Nonlinear Analog Behavioral Modeling of Microwave Devices and Circuits

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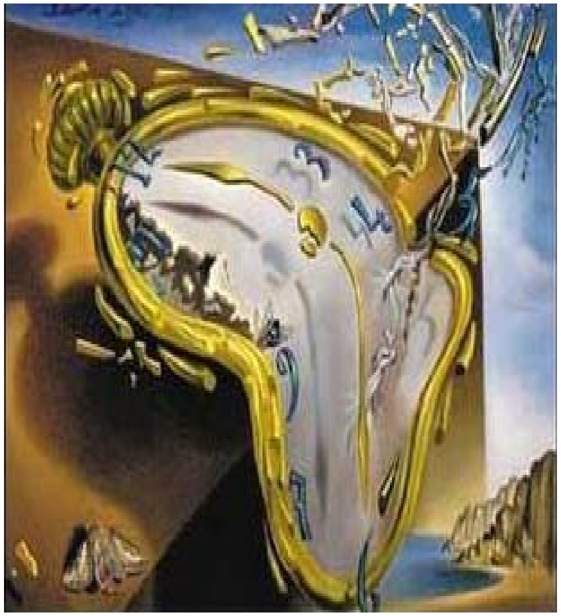
IEEE MTT-S DML Lecture #1
Bergen, Norway
May 7, 2010



Acknowledgement

Norway IEEE MTT/AP Chapter

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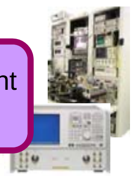
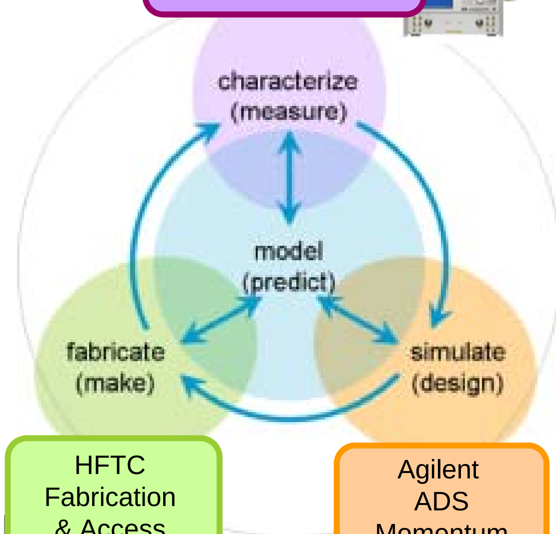
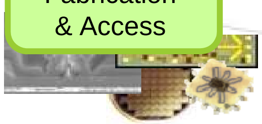
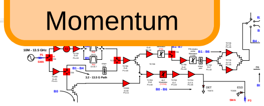




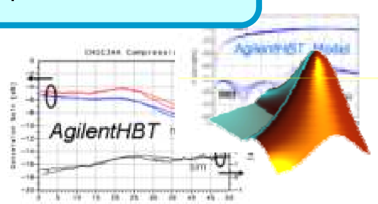
HFTC Fabrication & Access



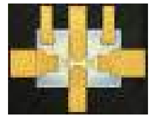
Agilent ADS Momentum



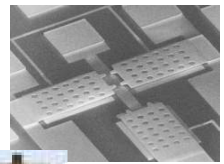
HFTC Model & Measurement IP
analytical empirical behavioral



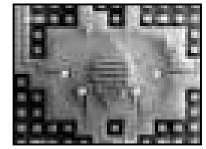
Integrated Diodes



MEMS



Liquid metal switches



Internal and external technology

GaN

GaAs

InP

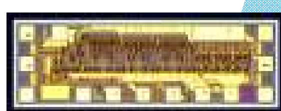
Diodes

Thin Film

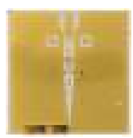
Hyperabrupt Diodes

pHEMT & FET ICs

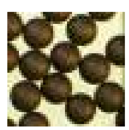
HBT ICs



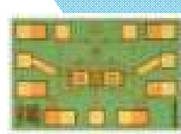
Thin Film

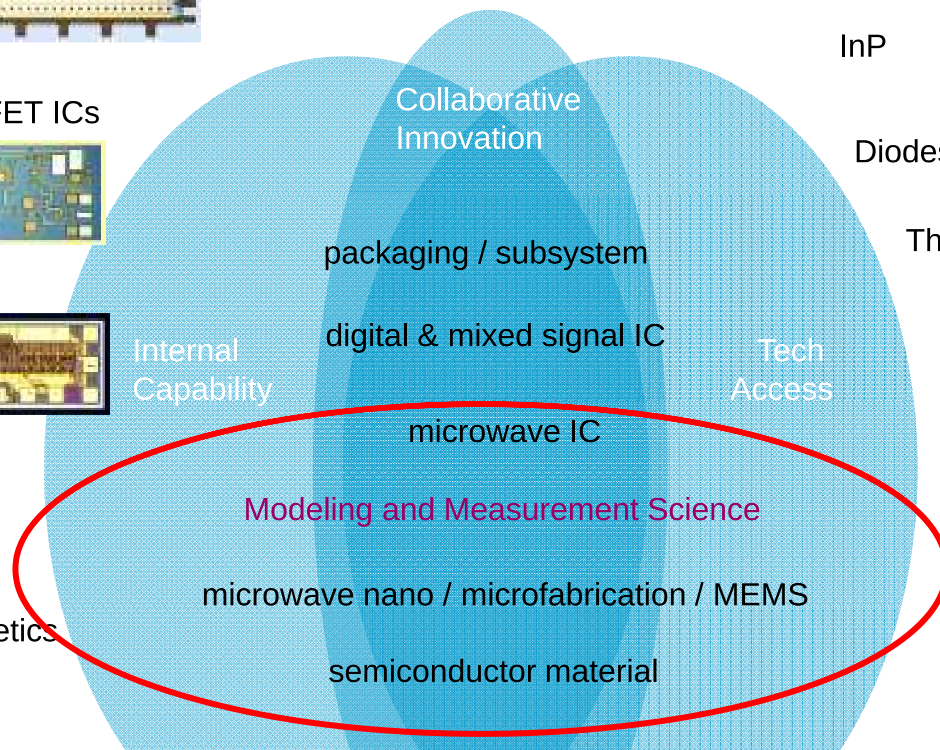


Ferromagnetics



Semiconductor switches





Outline

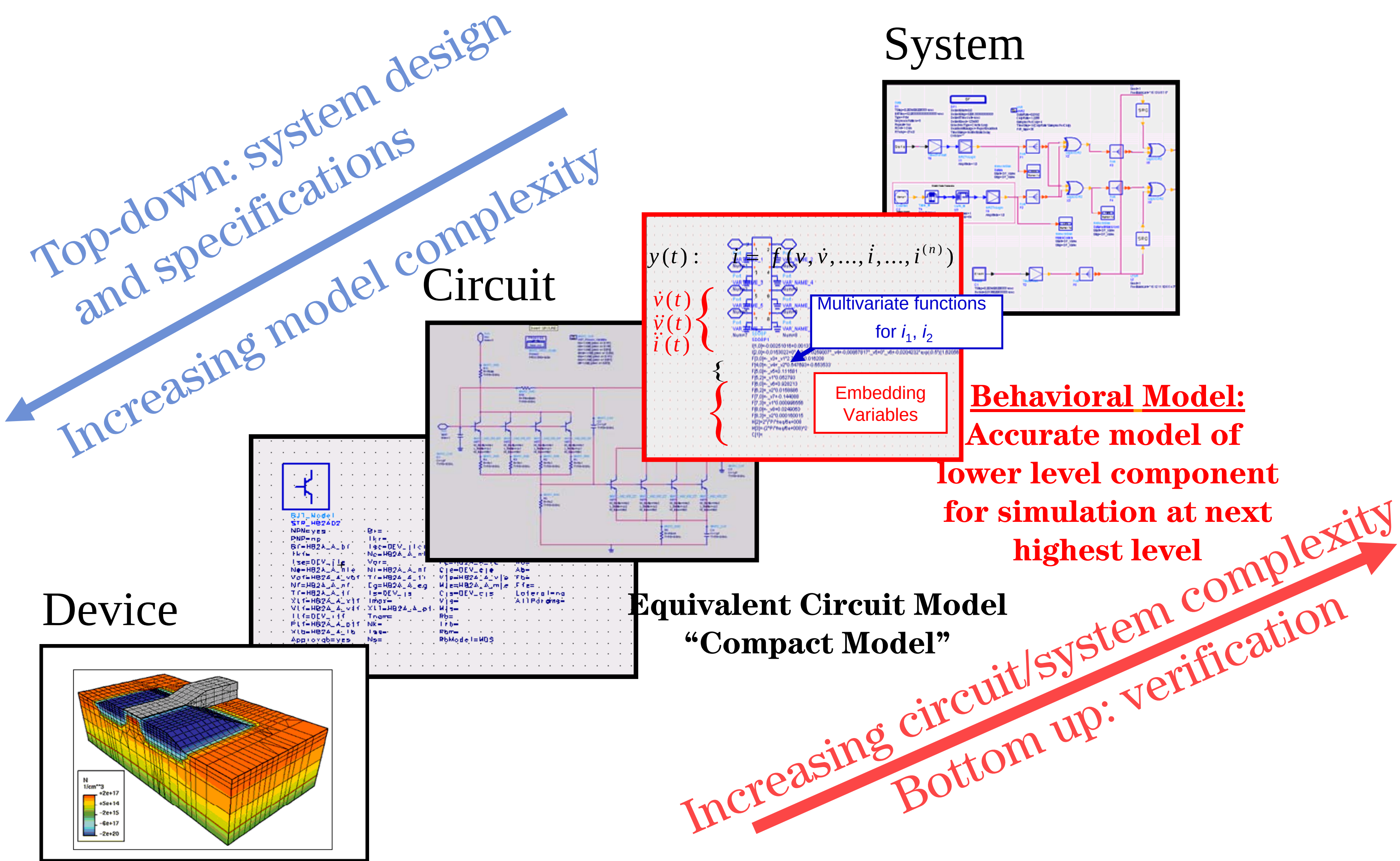
Introduction: Behavioral Models and NVNA

Functional Block Models

- Nonlinear Time Series
- X-parameters (PHD Model) in the Frequency Domain
- Mixed Time-Frequency Methods

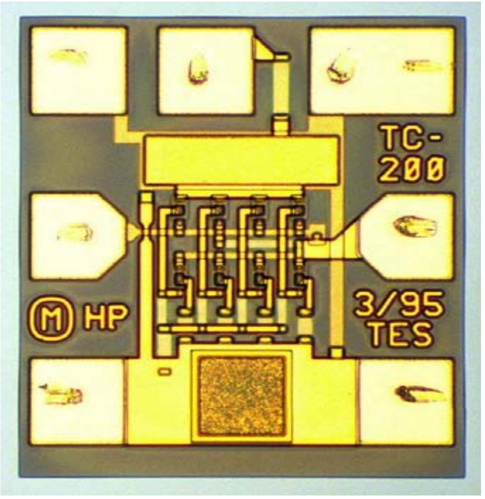
Summary and Conclusions

Introduction: Behavioral Modeling and Design Hierarchy



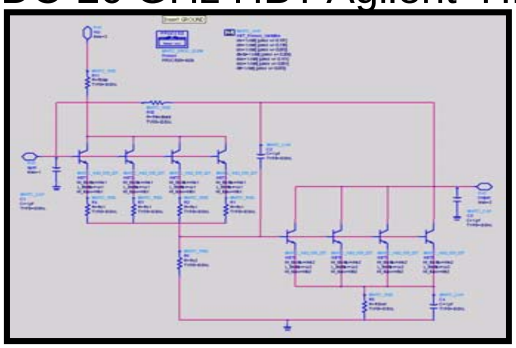
Measurement-Based and Simulation-Based Models

Actual Circuit



Amplifier or Mixer IC

DC-20 GHz HBT Agilent HMMC 5200 amp [2]



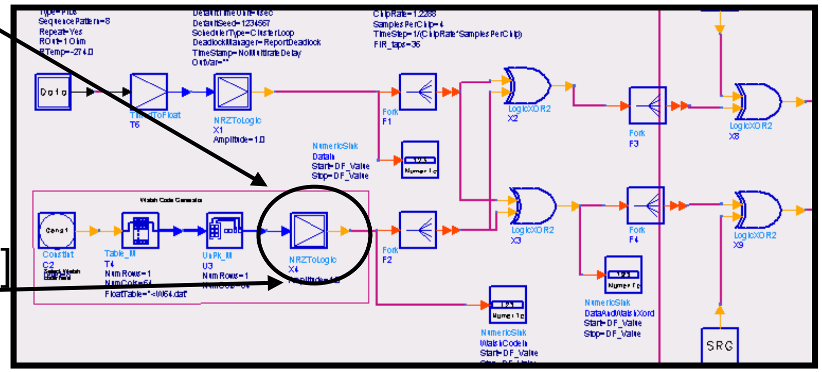
Detailed Circuit Model (SPICE/ADS) of IC

Generate Behavioral Model

Measurement-Based Model

- Ckt. model may not exist
- Ckt. models may be inaccurate
- Completely protect design IP

Design of Module or Instrument Front End



Simulation-Based Model

- Simulation speedup
- Design system before building/buying IC
- Completely protect design IP

Simple for Linear Ckts: S-parameters

S-parameters as simplest behavioral model

- Easy to measure at high frequencies
  - measure voltage traveling waves with a (linear) vector network analyzer (VNA)
  - don't need shorts/opens which can cause devices to oscillate or self-destruct
- Relate to familiar measurements (gain, loss, reflection coefficient ...)
- Can cascade S-parameters of multiple devices to predict system performance
- Can import and use S-parameter files in electronic-simulation tools (e.g. ADS)
- BUT: No harmonics, No distortion, No nonlinearities, ...

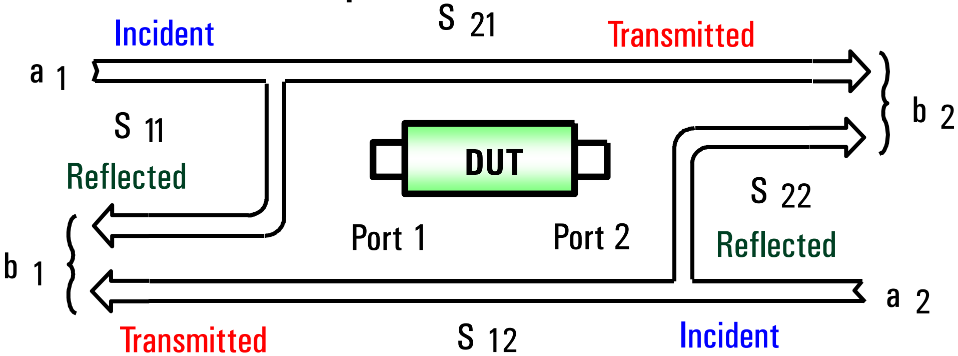
  Invalid for nonlinear devices excited by large signals, despite *ad hoc* attempts

Linear Simulation:
Matrix Multiplication

S-parameters

$$b_1 = S_{11}a_1 + S_{12}a_2$$
$$b_2 = S_{21}a_1 + S_{22}a_2$$

Measure with linear VNA:
Small amplitude sinusoids



Model Parameters:
Simple algebra

$$S_{ij} = \left. \frac{b_i}{a_j} \right|_{\substack{a_k = 0 \\ k \neq j}}$$

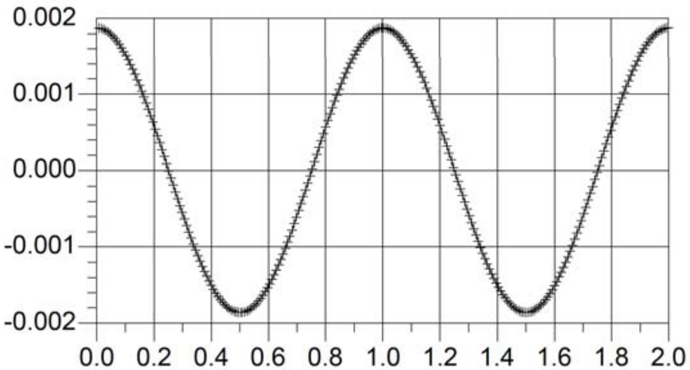
Three Components of Behavioral Modeling

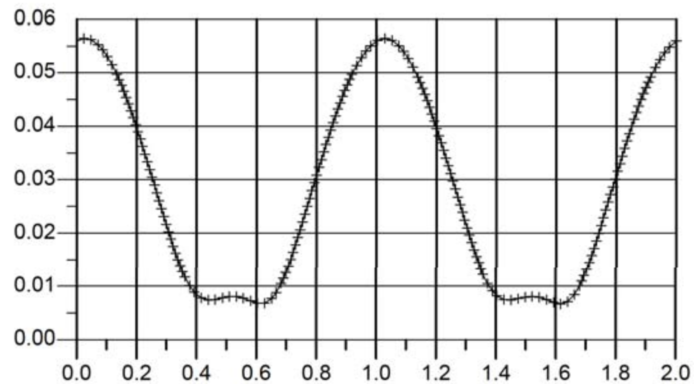
1. Model Formulation
   - Nonlinear ODEs in Time Domain (e.g. Transient Analysis; all others)
   - NL Spectral Map in Freq. Domain (e.g. Harmonic Balance) X-params
   - Mixed Domains (e.g. ODE-Coupled Envelopes in Circuit Env. Analysis)

2. Experiment Design
   - Stimulus needed to excite relevant dynamics

3. Model Identification
   - Procedure to determine model “parameters”

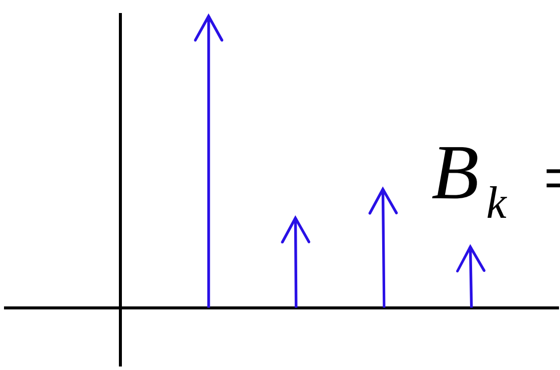
Model Formulation: Time & Freq. Domains [1,6]



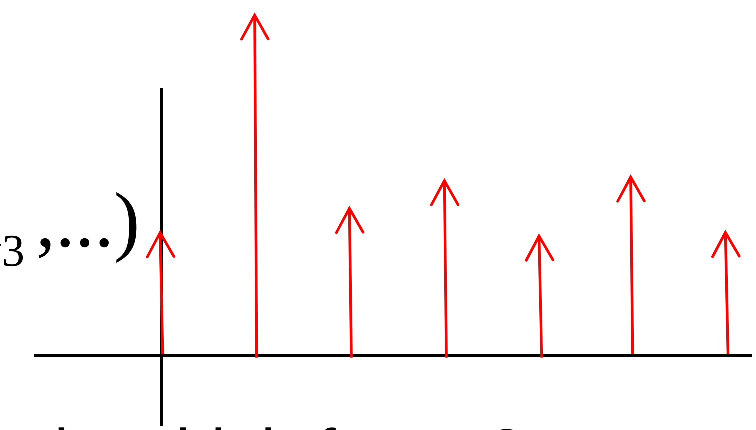


$$I(t) = F(V(t), \dot{V}(t), \ddot{V}(t), \ldots, \dot{I}(t), \ldots)$$

Natural for strongly nonlinear low-order (lumped) systems



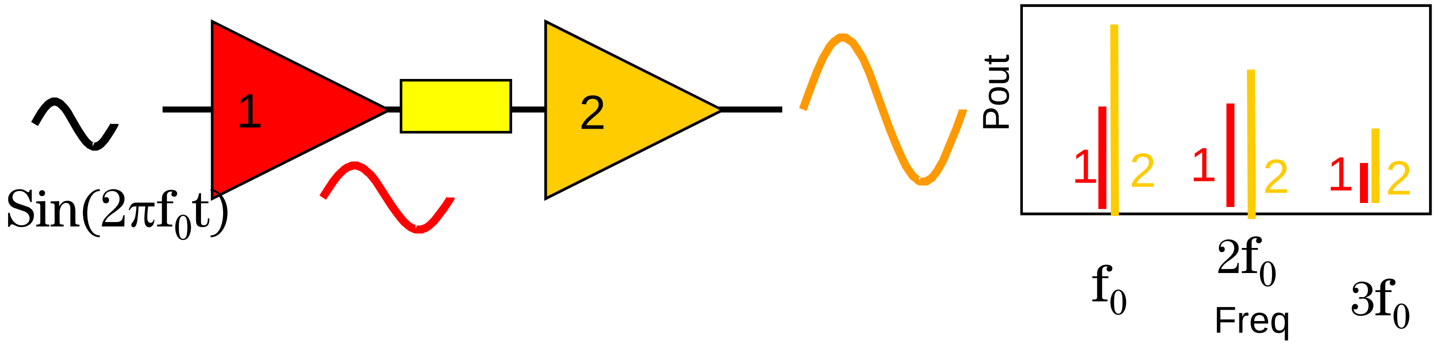
$$B_k = F_k(A_1, A_2, A_3, \ldots)$$



Freq. Domain natural for low-distortion, high-freq. ICs

Formulate model eqs. in language native to appropriate simulator

Wanted: Cascadability of *Nonlinear Components*

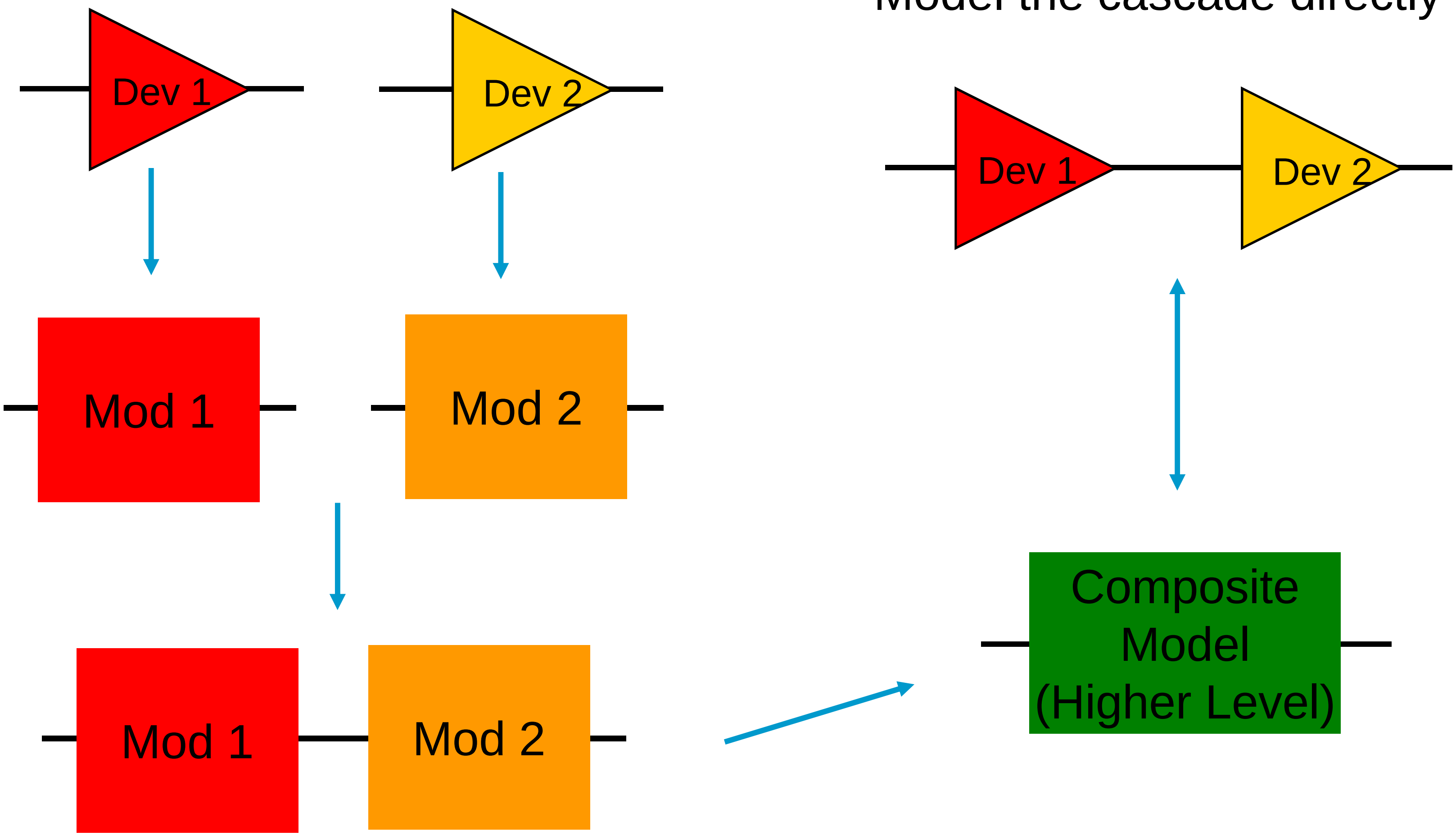


Predict signal and harmonics (magnitude and phase) through chains of *cascaded* nonlinear components under drive

- Inter-stage mismatch is important to final results
  – Can not infer these effects from VNA measurements (even "Hot S_{22}")
- Required for communication circuits and module design
- Linear S-parameter theory doesn't apply!
  Most previous attempts to generalize S-parameters to nonlinear case are wrong!

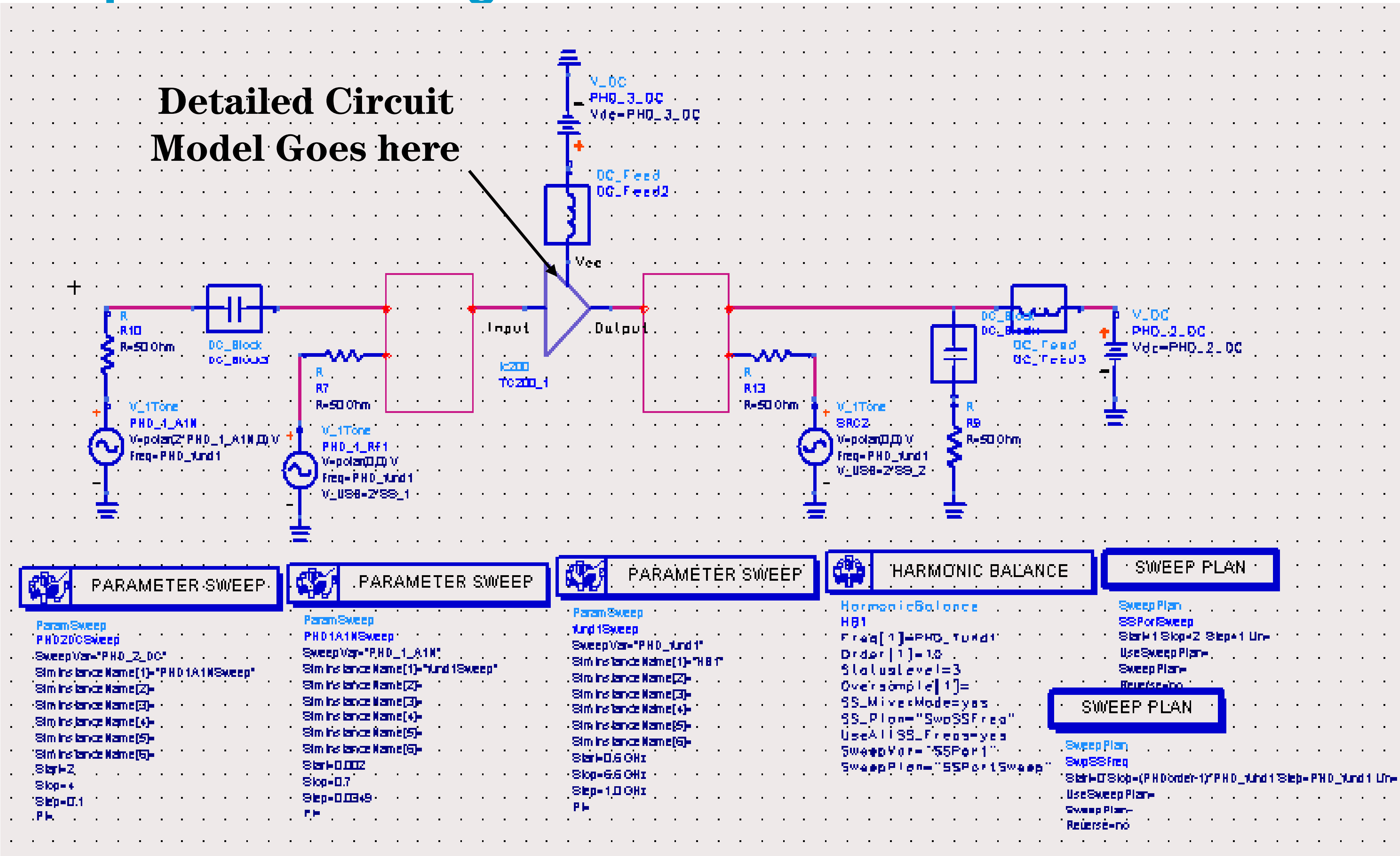
Wanted: Hierarchical Modeling

Model the cascade directly



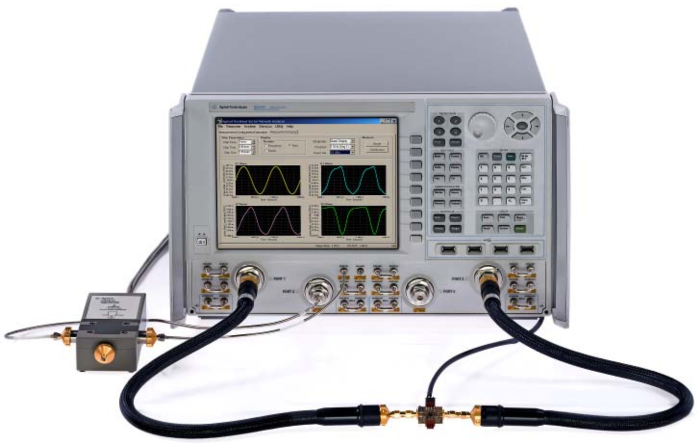
A cascade of many models *reduced to one*

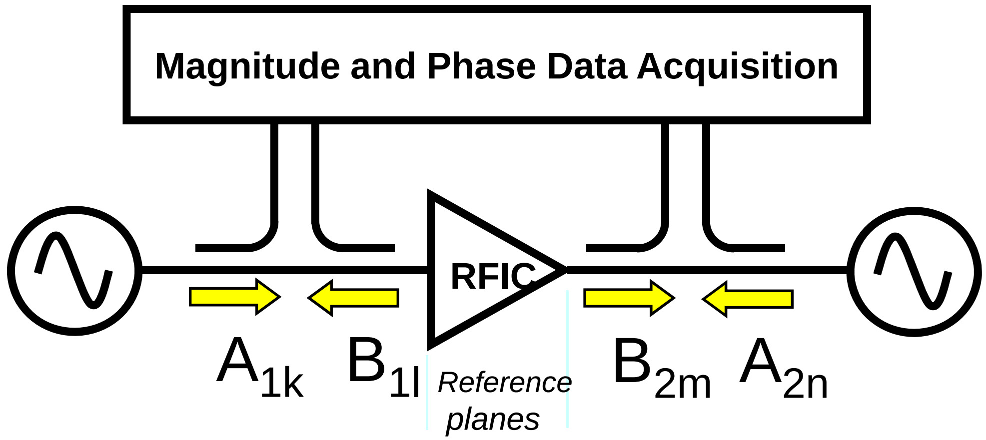
Experiment Design: Simulation



Experiment Design: Measurement

Nonlinear Vector Network Analyzer [9,14] (NVNA)





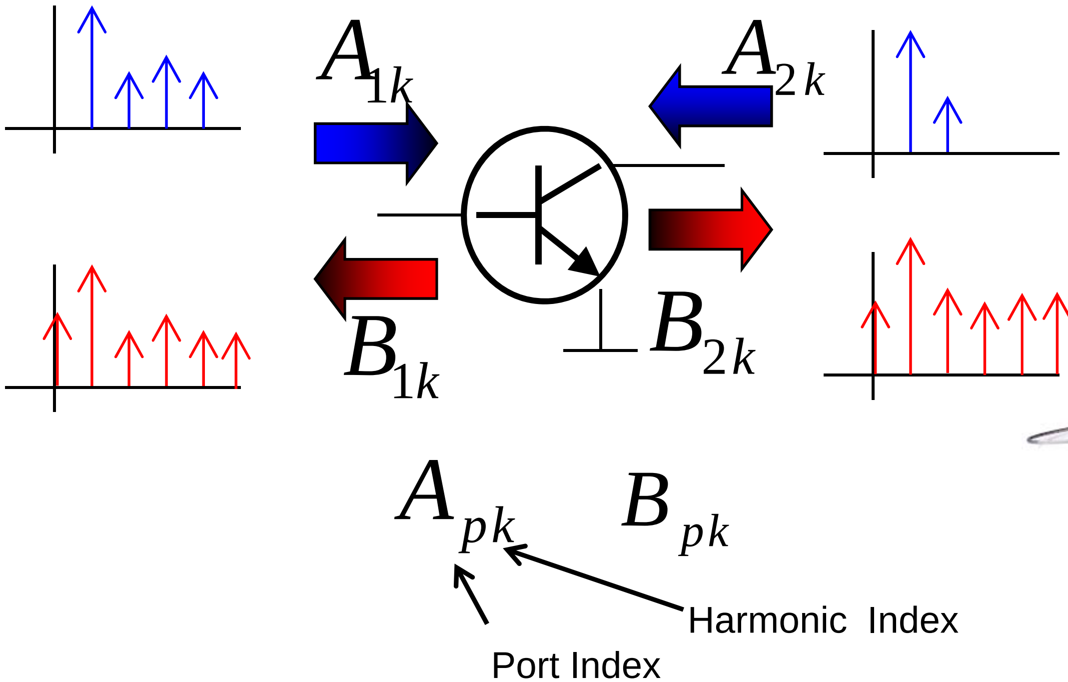
New phase calibration standard

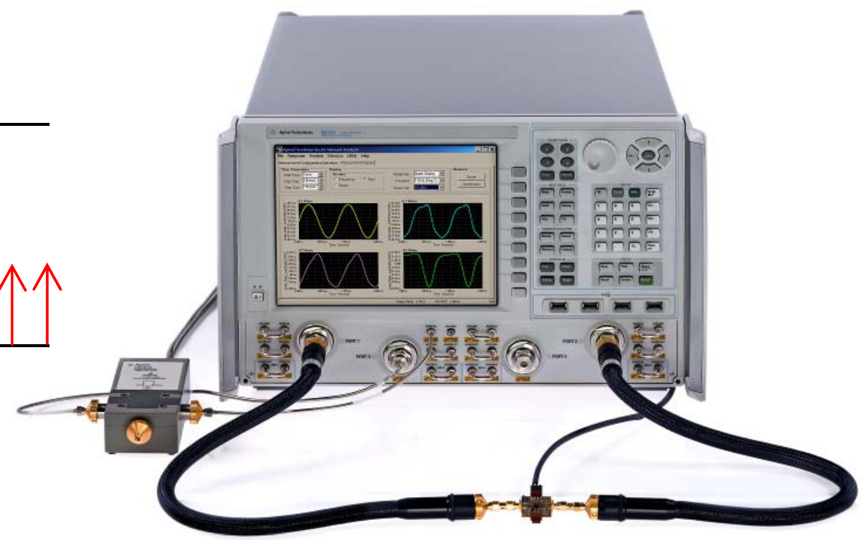
Calibrated magnitude *& phase* of harmonics/IMD

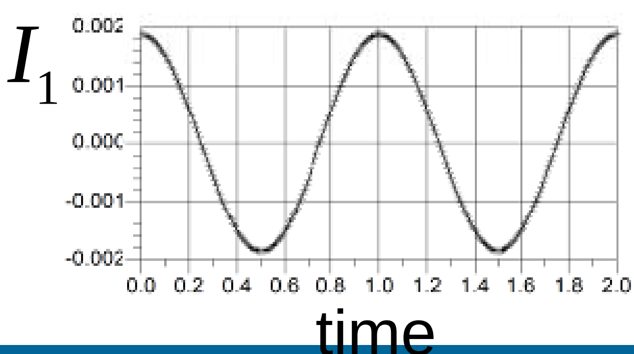
Measures under realistic large-signal conditions

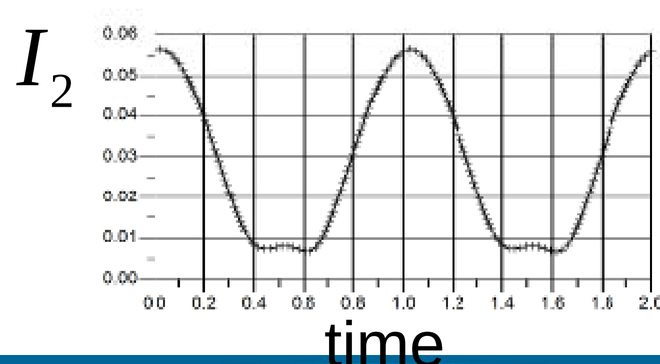
Based on Standard Agilent PNA Hardware
And custom reference generator

Introduction: NVNA measurements complex spectra and waveforms



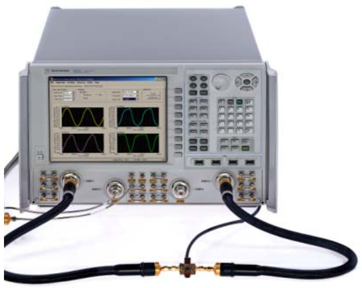






Nonlinear Vector Network Analyzer (NVNA) [14]:

Network Analyzer

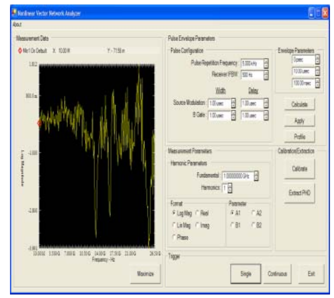


\+

Phase Reference

\+

Meas. Science
Algorithms & Software

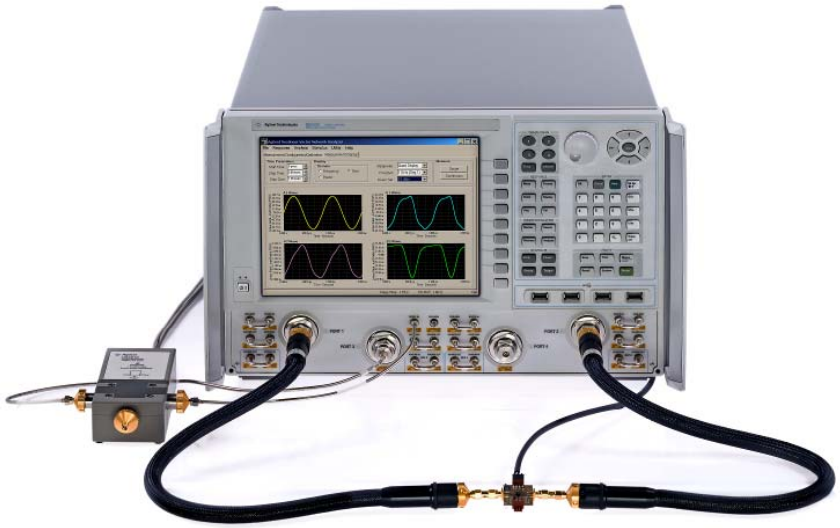


= NVNA

NVNA = PNA-X + Phase Reference (custom InP IC)
+ Application SW and calibration (mag and phase)
two internal sources, internal switches, and an internal broadband combiner

NVNA measures *Magnitude and Phase* of all relevant frequency components (cross-frequency coherence) necessary to measure X-parameters!

Nonlinear Vector Network Analyzer (NVNA) [14]



- Vector (amplitude/phase) corrected nonlinear measurements from 10 MHz to 50 GHz
  - Calibrated absolute amplitude and relative phase (cross-frequency relative phase) of measured spectra traceable to standards lab
  - 50 GHz of vector corrected bandwidth for time domain waveforms of voltages and currents of DUT
  - Multi-Envelope domain measurements for measurement and analysis of memory effects
  - X-parameters: Extension of Scattering parameters into the nonlinear region providing unique insight into nonlinear DUT behavior. Efficient measurements with phase control. External instrument control, pulsed, triggered measurements
  - X-parameter MDIF file read by ADS XnP component or nonlinear simulation and design.
  - X-parameter generation from detailed schematics within ADS simulator.
- Standard VNA HW with Nonlinear features & capability

Outline

Introduction: Behavioral Models and NVNA

Functional Block Models

- Nonlinear Time Series
- X-parameters (PHD model) in the Frequency Domain
- Mixed Time-Frequency

Summary and Conclusions

Nonlinear Time Series method of Behavioral Modeling [1,6]

Dynamical Systems & State Space

The dynamics of the nonlinear system can be assumed to be described by a system of nonlinear ODEs

$$y^{(n)}(t) = f(y^{(n-1)}, \ldots y, x, \dot{x}, \ldots x^{(m)})$$

Order of time derivative

$$\dot{\vec{u}}(t) = \vec{f}\left(\vec{u}(t), \vec{x}(t)\right)$$ Vector of State Equations

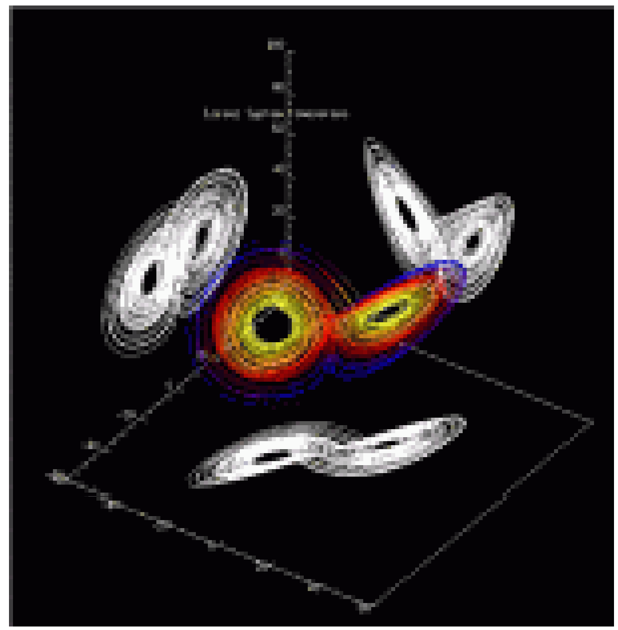
$$y(t) = h\left(\vec{u}(t), \vec{x}(t)\right)$$ Scalar output *y*(*t*)

The sampled solution of the ODE, y(t), is a time-series

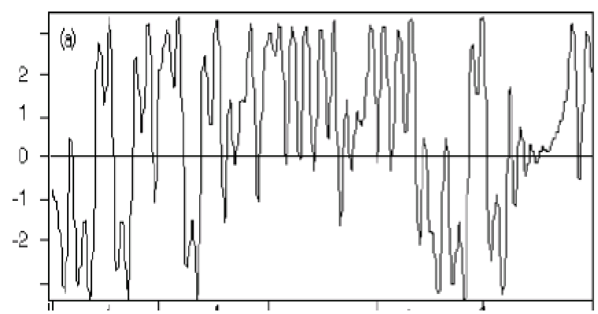
The solution of the dynamical equations for state variables, u(t), is a time-parameterized trajectory in Phase Space

Phase Space and Time Series

Lorenz system



The multi-dimensional space spanned by the state variables is known as phase space

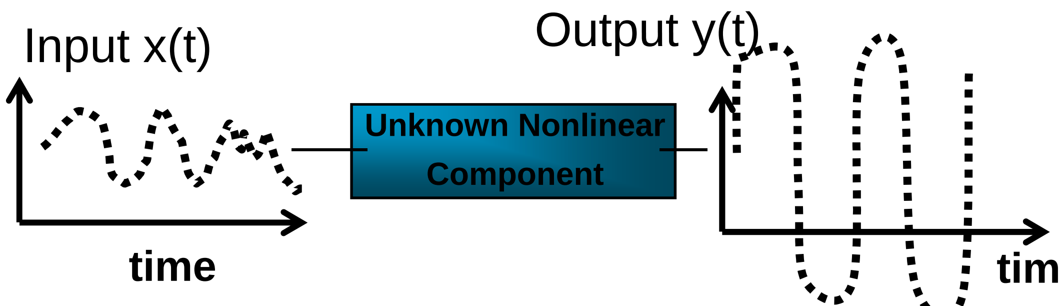


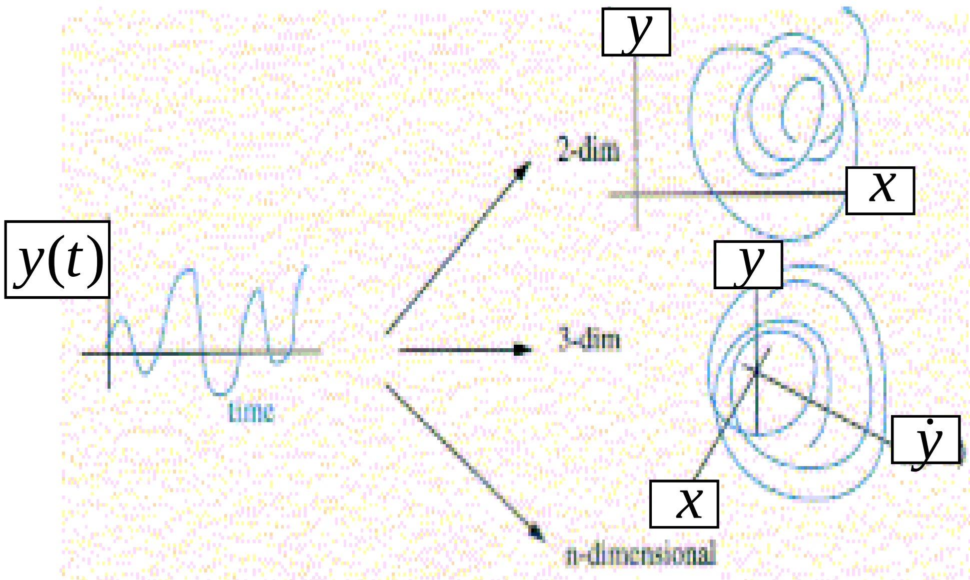
Any measurable output is a projection of this trajectory versus time:
a Time Series

Nonlinear Time Series (NLTS) Phase Space Reconstruction by Embedding

NLTS Behavioral Modeling is *"inverse"* of solving known ODEs

Start from input & output time series and *discover dynamics*





Stimulate System with drive x(t)

Record Time Series output y(t)

Embed drive x(t) & response y(t)

Stop when trajectory single valued

This results in the *Nonlinear ODE*:

$$f\,(y(t), \dot{y}(t), x(t), \ldots) = 0$$

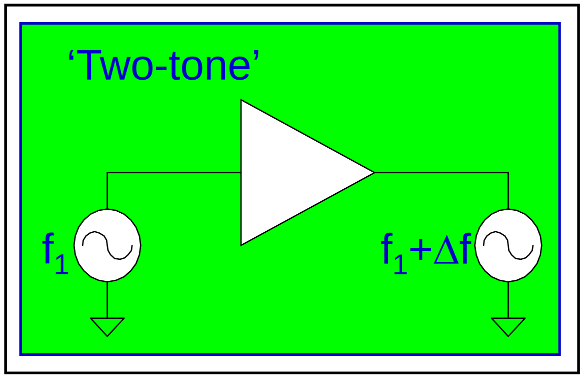
Approximate f with smooth function

Attach ODE Model to Circuit Simulator

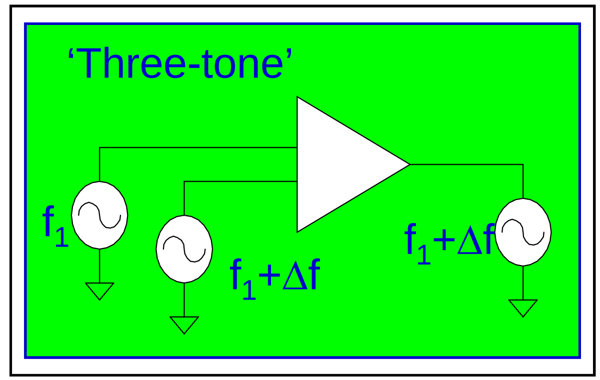
Excitation Designs

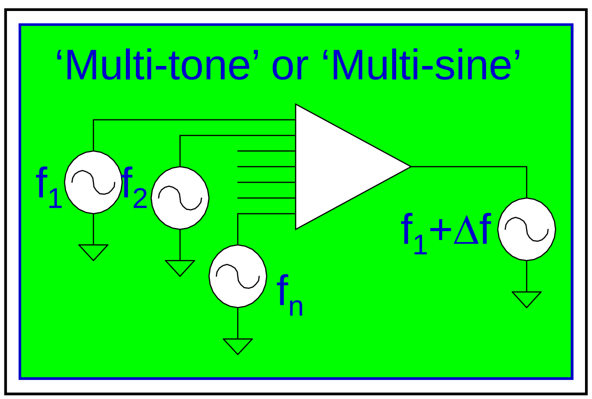
Goal: stimulate all *relevant* (observable) dynamics

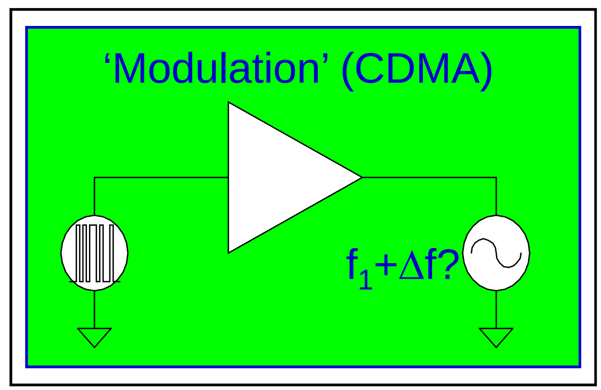
Sweep Power and Frequency to “cover phase space”



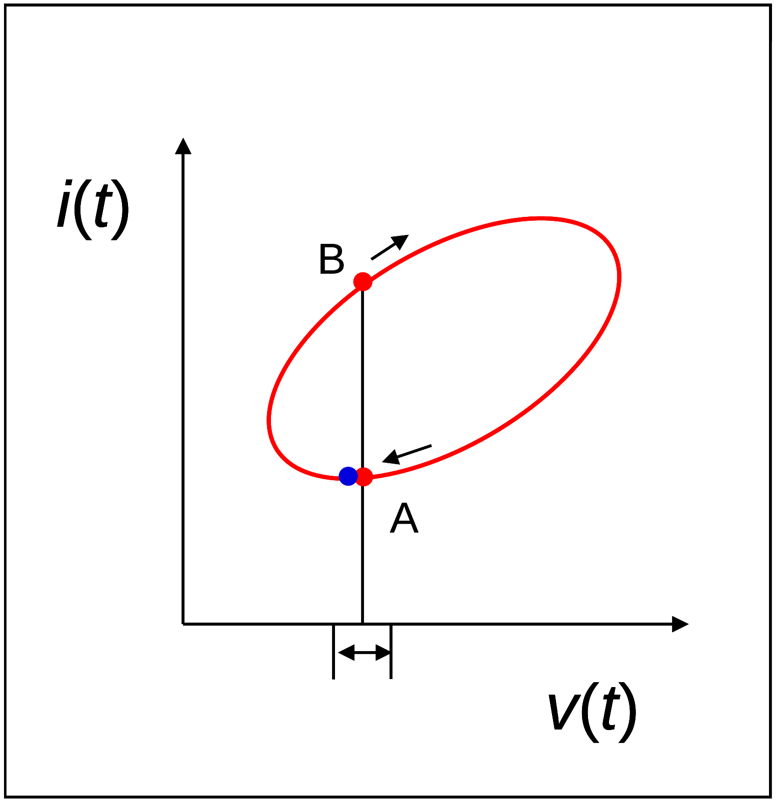
Used for models

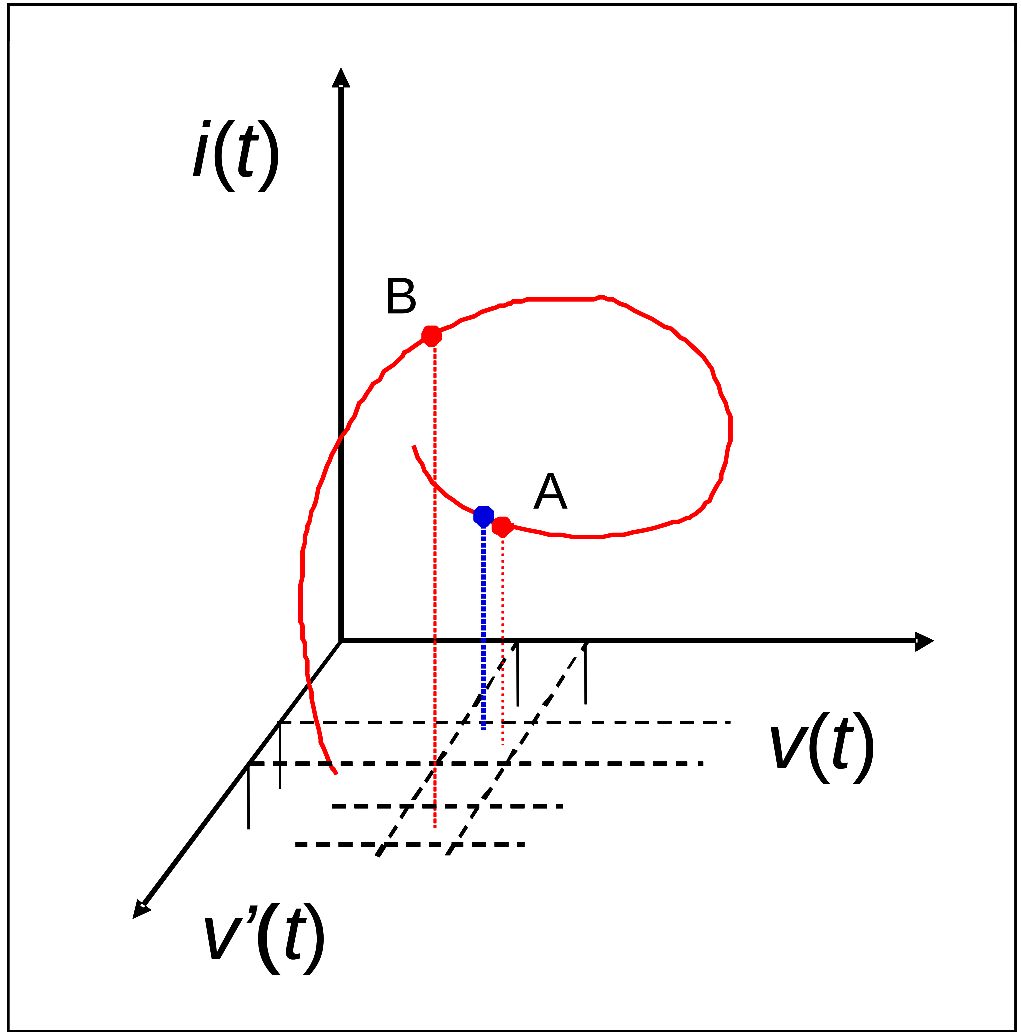






Embedding: Building up phase space to define ODE





$$i(t) \neq i(v(t))$$

$$i(t) = i(v(t), \dot{v}(t))$$

Model Identification: Nonlinear Time Series (NLTS)

Stimulate / Excite System
Sufficiently complex stimulus

$$x(t) \rightarrow [x(t), \dot{x}(t), \ldots, x^{(m)}(t)]$$
$$y(t) \rightarrow [y(t), \dot{y}(t), \ldots, y^{(n)}(t)]$$

Embed:
Create auxiliary variables
(represent waveform)

$$\begin{array}{cccccccc} x(t_1) & \dot{x}(t_1) & \ldots & x^{(m)}(t_1) & y(t_1) & \dot{y}(t_1) & \ldots & y^{(n)}(t_1) \\ x(t_2) & \dot{x}(t_2) & \ldots & x^{(m)}(t_2) & y(t_2) & \dot{y}(t_2) & \ldots & y^{(n)}(t_2) \\ . & . & \ldots & . & . & . & \ldots & . \\ x(t_p) & \dot{x}(t_p) & \ldots & x^{(m)}(t_p) & y(t_p) & \dot{y}(t_p) & \ldots & y^{(n)}(t_p) \end{array}$$

Sample data:

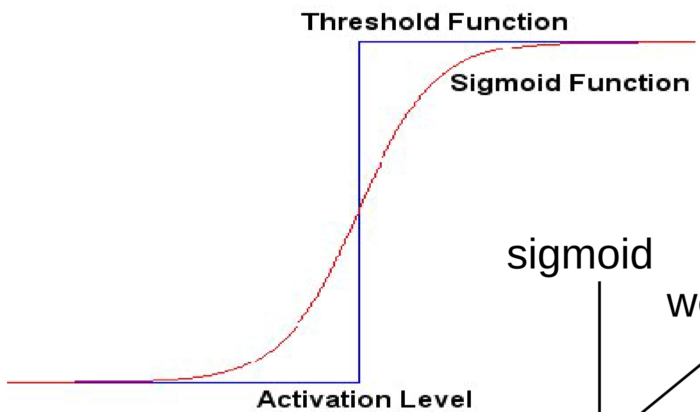
at high frequency
(or envelope;
hard if multiple timescales)

$$y^{(n)} = f(y^{(n-1)}, \ldots y, x, \dot{x}, \ldots x^{(m)})$$

Fit:
Nonlinear function f

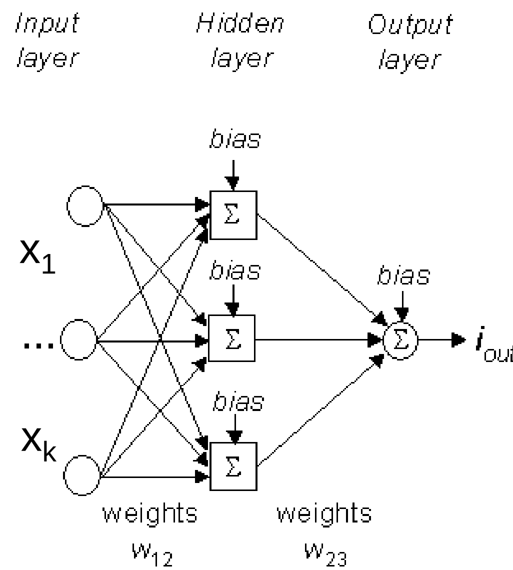
Function approximation Artificial Neural Networks

An ANN is a parallel processor made up of simple, interconnected processing units, called *neurons*, with weighted connections.



sigmoid — weights — biases

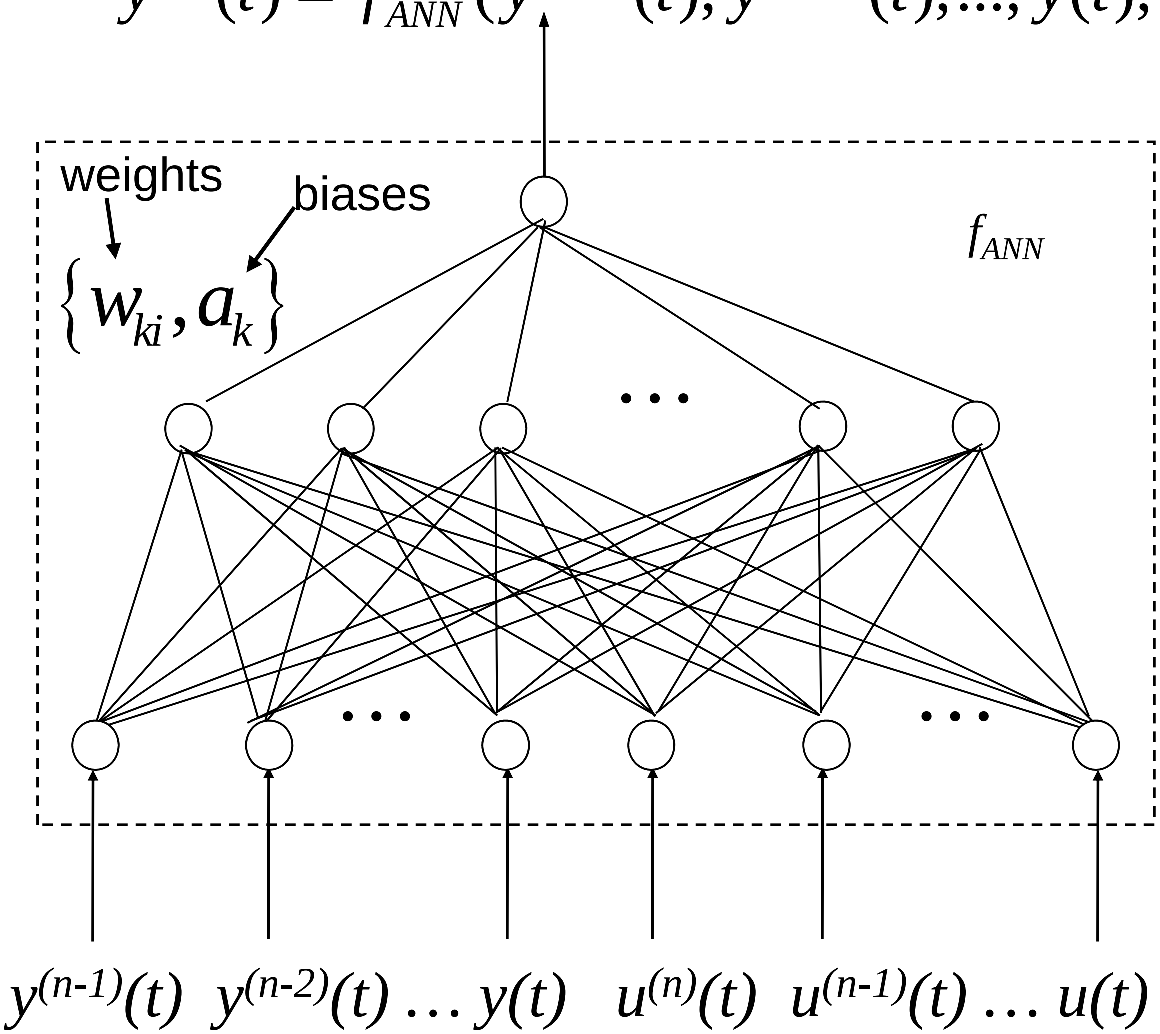
$$F(x_1,\ldots,x_K)=\sum_{i=1}^{I} v_i s\left(\sum_{k=1}^{K} w_{ki}x_k + a_i\right)+b$$



- Universal Approximation Theorem: Fit "any" nonlinear function of any # of variables
- Infinitely differentiable: *better for distortion than naïve splines or low-order polynomials*.
- Easy to train (fit) using standard third-party tools (MATLAB)
- Easy to train on scattered data

Function approximation: Artificial Neural Networks

$$y^{(n)}(t) = f_{ANN}(y^{(n-1)}(t), y^{(n-2)}(t), \ldots, y(t), u^{(n)}(t), u^{(n-1)}(t), \ldots, u(t))$$



"Dynamic Neural Network"

$\{w_{ki}, a_k\}$ Obtained by *Training*

Can also define *f* by polynomials,
radial basis functions,
lookup tables etc.

Model Implementation: ODE in circuit simulator

(after Zhang and Xu in [6])

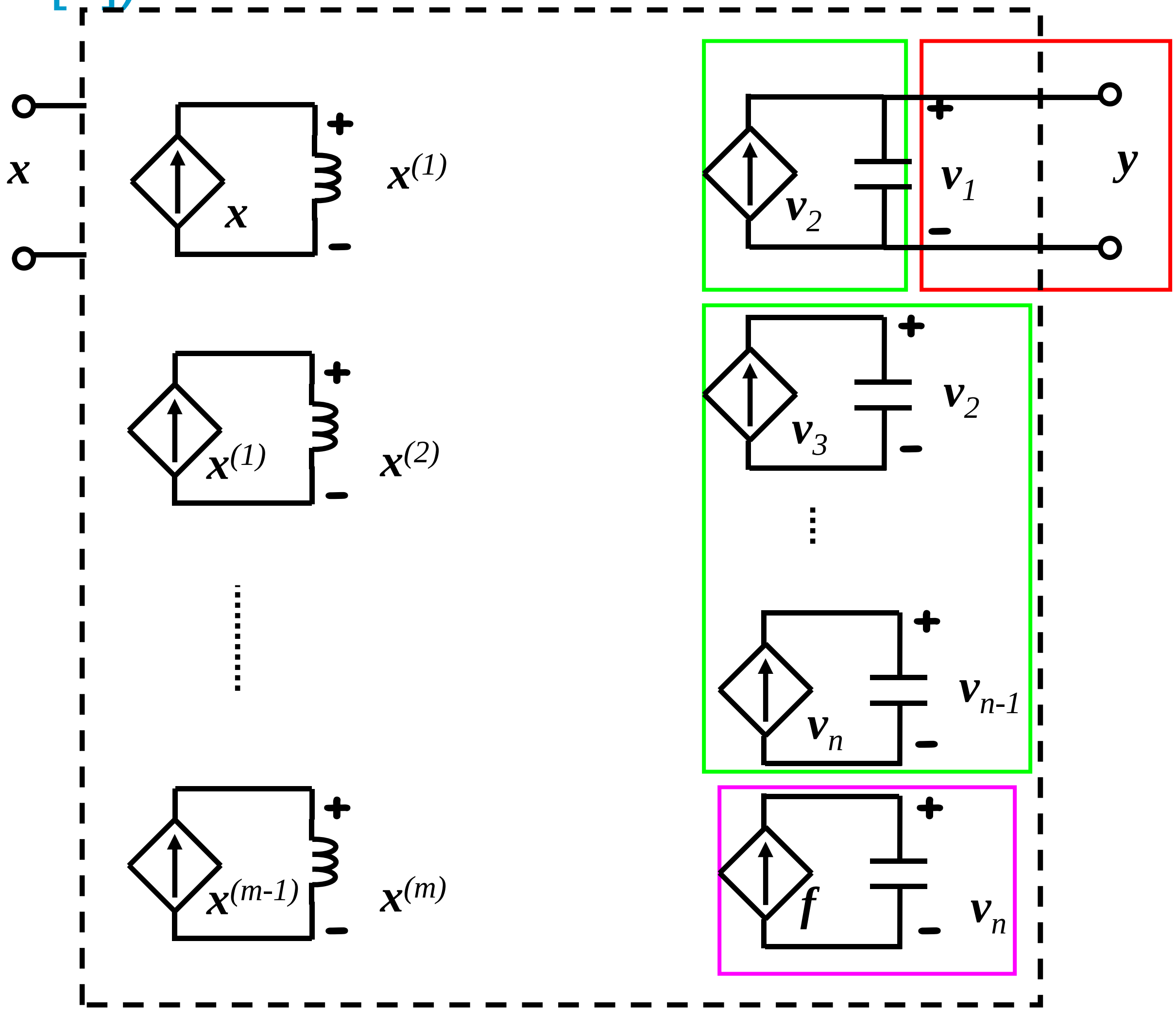
$$y^{(n)} = f(y^{(n-1)},\ldots y, x, \dot{x},\ldots x^{(m)})$$

$$v_1 = y$$

$$\dot{v}_1 = v_2$$

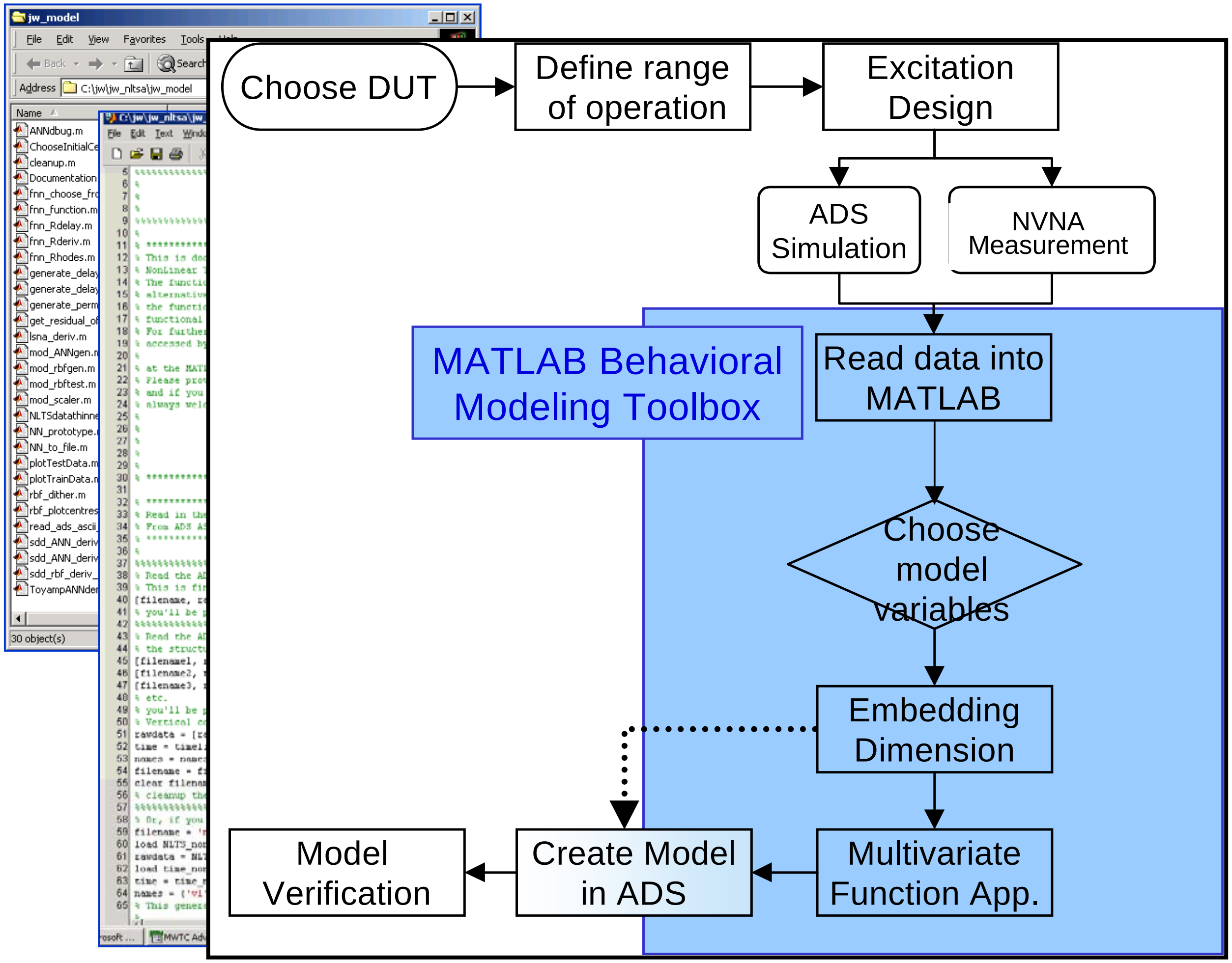
$$\vdots$$

$$\dot{v}_{n-1} = v_n$$



$$\dot{v}_n = f(v_{n-1}, v_{n-2},\ldots, v, x, \dot{x},\ldots, x^{(m)})$$

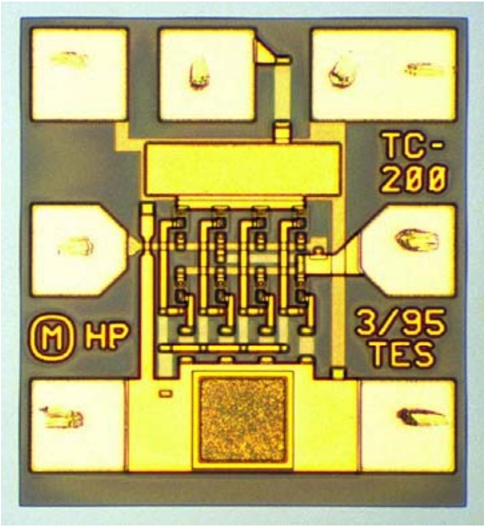
NLTSA modeling flow

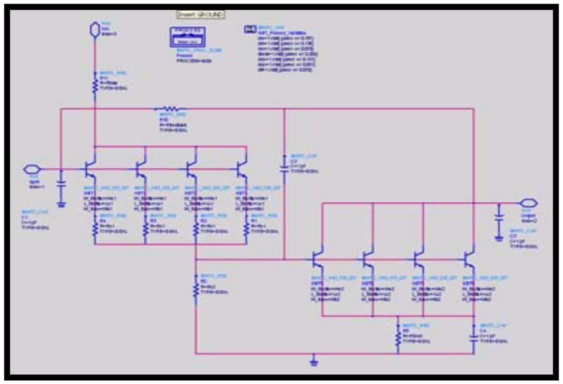


- *MATLAB* Toolbox, plus 3rd-party software
- '*NLTSfile*' structure
- ADS/NVNA-*MATLAB* interfaces
- ADS templates for
  - simulation
  - data display
  - model verification
- Model as SDD in ADS

Example: GaAs HBT MMIC

Actual Circuit





Detailed ckt model

DC-20 GHz GaAs HBT

(Agilent HMMC 5200 Amp)

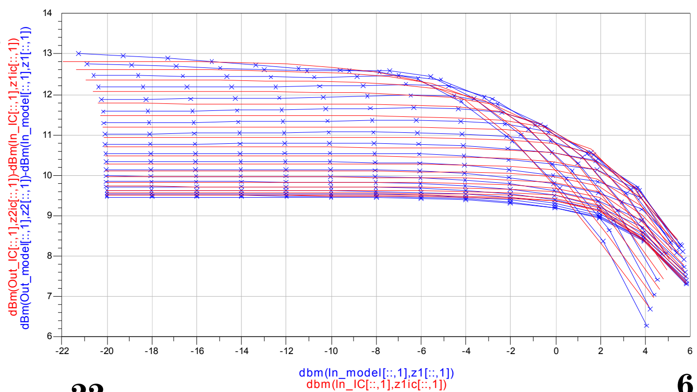
Series-Shunt Amplifier

Gain: 9.5 dB @ 1.5GHz

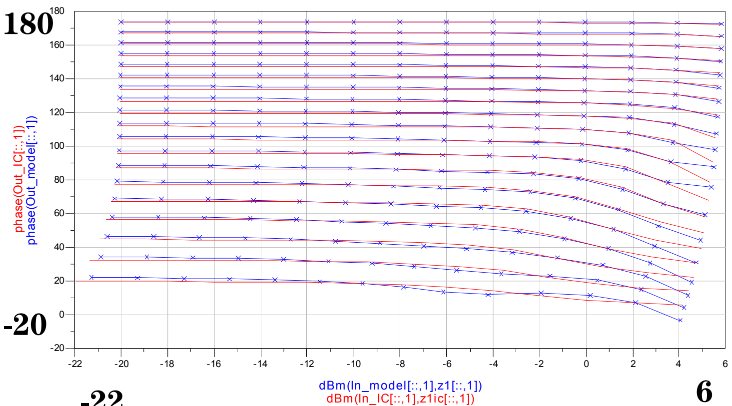
Results: NLTS Accuracy and Speed [1,6]

NLTS Behavioral model | Circuit model data

Fundamental Gain



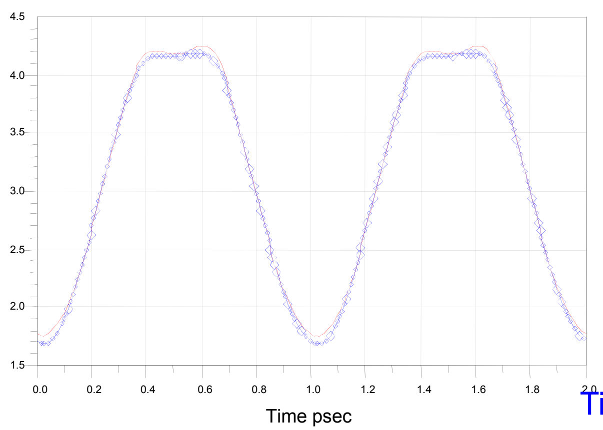
Fundamental Phase

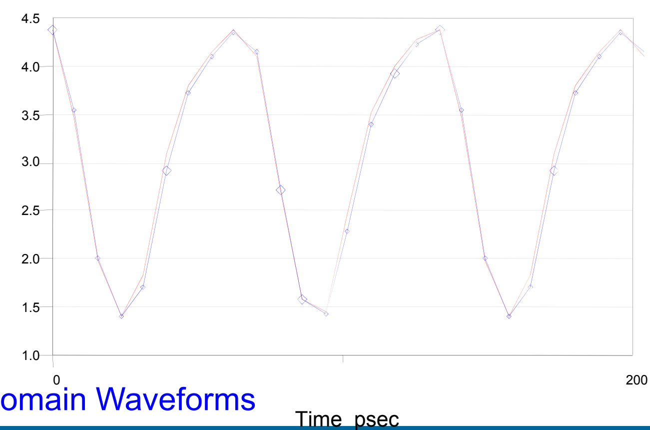


1 - 19 GHz

$$I_i(t) = f_i(\dot{I}_i, V_1(t), V_2(t), \dot{V}_1(t), \dot{V}_2(t), V_1^{(2)}(t), V_2^{(2)}(t))$$

19 neurons



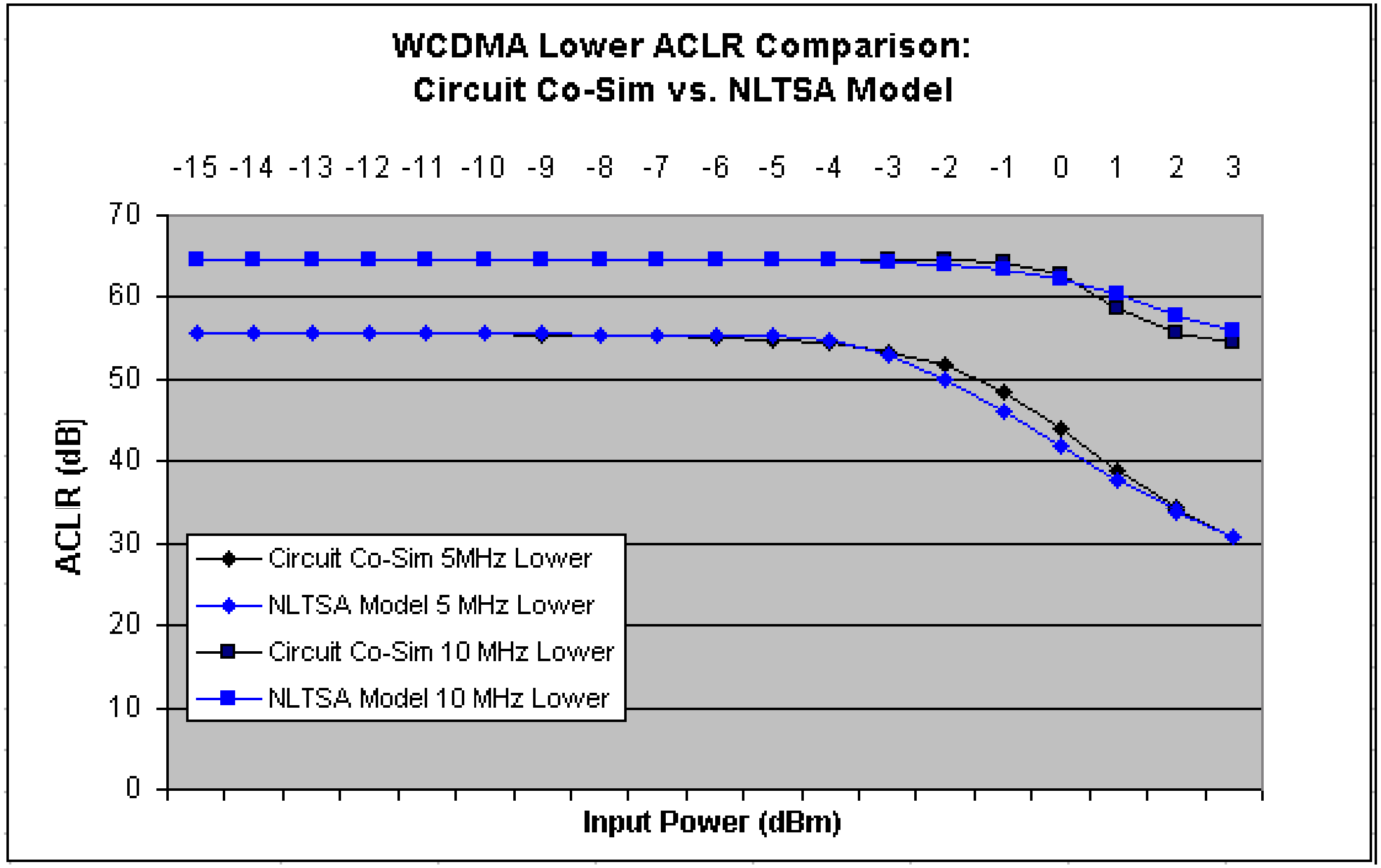


Time Domain Waveforms

229.68 seconds

11315.67 seconds

Circuit Co-Simulation vs. NLTSA Model
Results 3GPP WCDMA (lower) ACLR



3GHz WCDMA

Model generated from only sinusoidal signals

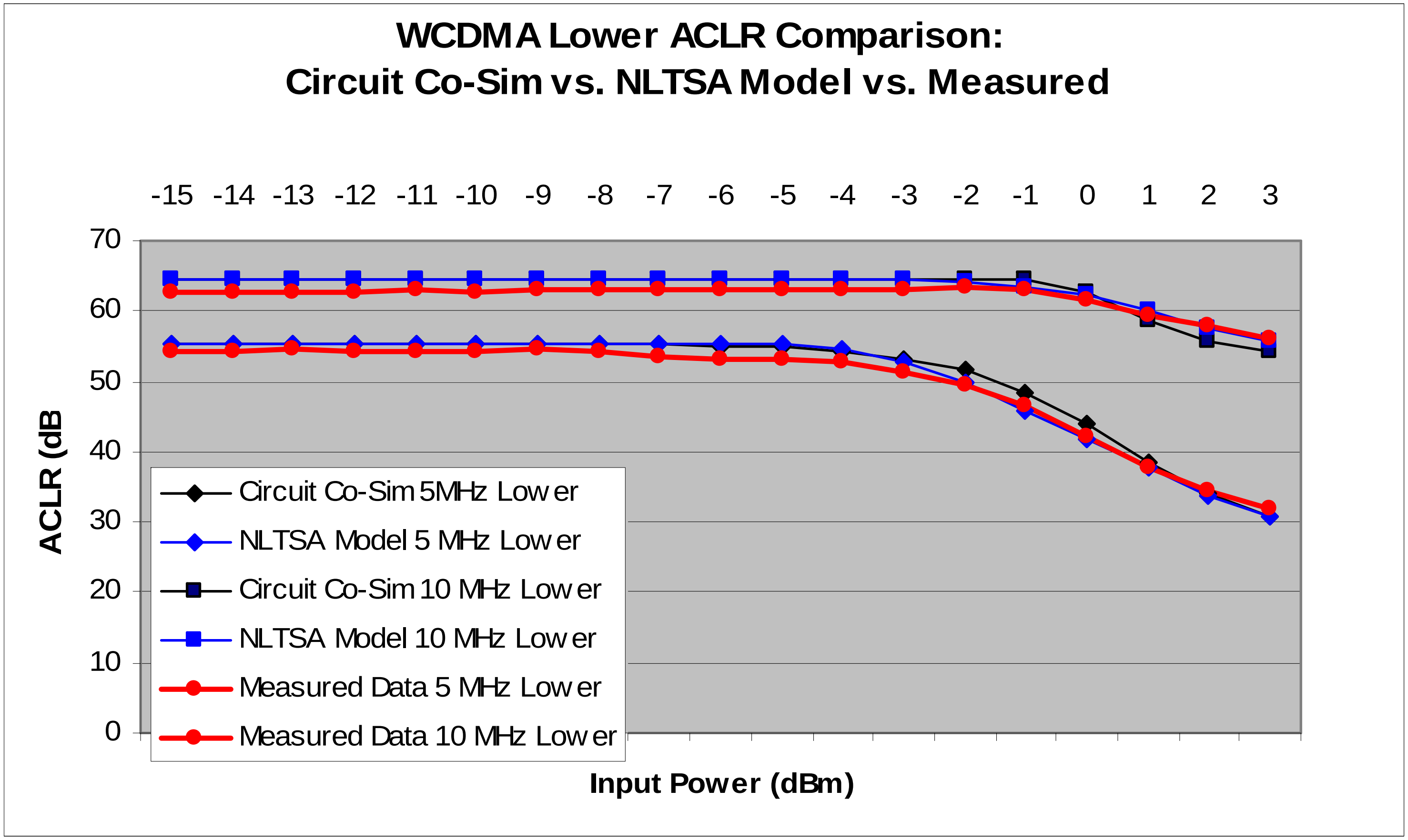
294 sec/pt NLTS

1532 sec/pt Ckt.

40 neuron model

Courtesy Greg Jue

Circuit Co-Simulation vs. NLTSA Behavioral Model Results vs. Measured 3GPP WCDMA (lower) ACLR



3GHz simulated

2.4GHz meas

Model is also *cascadable*

Model works in TA, HB, Envelope

Outline

Introduction: Behavioral Models and NVNA

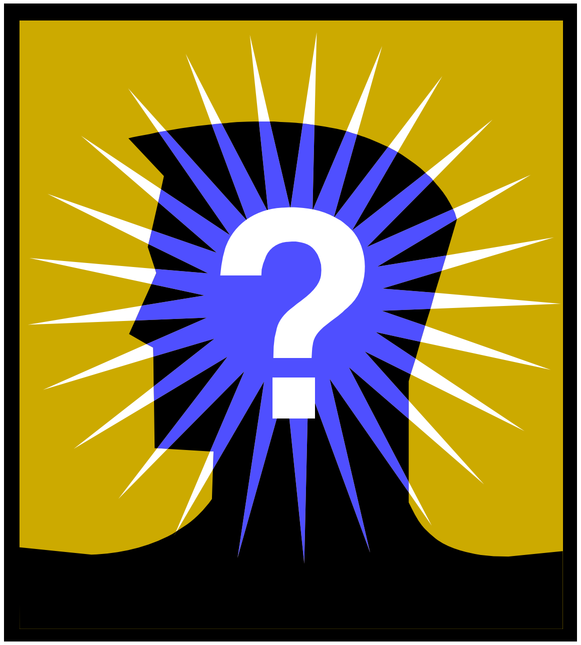
Functional Block Models

- Nonlinear Time Series
- X-parameters (PHD Model) in the Frequency Domain
- Mixed Time-Frequency Methods

Summary and Conclusions

X-parameters (PHD model): a *nonlinear* paradigm

"Is there an analogue with linear S-parameters to help with the nonlinear problem?"



Frequency Domain description is natural for high-frequency, distributed systems

Natural for Harmonic Balance Algorithms and NVNA data

Arbitrarily Nonlinear; Not limited to Volterra Theory

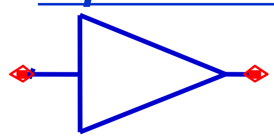
X-Parameters: The Nonlinear Paradigm

X-parameters are the mathematically correct superset of S-parameters, applicable to both large-signal and small-signal conditions, for linear and nonlinear components. *The math exists!*

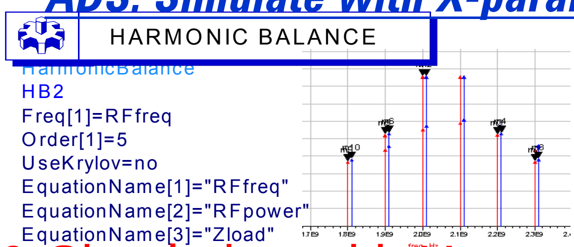
We can measure, model, & simulate with X-parameters
Each part of the puzzle has been created
The pieces now fit together seamlessly

NVNA: Measure X-params

X-parameter block



ADS: Simulate with X-params



Interoperable Nonlinear Measurement, Modeling & Simulation with X-params

"X-parameters have the potential to do for characterization, modeling, and design of nonlinear components and systems what linear S-parameters do for linear components & systems"

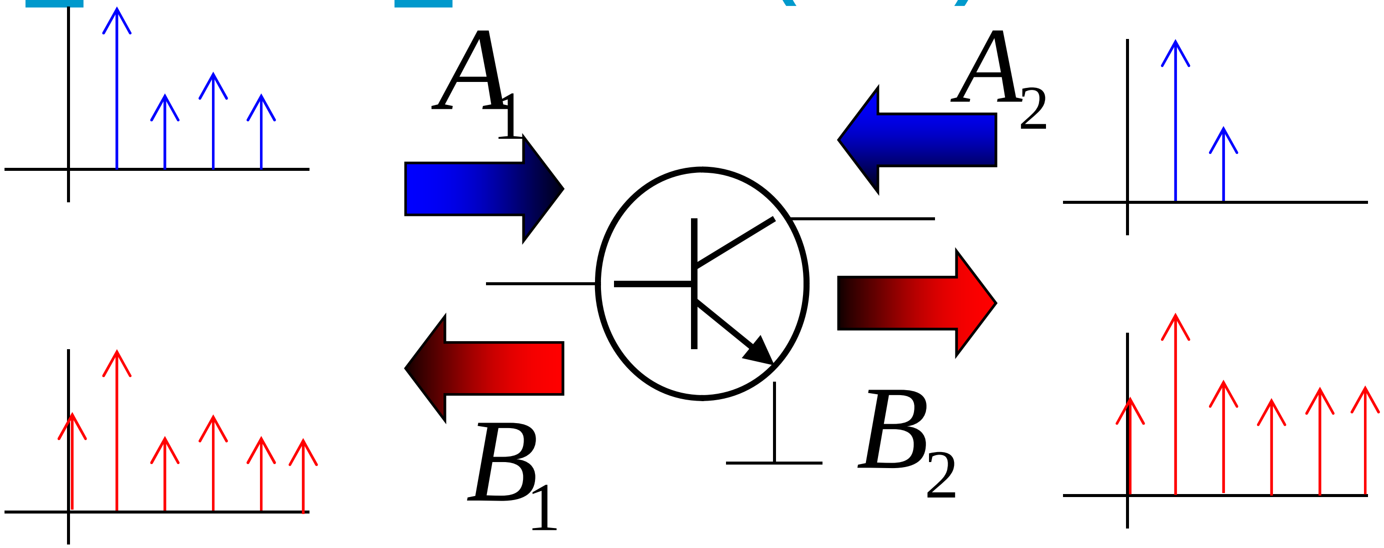
X-Parameters: Why They are Important:

Predict performance of cascaded NL components

Cascaded Nonlinear Amplifiers:
X-parameters enable nonlinear simulation from measured data in the presence of mismatch

- Unambiguously identifiable from a simple set of measurements
- Extremely accurate for high-frequency, distributed nonlinear systems
- Fully nonlinear vector quantities (Magnitude *and* phase of all harmonics)
- Cascadable (correct behavior in mismatched environment)

X-parameters come from the Poly-Harmonic Distortion (PHD) Framework [3-6,12]



$$B_{1k} = F_{1k}(DC, A_{11}, A_{12}, \ldots, A_{21}, A_{22}, \ldots)$$
$$B_{2k} = F_{2k}(DC, A_{11}, A_{12}, \ldots, A_{21}, A_{22}, \ldots)$$

Port Index ↗ ↖ Harmonic (or carrier) Index

Spectral map of complex *large* input phasors to *large* complex output phasors

Black-Box description holds for transistors, amplifiers, RF systems, etc.

X-parameters: Simplest Case - driven with single large tone at port 1 [1] (derivation in lecture 2)

$$B_{e,f} = F_{e,f}(DC, A_{11}, A_{12}, \ldots, A_{21}, A_{22}, \ldots)$$

Concept: simplify general nonlinear spectral mapping by spectral linearization

$$B_{e,f} = X_{ef}^{(F)}(|A_{11}|)P^{f} + \sum_{g,h} X_{ef,gh}^{(S)}(|A_{11}|)P^{f-h} \cdot A_{gh} + \sum_{g,h} X_{ef,gh}^{(T)}(|A_{11}|) \quad P^{f+h} \cdot A_{gh}^{*}$$

Perfectly matched response

Mismatch terms: linear in A_{gh}

Mismatch terms: linear in A_{gh}^{*}

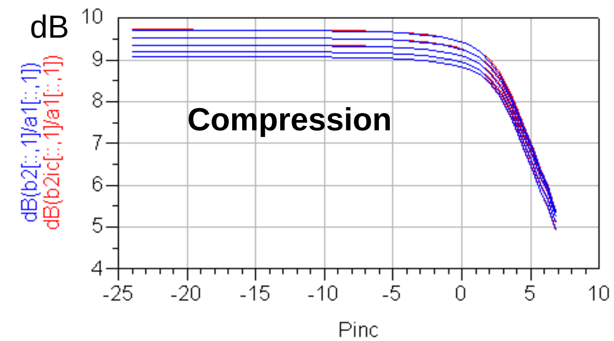
Not both g and h =1 in sums

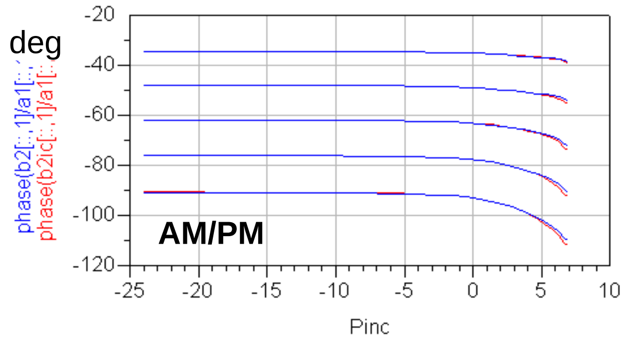
$$P = e^{j\varphi(A_{11})}$$

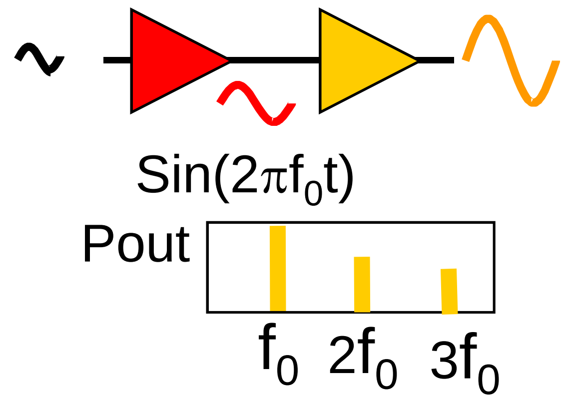
Phase terms come from time-invariance:

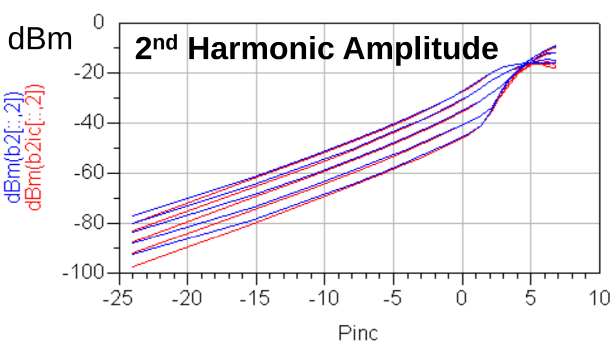
“Output of delayed input is just the delayed output”

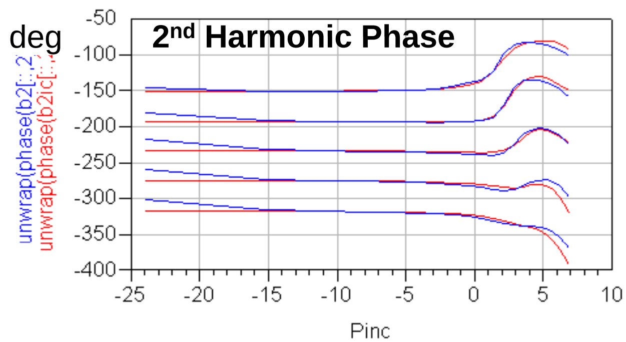
X-parameter Results: Cascadability of Nonlinear Blocks







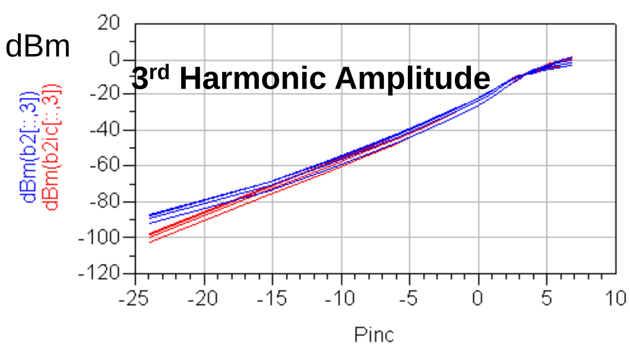


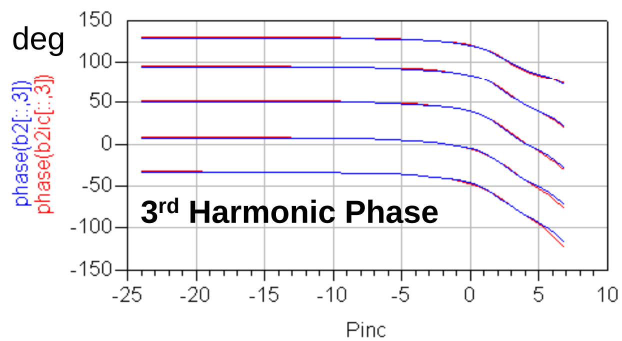


Cascaded PHD models

Cascaded Ckt. Models

0.6GHz – 6.0GHz

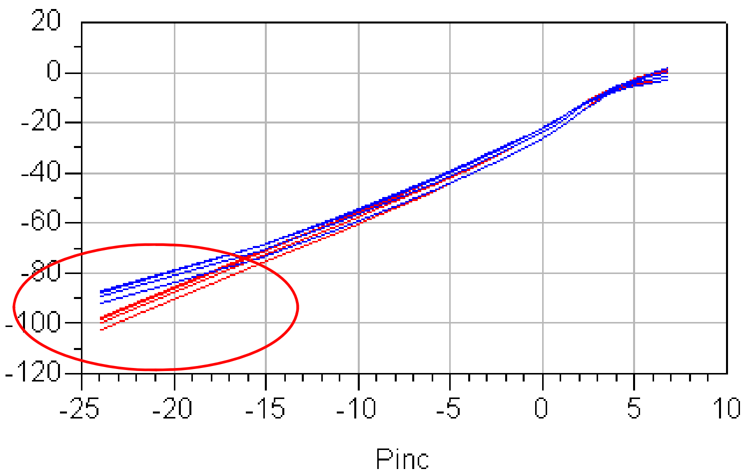


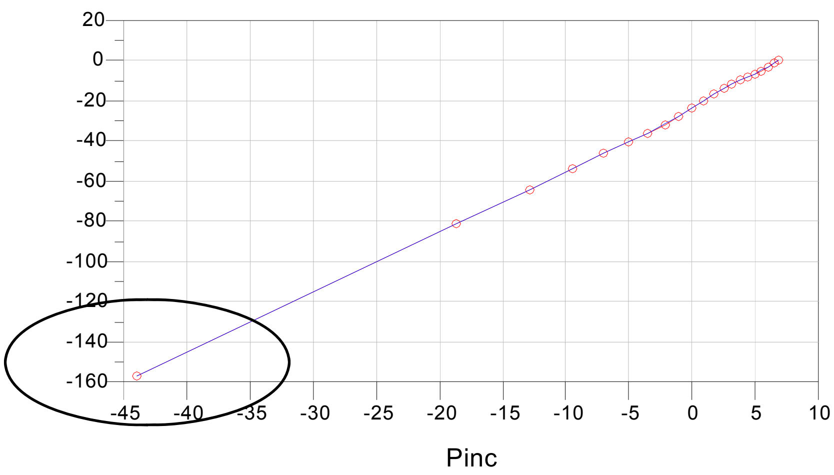


Does for distortion of *nonlinear components* what S-parameters do for linear components

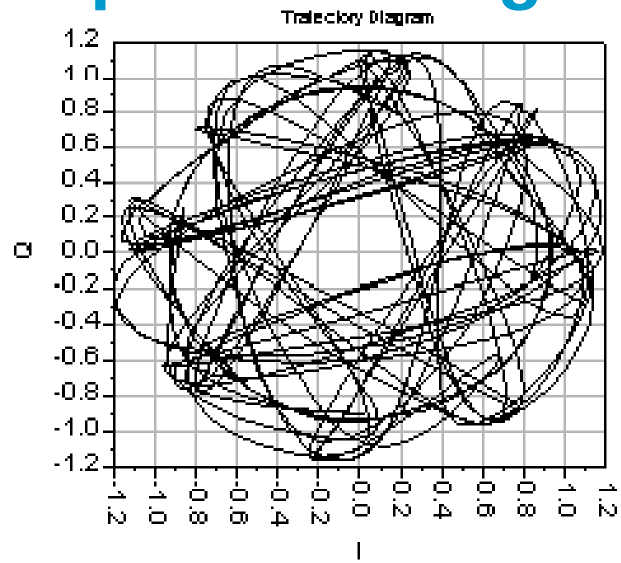
Improved Asymptotic Behavior

Volterra Theory Constraints Added for
Improved asymptotic behavior at low power



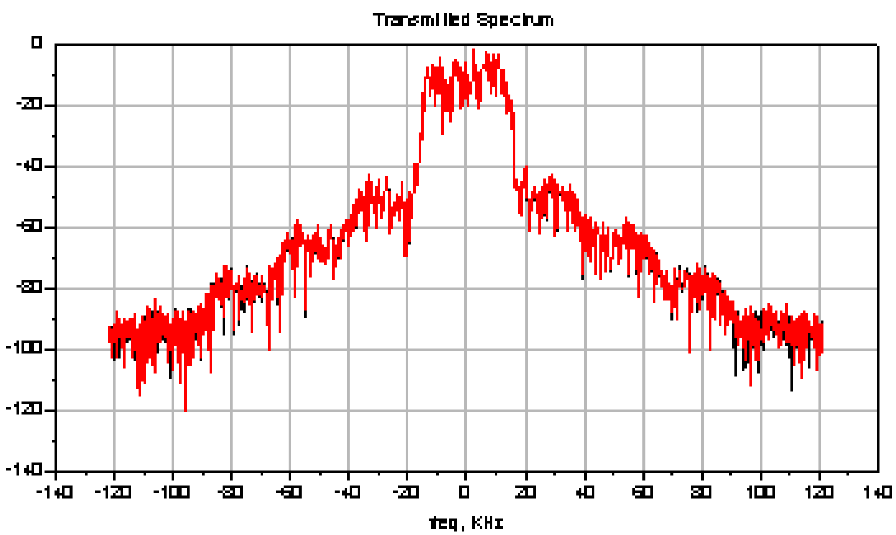


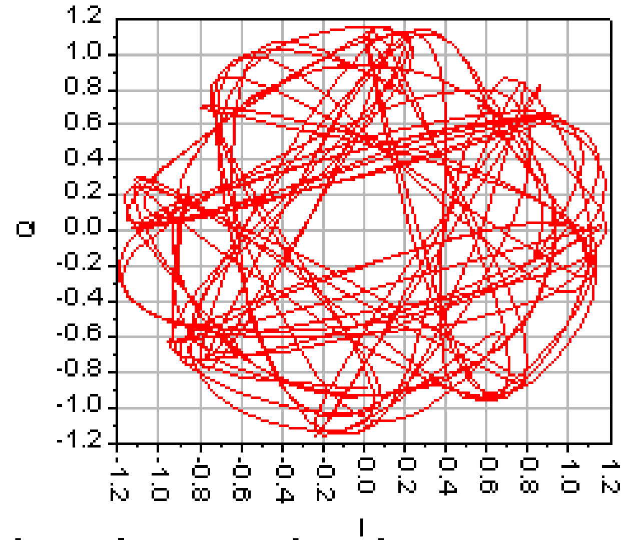
X-parameters: HMMC 5200 Response to Digital Modulation



Circuit Model

X-parameters generated from ckt model

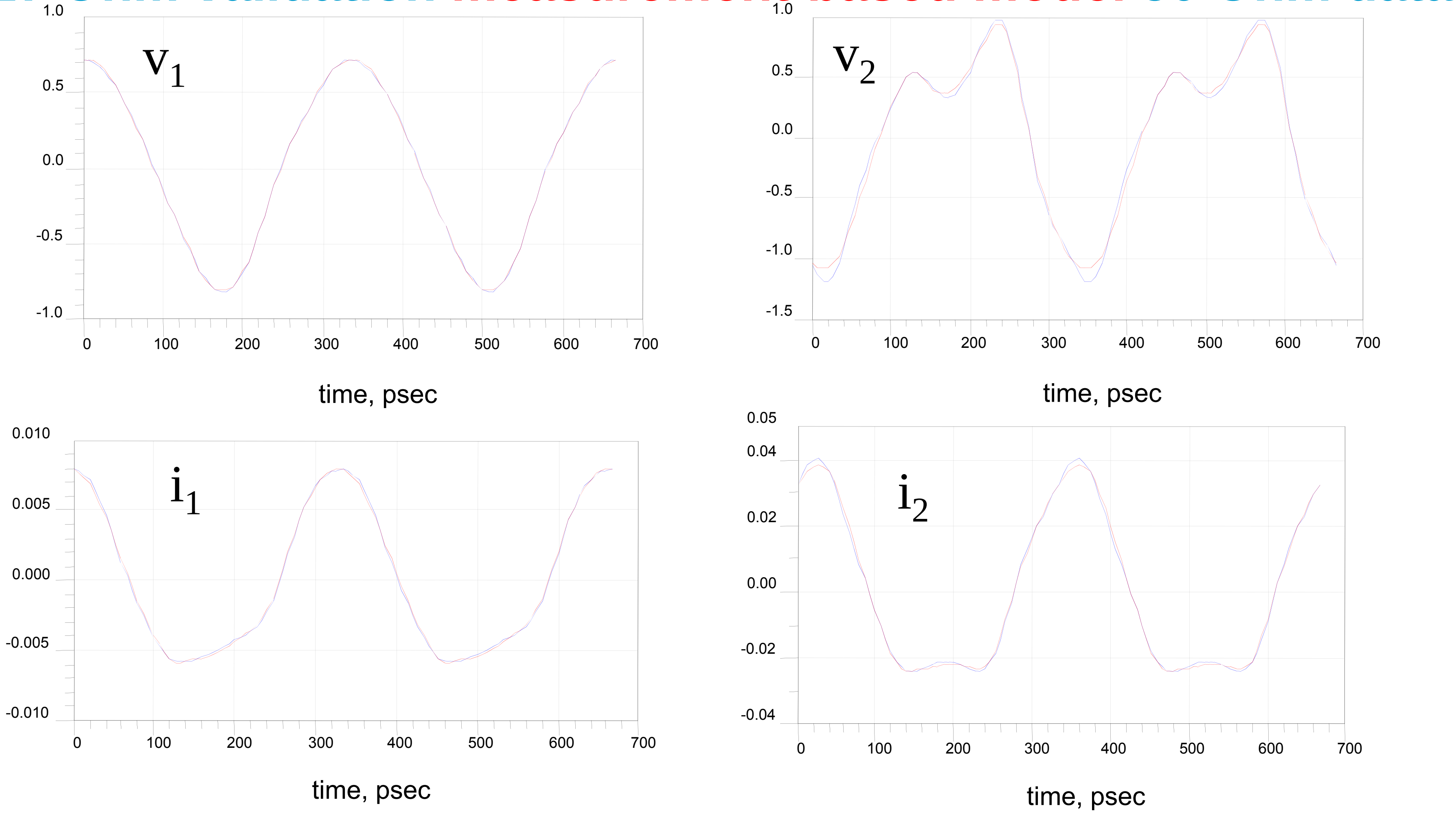




Excellent Results from Simple Excitations

X-parameter Results: Transportability

27 Ohm validation measurement-based model 50 Ohm data



Measurement-Based X-parameter Model

Independent NVNA Data

Rough Comparison of Methods and Applicability

NLTSA

Works in TA, HB, Envelope

Excellent for strongly nonlinear, but lumped (low order ODE) systems

Training non-algorithmic

Experiment design not fully solved

Not as robust for convergence

Scales well with complexity

Great gains in simulation speed

X-Parameters

Frequency Domain natural for highly linear, distributed, broad-band ckts

Experiment Design completely solved

Highly automated Model Identification

Works in HB & Envelope

Very robust for convergence

Always accurate if sampled densely

Complexity increases rapidly for multiple tones

Outline

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Summary and Conclusions

Envelope Domain for Long-Term Memory [7,8]

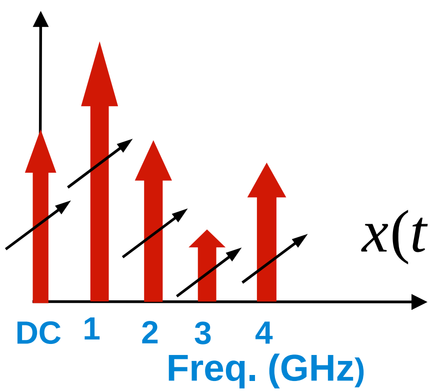
Applies to systems under large-signal modulated drives

Time-varying spectra for all inputs, outputs, & state variables

Perfectly suited for Circuit Envelope Analysis

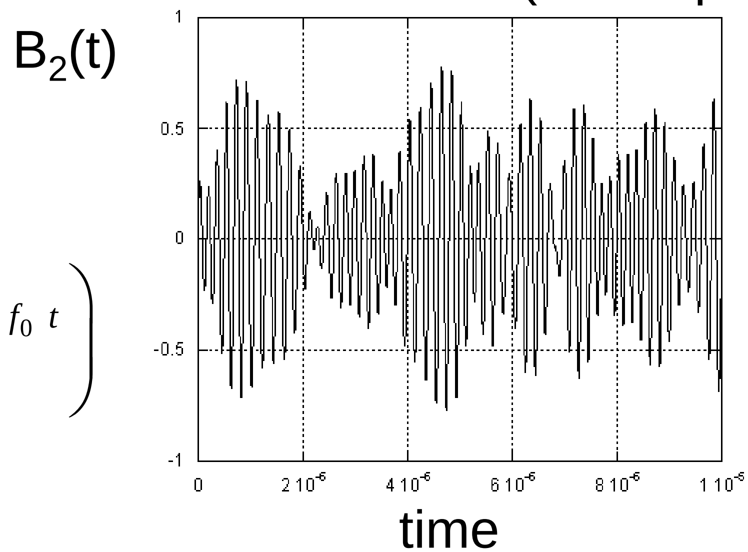
Well-matched for data from Nonlinear Vector Network Analyzer

Time-varying spectrum



$$x(t) = \mathrm{Re}\left(\sum_{h=0}^{H} X_h(t) \quad e^{j\ 2\pi\ h\ f_0\ t} \right)$$

Time Domain (envelope)



$X_h(t)$ set of *complex* (amplitude and phase) *waveforms* at each harmonic index h
Modeling problem: *map input envelopes to output envelopes*

Envelope Domain for Long-Term Memory [7,8]

Merge Frequency and Time Domains

Spectral mapping $B_{pk} = X_{pk}^{(F)}(A_{11}, A_{12}, \ldots, A_{21}, A_{22}, \ldots)$
$\rightarrow$ a differential equation *in the envelope domain*

$$\hat{B}_k = f_k(\hat{B}_k^{(1)}(t), \ldots, \hat{B}_k^{(n)}(t), \hat{A}_l(t), \hat{A}_l^{(1)}(t), \ldots, \hat{A}_k(t), \ldots, \hat{A}_k^{(m)}(t))$$

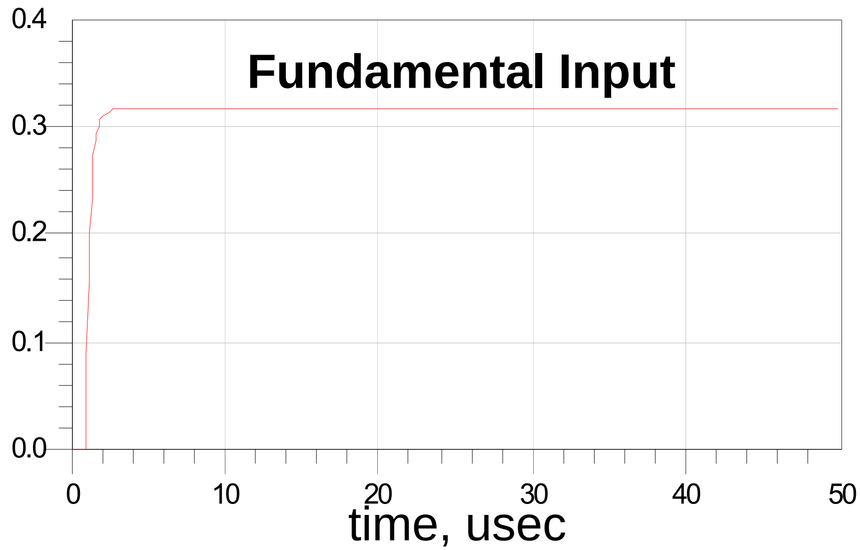
Order of time derivative
Envelope or carrier index

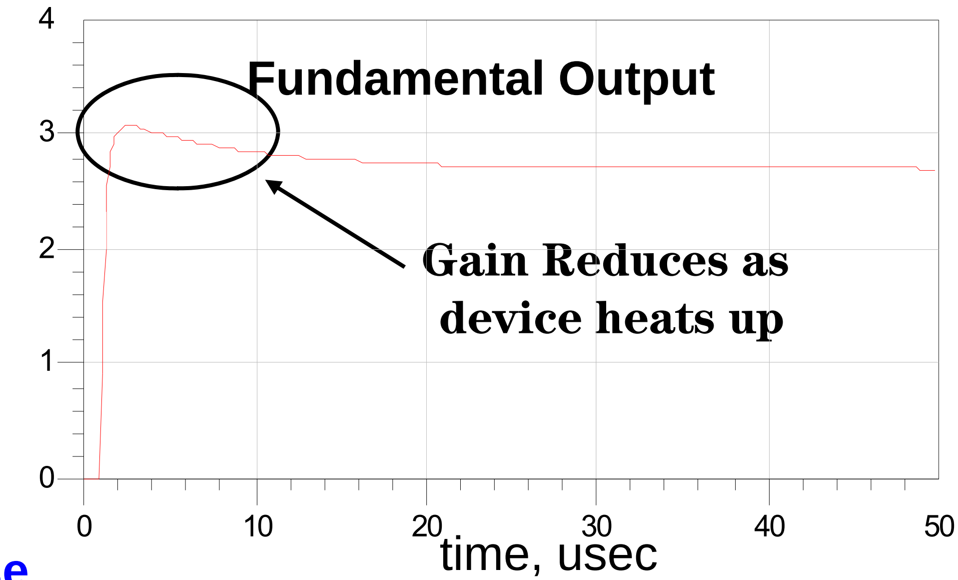
Example:

$$\hat{B}_{21}(t) = f_{21}(\hat{B}_{20}(t), \hat{A}_{11}(t))$$

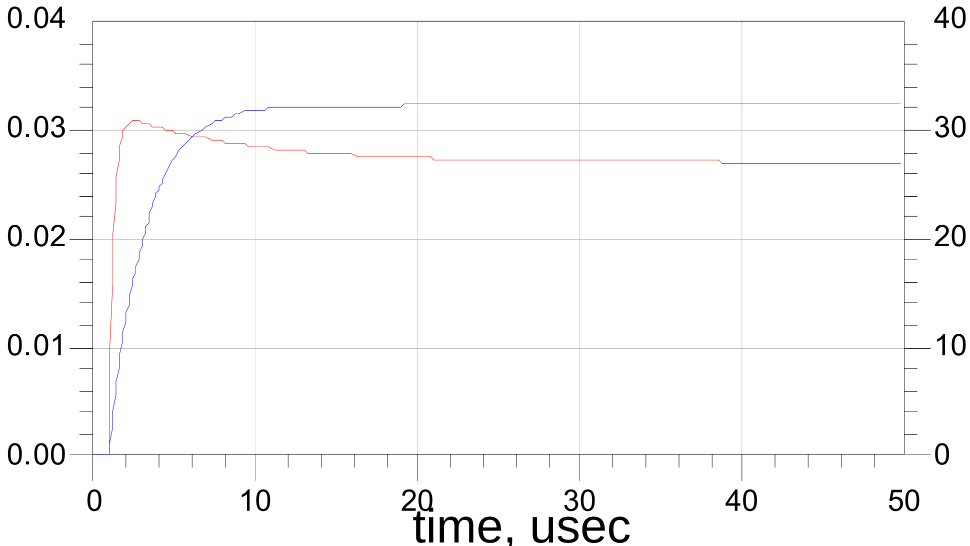
$$\frac{d\hat{B}_{20}(t)}{dt} = g(\left\langle \left|\hat{A}_{11}(t)\right|^2 \right\rangle, \hat{B}_{21}(t))$$

Envelope Model: Amplifier with Self-Heating [8]





Third Harmonic Output Mag & Phase



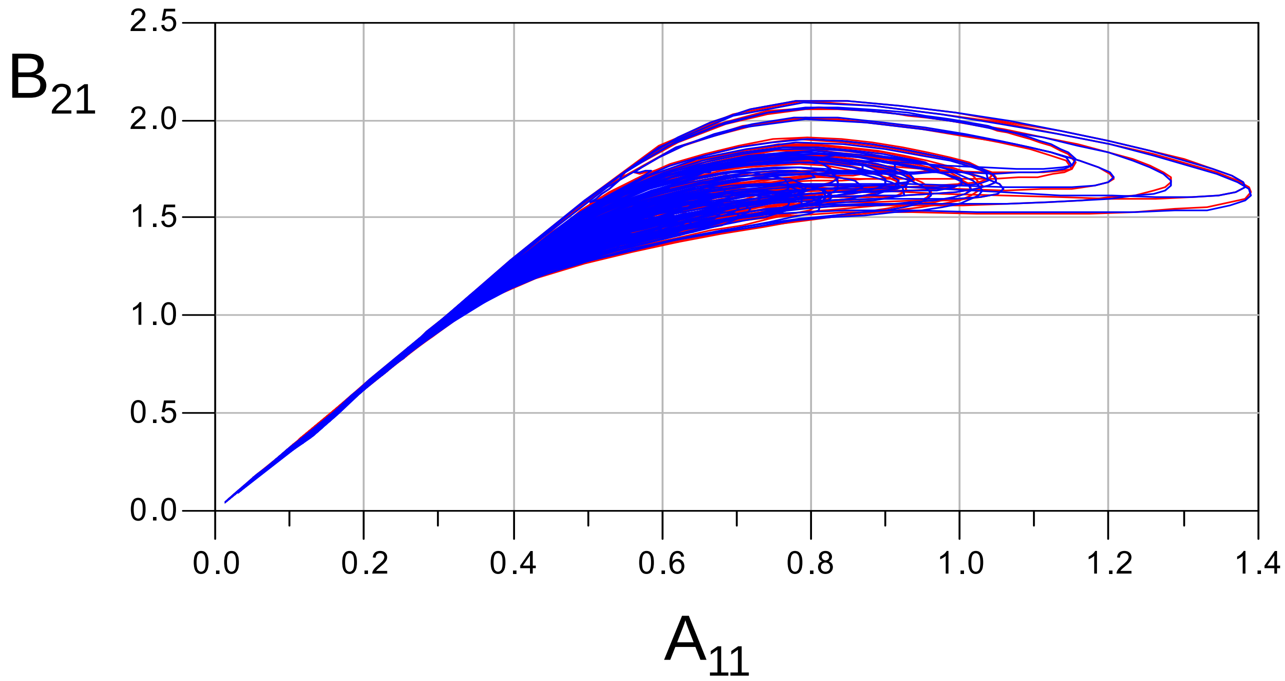
Pulsed RF signal at 1GHz:
Thermal Time Const. 10usec

Systematic approach to identifying "hidden" state variables for long-term memory IMS2007 [13]

Dynamic Long-Term Memory PHD Models Envelope Differential Equations in ADS [7,8,13]

Verspecht et al in *2007 International Microwave Symposium Digest* [13]

X-parameters with dynamic memory (red)
compared to circuit-level model (blue)



Conclusions

Powerful nonlinear device & behavioral modeling approaches in time, frequency, and mixed domains have been presented

- X-parameters are mature. Commercial solutions to measure, model, and simulate are available, supported, and expanding (see lecture 2).
- Time-domain (NLTSA) techniques could become practical soon.
- Envelope domain (dynamic X-parameters) is attractive for memory.

Emergence of commercially available Large-Signal HW & SW

- e.g. NVNA on modern PNA-X platform [9,14]
- e.g. nonlinear simulators with built-in XnP components & X-param analysis

Great opportunity for applications

- Specification of active components by *X-parameters*
- Device and behavioral modeling applications of NVNA measurements
- Stability analysis and matching power amplifiers under drive
- Active Signal Integrity

References

[1] J. Wood, D. E. Root, N. B. Tufillaro, "A behavioral modeling approach to nonlinear model-order reduction for RF/microwave ICs and systems," *IEEE Transactions on Microwave Theory and Techniques,* Vol. 52, Issue 9, Part 2, Sept. 2004 pp. 2274-2284

[2] Agilent HMMC-5200 DC-20 GHz HBT Series-Shunt Amplifier, Data Sheet, August 2002.

[3] J. Verspecht, M. Vanden Bossche, F. Verbeyst, "Characterizing Components under Large Signal Excitation: Defining Sensible `Large Signal S-Parameters'?!," in *49th IEEE ARFTG Conference Dig*., Denver, CO, USA, June 1997, pp. 109-117.

[4] J. Verspecht, D.E. Root, J. Wood, A. Cognata, "Broad-Band, Multi-Harmonic Frequency Domain Behavioral Models from Automated Large-Signal Vectorial Network Measurements," in *2005 IEEE MTT-S International Microwave Symposium Digest*, Long Beach, CA, USA, June 2005.

[5] D. E. Root, J. Verspecht, D. Sharrit, J. Wood, and A. Cognata, "Broad-Band Poly-Harmonic Distortion (PHD) Behavioral Models from Fast Automated Simulations and Large-Signal Vectorial Network Measurements", *IEEE Transactions on Microwave Theory and Techniques* Vol. 53. No. 11, November, 2005 pp. 3656-3664

[6] J. Wood, D. E. Root, editors, **Fundamentals of Nonlinear Behavioral Modeling for RF and Microwave Design**, 1st ed. Norwood, MA, USA, Artech House, 2005.

[7] Root et al US Patent Publication # US2005102124 AA, Published 2005

[8] D. E. Root, D. Sharrit, J. Verspecht, "Nonlinear Behavioral Models with Memory: Formulation, Identification, and Implementation," *2006 IEEE MTT-S International Microwave Symposium Workshop (WSL) on* Memory Effects in Power Amplifiers

[9] Blockley et al *2005 IEEE MTT-S International Microwave Symposium Digest*, Long Beach, CA, USA, June 2005.

[10] Jan Verspecht Patent US 7,038,468 B2 (issued May 2, 2006 based on a provisional patent 60/477,349 filed on June 11, 2003)

[11] Soury et al *2005 IEEE International Microwave Symposium Digest* pp. 975-978

[12] J. Verspecht and D. E. Root, "Poly-Harmonic Distortion Modeling," *in IEEE Microwave Theory and Techniques Microwave Magazine*, June, 2006.

[13] J. Verspecht, D. Gunyan, J. Horn, J. Xu, A. Cognata, and D.E. Root, "Multi-tone, Multi-Port, and Dynamic Memory Enhancements to PHD Nonlinear Behavioral Models from Large-Signal Measurements and Simulations," *2007 IEEE MTT-S Int. Microwave Symp. Dig.*, Honolulu, HI, USA, June 2007.

[14] Horn et al 2008 Power Amplifier Symposium, Orlando, Jan. 2008