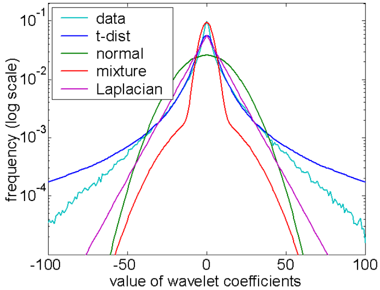
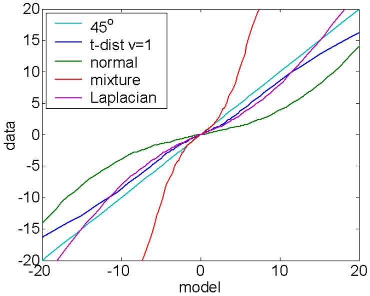
Empirical Partial Bayes Strategy (EPB)

- It is EPB because θ contains both hyper-parameter and nuisance parameter (e.g. Noise Covariance Matrix).
- The *Chicken-and-Egg* problem:
  1. estimate $\boldsymbol{d} = \boldsymbol{W}\boldsymbol{f}$ from θ and $\boldsymbol{y}_{\text{obs}}$ using posterior mean.
  2. estimate θ from $\boldsymbol{d} = \boldsymbol{W}\boldsymbol{f}$ using maximum likelihood.
- Use Expectation-Maximization (EM) algorithm to iterate between these two steps!

Histogram of Wavelet Coefficients
Comparison of Models



log-histogram and models



QQ plot

Wavelet and Sampling Models

Wavelet Model (prior)

$$\boldsymbol{d} = \boldsymbol{W}\boldsymbol{f}$$
$$\boldsymbol{d}_k | q_k \sim \mathcal{N}(\boldsymbol{0}, \boldsymbol{\Sigma}_d / q_k)$$
$$q_k \sim \chi^2_\nu / \nu$$

Noise Model (likelihood)

$$\boldsymbol{y} = \boldsymbol{f} + \boldsymbol{e}$$
$$\boldsymbol{w} = \boldsymbol{W}\boldsymbol{y}$$
$$\boldsymbol{w}_k | \boldsymbol{d}_k \sim \mathcal{N}(\boldsymbol{d}_k, \boldsymbol{\Sigma}_w)$$

Marginal Likelihood Conditioned On q_k

$$\boldsymbol{w}_k | q_k \sim \mathcal{N}(0, \boldsymbol{\Sigma}_d / q_k + \boldsymbol{\Sigma}_w)$$

Sampling

$$\boldsymbol{y} = \begin{bmatrix} \boldsymbol{y}_{\text{obs}} \\ \boldsymbol{y}_{\text{mis}} \end{bmatrix}$$

parameters

$$\boldsymbol{\theta} = \{\boldsymbol{\Sigma}_d, \boldsymbol{\Sigma}_w, \nu\}$$

complete data

$$\boldsymbol{x} = \{\boldsymbol{d}, \boldsymbol{w}, \boldsymbol{q}\}$$
$$(\boldsymbol{q} = \{q_1, \ldots, q_K\})$$

Empirical Bayes & EM Algorithm

E-Step

impute sufficient statistics

updated θ

$E[\boldsymbol{w}_k | \boldsymbol{y}_{\text{obs}}, \theta]$,
$E[\boldsymbol{d}_k | \boldsymbol{y}_{\text{obs}}, \theta]$,
etc...

M-Step

estimate parameters

- impute $\boldsymbol{x}$ from $\boldsymbol{y}_{\text{obs}}$ and θ using posterior mean.
- estimate θ from $\boldsymbol{y}_{\text{obs}}$ (and $\boldsymbol{x}$) using maximum likelihood.

$$\underbrace{\log p(\boldsymbol{y}_{\text{obs}} | \boldsymbol{\theta}^{\text{new}})}_{\text{log likelihood of } \boldsymbol{\theta}^{\text{new}}} \geq \log p(\boldsymbol{y}_{\text{obs}} | \boldsymbol{\theta}^{\text{prev}})$$

EM Algorithm: M-Step

$$\boldsymbol{\theta}^{\text{new}} = \arg\max_{\boldsymbol{\theta}} \underbrace{E\left[\log p(\boldsymbol{x}|\boldsymbol{\theta})|\boldsymbol{y}_{\text{obs}}, \boldsymbol{\theta}^{\text{prev}}\right]}_{Q(\boldsymbol{\theta};\boldsymbol{\theta}^{\text{prev}})}$$

Assuming that ν is known...

$$Q(\boldsymbol{\theta};\boldsymbol{\theta}^{\text{prev}}) = -\frac{1}{2}\sum_k E\Big[\log|\Sigma_w| + (\boldsymbol{w}_k - \boldsymbol{d}_k)^T\Sigma_w^{-1}(\boldsymbol{w}_k - \boldsymbol{d}_k)$$

$$+ \log|\Sigma_d| + q_k \boldsymbol{d}_k^T \Sigma_d^{-1} \boldsymbol{d}_k \Big| \boldsymbol{y}_{\text{obs}}, \boldsymbol{\theta}^{\text{prev}}\Big] + \text{constant}$$

Hyper- and Nuisance Parameters

$$\Sigma_d^{\text{new}} = K^{-1}\sum_k E\left[q_k \boldsymbol{d}_k \boldsymbol{d}_k^T \middle| \boldsymbol{y}_{\text{obs}}, \boldsymbol{\theta}^{\text{prev}}\right]$$

$$\Sigma_w^{\text{new}} = K^{-1}\sum_k E\left[\boldsymbol{w}_k \boldsymbol{w}_k^T - \boldsymbol{w}_k \boldsymbol{d}_k^T - \boldsymbol{d}_k \boldsymbol{w}_k^T + \boldsymbol{d}_k \boldsymbol{d}_k^T \middle| \boldsymbol{y}_{\text{obs}}, \boldsymbol{\theta}^{\text{prev}}\right]$$

EM Algorithm: E-Step

noisy wavelet coefficients

$$\hat{\boldsymbol{w}}_{k|\boldsymbol{q}} = E[\boldsymbol{w}_k|\boldsymbol{q}, \boldsymbol{y}_{\mathrm{obs}}, \theta] = \boldsymbol{W}_k E[\boldsymbol{y}_{\mathrm{com}}|\boldsymbol{q}, \boldsymbol{y}_{\mathrm{obs}}, \theta]$$

$$\hat{\boldsymbol{w}}_k = E\left[\boldsymbol{w}_k \middle| \boldsymbol{y}_{\mathrm{obs}}, \theta\right] = E\left[\hat{\boldsymbol{w}}_{k|\boldsymbol{q}} \middle| \boldsymbol{y}_{\mathrm{obs}}, \theta\right] = \int_0^\infty \hat{\boldsymbol{w}}_{k|\boldsymbol{q}} p(\boldsymbol{q}|\boldsymbol{y}_{\mathrm{obs}}, \nu) d\boldsymbol{q}$$

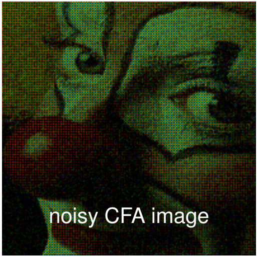
ideal wavelet coefficients

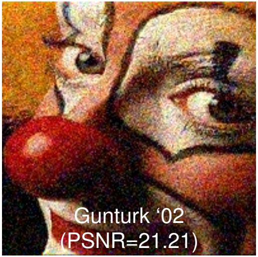
$$\begin{aligned}\hat{\boldsymbol{d}}_{k|\boldsymbol{q}} &= E[\boldsymbol{d}_k|\boldsymbol{q}, \boldsymbol{y}_{\mathrm{obs}}, \theta] = E\left[E[\boldsymbol{d}_k|\boldsymbol{w}, \boldsymbol{q}, \boldsymbol{y}_{\mathrm{obs}}, \theta]\middle|\boldsymbol{q}, \boldsymbol{y}_{\mathrm{obs}}, \theta\right] \\ &= (\Sigma_w^{-1} + \Sigma_d^{-1} q_k)^{-1} \Sigma_w^{-1} \hat{\boldsymbol{w}}_{k|\boldsymbol{q}}\end{aligned}$$

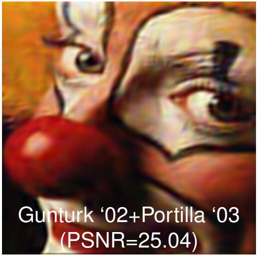
$$\hat{\boldsymbol{d}}_k = E\left[\boldsymbol{d}_k \middle| \boldsymbol{y}_{\mathrm{obs}}, \theta\right] = E\left[\hat{\boldsymbol{d}}_{k|\boldsymbol{q}} \middle| \boldsymbol{y}_{\mathrm{obs}}, \theta\right] = \int_0^\infty \hat{\boldsymbol{d}}_{k|\boldsymbol{q}} p(\boldsymbol{q}|\boldsymbol{y}_{\mathrm{obs}}, \nu) d\boldsymbol{q}$$

Experimental Results: Interpolation + 10% noise

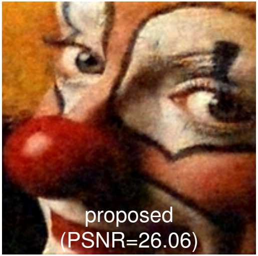

original


noisy CFA image


Gunturk ‘02 (PSNR=21.21)


Gunturk ‘02+Portilla ‘03 (PSNR=25.04)


Hirakawa ’06 (PSNR=25.25)


proposed (PSNR=26.06)

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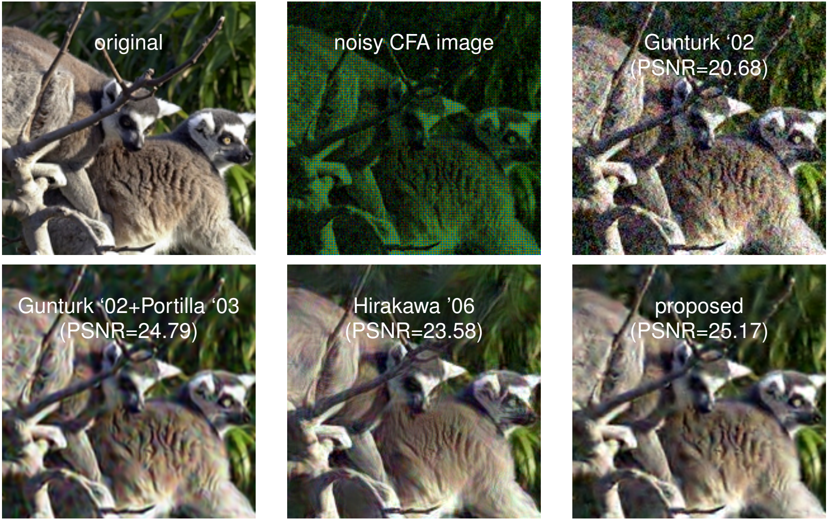

Summary

- Combine sophisticated Wavelet Models with the Missing Data treatment.
- Empirical Partial Bayes using EM Algorithm.
  - Posterior Mean Estimation of Noisy and Clean Wavelet Coefficients.
  - Maximum Likelihood Estimation of Hyper- and Nuisance Parameters.
- Experimental Results:
  - Image quality better than treating denoising and interpolation independently.
  - Visible improvement over previous methods.

Outline

Color Image Acquisition

What are the fundamental limitations to DSP imposed by the acquisition hardware?

- Types of data losses?
  - Spatial resolution (e.g. 6 megapixel)
  - Spectral resolution (e.g. red, green, blue)
  - Quantization (e.g. 24-bit color)
  - Temporal resolution (e.g. frame rate)
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- Given natural scene statistics models, can we quantitatively analyze the information loss?
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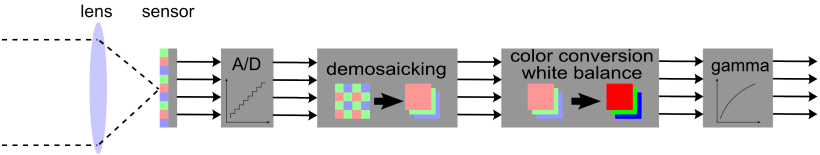
Digital Camera Image Processing Pipeline



Observations:

- CFA represents one of the very first steps in acquisition.
- Subsequent steps process sensor data acquired through CFA.
- We see diminishing return in image quality for additional complexity in algorithm.

Goal:

- Design a new CFA pattern that preserves the integrity of the signal.
- ... should yield better computation-quality trade-offs.
- ... should enhance the performance bounds.

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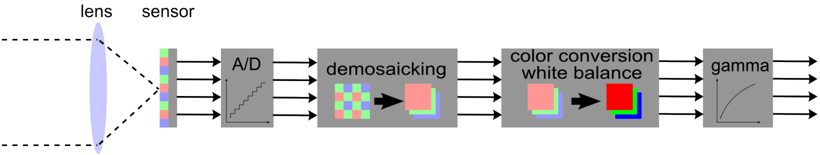

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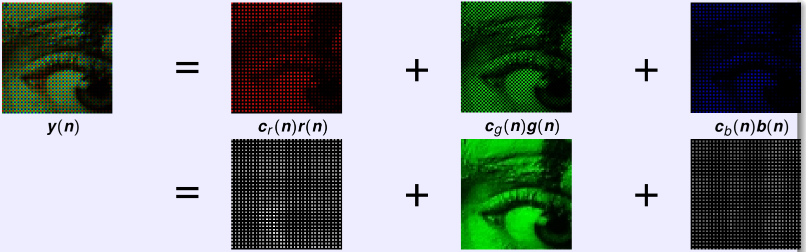
CFA Image & Difference Image



$y(n) = c_r(n)r(n) + c_g(n)g(n) + c_b(n)b(n)$

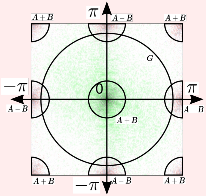
$= c_r(n)(r(n)-g(n)) + g(n) + c_b(n)(b(n)-g(n))$

$$y(n) = c_g(n)g(n) + c_r(n)r(n) + c_b(n)b(n)$$

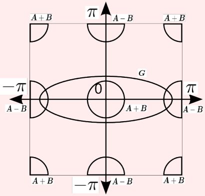
$$= g(n) + \underbrace{c_r(n)\alpha(n)}_{\text{sampling}} + \underbrace{c_b(n)\beta(n)}_{\text{sampling}}$$

CFA Image & Difference Image—Fourier Transform

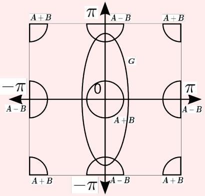
$$y(\boldsymbol{n}) = g(\boldsymbol{n}) + c_r(\boldsymbol{n})\alpha(\boldsymbol{n}) + c_b(\boldsymbol{n})\beta(\boldsymbol{n})$$
$$\mathcal{F}\{y\} = \mathcal{F}\{g\} + \mathcal{F}\{c_r\alpha\} + \mathcal{F}\{c_b\beta\}$$



global Fourier feature



presumed locally vertical Fourier feature



presumed locally horizontal Fourier feature

- No global solution to recovering the image signal
- Need additional assumptions about the signal
- Motivates nonlinear processing driven by local statistics
- Nonlinearity affects noise characterization

CFA Image & Difference Image—Fourier Transform

$$y(\mathbf{n}) = g(\mathbf{n}) + c_r(\mathbf{n})\alpha(\mathbf{n}) + c_b(\mathbf{n})\beta(\mathbf{n})$$
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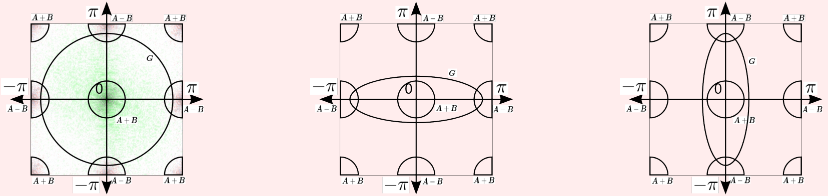

global Fourier feature

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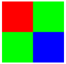
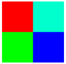
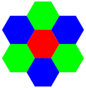
Analysis of Data Loss in Image Acquisition



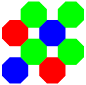
Bayer Pattern[a]



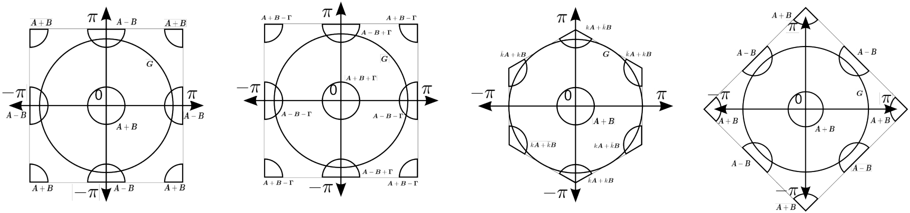
4 Color Pattern[b]



Hexagon[c]



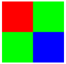
Octagon/Quincunx[d]



Aliased (overlapped) regions mean lost data!

[a] B.E. Bayer, "Color imaging array," U.S. Patent 3 971 065, 1976.
[b] www.sony.net/sonyinfo/news/press_archive/200307/03-029E
[c] R.M. Mersereau, "The processing of hexagonally sampled two-dimensional signals," Proceedings of the IEEE Vol.67, No.6, 1979
[d] home.fujifilm.com/pma2000/sprccd.html

Analysis of Data Loss in Image Acquisition



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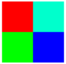

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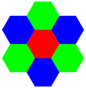
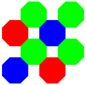


Hexagon[c]



Octagon/Quincunx[d]

Theorem (Hirakawa & Wolfe 2008)

No choice of pure-color CFA (Bayer, Hexagonal, Octagonal, etc.) will admit the maximal spectral radius at baseband.

Proof follows from the theory of (sampling) lattices.

[a] B.E. Bayer, “Color imaging array,” U.S. Patent 3 971 065, 1976.
[b] www.sony.net/sonyinfo/news/press_archive/200307/03-029E
[c] R.M. Mersereau, “The processing of hexagonally sampled two-dimensional signals,” Proceedings of the IEEE Vol.67, No.6, 1979
[d] home.fujifilm.com/pma2000/sprccd.html

AM Radio!

- Amplitude modulation (AM) radio uses modulation of signal $\boldsymbol{x}(\boldsymbol{n})$ by carrier frequency $\boldsymbol{c}(\boldsymbol{n})$ via multiplication in the time domain:

$$\boldsymbol{y}(\boldsymbol{n}) = \boldsymbol{x}(\boldsymbol{n})\boldsymbol{c}(\boldsymbol{n}).$$

- The partitioning in the frequency domain allows transmission of multiply speech/music signals to be carried over the same media.

Color Filter Array Sampling

Sensor Data Model

$$y(n) = c_g(n)g(n) + c_r(n)r(n) + c_b(n)b(n)$$

Simplification

Impose convex combination constraint: $c_g(n) + c_r(n) + c_b(n) = 1$. Then

$$\begin{aligned} y(n) =& (1 - c_r(n) - c_b(n))g(n) + c_r(n)r(n) + c_b(n)b(n) \\ =& g(n) + c_r(n)(r(n) - g(n)) + c_b(n)(b(n) - g(n)) \\ =& g(n) + \underbrace{c_r(n)\alpha(n)}_{\text{amplitude modulation}} + \underbrace{c_b(n)\beta(n)}_{\text{amplitude modulation}} \end{aligned}$$

Color Filter Array Sampling

Sensor Data Model

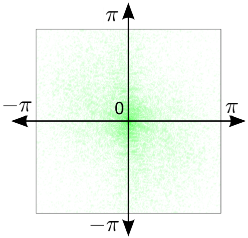
$$\boldsymbol{y}(\boldsymbol{n}) = \boldsymbol{c}_g(\boldsymbol{n})\boldsymbol{g}(\boldsymbol{n}) + \boldsymbol{c}_r(\boldsymbol{n})\boldsymbol{r}(\boldsymbol{n}) + \boldsymbol{c}_b(\boldsymbol{n})\boldsymbol{b}(\boldsymbol{n})$$

Simplification

Impose convex combination constraint: $\boldsymbol{c}_g(\boldsymbol{n}) + \boldsymbol{c}_r(\boldsymbol{n}) + \boldsymbol{c}_b(\boldsymbol{n}) = 1$. Then

$$\begin{aligned}
\boldsymbol{y}(\boldsymbol{n}) =& (1 - \boldsymbol{c}_r(\boldsymbol{n}) - \boldsymbol{c}_b(\boldsymbol{n}))\boldsymbol{g}(\boldsymbol{n}) + \boldsymbol{c}_r(\boldsymbol{n})\boldsymbol{r}(\boldsymbol{n}) + \boldsymbol{c}_b(\boldsymbol{n})\boldsymbol{b}(\boldsymbol{n}) \\
=& \boldsymbol{g}(\boldsymbol{n}) + \boldsymbol{c}_r(\boldsymbol{n})(\boldsymbol{r}(\boldsymbol{n}) - \boldsymbol{g}(\boldsymbol{n})) + \boldsymbol{c}_b(\boldsymbol{n})(\boldsymbol{b}(\boldsymbol{n}) - \boldsymbol{g}(\boldsymbol{n})) \\
=& \boldsymbol{g}(\boldsymbol{n}) + \underbrace{\boldsymbol{c}_r(\boldsymbol{n})\alpha(\boldsymbol{n})}_{\text{amplitude modulation}} + \underbrace{\boldsymbol{c}_b(\boldsymbol{n})\beta(\boldsymbol{n})}_{\text{amplitude modulation}}
\end{aligned}$$

Spatio-Spectral Sampling



It's a Sphere Packing problem!

- Recall spectral support of the green image.
- Green image spectrum does not occupy frequency regions far away from the origin.
- Main Idea: Use $\boldsymbol{c}_r$ and $\boldsymbol{c}_b$ to modulate $\alpha(\boldsymbol{n})$ and $\beta(\boldsymbol{n})$ away from origin!

Design of Color Filter Array

We design $\mathbf{c}_r$ and $\mathbf{c}_b$ in the 2D Fourier domain:

1. Pick carrier frequencies $\{\tau_k \in \mathbb{R}^2 : \|\tau_k\|_\infty = \pi\}$.
2. Pick corresponding weights $\{s_i, t_i \in \mathbb{C}\}$.
3. Set $\mathbf{C}_r(\omega) = s_0 + \sum_k s_k\delta(\omega - \tau_k) + \bar{s}_k\delta(\omega + \tau_k)$.
4. Set $\mathbf{C}_b(\omega) = t_0 + \sum_k t_k\delta(\omega - \tau_k) + \bar{t}_k\delta(\omega + \tau_k)$.
5. Take inverse Fourier transform: $\mathbf{c}_r = \mathcal{F}^{-1}\{\mathbf{C}_r\}$, $\mathbf{c}_b = \mathcal{F}^{-1}\{\mathbf{C}_b\}$.
6. $\mathbf{c}_g = 1 - \mathbf{c}_r - \mathbf{c}_b$.

$$\mathcal{F}\{\mathbf{c}_r \cdot \alpha\}(\omega) = s_0\mathcal{F}\{\alpha\}(\omega) + \sum_k s_k\mathcal{F}\{\alpha\}(\omega - \tau_k) + \bar{s}_k\mathcal{F}\{\alpha\}(\omega + \tau_k)$$

$$\mathcal{F}\{\mathbf{c}_b \cdot \beta\}(\omega) = t_0\mathcal{F}\{\beta\}(\omega) + \sum_k t_k\mathcal{F}\{\beta\}(\omega - \tau_k) + \bar{t}_k\mathcal{F}\{\beta\}(\omega + \tau_k)$$

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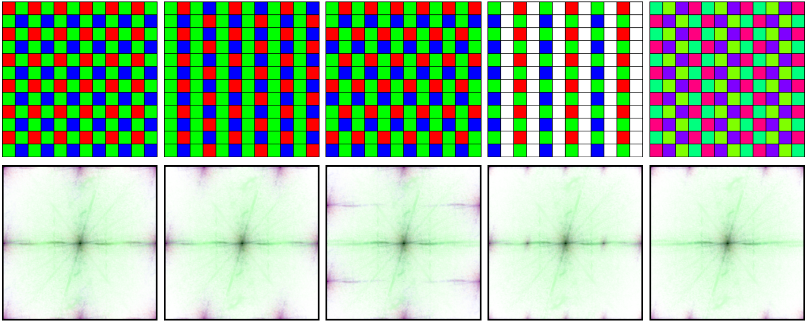
CFA Image & Difference Image—Fourier Transform



Bayer Yamanaka Lukac Kodak Hirakawa

Minimizing Data Loss in Image Acquisition

Disadvantages to traditional CFA design

- aliasing occurs when one signal "contaminates" another signal.
- anti-aliasing reduces resolution.
- "un-doing" aliasing is an ill-posed problem $\Rightarrow$ additional assumption and complexity! (e.g. directionality).

Benefits to spatio-spectral CFA design

- minimize data loss $\Rightarrow$ improved image quality
- not sensitive to directions $\Rightarrow$ completely linear fast reconstruction method
- low-complexity, low-power, low-memory
- improvements for noise and video (panchromatic)

Main idea: with sufficient partitioning in Fourier domain, a very crude filter will suffice for reconstruction.

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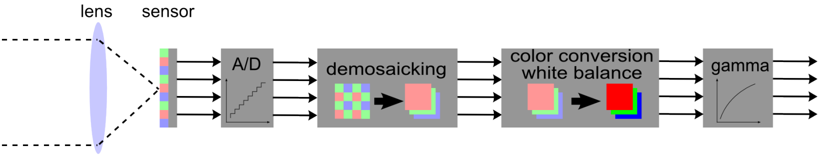
Efficient Linear Demosaicking for Spatio-Spectral CFA



Demosaicking as AM Demodulation

$$\hat{\boldsymbol{x}}(\boldsymbol{y}) = \underbrace{\begin{bmatrix} 1\text{-}c_r(\boldsymbol{n}) & 1 & \text{-}c_b(\boldsymbol{n}) \\ \text{-}c_r(\boldsymbol{n}) & 1 & \text{-}c_b(\boldsymbol{n}) \\ \text{-}c_r(\boldsymbol{n}) & 1 & 1\text{-}c_b(\boldsymbol{n}) \end{bmatrix}}_{\text{pixel-wise operation}} \underbrace{\begin{bmatrix} h * \{\theta_\alpha y\} \\ y \\ h * \{\theta_\beta y\} \end{bmatrix}}_{\text{spatial processing}}$$

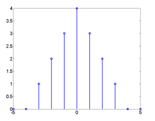
- matrix operator combined with color correction offline $\Rightarrow$ comes for free!
- spatial processing is the only demosaicking cost.

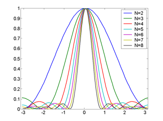
$$h * \{\theta_\alpha y\} = h'(n_2) * \{\theta'_\alpha(n_2) y'(\boldsymbol{n})\}$$
$$h * \{\theta_\beta y\} = h'(n_2) * \{\theta'_\beta(n_2) y'(\boldsymbol{n})\}$$
$$y' = h'(n_1) * \{(\text{-}1)^{n_1} y\}$$

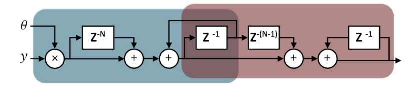
Efficient Linear Demosaicking for Spatio-Spectral CFA

- with sufficient partitioning in Fourier domain, a very crude filter will suffice for reconstruction.
- so, can we use cheap filters? YES!
- we rival state-of-the-art demosaicking with
  - only 10 add operations per full-pixel reconstruction!
  - no nonlinear elements such as *if-then* or *greater-than*.
  - no multiplier (except $\theta \in \{0, \pm 1\}$)

Triangle Filter h'

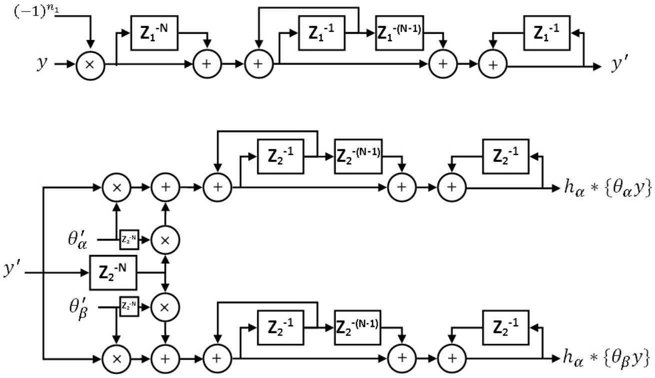






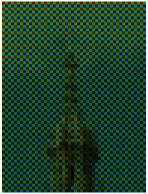
4 adds, $2N+1$ delay lines

Efficient Linear Demosaicking for Spatio-Spectral CFA

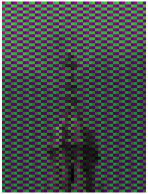


- $\theta'_\alpha, \theta'_\beta \in \{0, \pm 1\} \Rightarrow$ savings with $\theta'_\alpha y' = 0$ and $\theta'_\beta y' = 0$
- Z_1 line buffers, Z_2 registers (ASIC)
- 10 adds, $2N + 1$ line buffers, $3N + 2$ registers

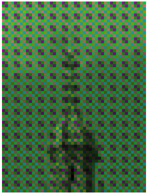
Example of Data Acquisition and Reconstruction



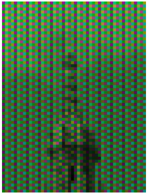
Bayer Pattern[a]



Pattern A



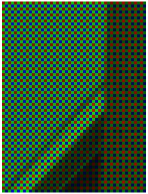
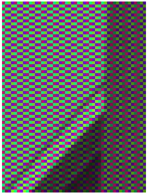
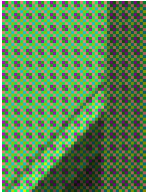
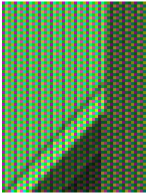
Pattern B



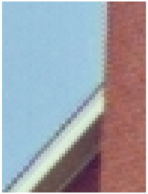
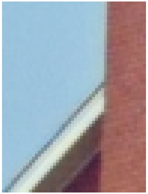
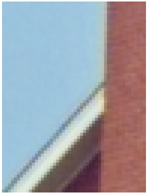
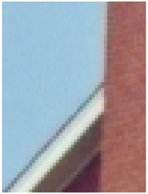
Pattern C

[a] Gunturk *et al*, *"Demosaicking: Color Filter Array Interpolation," IEEE Signal Processing Magazine, Jan. 2005*

Example of Data Acquisition and Reconstruction

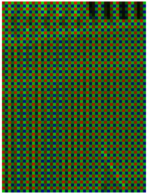





Bayer Pattern[a] | Pattern A | Pattern B | Pattern C

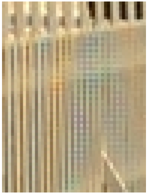





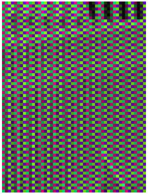
[a] Gunturk *et al*, *"Demosaicking: Color Filter Array Interpolation," IEEE Signal Processing Magazine, Jan. 2005*

Example of Data Acquisition and Reconstruction

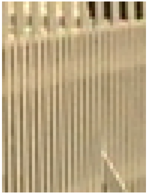


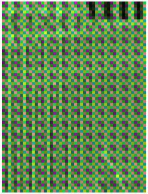
Bayer Pattern[a]



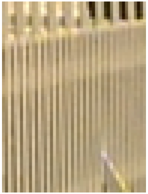


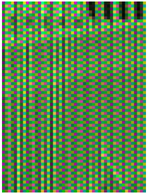
Pattern A



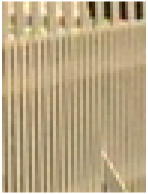


Pattern B





Pattern C



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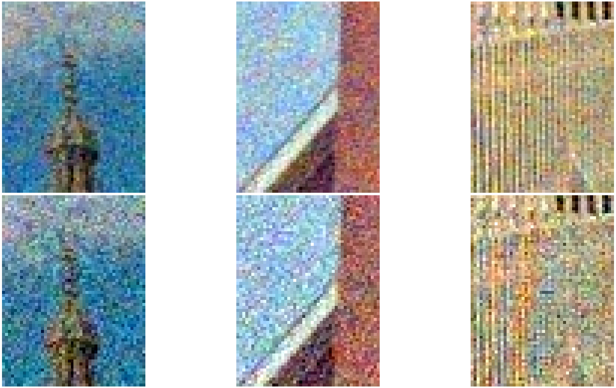
Example of Noisy Sensor

Spatio-Spectral Linear Demosaicking

Bayer Nonlinear Demosaicking



Outline

Motivation

natural scene statistics

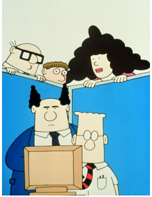
digital camera & hardware

DATA LOST HERE!!

$\Downarrow$

impose limits on DSP

signal & image processing



display device, human vision

DATA LOST HERE!!

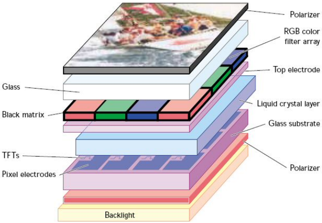
$\Downarrow$

impose limits on vision

What limitations does hardware impose on visual perception?

- Types of data losses? Resolutions in Spatial, Spectral, Quantization, Temporal...
- Given human visual system models and what we know about the signal, can we quantify information loss?
- Can we design a hardware that minimizes information loss?

Color Filter Array & Display Device



LCD (Wandell '99)

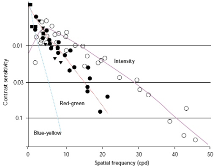
Observations:

- CFA represents one of the very last steps in display device.
- Human visual system processes image data displayed via CFA.

Goal:

- Design a new CFA pattern that preserves the integrity of the signal.
- ... should yield better resolution-quality trade-offs.
- ... should enhance the performance bounds.

Quick Review: Luminance-Chrominance (L-C)

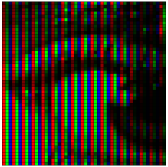


CSF (Wandell '99)

- Visual spatial processing organized as parallel channels (components) in the nervous system.
- luminance ("intensity") & chrominance ("red-green" and "blue-yellow")
- The contrast sensitivity functions (CSF) reveal that the passband structure in vision.

$$\underbrace{\mathcal{W}\{\boldsymbol{x}\}}_{\text{perceived}} = \begin{bmatrix} h_1(\boldsymbol{n}) * y_1(\boldsymbol{n}) \\ h_2(\boldsymbol{n}) * y_2(\boldsymbol{n}) \\ h_3(\boldsymbol{n}) * y_3(\boldsymbol{n}) \end{bmatrix} = \underbrace{\begin{bmatrix} h_1(\boldsymbol{n})* \\ h_2(\boldsymbol{n})* \\ h_3(\boldsymbol{n})* \end{bmatrix}}_{\text{CSF}} \underbrace{\boldsymbol{M} \overbrace{\boldsymbol{x}(\boldsymbol{n})}^{\text{RGB color}}}_{\text{L-C color}}$$

Color Filter Array & Display Stimuli



Stimulus = Projection

$$\underbrace{\mathbf{v}(\mathbf{n}) = \begin{bmatrix} v_r(\mathbf{n}) \\ v_g(\mathbf{n}) \\ v_b(\mathbf{n}) \end{bmatrix}}_{\text{displayed image}} = \underbrace{\begin{bmatrix} c_r(\mathbf{n}) \\ c_g(\mathbf{n}) \\ c_b(\mathbf{n}) \end{bmatrix}}_{\text{CFA}} \underbrace{u(\mathbf{n})}_{\text{stimulus}}$$

$$= \begin{bmatrix} c_r(\mathbf{n}) \\ c_g(\mathbf{n}) \\ c_b(\mathbf{n}) \end{bmatrix} \underbrace{\begin{bmatrix} d_r(\mathbf{n}) & d_g(\mathbf{n}) & d_b(\mathbf{n}) \end{bmatrix}}_{\text{projection matrix}} \underbrace{\begin{bmatrix} x_r(\mathbf{n}) \\ x_g(\mathbf{n}) \\ x_b(\mathbf{n}) \end{bmatrix}}_{\text{color image}}$$

Stimulus $u(\mathbf{n})$ controls the intensity of color $\mathbf{c}(\mathbf{n})$.

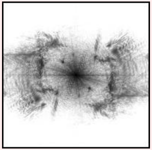
Color Filter Array & Display Stimuli

Stimulus

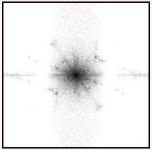
$$u(\boldsymbol{n}) = \underbrace{\begin{bmatrix} d_r(\boldsymbol{n}) & d_g(\boldsymbol{n}) & d_b(\boldsymbol{n}) \end{bmatrix} \boldsymbol{M}^{-1}}_{\boldsymbol{\phi}^T = \text{projection in L-C}} \underbrace{\boldsymbol{M} \begin{bmatrix} x_r(\boldsymbol{n}) \\ x_g(\boldsymbol{n}) \\ x_b(\boldsymbol{n}) \end{bmatrix}}_{\boldsymbol{y} = \text{image in L-C}}$$

$$= \underbrace{\phi_1(\boldsymbol{n}) y_1(\boldsymbol{n})}_{\text{luminance}} + \underbrace{\phi_2(\boldsymbol{n}) y_2(\boldsymbol{n}) + \phi_3(\boldsymbol{n}) y_3(\boldsymbol{n})}_{\text{chrominance}}$$

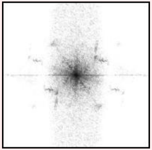
Fourier Transform



luminance $(\mathcal{F} y_1)$



chrominance $(\mathcal{F} y_2)$

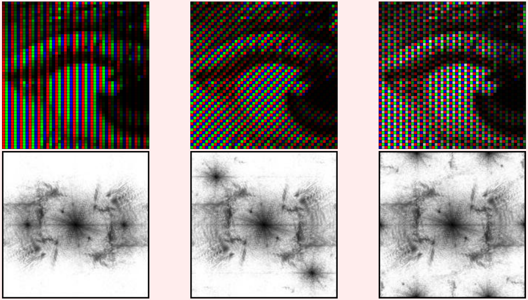


chrominance $(\mathcal{F} y_3)$

Aliasing in Display Stimuli

$$u(\boldsymbol{n}) = \phi_1(\boldsymbol{n})y_1(\boldsymbol{n}) + \phi_2(\boldsymbol{n})y_2(\boldsymbol{n}) + \phi_3(\boldsymbol{n})y_3(\boldsymbol{n})$$

Idea: When ϕ_i is a sinusoid, $\phi_i y_i$ is a modulation and ϕ_i called carrier.



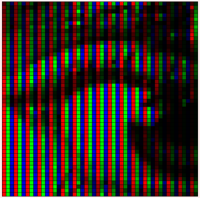
Vertical Diagonal Pollack '06

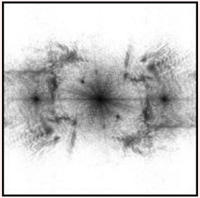
Idea: Spectral overlap is called aliasing $\Rightarrow$ lost information!

Oversampling To Overcome Aliasing

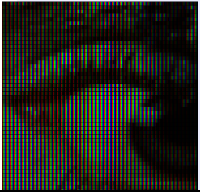
Aliasing is eliminated with oversampling, but it increases pixel count.

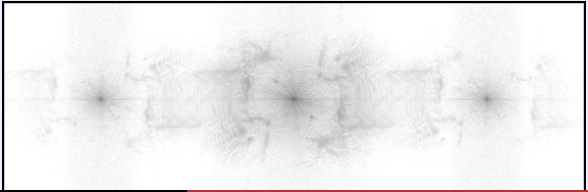
Vertical (2500 subpixels)





Oversampled by 3 (7500 subpixels)





Quick Review: Amplitude Modulation

$$\underbrace{u(\boldsymbol{n})}_{\text{coded signal}} = \sum_i \underbrace{\phi_i(\boldsymbol{n})}_{\text{carrier frequency}} \underbrace{y_i(\boldsymbol{n})}_{\text{signal}}$$

$$\underbrace{\hat{y}_i(\boldsymbol{n})}_{\text{reconstruction}} = \underbrace{h_i(\boldsymbol{n}) * \{\psi_i(\boldsymbol{n})u(\boldsymbol{n})\}}_{\text{de-modulation}}$$

The partitioning in the frequency domain allows transmission of multiply speech/music signals to be carried over the same media.

Motivation

Ultimately, want $\mathcal{W}\{\boldsymbol{x}\} \approx \mathcal{W}\{\boldsymbol{v}\}$, where observed image is...

$$\mathcal{W}\{\boldsymbol{v}\} = \begin{bmatrix} h_1 * \\ h_2 * \\ h_3 * \end{bmatrix} \underbrace{\boldsymbol{M} \begin{bmatrix} c_r(\boldsymbol{n}) \\ c_g(\boldsymbol{n}) \\ c_b(\boldsymbol{n}) \end{bmatrix}}_{\psi(\boldsymbol{n})} u(\boldsymbol{n}) = \underbrace{\begin{bmatrix} h_1 * \\ h_2 * \\ h_3 * \end{bmatrix} \psi(\boldsymbol{n})}_{\text{de-modulation}} \underbrace{\overbrace{\phi(\boldsymbol{n})^T}^{\boldsymbol{d}(\boldsymbol{n})\boldsymbol{M}^{-1}} \boldsymbol{y}(\boldsymbol{n})}_{\text{modulation}}$$

- This is Amplitude Modulation and De-Modulation!!!
- Idea 1: Design ϕ_i and ψ_i such that $\mathcal{W}\{\boldsymbol{v}\}$ is an amplitude demodulation. We “borrow” the convolution filters from the observer’s eye.
- Idea 2: Parameterize ϕ_i and ψ_i in Fourier domain explicitly such that we achieve partitioning (i.e. no aliasing).

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Fourier Domain CFA Design

We design the carrier frequencies $\phi_i(\boldsymbol{n}) = k_i\psi_i(\boldsymbol{n})$ such that:

- $h_i(\boldsymbol{n}) * \{\psi_i(\boldsymbol{n})\phi_j(\boldsymbol{n})\} = 1$ when $i = j$.
- $h_i(\boldsymbol{n}) * \{\psi_i(\boldsymbol{n})\phi_j(\boldsymbol{n})\} = 0$ when $i \neq j$.
- $u = \phi_1 y_1 + \phi_2 y_2 + \phi_3 y_3$ is alias free.
- $u \geq 0$.

We choose $\phi(\boldsymbol{n})$ in the 2D Fourier domain:

1. Set $\phi_1(\boldsymbol{n}) = 1$.
2. Pick carrier frequencies $\{\tau_k \in \mathbb{R}^2 : \|\tau_k\}_\infty = \pi\}$.
3. Pick corresponding weights $\{s_k, t_k \in \mathbb{C}^2\}$.
4. Set $\Phi_2(\omega) = \sum_k s_k\delta(\omega - \tau_k) + \bar{s}_k\delta(\omega + \tau_k)$.
5. Set $\Phi_3(\omega) = \sum_k t_k\delta(\omega - \tau_k) + \bar{t}_k\delta(\omega + \tau_k)$.
6. Take inverse FFT: $\phi_2 = \mathcal{F}^{-1}\{\Phi_2\}$, $\phi_3 = \mathcal{F}^{-1}\{\Phi_3\}$.

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Fourier Analysis of Stimuli

Fourier Transform of Stimuli

$$
\begin{aligned}
\mathcal{F}\{u\} =& \mathcal{F}\{\phi^T \boldsymbol{y}\} \\
=& \mathcal{F}\{y_1\}(\omega) + \sum_k \{s_i \mathcal{F}\{y_2\} + t_i \mathcal{F}\{y_3\}\}(\omega - \tau_k) \\
&+ \sum_k \{\bar{s}_i \mathcal{F}\{y_2\} + \bar{t}_i \mathcal{F}\{y_3\}\}(\omega - \tau_k)
\end{aligned}
$$

By choosing the carriers τ_k away from the baseband (high frequency),

- The chances of $\phi_2 y_2$ and $\phi_3 y_3$ overlapping with $\phi_1 y_1$ is minimized.
- $\phi_1\psi_2, \phi_1\psi_3, \phi_2\psi_1, \phi_3\psi_1$ fall outside of the passband for h_1, h_1, h_2, h_3, respectively.

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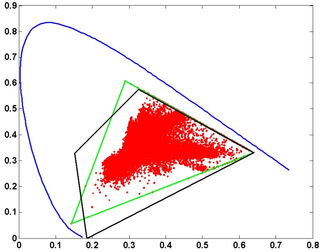
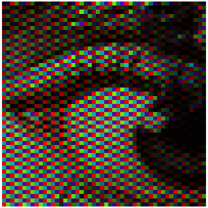
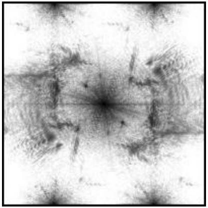
An Example of New CFA



x-y chromaticity plot



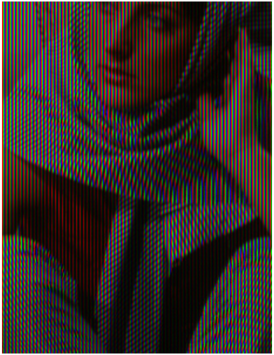
proposed CFA



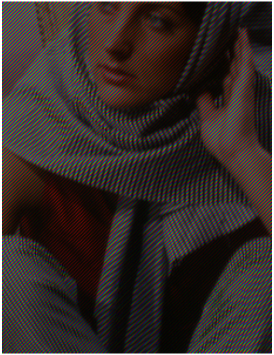
stimulus

- black=display device; green=RGB; red=real image data.
- Not unique to the above—offers much flexibility in design!
- Larger gamut, but does not cover all of RGB.

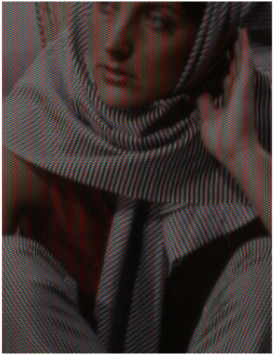
Display Example



Striped



Diagonal



Proposed

Outline

Summary

- Color Image Acquisition: avoidance of information loss.
  - Examined the aliasing inherent in CFA patterns.
  - Theorem: suboptimality of pure-color CFA patterns.
  - Designed a new way to capture color image data using CFA as a modulation operator.
- Color Image Processing: representation of sampled data in transform domain.
  - Combine sophisticated wavelet models with missing data treatment.
  - Empirical partial Bayes using EM Algorithm.
  - L^2 estimation of clean wavelet coefficient.
  - MLE of parameters
- Color Image Display: avoidance of visual information loss.
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Thank you!

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