

Computational Electromagnetics for Circuits Simulations--the Challenges

W.C. Chew,¹ M.K. Li,¹ and L.J. Jiang²

**¹CCML, Department of Electrical and Computer Engineering
University of Illinois, Urbana, IL 61801-2991**

²IBM TJ Watson Research Center, Yorktown Heights, NY 10598

Future Direction in IC and Packaging Design Workshop

EPEP 2005

October 23, 2005

Austin, TX

Acknowledgements

- **Junsheng Zhao, Chris Pan, Yunhui Chu, I-Ting Chiang, Yuan Liu, Zhiguo Qian, Andrew Hesford, Greg Sorenson, Shinichiro Ohnuki, Hsueh-Yung Chao**

Outline

- Solving Maxwell's equation in the “twilight zone”?
- Solving Laplace's Equation
 - Center of Charge Method
 - Layered Medium Modeling
- Solving problems in the low-frequency regime
- Contact region modeling
- A new broadband fast solver
- Equivalence Principle Algorithm (EPAL)
- Conclusions

Electromagnetics at Low Frequency

■ Decoupling of Electric and Magnetic Fields at DC

The Maxwell's equations can be written at zero frequency as follows:

$$\nabla \times \mathbf{E} = 0, \quad \nabla \cdot \epsilon \mathbf{E} = \rho = \lim_{\omega \rightarrow 0} \nabla \cdot \mathbf{J} / i\omega$$

$$\nabla \times \mathbf{H} = \mathbf{J}, \quad \nabla \cdot \mu \mathbf{H} = 0$$

It is easy to see that the electric and magnetic fields are decoupled at DC and the current can be decomposed as

$$\mathbf{J} = \mathbf{J}_{sol} + \mathbf{J}_{irr}, \quad \mathbf{J}_{irr} \sim O(\omega), \quad \mathbf{J}_{sol} \gg \mathbf{J}_{irr}, \quad \omega \rightarrow 0$$

divergence-free



\mathbf{H}

curl-free



\mathbf{E}

Another POV

- Helmholtz system when k tends to zero:

$$\left(\nabla^2 + k^2\right)\phi = s$$

$$\nabla^2\phi = s$$

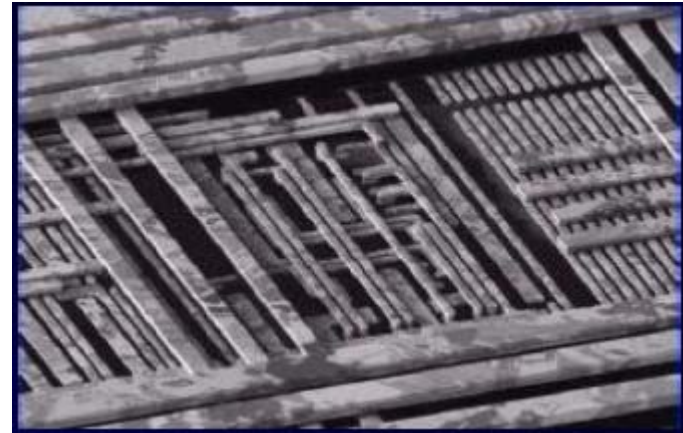
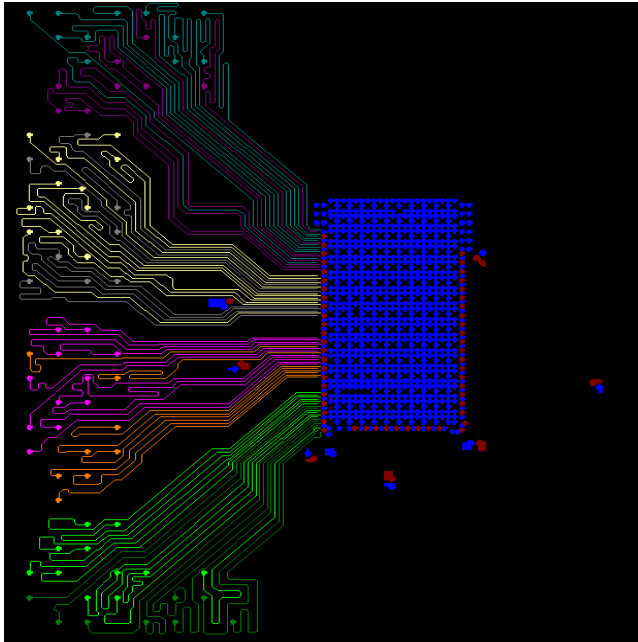
- Electromagnetic system when k tends to zero:

$$\nabla \times \nabla \times \mathbf{E} - k^2 \mathbf{E} = i\omega\mu\mathbf{J}$$

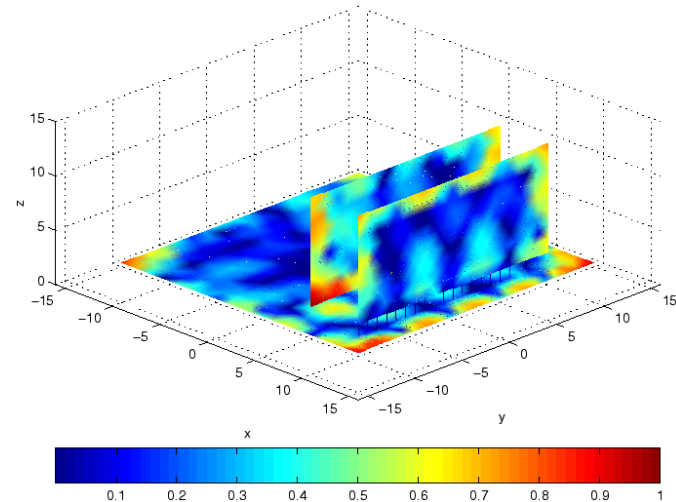
$$\nabla \times \nabla \times \mathbf{E} = i\omega\mu\mathbf{J}$$

- A singular perturbation problem!

Multi-Scale Problem--Vertically



- **Multi-scale**
- **DC to tens of GHz**
- **Stable and fast solvers**



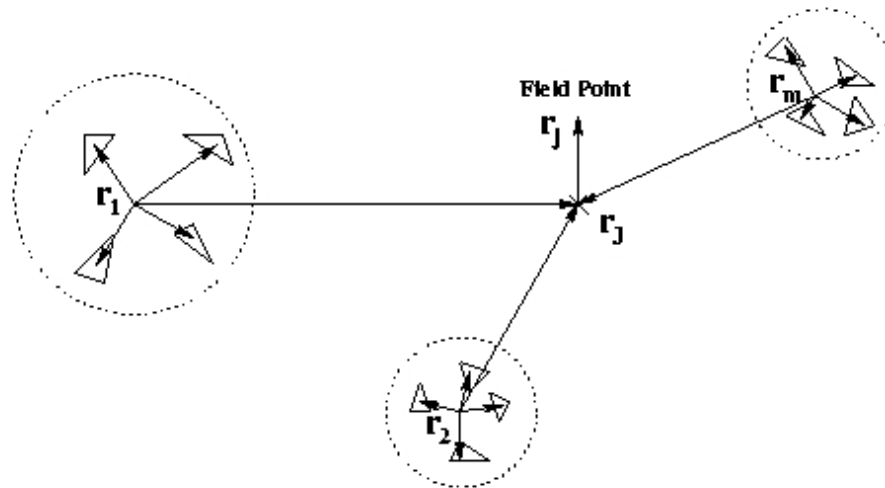


Static Problems—Laplace's Equation

Some Popular Methods

- **Random walk**
- **Static Fast Multipole for Homogeneous Media**

Center of Charge Method (CCM) for Laplace's Equation



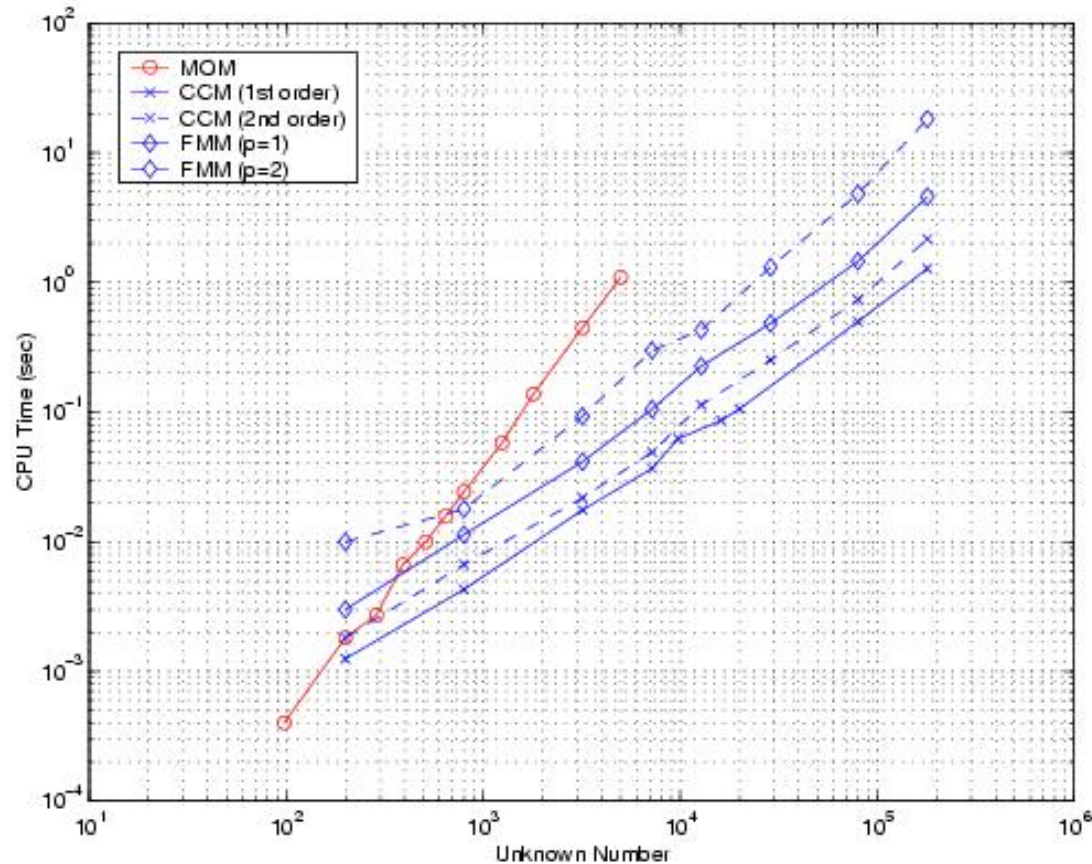
First Order

$$\Phi(\mathbf{r}_j) \approx \frac{\sum_i \int_{s_i} \sigma_i(\mathbf{r}') dr'}{|\mathbf{r}_j - \mathbf{r}_I|} \quad \mathbf{r}_I = \frac{\sum_i Q_i \cdot \mathbf{r}_i}{\sum_i Q_i}$$

Second Order

$$\Phi(\mathbf{r}_j) \approx \sum_m \frac{Q_{Im}}{|\mathbf{r}_J - \mathbf{r}_{Im}|} - \sum_m \frac{Q_{Im}(\mathbf{r}_J - \mathbf{r}_{Im})}{|\mathbf{r}_J - \mathbf{r}_{Im}|^3} \cdot (\mathbf{r}_j - \mathbf{r}_J)$$

CCM CPU Time



CCM can be as much as 10 times faster than FASTCAP type algorithm

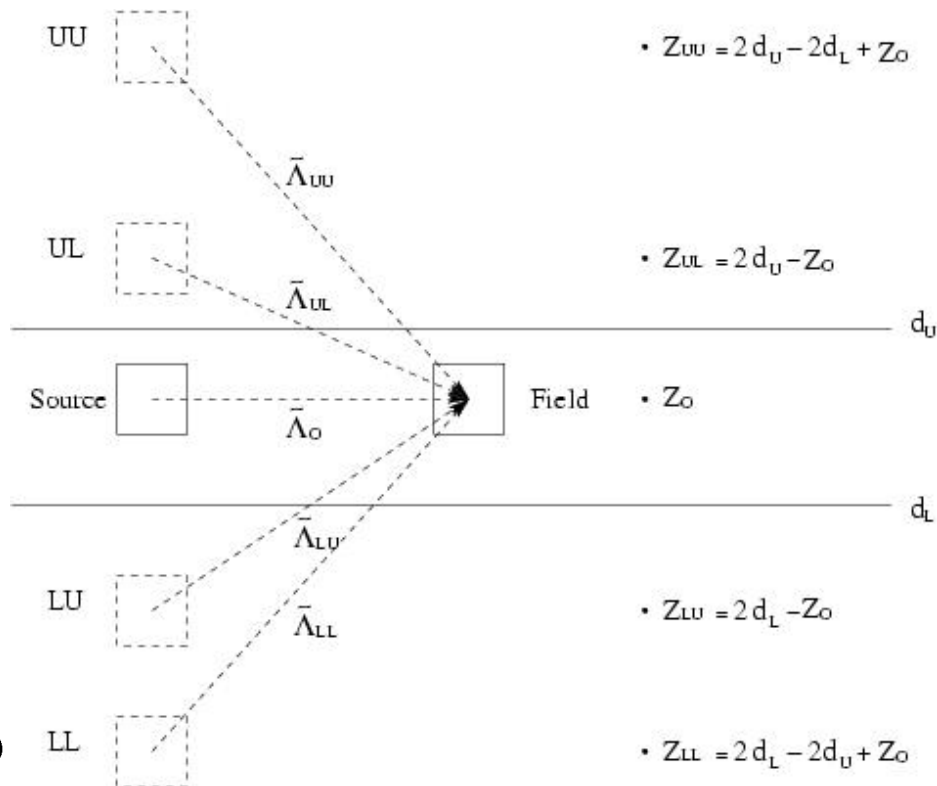
Multiple-Reflections in the Multilayer Green's Function (Laplace Problem)

Outgoing to local multipole translator

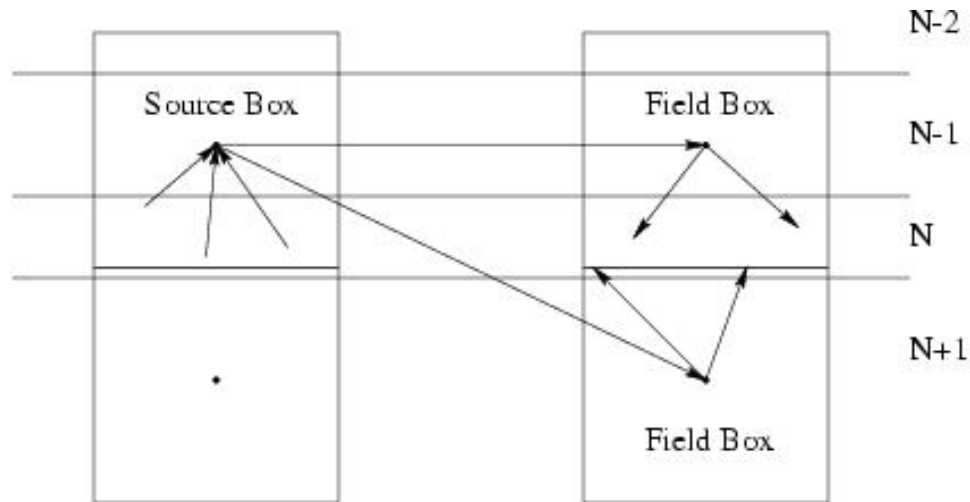
$$\alpha_{nm}^{jk}(\mathbf{r}_D) = \tilde{\Lambda}_o + \tilde{\Lambda}_{UU} + \tilde{\Lambda}_{UL} + \tilde{\Lambda}_{LU} + \tilde{\Lambda}_{LL}$$

Closed form of the outgoing to local multipole translator

$$\tilde{\Lambda}_{imag} = \sum_{s=0}^{N_{imag}} c_s B_{nm}^{jk} \left(\frac{a}{r}\right)^{j+n+1} Y_{(j+n)(m-k)}(\cos \theta, \phi)$$



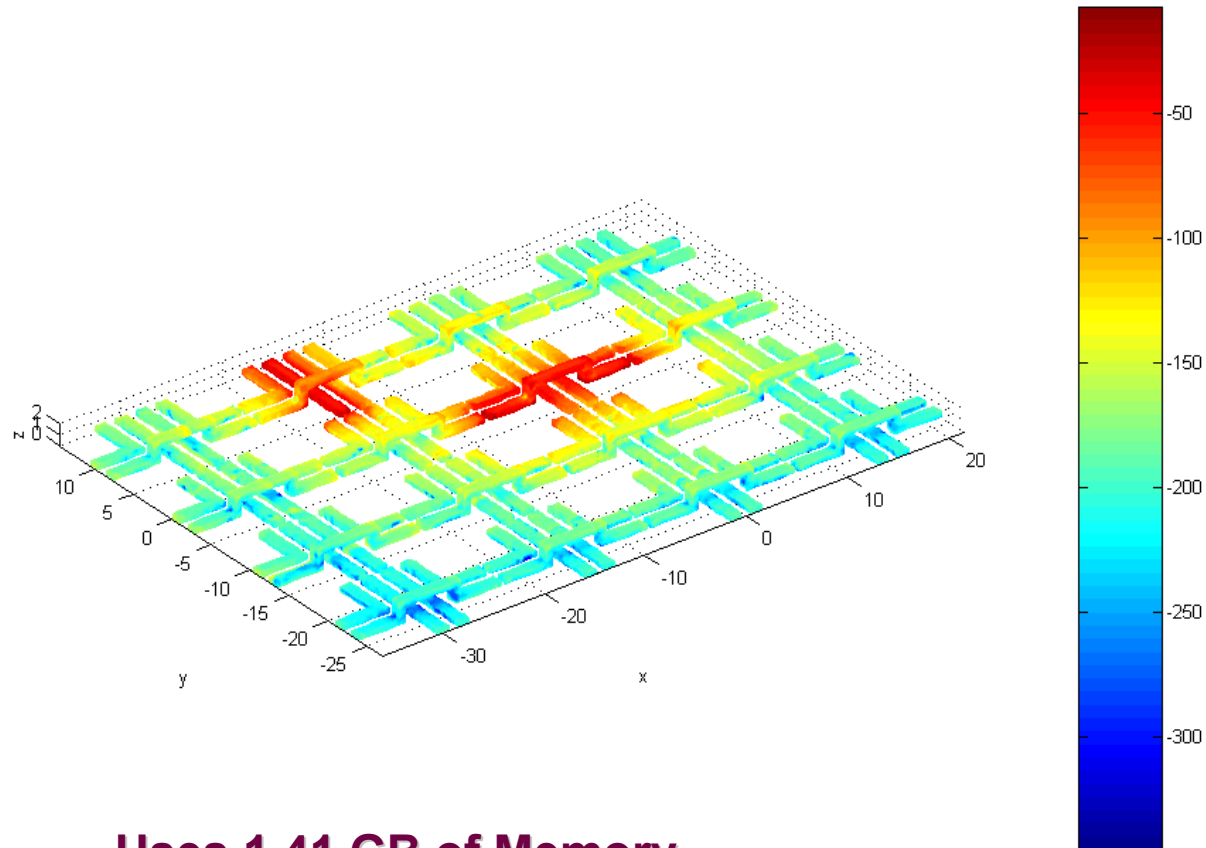
Multiple-Reflection in the Multilayers (Laplace Problem)



- ❑ Special box positions for the outgoing to local translator
- ❑ The substrate parameter is still used for the translation
- ❑ Closed forms of the translators are still valid

Solving Laplace's Equation

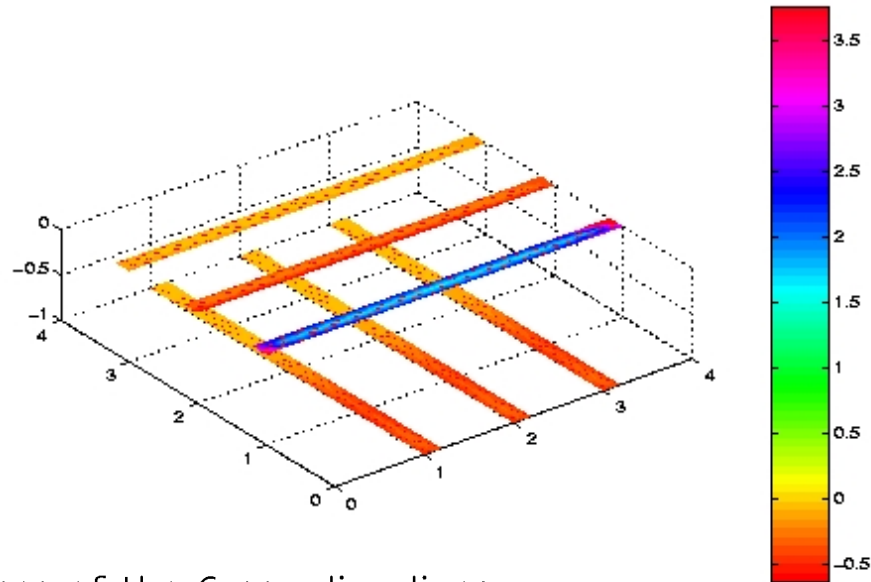
(1,204,352 unknowns)



Uses 1.41 GB of Memory

Six Coupling Lines—Acceleration with Discrete Complex Image Method (DCIM)

Charge Distribution of the Six Coupling Lines

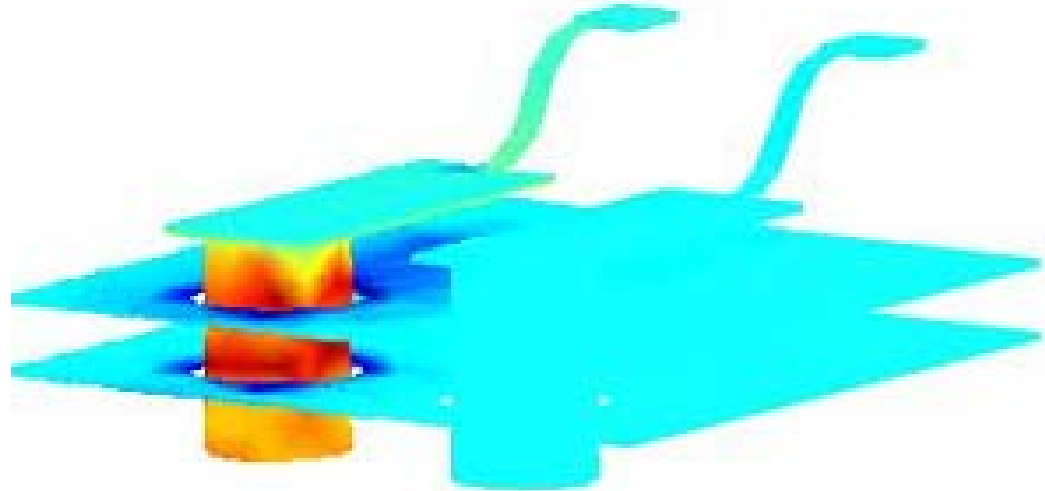
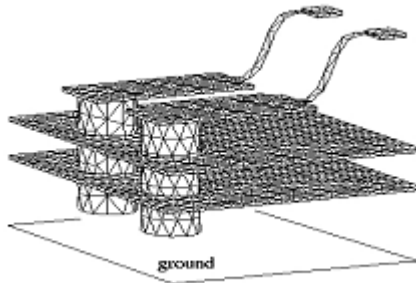
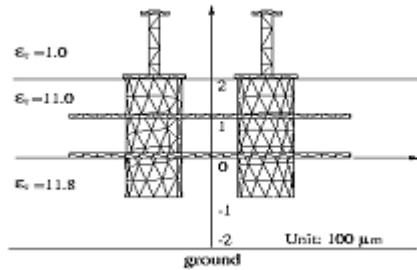


Capacitances of the 6 coupling lines

	C_{11} (pF/m)	C_{16} (pF/m)	C_{66} (pF/m)
Direct Method	47.0516	-5.01224	62.311
DCIM Method	47.1664	-5.01811	62.264

	FMA Setup CPU (sec)	MoM Setup CPU (sec)
Old Method	4880.12	115.526
New Method	22.152	4.376
Speed Up	220.3	26.4

Two Signal Lines—DCIM Acceleration



Setup time (sec) for the two signal lines

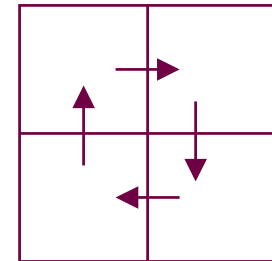
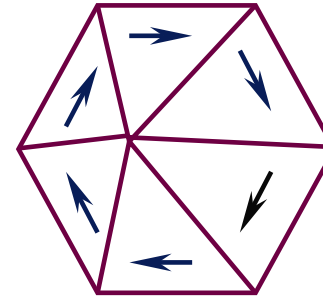
	FMA Setup CPU Time	MOM Setup CPU Time
Direct Method	15104.2	1366.49
DCIM Method	342.712	31.496
Speed Up	44.07	43.39

Solving Low Frequency Problems

Involves full Maxwell's equations
In electromagnetics, LF is decided by
size/wavelength ratio

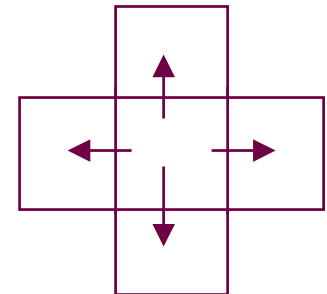
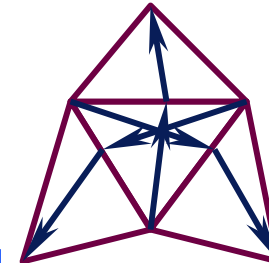
Low Frequency—Quasi-Helmholtz Decomposition

– **Loop Basis:** divergence free



– **Star Basis:** quasi-curl-free.

– **Tree Basis:** RWG basis with the basis along a cut removed.



The cut prevents the rest of the RWG basis (tree basis) from forming any loop.

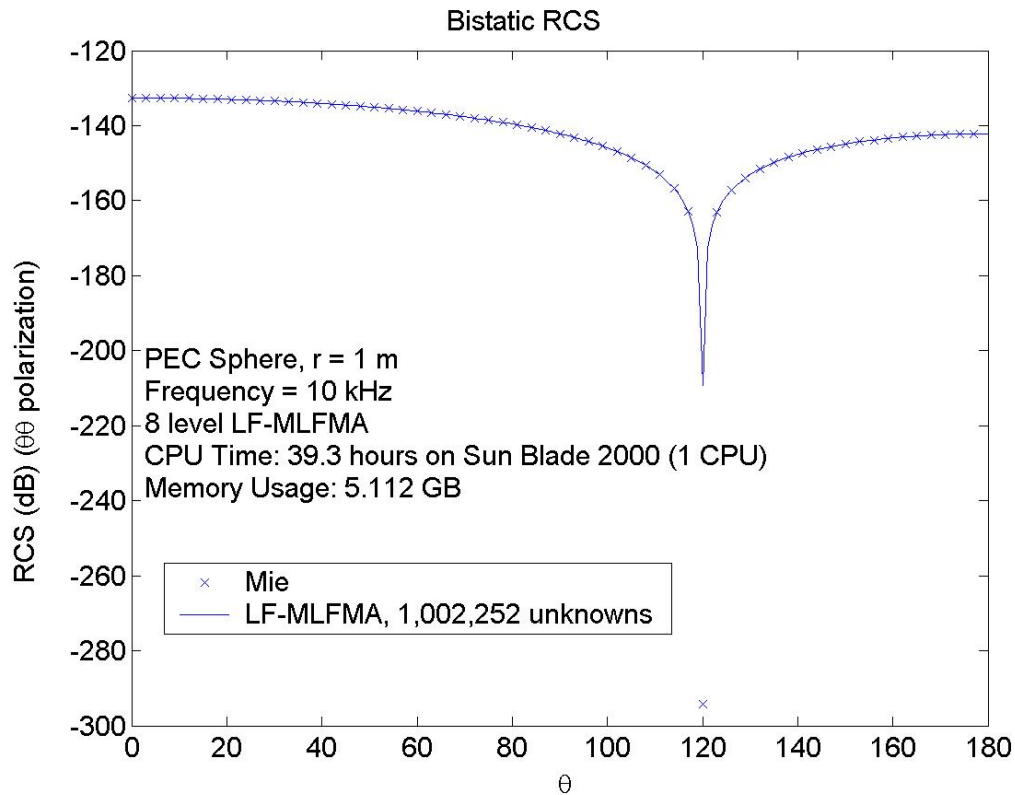
- ☞ LS or LT formulation isolates the contribution of vector potential and scalar potential.
- ☞ Information of vector potential will not be lost due to machine precision.

Acceleration Methods in Low Frequency

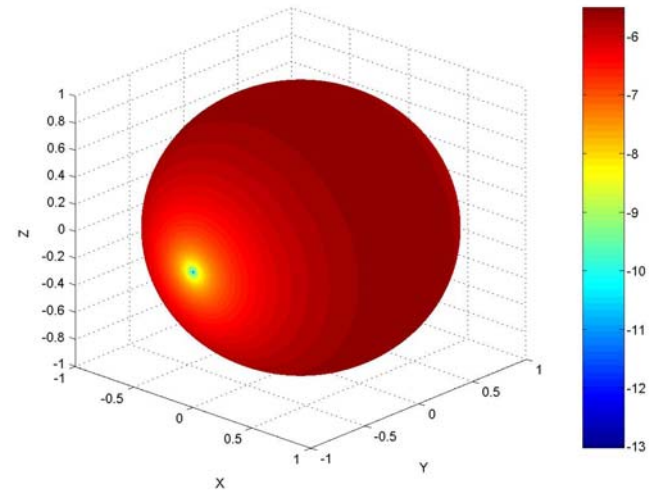
- **FFT**
- **Fast Multipole Algorithm**
- **Stabilization is necessary for the fast multipole method**
 - **Stabilized via frequency normalization technique**

Large-Scale Low-Frequency Computation with LF-FMA Acceleration

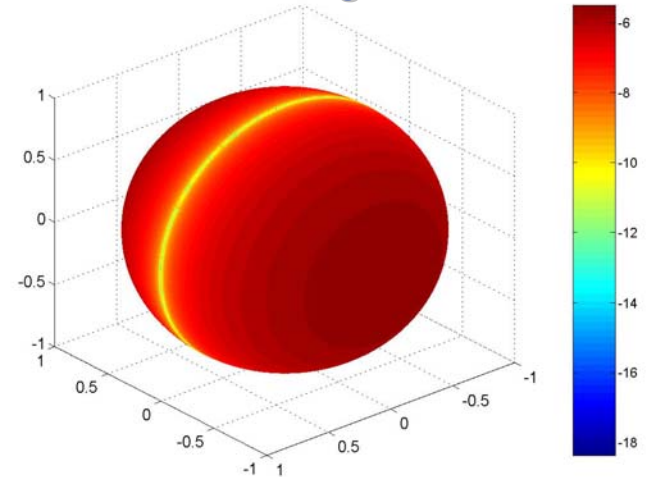
A PEC Sphere with Over One Million RWG Unknowns



Current

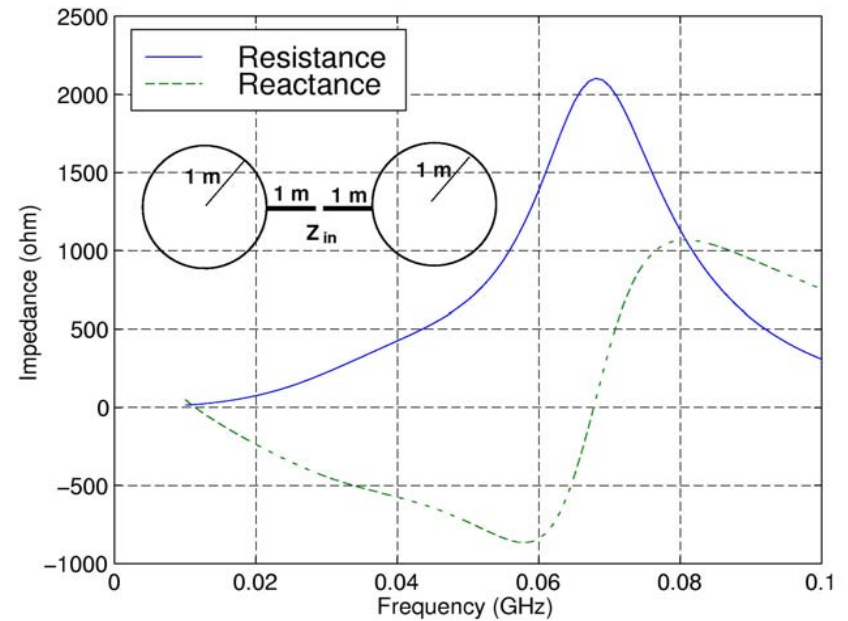
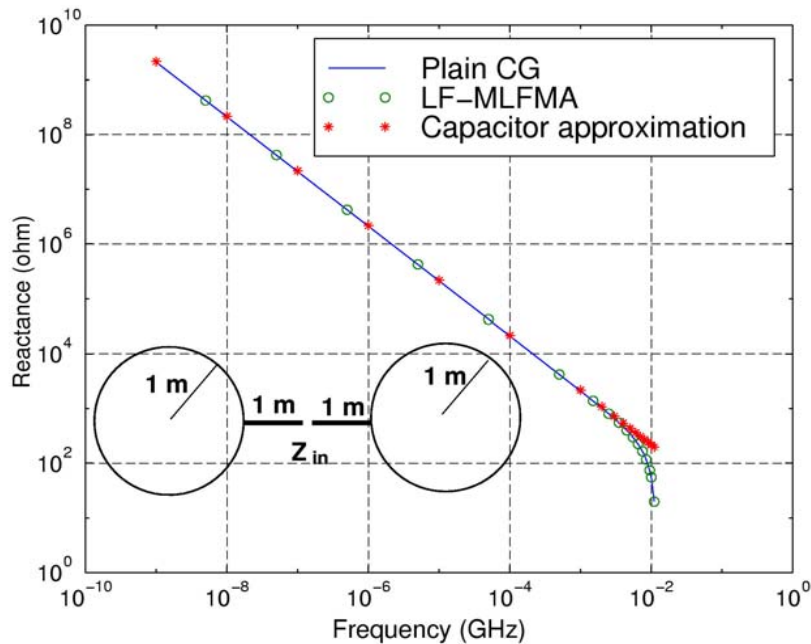


Charge



Hertzian Dipole from Zero to Microwave Frequencies

- Input admittance/impedance of a Hertzian dipole at very low frequencies and at higher frequencies



Low-Frequency Fast Multipole Algorithm

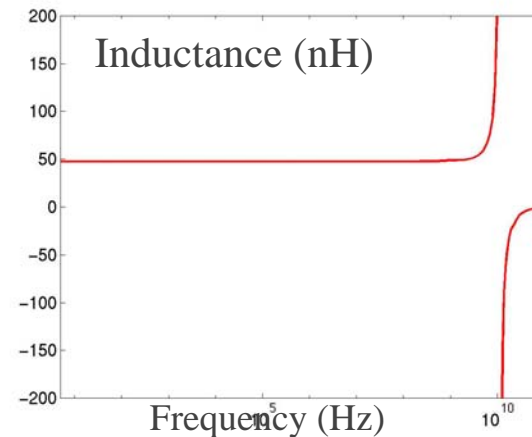
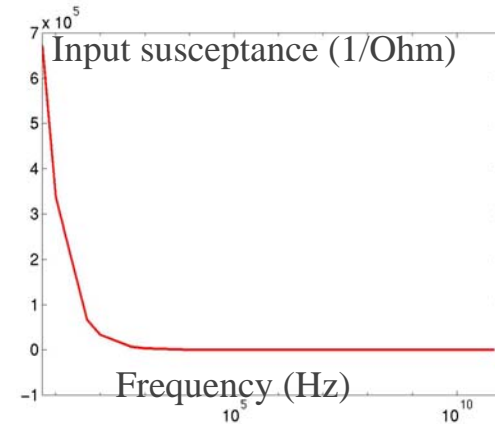
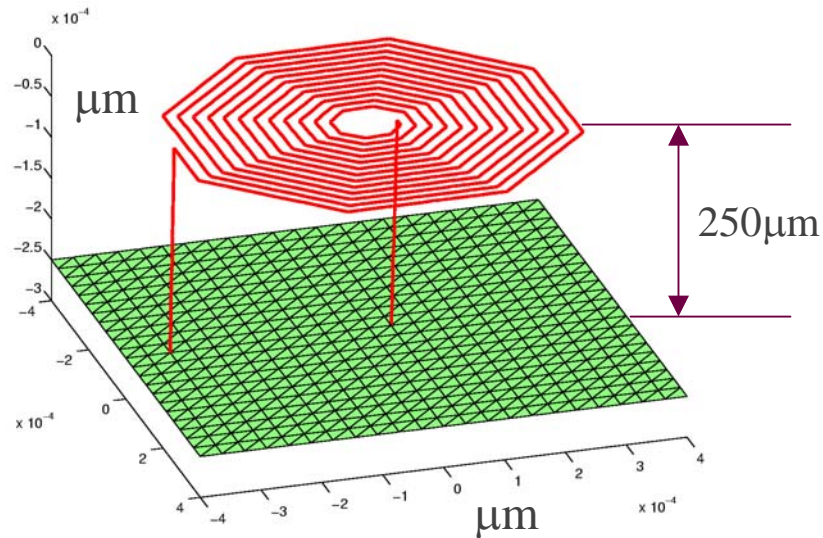
■ Wire Spiral Inductor with Ground Plane

12 Turns

Radius of wire = $2.76 \mu\text{m}$

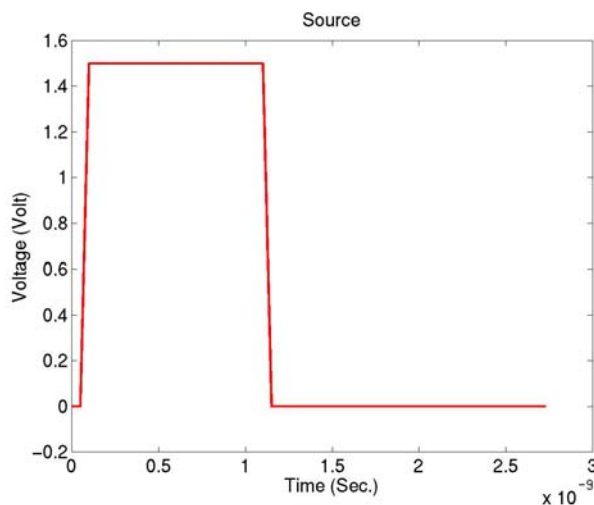
Track spacing = $24 \mu\text{m}$

Diameter of the inductor = $680 \mu\text{m}$



Cross-talk Simulation Suggested by Mazumder at Intel

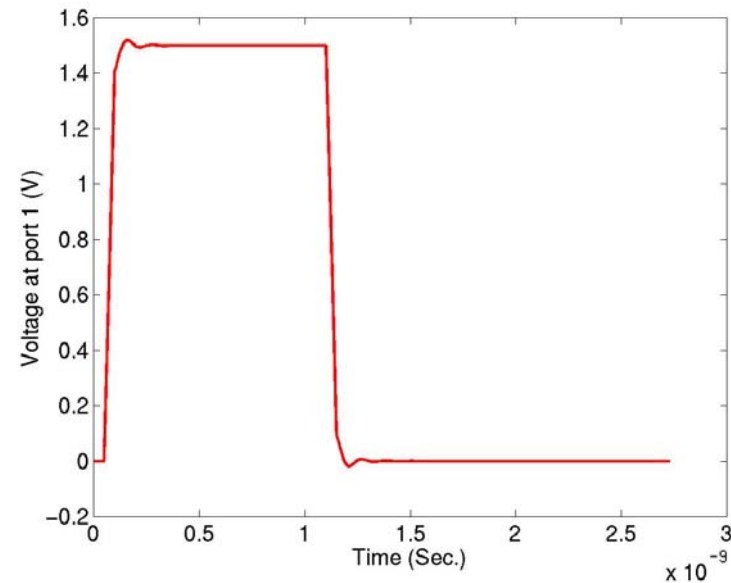
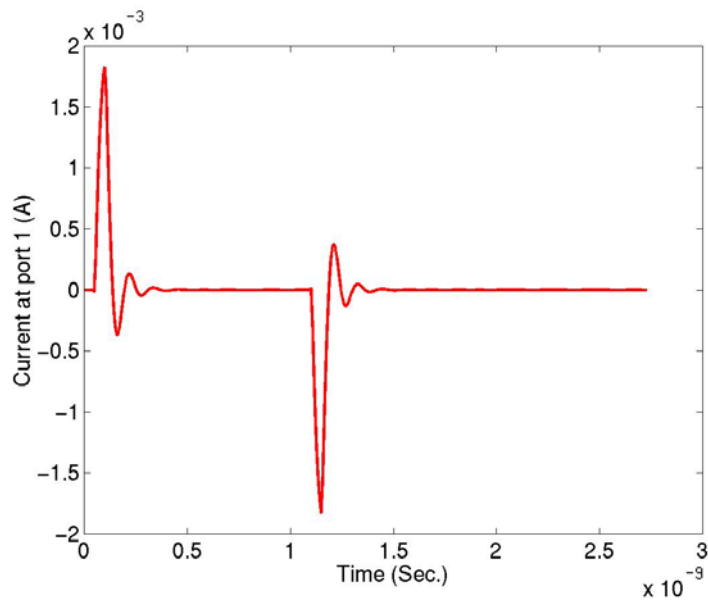
■ Setup for the Simulation



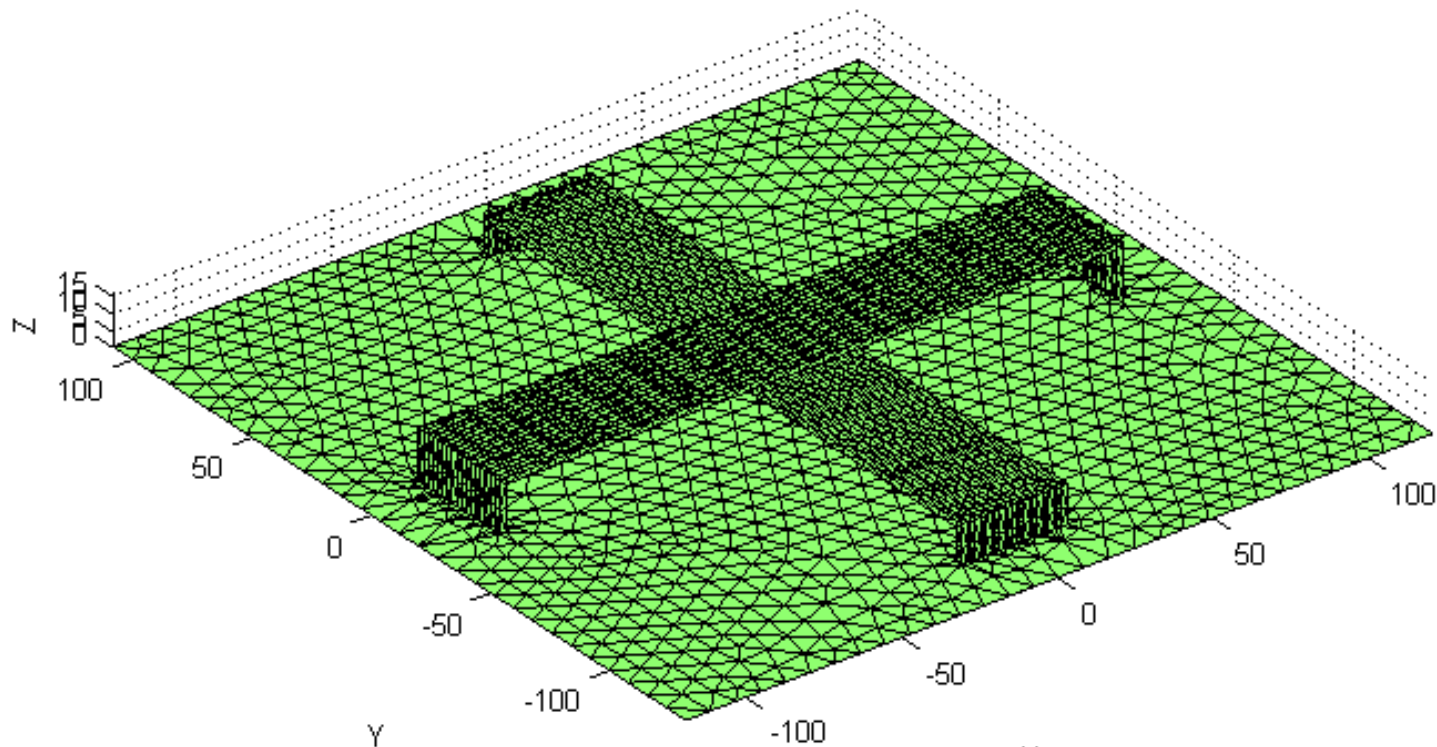
- Resistor: 50 (ohm)**
- Capacitance: 50E-15 Farad**
- Radius of the wire: 0.4 μm**
- Length of the wires: 2000 μm**
- Distant between axes of wires: 1.6 μm**
- Conductivity of the wire: 2.38E7**
- Conductivity of the plate: 2.893E7**
- Thickness of the plate: 2.0 μm**
- Height of the wire: 60 μm**
- Rising/falling time: 50 ps**

Cross-talk Simulation

■ Current and Voltage at Port 1

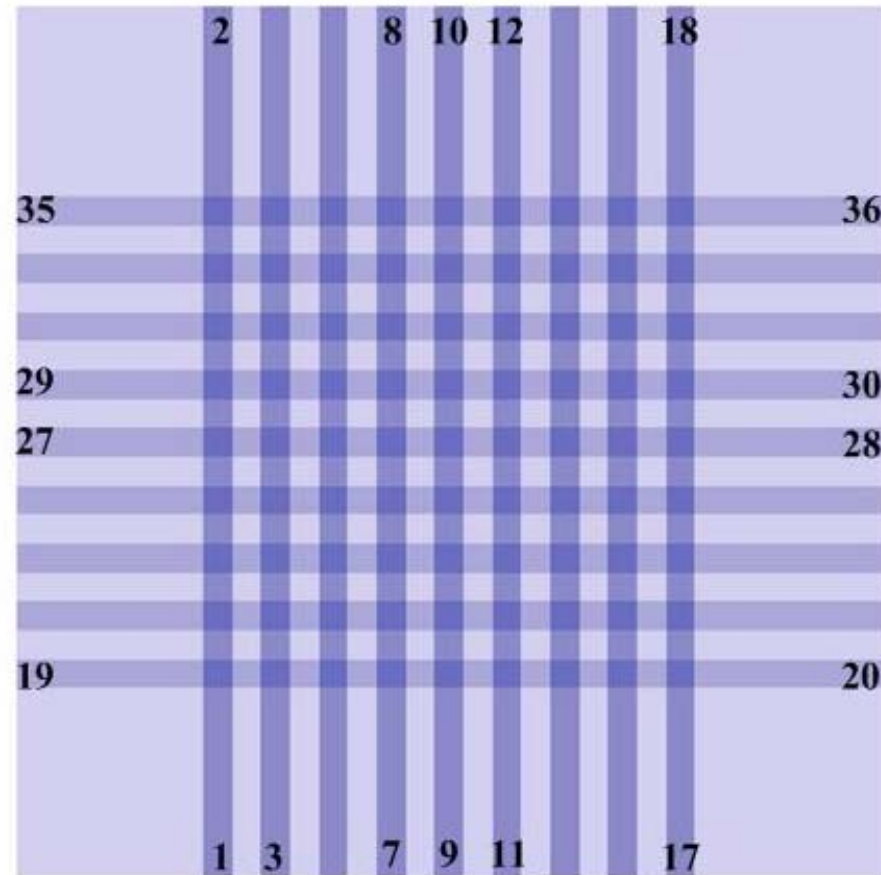


Example 3: a more complicated crisscross line structure with impedance load

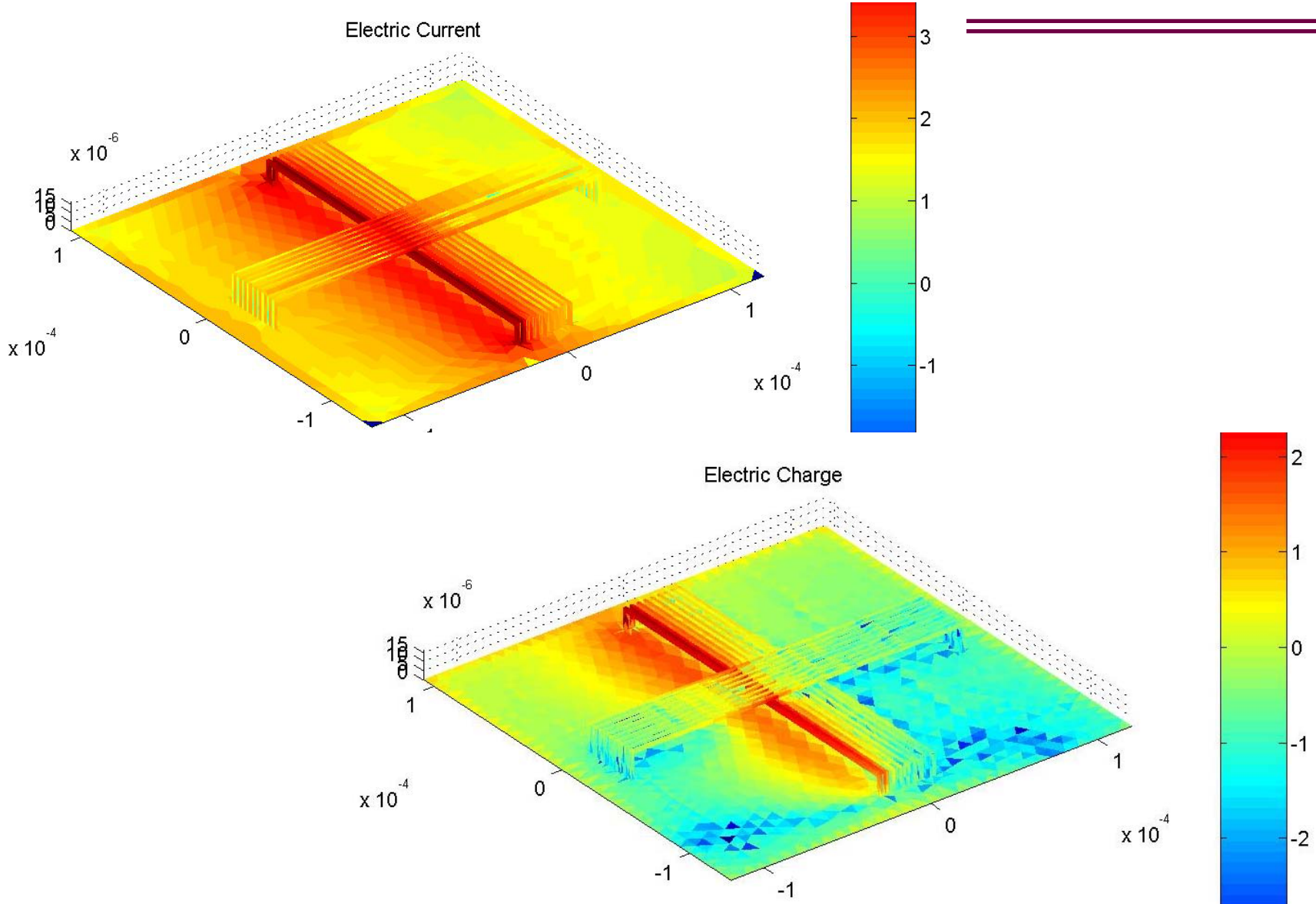


- Resistor: 50 (ohm)**
- Side width of the wire: 2 um**
- Length of the wires: 200 um**
- Distant between axes of wires: 4 um**
- Height of the axes of upper layer wires: 11 um**
- Height of the axes of upper layer wires: 15 um**
- Rising/falling time: 20 ps**

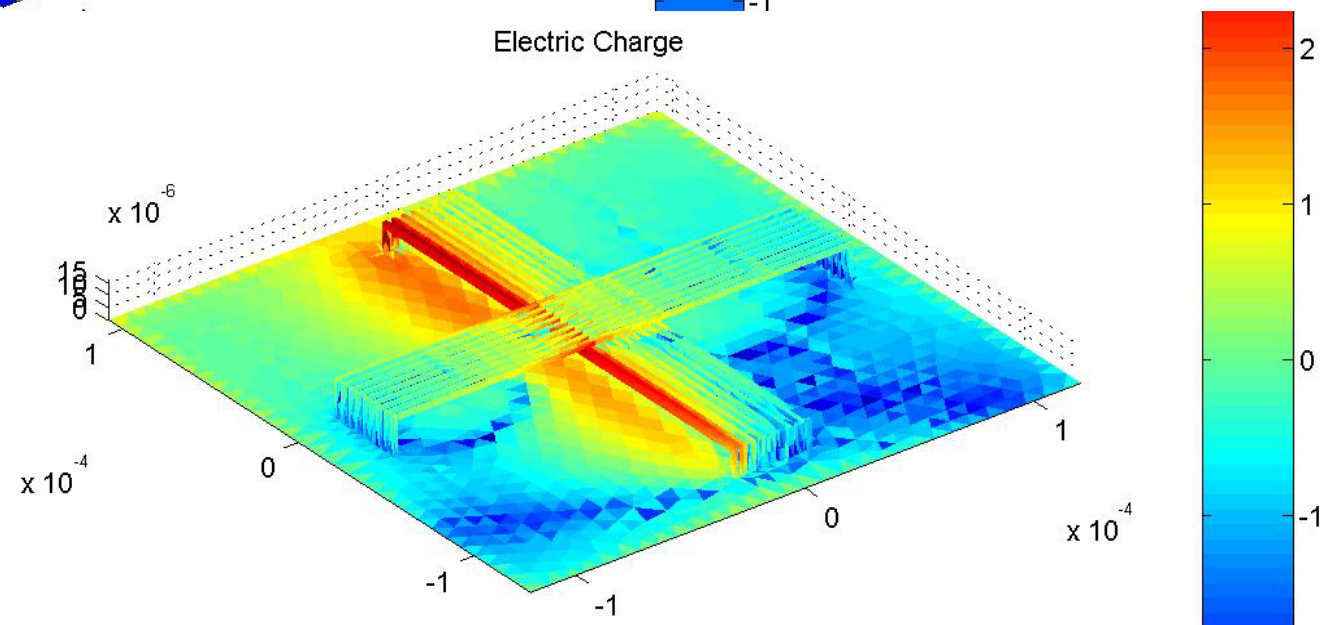
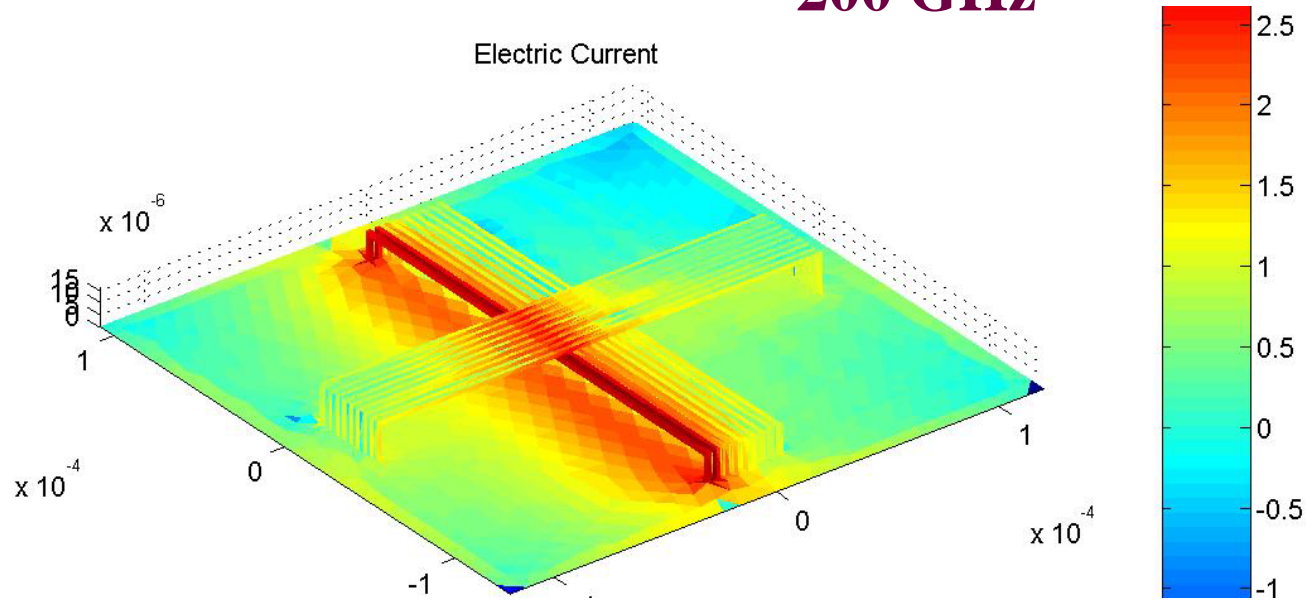
Port definition



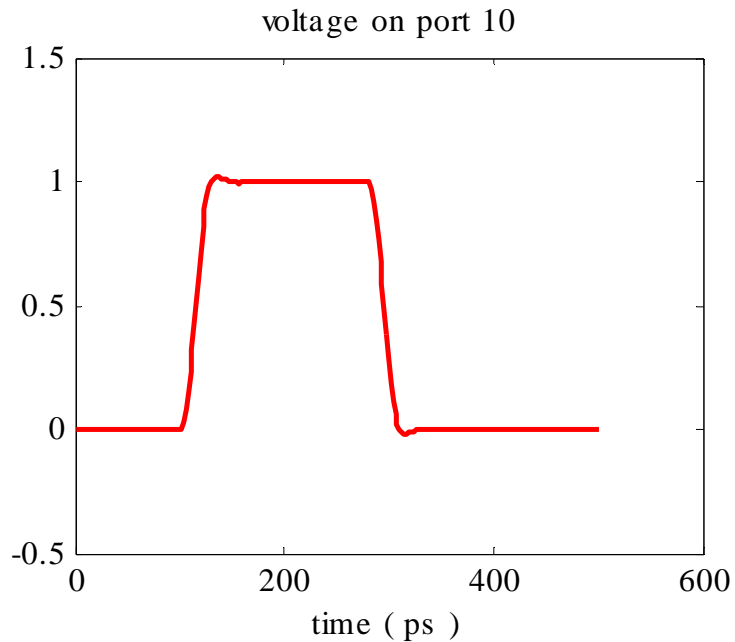
Electric current and electric charge distribution at 10 GHz



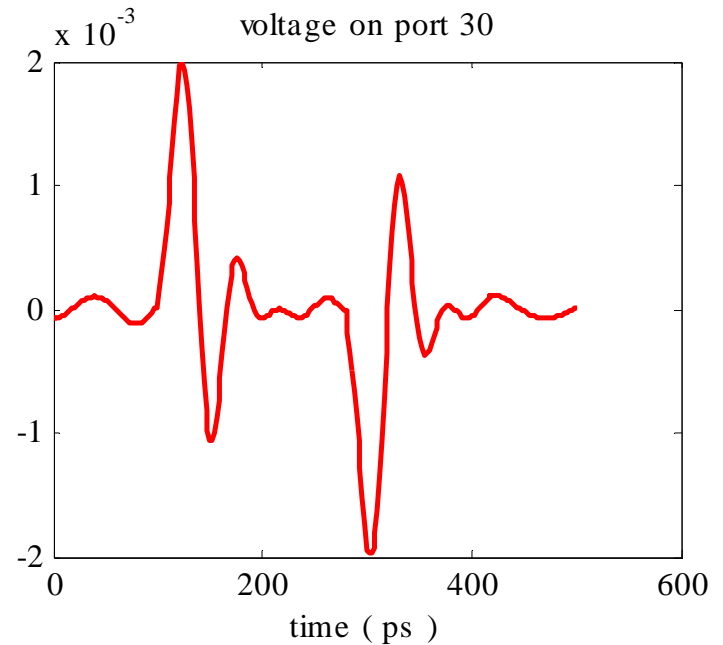
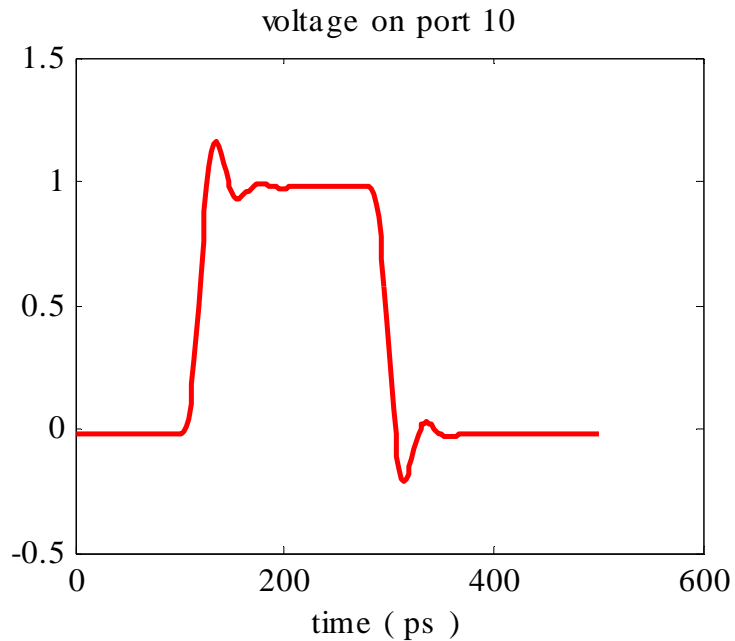
Electric current and electric chargedistribution at 200 GHz



Voltage response at port 10 and port 30 when port 9 is excited

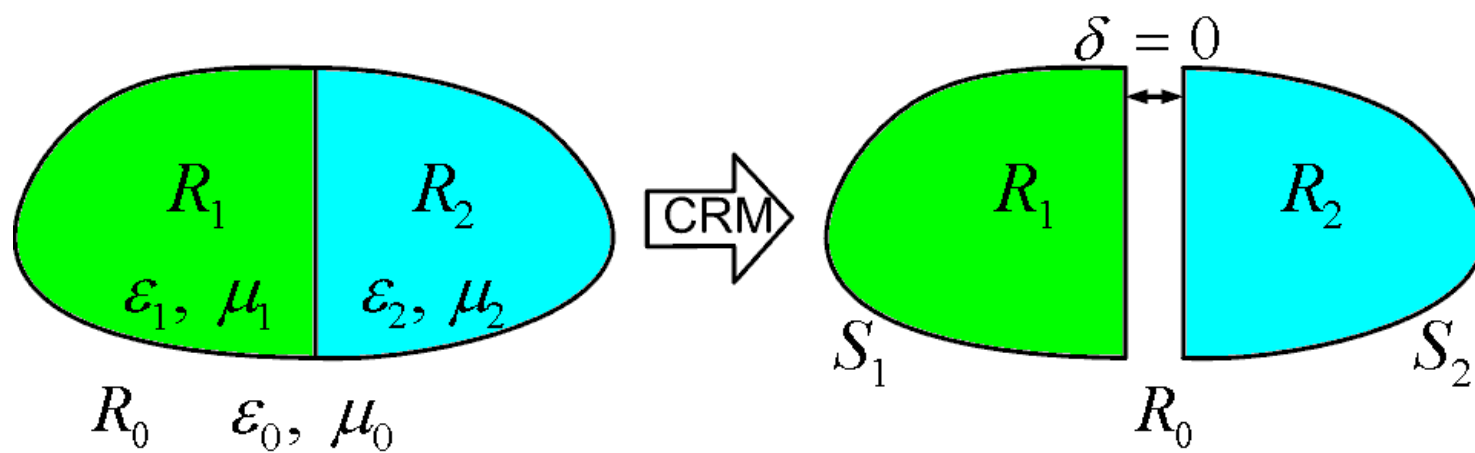


Voltage response at port 30 when all the driven ports of the higher layer are excited



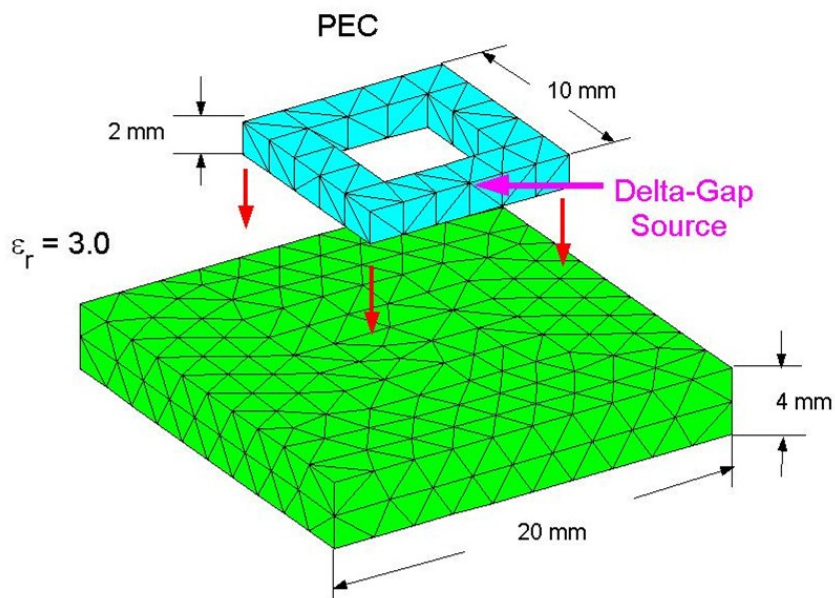
Surface Integral Equation (SIE) Multi-Region Problem

Contact-Region Modeling and PMCHWT Formulation

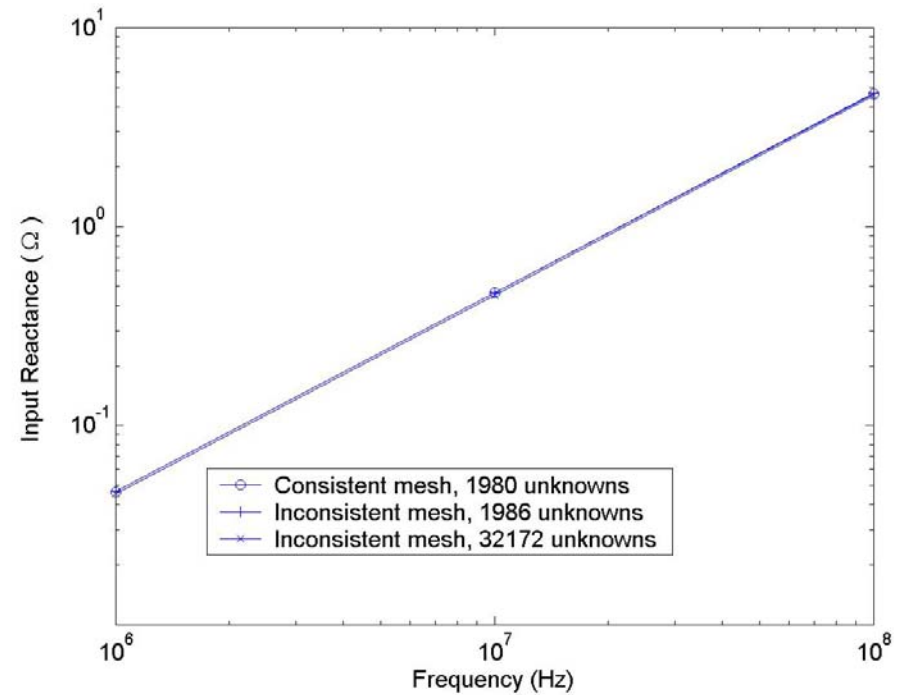


CRM--Numerical Results

An Inductance Structure



Input Reactance



CRM--Numerical Results

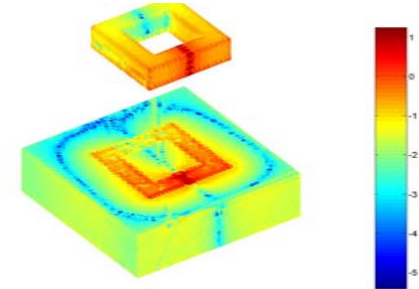
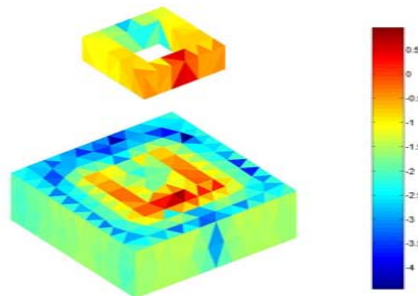
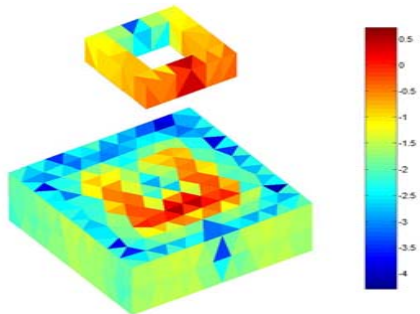
Validation of CRM

Inconsistent mesh
 1980 unknowns

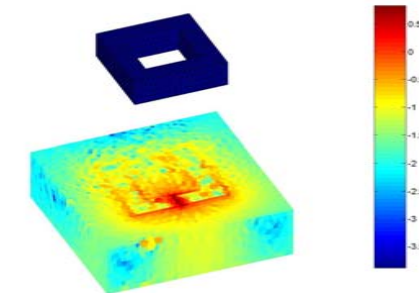
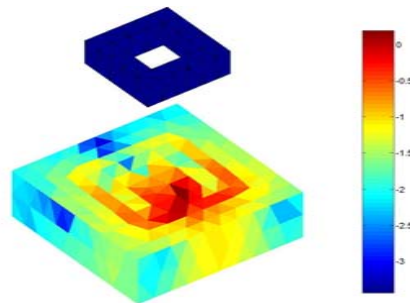
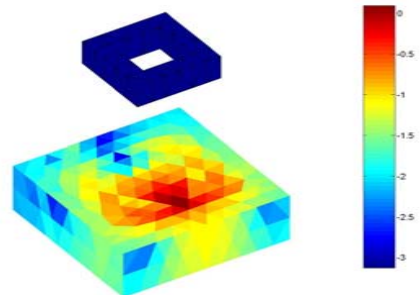
Consistent mesh
 1986 unknowns

Inconsistent mesh
 32172 unknowns

Electric Charge



Magnetic Current



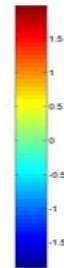
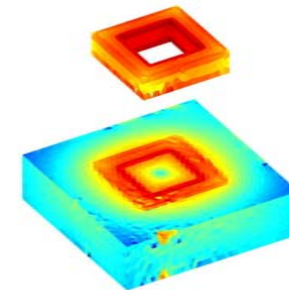
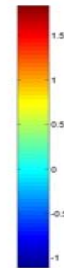
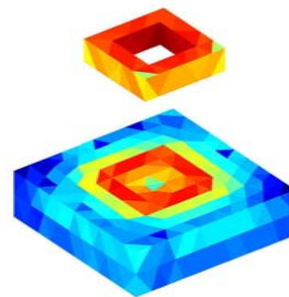
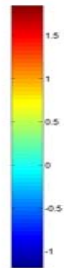
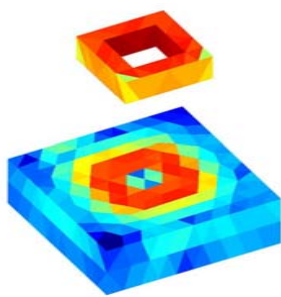
CRM--Numerical Results

Inconsistent mesh
 1980 unknowns

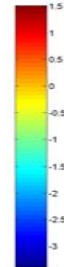
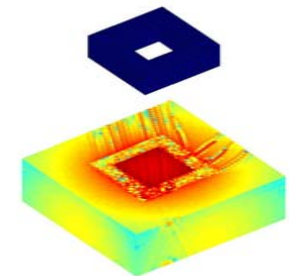
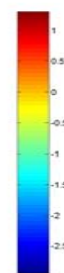
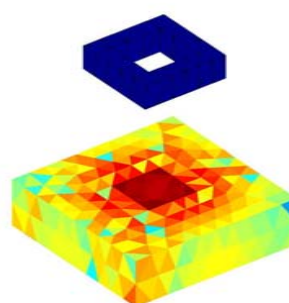
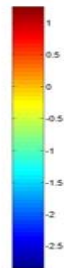
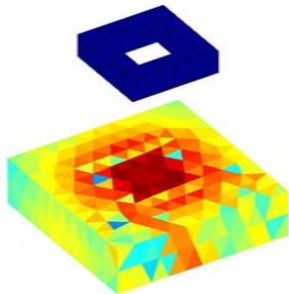
Consistent mesh
 1986 unknowns

Inconsistent mesh
 32172 unknowns

**Electric
 Current**



**Magnetic
 Charge**



**Total CPU
 Time**

38 min.

41 min.

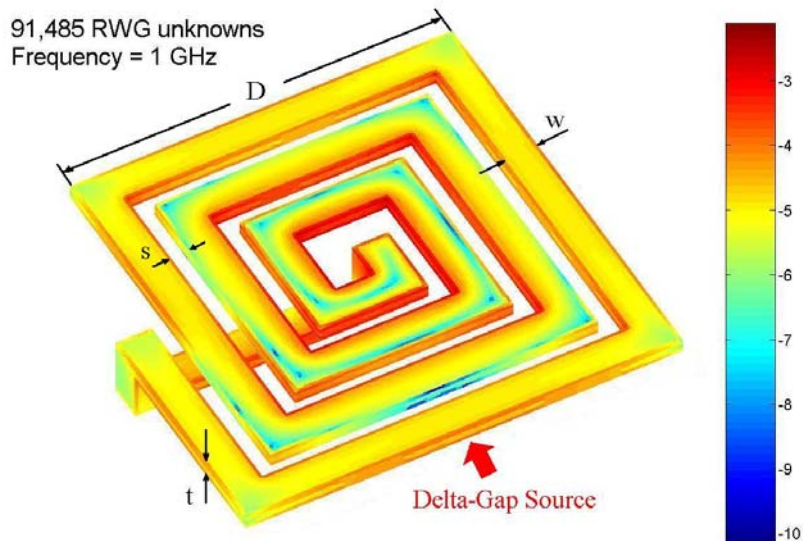
858 min.

Large-Scale Low-Frequency Computation (cont'd)

On-Chip Spiral Inductor

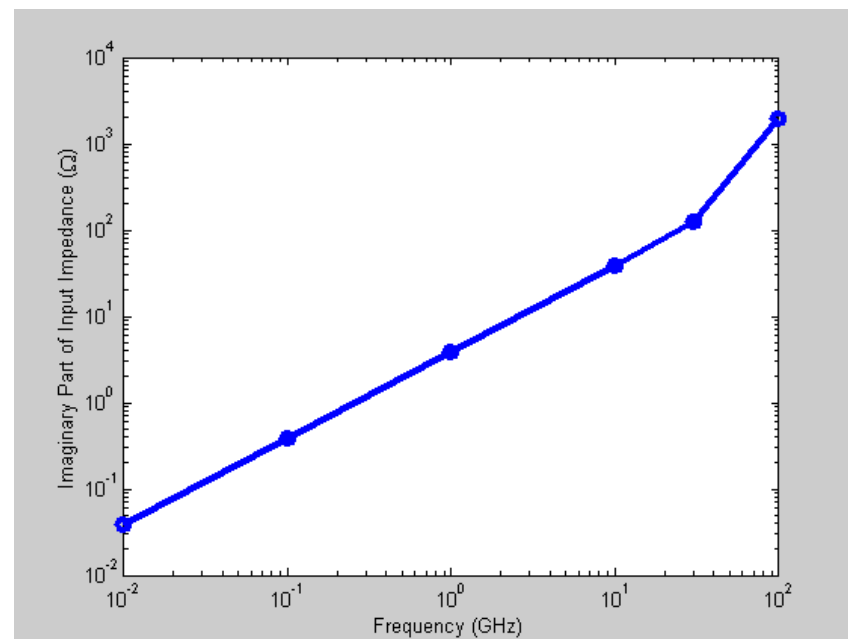
Current Magnitude Distribution in Logarithmic Scale at 1 GHz with 91,485 RWG Unknowns

$D=95 \mu\text{m}$, $w = 10 \mu\text{m}$, $t=3 \mu\text{m}$, $s = 5 \mu\text{m}$



Computed on a Sun Blade 2000 (1 CPU)
Total CPU Time: 5.25 hours
Total Memory Usage: 599 Mb

Imaginary Part of Input Impedance with 9,894 RWG Unknowns

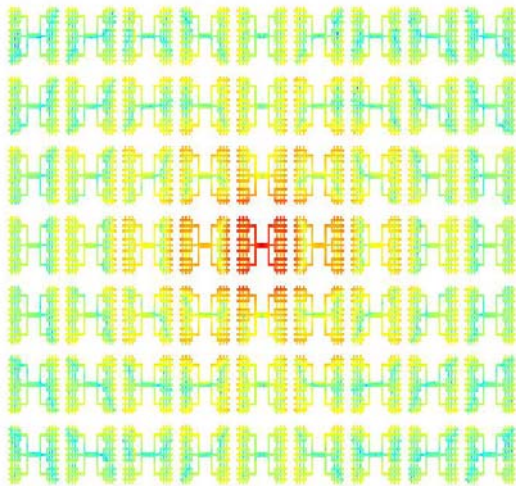


CPU Time/Freq. Pt. : 25 minutes
Total Memory Usage: 155 Mb

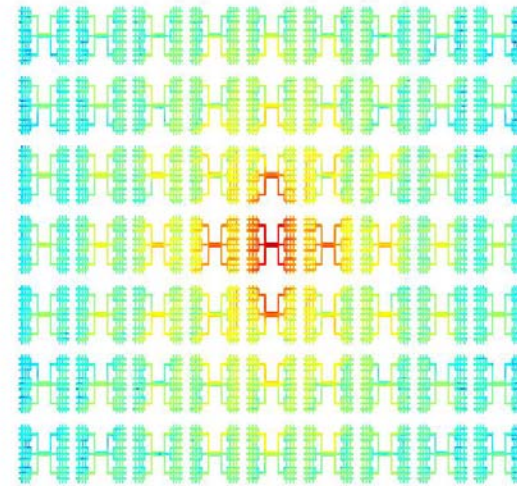
Numerical Results (cont'd)

939,897 Unknowns

Current



Charge

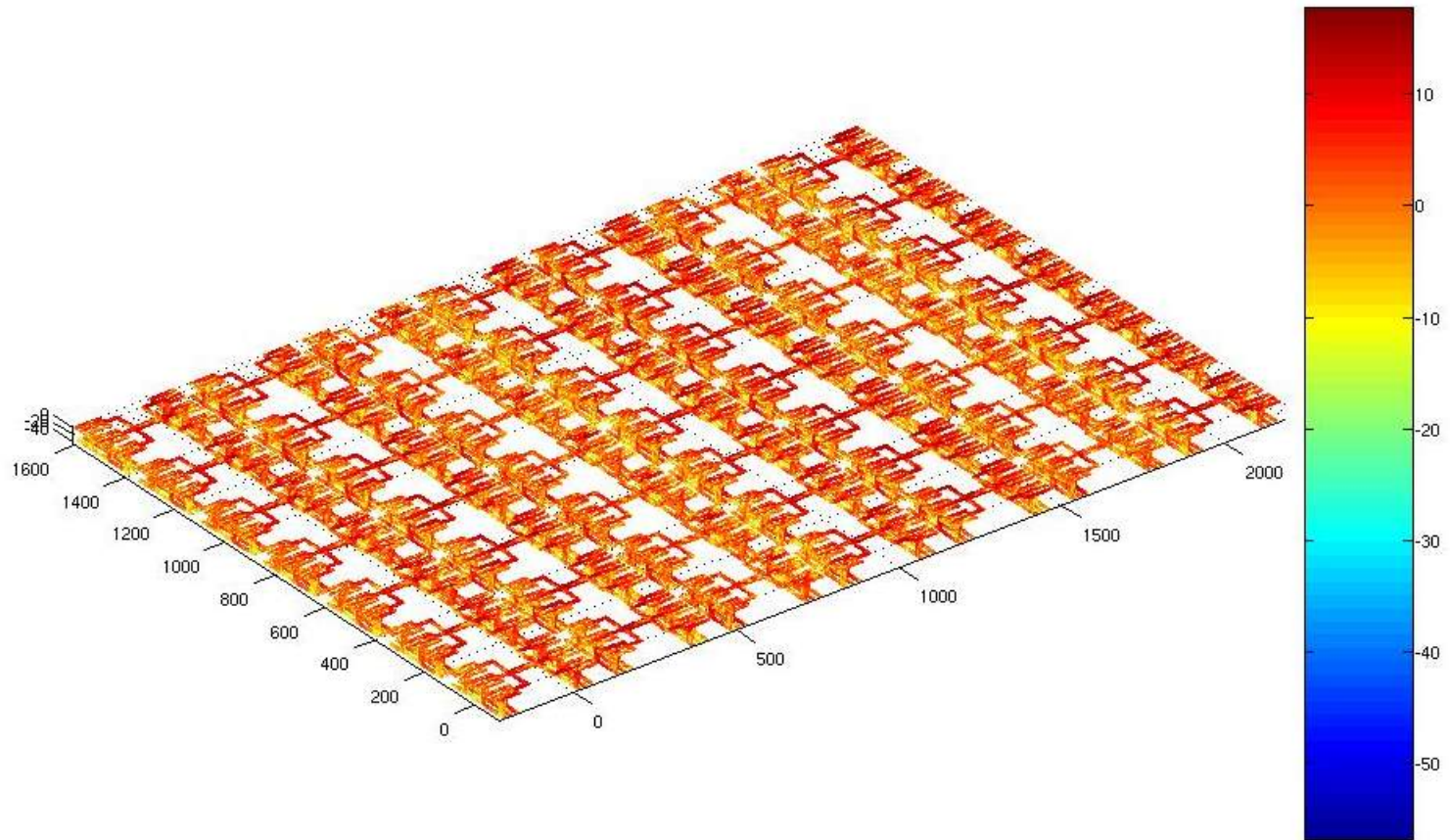


7 x 9 Array

Mixed Form Fast Multipole Algorithm

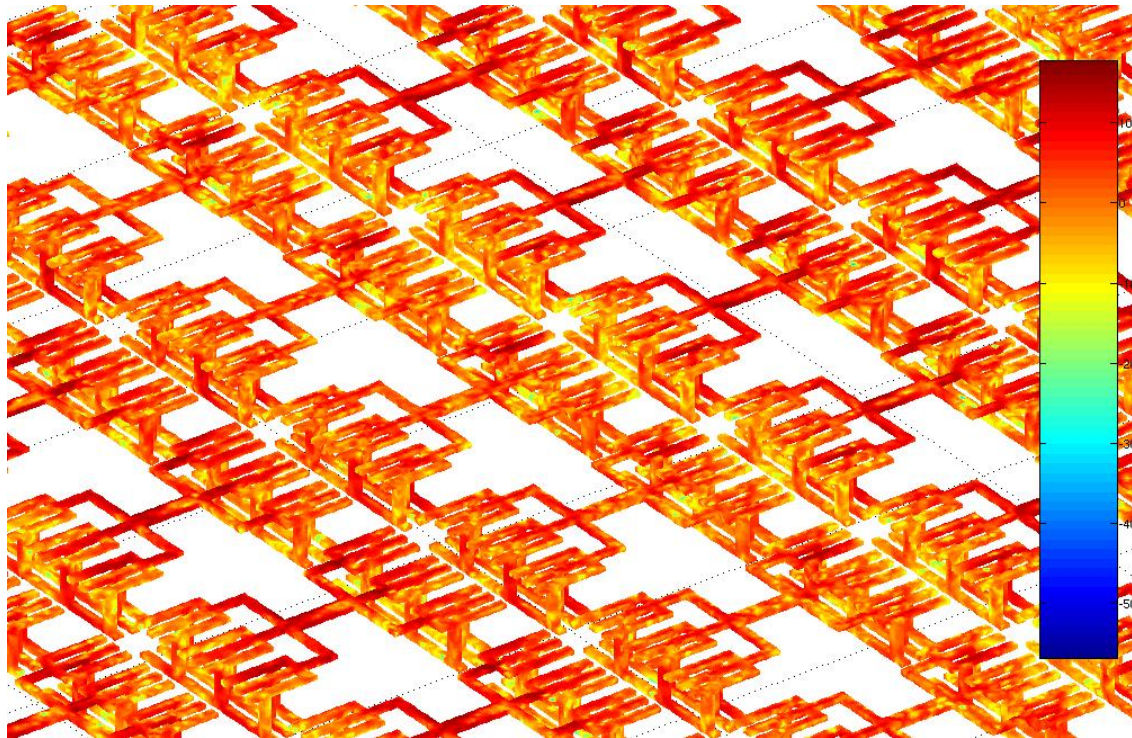
$$\begin{aligned}
 [\alpha_{LL'}(\mathbf{r}_{ji})]_{L \times L'} &= [\beta_{LL_1}(\mathbf{r}_{jJ_1})]_{L \times L_1} && \text{Low frequency} \\
 &\cdot [\beta_{L_1L_2}(\mathbf{r}_{J_1J_2})]_{L_1 \times L_2} \cdot [\beta_{L_2L_3}(\mathbf{r}_{J_2J_3})]_{L_2 \times L_3} \\
 &\cdot [D]_{S_4 \times L_3}^\dagger && \text{Transformer} \\
 &\cdot \text{diag} [e^{i\mathbf{k} \cdot \mathbf{r}_{J_3J_4}}]_{S_4 \times S_4} \cdot [I]_{S_5 \times S_4}^T \\
 &\cdot \text{diag} [e^{i\mathbf{k} \cdot \mathbf{r}_{J_4J_5}}]_{S_5 \times S_5} \\
 &\cdot \text{diag} [\tilde{T}(\Omega_{s_5}, \mathbf{r}_{J_5I_5})w_{s_5}]_{S_5 \times S_5} && \text{Mid frequency} \\
 &\cdot \text{diag} [e^{i\mathbf{k} \cdot \mathbf{r}_{I_5I_4}}]_{S_5 \times S_5} \cdot [I]_{S_5 \times S_4} \\
 &\cdot \text{diag} [e^{i\mathbf{k} \cdot \mathbf{r}_{I_4I_3}}]_{S_4 \times S_4} \\
 &\cdot [D]_{S_4 \times L_3} && \text{Transformer} \\
 &\cdot [\beta_{L_3L_2}(\mathbf{r}_{I_3I_2})]_{L_3 \times L_2} \cdot [\beta_{L_2L_1}(\mathbf{r}_{I_2I_1})]_{L_2 \times L_1} \\
 &\cdot [\beta_{L_1L'}(\mathbf{r}_{I_1i})]_{L_1 \times L'} && \text{Low frequency}
 \end{aligned}$$

Full Band MF-FMA



7 x 7 fork structure

Full Band MF-FMA



Incident Wave: 1 MHz

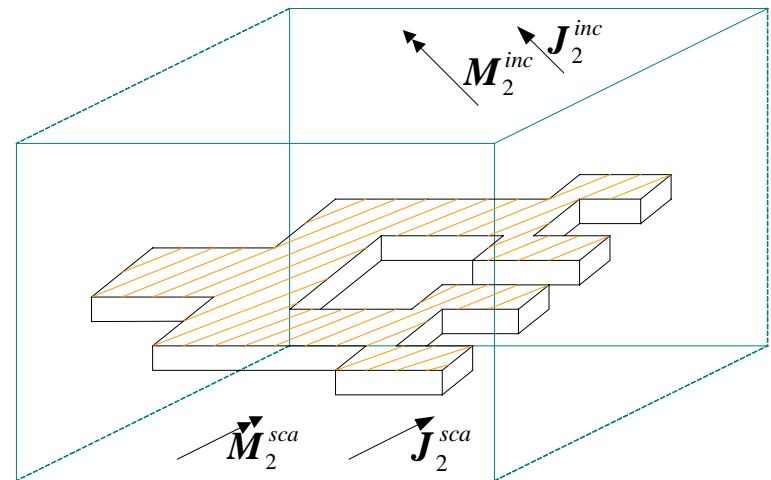
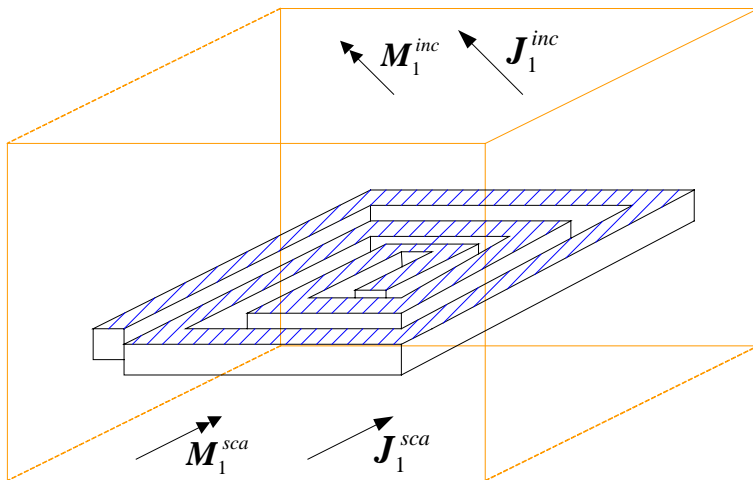
$\theta=45\text{deg}$

$\Phi=45\text{deg}$

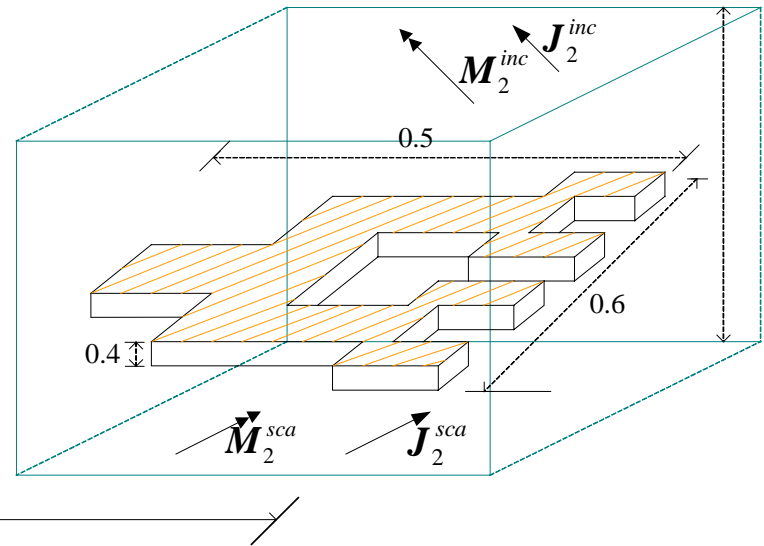
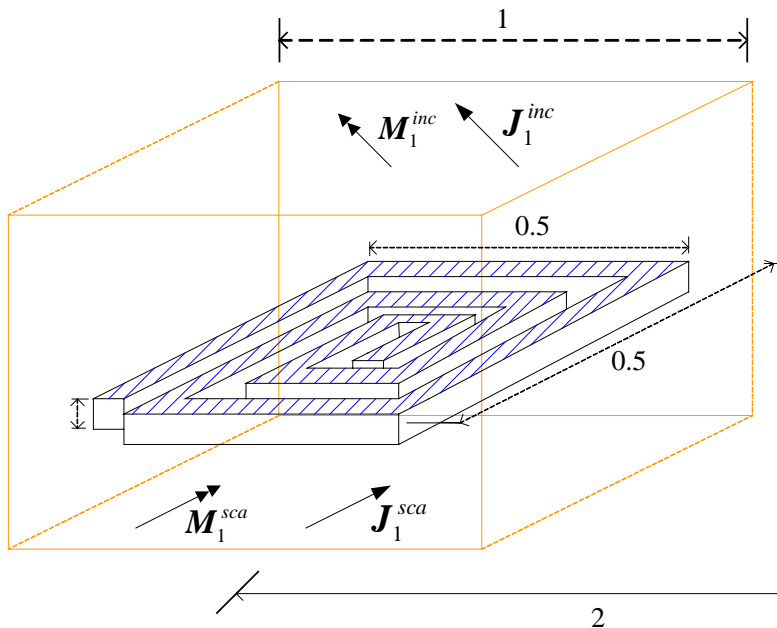
No of triangles: 487,354

No of unknowns: 731,031

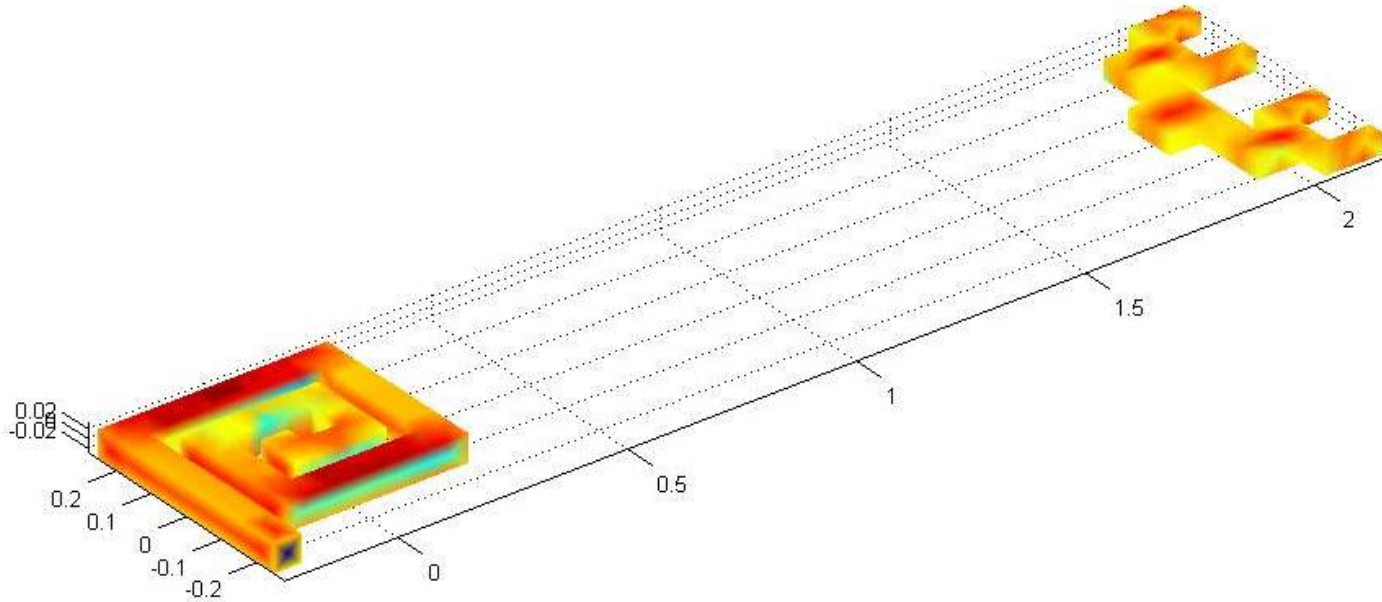
One Example of Equivalence Principle Algorithm



One Example of Equivalence Principle Algorithm (cont'd)

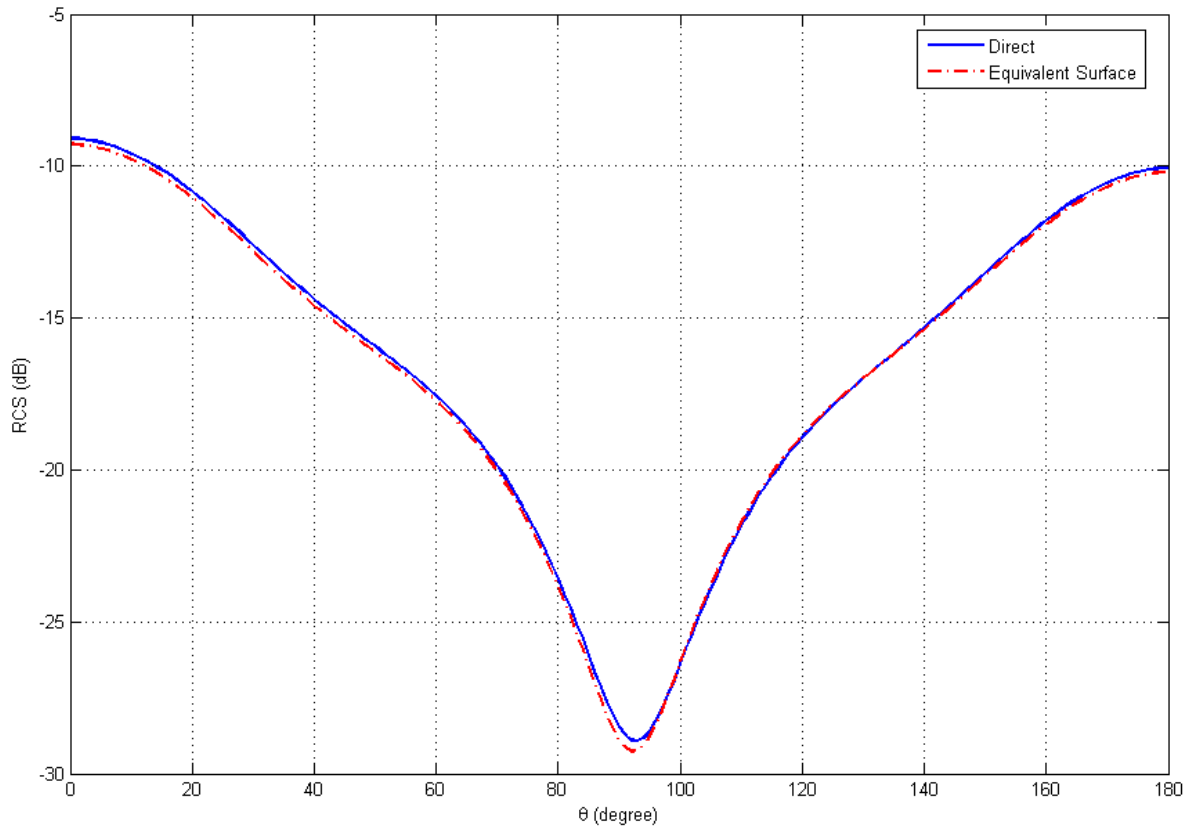


One Example of Equivalence Principle Algorithm (cont'd)



Frequency:	100 MHz	
Left Conductor:	0.5 x 0.5 x 0.06 (m)	306 Unknowns (84/λ)
Right Conductor:	0.5 x 0.6 x 0.04 (m)	372 Unknowns (142/λ)
Huygens' Surface:	1.0 x 1.0 x 1.0 (m)	144 Unknowns (25/λ)

One Example of Equivalence Principle Algorithm (cont'd)



Frequency:

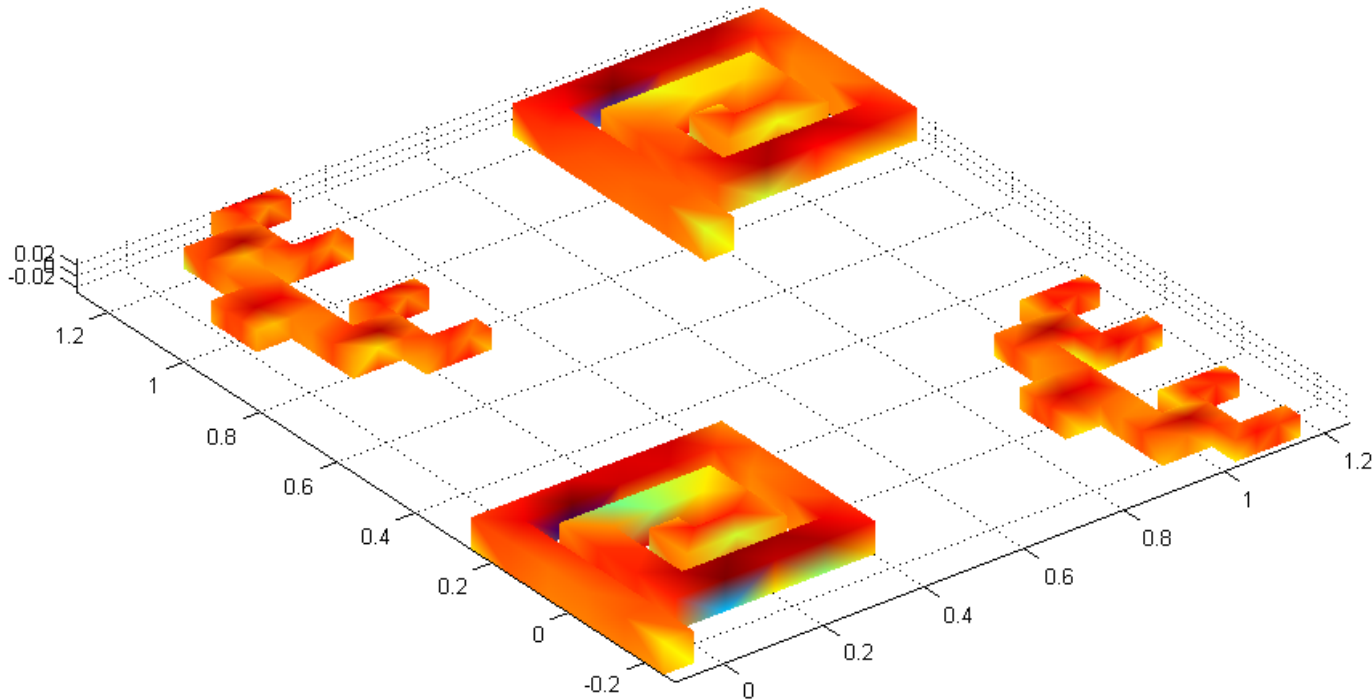
0.1 GHZ

Incident wave:

$\theta=0$ degree

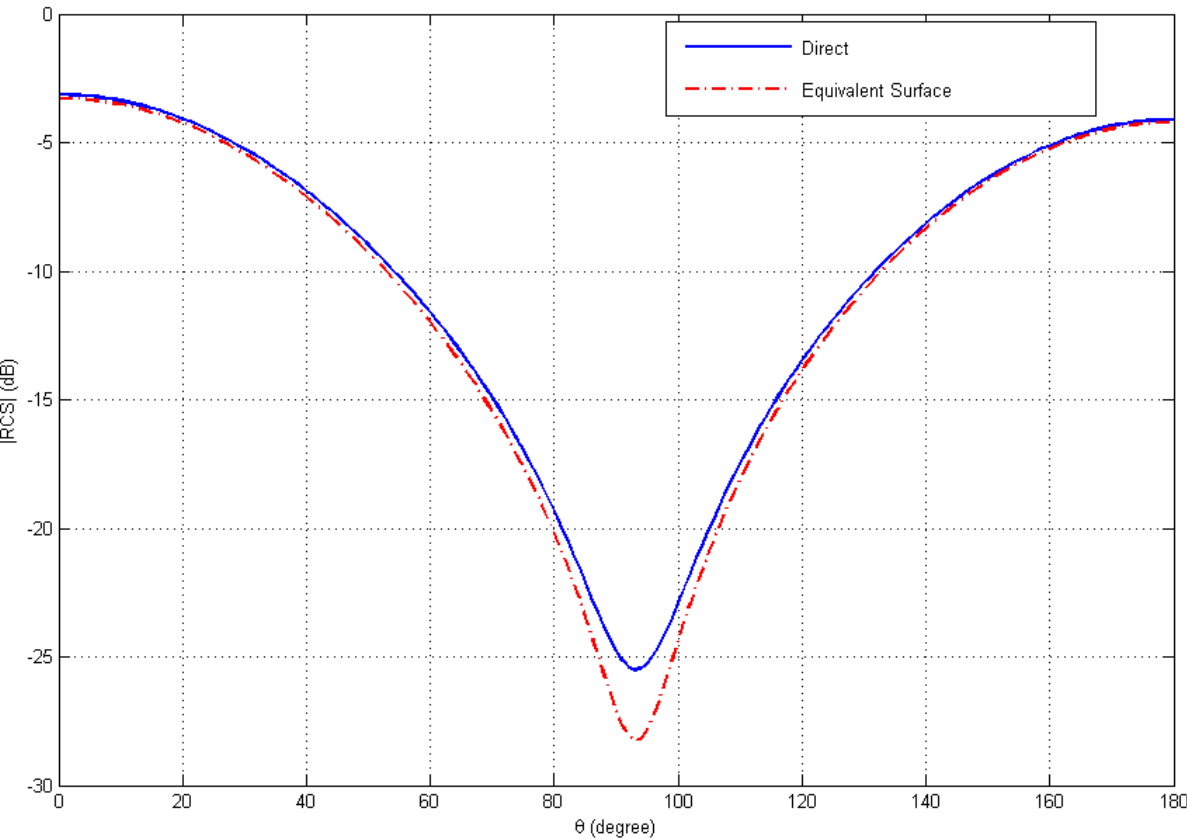
$\Phi=0$ degree

Second Example of Equivalence Principle Algorithm (cont'd)



Frequency:	100 MHz	
Coil Conductor:	0.5 x 0.5 x 0.06 (m)	306 Unknowns (84/λ)
Fork Conductor:	0.5 x 0.6 x 0.04 (m)	372 Unknowns (142/λ)
Huygens' Surface:	1.0 x 1.0 x 1.0 (m)	144 Unknowns (25/λ)

Second Example of Equivalence Principle Algorithm (cont'd)



Frequency:

0.1 GHZ

Incident wave:

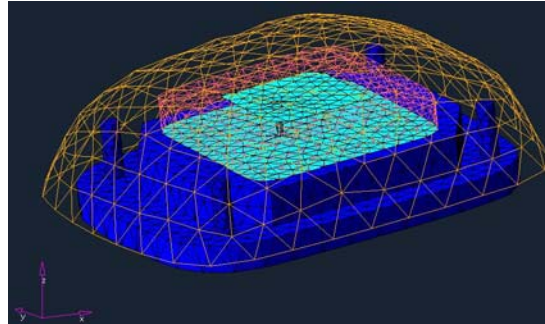
θ=0 degree

Φ=0 degree

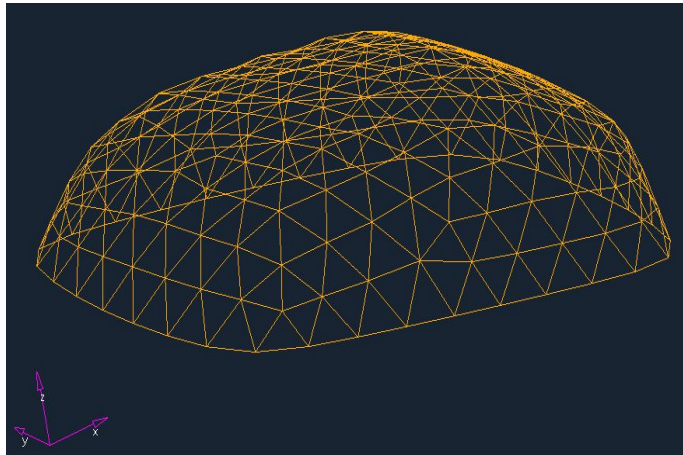
Antenna on Vehicle---Geometry and Mesh

Parameters for Substrate and Radome

Radome



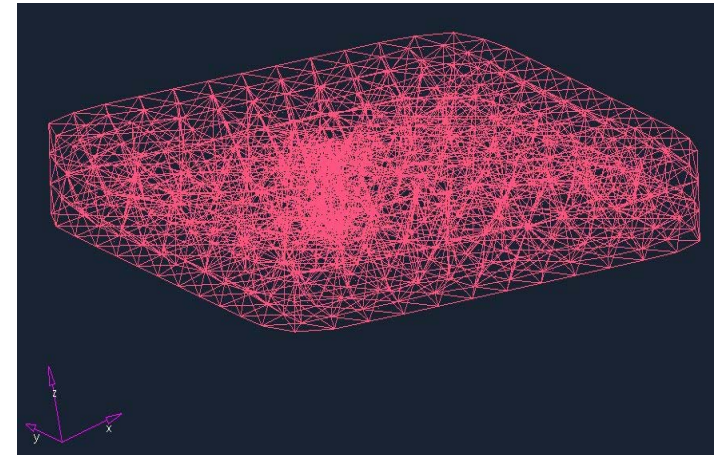
Substrate



$$\epsilon_r = 2.6$$

$$d = 1.0 \text{ mm}$$

$$Z_{TDS} = -4823.2j \text{ @ } 2.3325 \text{ GHz}$$

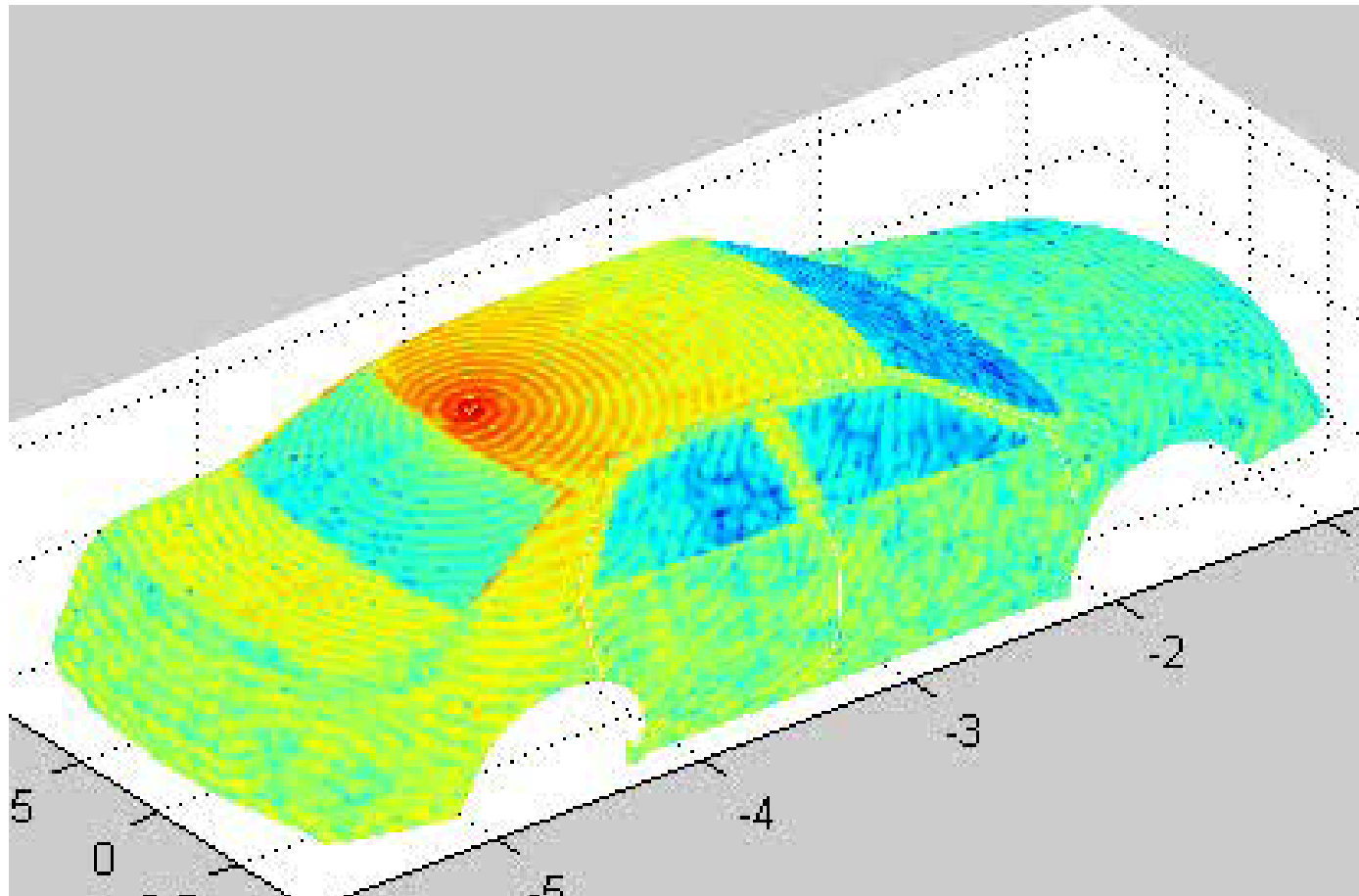


$$\epsilon_r = 21.7$$

$$d = 4.0 \text{ mm}$$

$$\text{Diameter of Probe} = 0.74 \text{ mm}$$

Animation of Current Distribution on Car



- Antenna generates a circularly polarized field and current on rooftop of the car.
- 2 GHz antenna on a 4.25 m size car is equivalent to 100 GHz on a 8 cm board

Conclusions

- **Need for easy modeling of complex structures.**
- **Contact region modeling allows for the inclusion of materials in low frequency modeling.**
- **Multiscale problems are the challenge problems of the future.**
- **Circuit physics and wave physics have to be captured simultaneously in a simulation.**
- **Mixed form FMA a new fast algorithm, will allow a seamless way to simulate structures all the way from static to microwave.**
- **Equivalence principle algorithm (EPAL) allows the decoupling of circuit physics from wave physics.**