Accounting for Variability and Uncertainty in Signal and Power Integrity Modeling

Andreas Cangellaris & Prasad Sumant

Electromagnetics Lab, ECE Department

University of Illinois

ECE ILLINOIS
Department of Electrical and Computer Engineering

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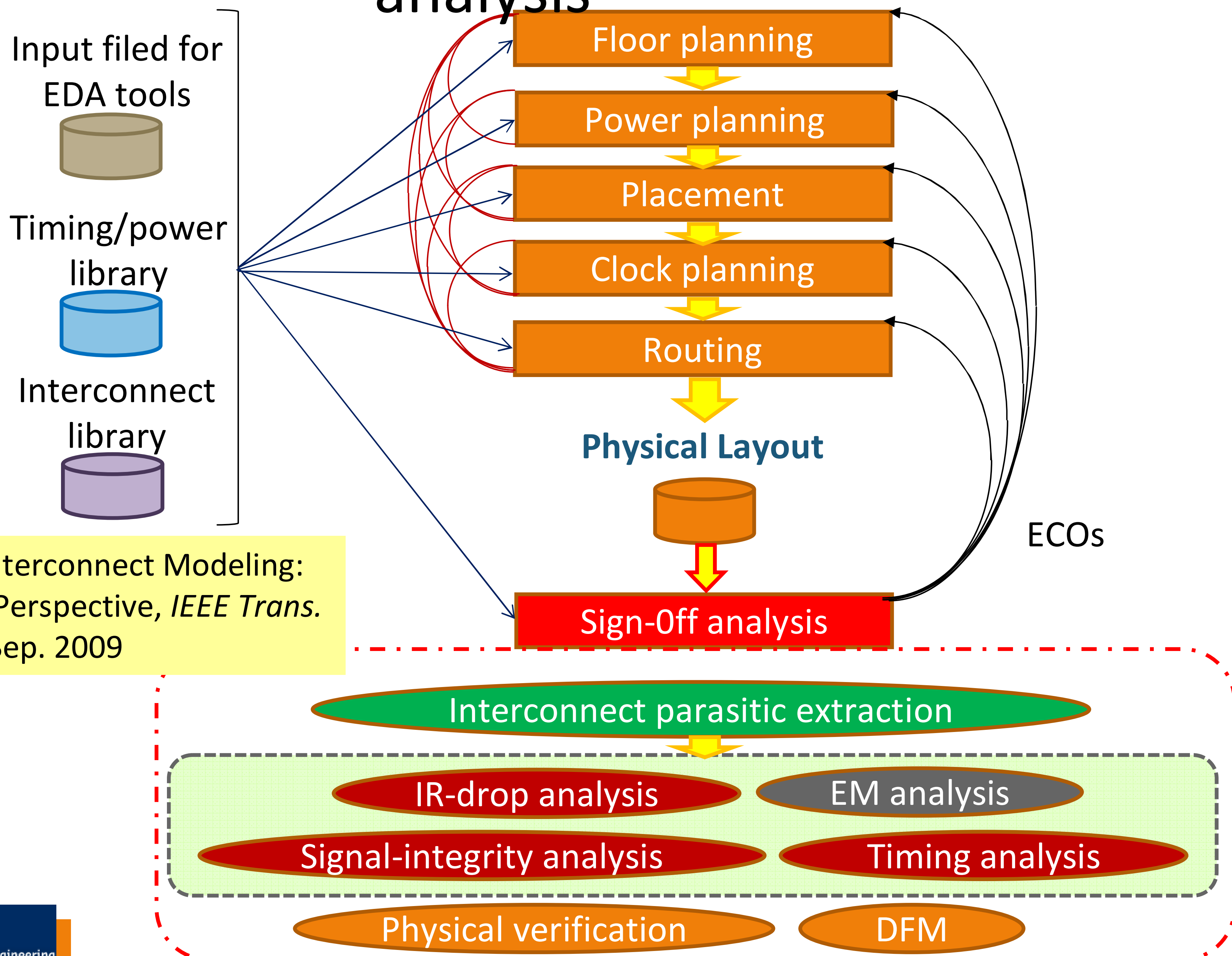
Agenda

- Are we using the might of EM CAD wisely?
  - Uncertainty and Variability (UV) in Signal/Power Integrity Modeling
- Accounting for UV in SI/PI modeling and simulation
  - Interconnect electrical modeling
  - Model order reduction in the presence of UV
- Closing Remarks

Are we using the might of EM CAD for signal integrity-aware design wisely?

Electromagnetic modeling/simulation pervasive in physical design and sign-off analysis



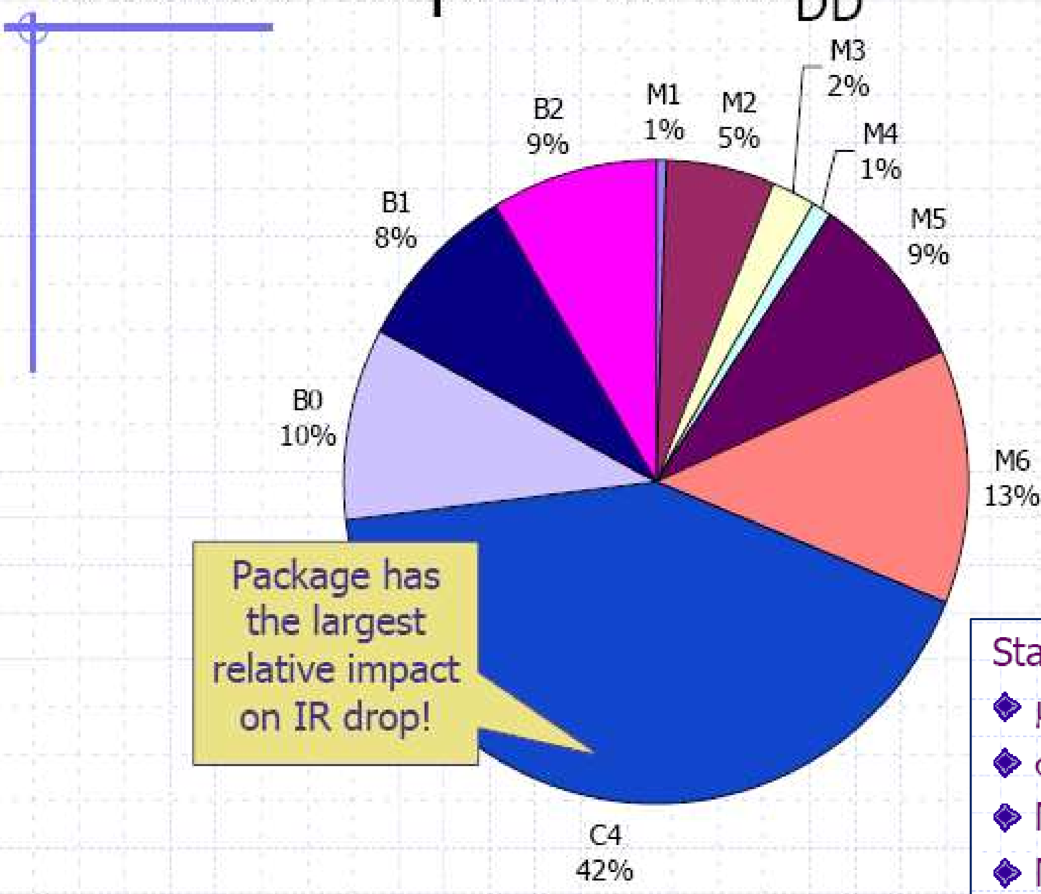
Kurokawa et al, Interconnect Modeling: A Physical Design Perspective, *IEEE Trans. Electron Devices,* Sep. 2009

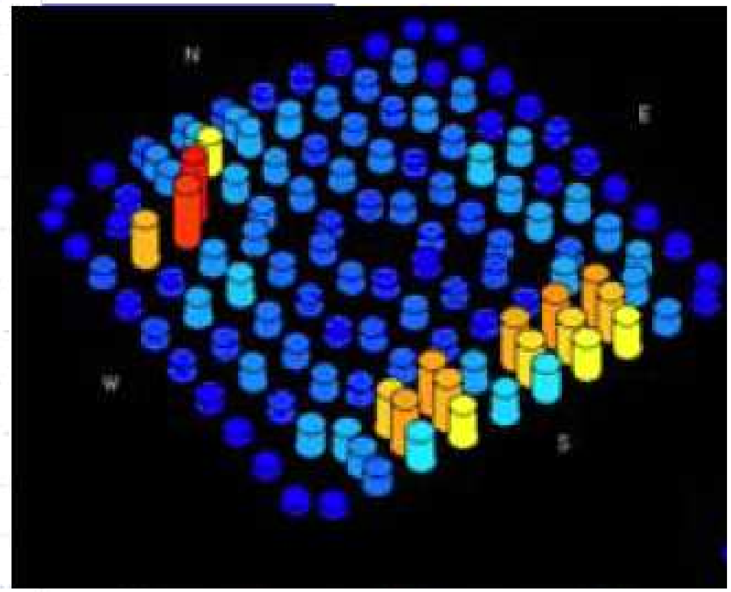
Power Integrity

- Package impedance
  - Design/layout-dependent
  - *R* impacted by manufacturing tolerances
- On-chip grid
  - *R* variability due to CMP
- On-chip decoupling
  - Available *C* dependent on operation, V_T, T_{ox}, …

Impact of Package *R* on IR drop(*)

Relative Impact on ΔV_{DD}

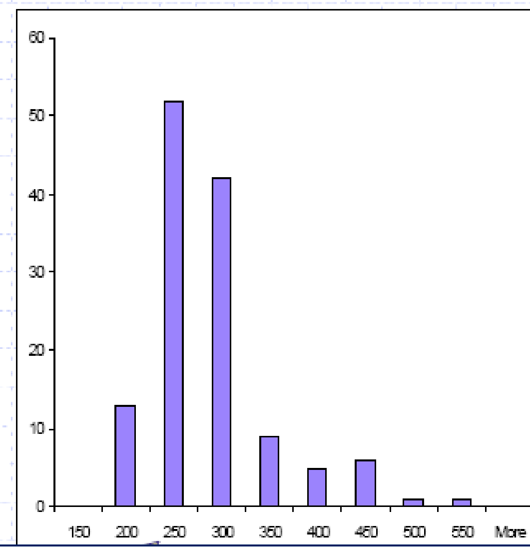




Statistics (mΩ):

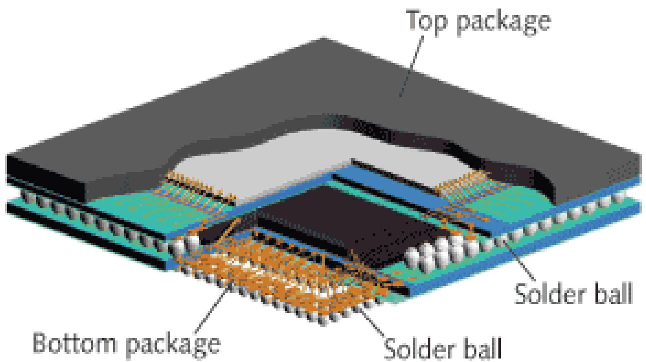
- μ=263.4
- σ=63.6 (24%)
- Min=157.5
- Max=519.6

- A 10% tolerance on resistivity is <u>insignificant</u> compared to the systematic variations!



(*)Sani Nassif, IBM

Variability and Uncertainty



Package-on-package

Stacked dies

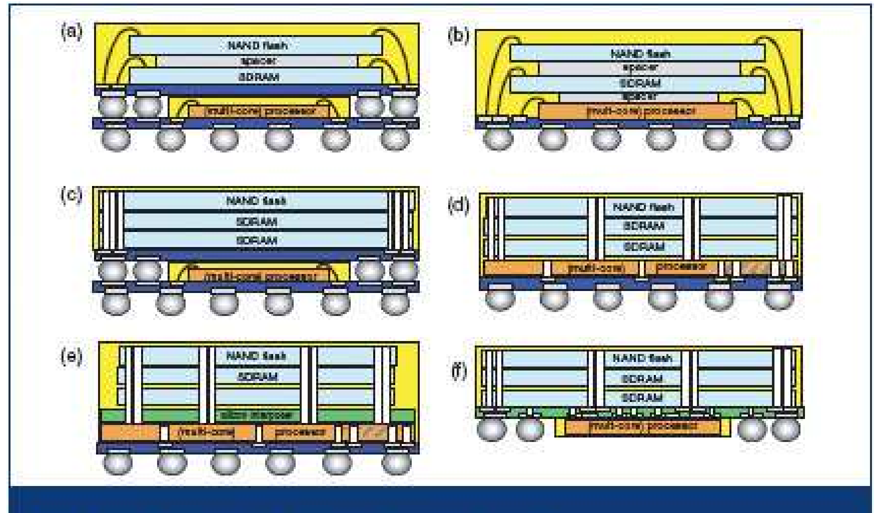
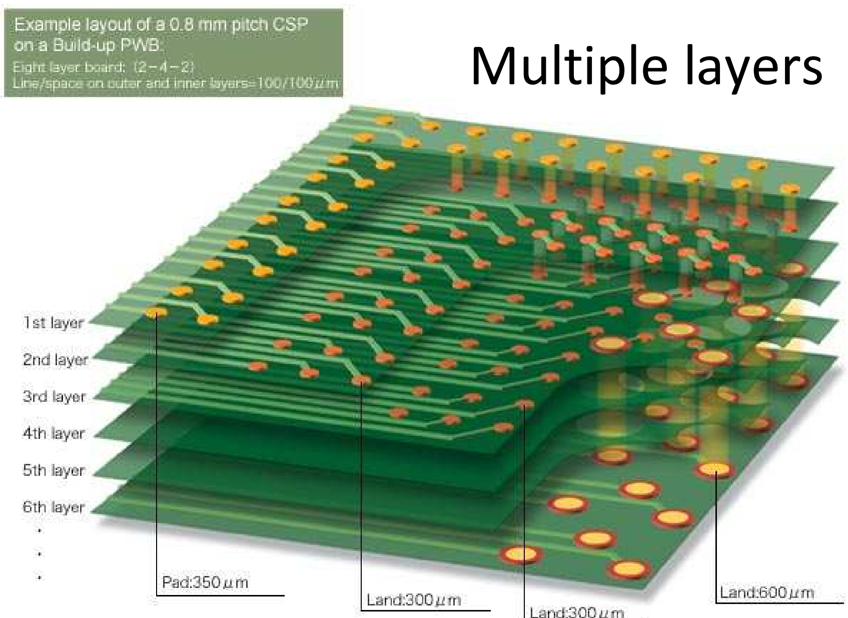


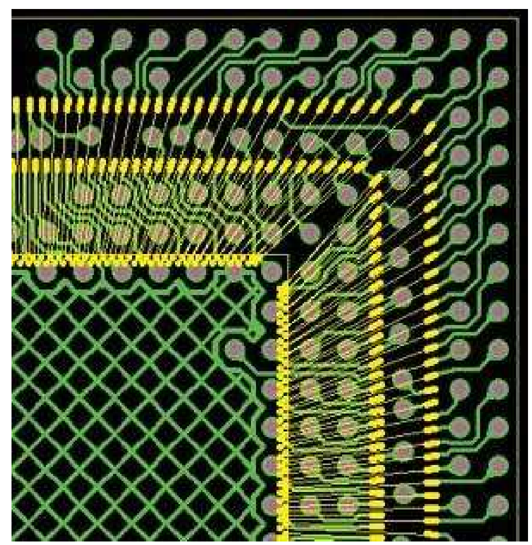
Figure 1. Various Stacked Die Solutions for Compute Engines

Source: Future-Fab International

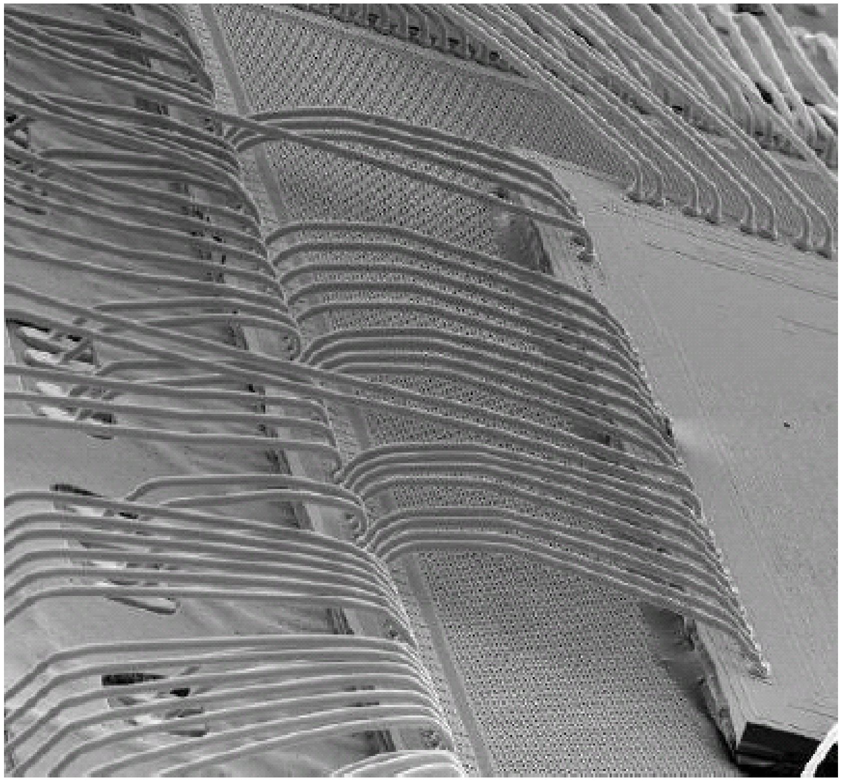
Multiple layers



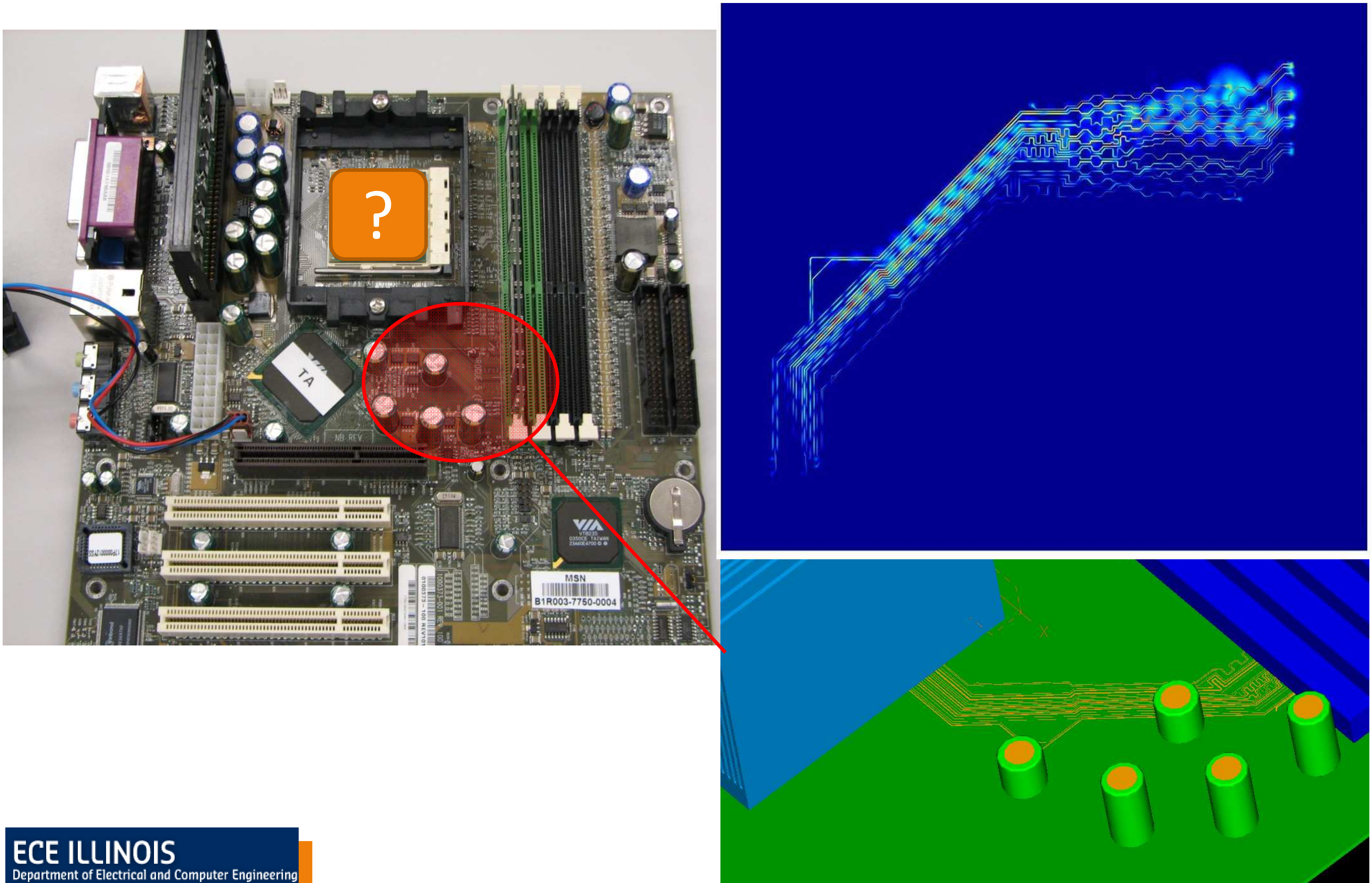
Source: TNCSI



Variability and Uncertainty

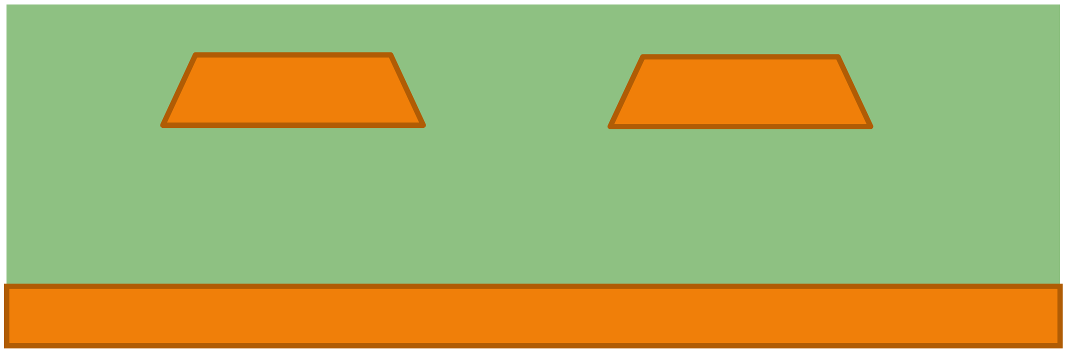


Variability and Uncertainty



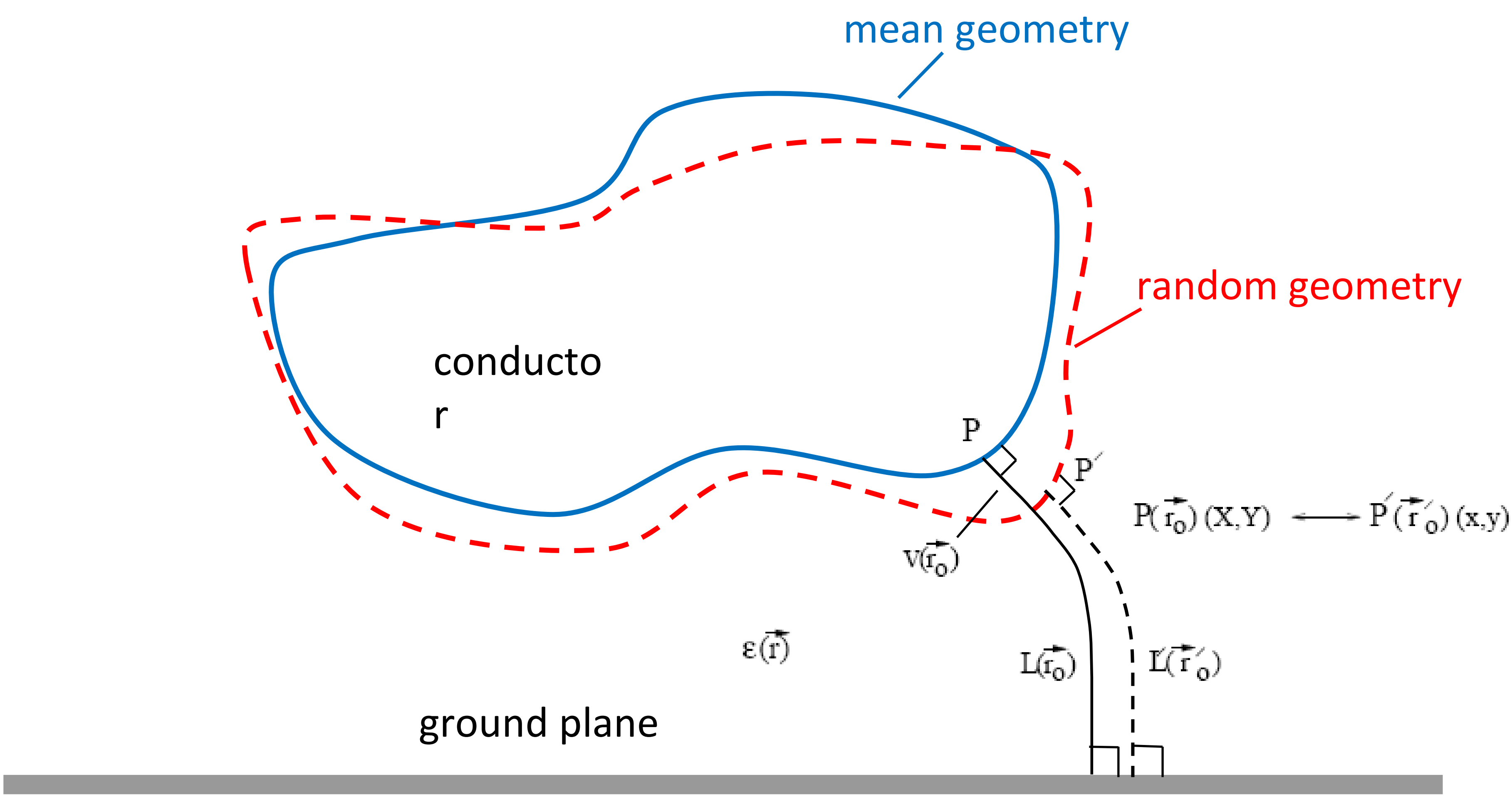
Accounting for SI/PI Modeling and Simulation

Interconnect Cross-Sectional Geometry



- Parameters
  - Trace width
  - Trace thickness
  - Trace shape
  - Pitch
  - Height above ground
  - Surface roughness
  - Substrate permittivity
  - Metallization conductivity
- Derivative quantities
  - Transmission-Line Modeling
    - R, L, C, G (per-unit-length)
    - Characteristics Impedance
    - Phase constant
    - Attenuation constant
  - Full-wave modeling
    - Metallization surface impedance

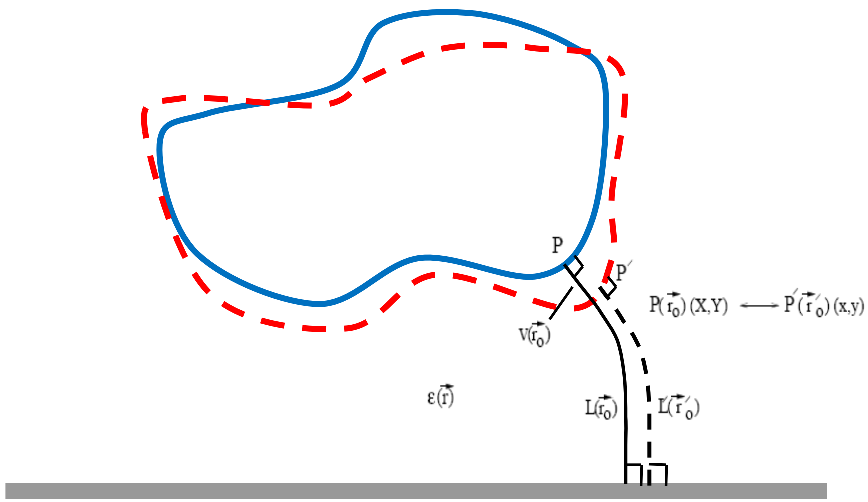
Transmission-Line Parameter Extraction



Mapping between random sample and mean geometry

Random sample

$$\int_{L'} \vec{E}' \cdot \vec{dl}' = V_0 \Rightarrow \int_{L'} \frac{\vec{D}'(\vec{r}') . \vec{dl}'}{\varepsilon(\vec{r}')} = V_0 \Rightarrow \left|\vec{D}_c\right| \int_{L'} \frac{\vec{dl}'}{\varepsilon(\vec{r}')} = V_0$$



Mean geometry

$$\left|\vec{D}_0'(\vec{r}_0')\right| = \left|\vec{D}_c\right| = \frac{V_0}{\int_{L'} \frac{\vec{dl}'}{\varepsilon(\vec{r}')}} \quad (1)$$

$$\left|\vec{D}(\vec{r}_0)\right| = \frac{V_0}{\int_{L} \frac{\vec{dl}}{\varepsilon(\vec{r})}} \quad (2)$$

Mapping relationship

$$\left|\vec{D}_0'(\vec{r}_0')\right| = Q\left|\vec{D}(\vec{r}_0)\right|$$

$$Q = \frac{\int_{L} \frac{\vec{dl}}{\varepsilon(\vec{r})}}{\int_{L'} \frac{\vec{dl}'}{\varepsilon(\vec{r}')}} \quad (3)$$

Electric flux density computation using solution on mean geometry

Position-dependent flux length/gap on mean geometry:

$$G(\vec{r}_0) = \varepsilon(\vec{r}_0)\frac{V_0}{\left|\vec{D}(\vec{r}_0)\right|} \Rightarrow \int_L \frac{1}{\varepsilon(\vec{r})} dl = \frac{G(\vec{r}_0)}{\varepsilon(\vec{r}_0)} \qquad \text{(exact)}$$

Flux length on random sample:

$$L'(\vec{r}_0) \approx L(\vec{r}_0) - v(\vec{r}_0)$$

$$\int_{L'} \frac{1}{\varepsilon(\vec{r}')} dl' \approx \int_{L-v} \frac{1}{\varepsilon(\vec{r})} dl \Rightarrow \int_{L'} \frac{1}{\varepsilon(\vec{r}')} dl' \approx \frac{G(\vec{r}_0)}{\varepsilon(\vec{r}_0)} - \frac{v(\vec{r}_0)}{\varepsilon(\vec{r}_0)}$$

Electric flux density on random sample using mean geometry:

$$\left|\vec{D}'(\vec{r}_0')\right| = \left|\vec{D}(\vec{r}_0)\right| \frac{G(\vec{r}_0)}{G(\vec{r}_0) - v(\vec{r}_0)}$$

Computation of capacitance of random sample

Capacitance on random sample:

$$C' = \int_{C_{rs}} \left| \overrightarrow{D'}(\vec{r'}) \right| dl'$$

$$C' = \int_{C_{mean}} \frac{G(\vec{r_0})}{G(\vec{r_0}) - v(\vec{r_0})} \left| \vec{D}(\vec{r_0}) \right| \left| J \right| dl$$

Where *J* represents the Jacobian, the map between the random and mean geometry:

$$J = \begin{bmatrix} \frac{\partial x}{\partial X} & \frac{\partial y}{\partial X} \\ \frac{\partial x}{\partial Y} & \frac{\partial y}{\partial Y} \end{bmatrix}$$

Representing uncertainty using polynomial chaos

Polynomial chaos expansion:

Orthogonal polynomials are random variables

$$u(\vec{r},\theta) = \sum_{i=0}^{\infty} \hat{a}_i(\vec{r})\Psi(\xi(\theta))$$

Coefficients: functions of space

Type of polynomials depends on distribution of input random variable

- Gaussian distribution, Hermite polynomial chaos

$$\Psi_0(\xi) = 1, \Psi_1(\xi) = \xi, \Psi_2(\xi) = \xi^2 - 1, \Psi_3(\xi) = \xi^3 - 3\xi, \Psi_4(\xi) = \xi^4 - 6\xi^2 + 3, \ldots$$

Truncated polynomial chaos expansion:

- Number of different input random variables: *n*
- Order of polynomials: *p*

$$u(\vec{r},\theta) = \sum_{i=0}^{N} \hat{a}_i(\vec{r})\Psi(\xi(\theta))$$

$$N + 1 = \frac{(n+p)!}{n!\,p!}$$

Computing stochastic capacitance

Displacement of random geometry from the mean geometry:

$$v(\vec{r},\theta)=\sum_{i=0}^{N} v_i(\vec{r})\Psi(\xi(\theta))$$

Use relationship between random and mean flux length:

$$\tilde{G}(\vec{r},\theta)=\bar{G}(\vec{r})-v(\vec{r},\theta)$$

Stochastic Electric Flux Density:

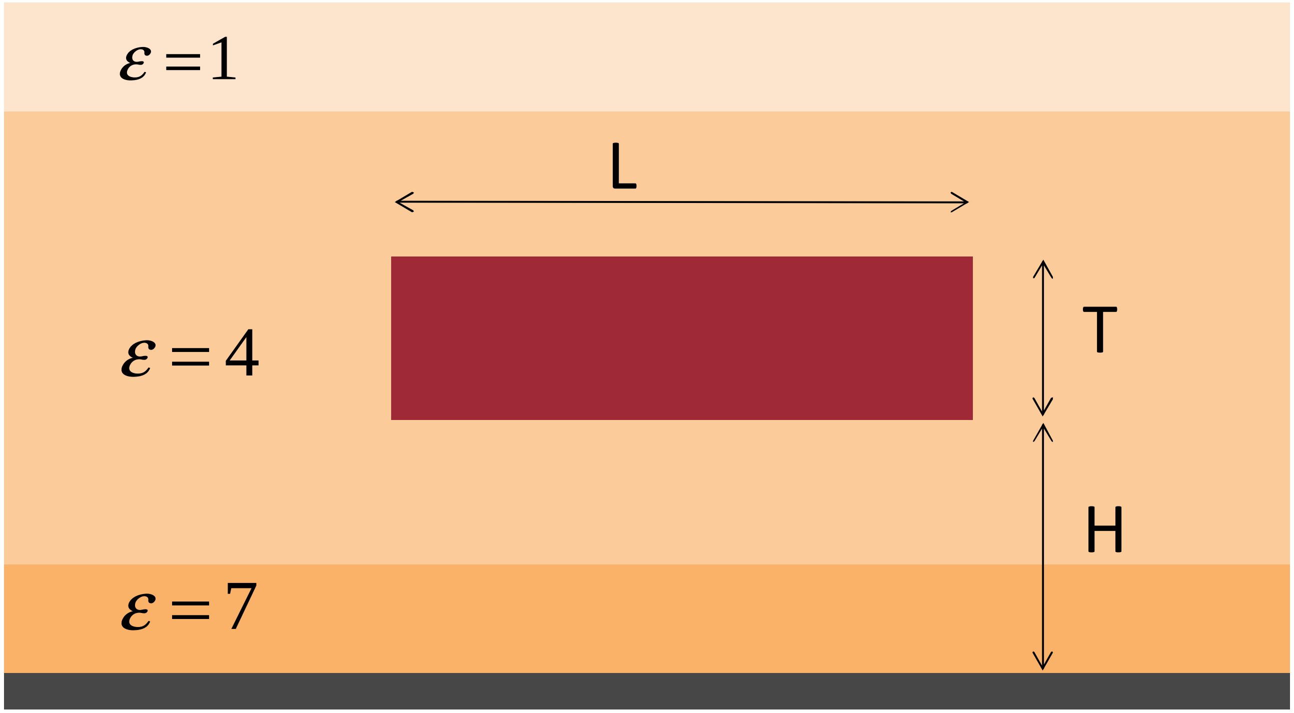
$$\left|\vec{D}'(\vec{r}')\right| \approx \left|\vec{D}_0(\vec{r})\right| \frac{G(\vec{r})}{G(\vec{r})-v(\vec{r})} \Rightarrow \left|\tilde{\vec{D}}'(\vec{r})\right| \approx \left(1+\frac{v(\vec{r},\theta)}{G(\vec{r})}+\frac{v^2(\vec{r},\theta)}{G^2(\vec{r})}+\frac{v^3(\vec{r},\theta)}{G^3(\vec{r})}+....\right)\left|\vec{D}_0(\vec{r})\right|$$

Stochastic Capacitance:

$$\tilde{C}=\sum_{i=0}^{N} C_i(\vec{r})\Psi(\xi(\theta))$$

$$\tilde{C} \approx \int_S \left(1+\frac{v(\vec{r},\theta)}{G(\vec{r})}+\frac{v^2(\vec{r},\theta)}{G^2(\vec{r})}+\frac{v^3(\vec{r},\theta)}{G^3(\vec{r})}+....\right)\left|J\right| \vec{D}_0.\vec{ds} \quad (*)$$

Single trace over ground plane



- The height of the conductor above the ground plane 'H' is uncertain.

Mean Geometry dimensions : L=1 um, T = 0.1 um, H= 0.2 um

$$H(\theta) = H_0(1 - \nu\xi(\theta))$$

-Where H_0 is the mean height

- ξ is a Gaussian random variable with mean 0 and variance 1

Single trace over ground plane

Displacement of random geometry from the mean geometry

$$v(\vec{r},\theta) = v\xi H_0$$

$$\tilde{C} \approx \int_S \left(1 + \frac{vH_0}{G(\vec{r})}\xi + \frac{v^2 H_0^{\ 2}}{G^2(\vec{r})}\xi^2 + \dots \right) |J| \left|\vec{D}_0\right| dl$$

Second-order Hermite polynomial chaos for stochastic capacitance

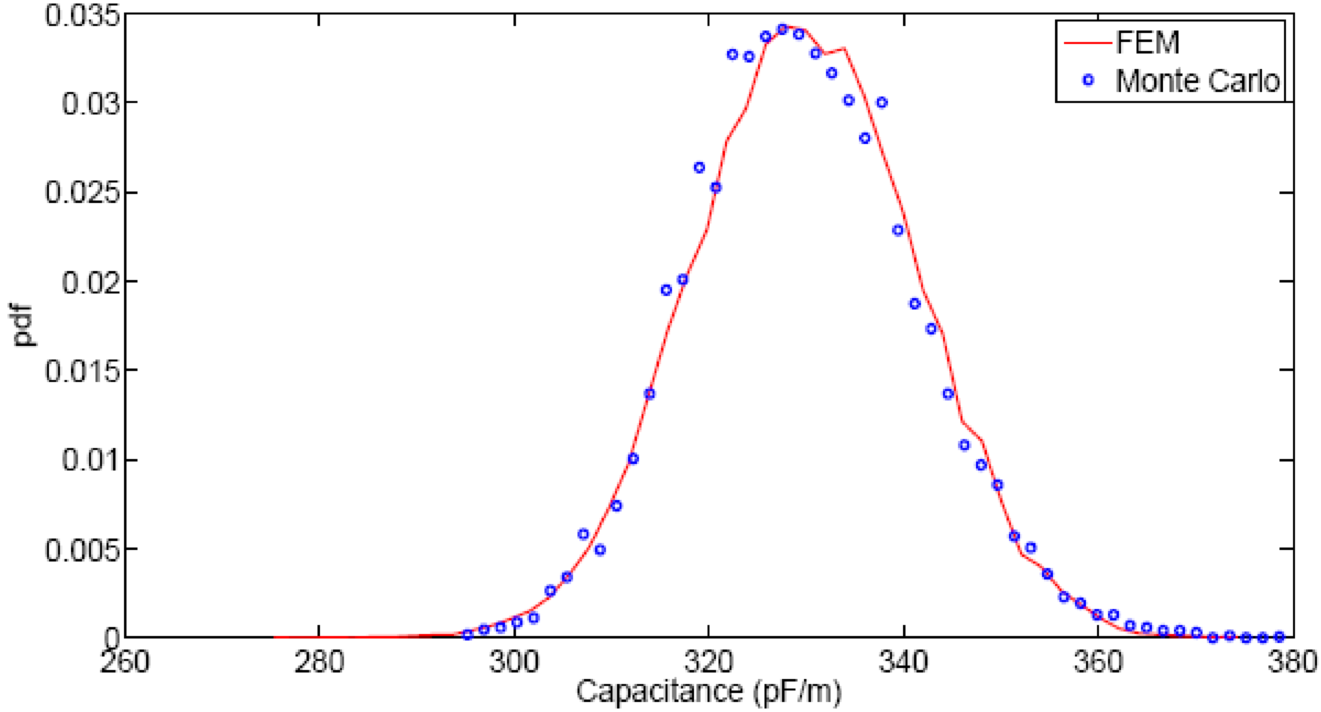
$$C = C_0 + C_1\xi + C_2(\xi^2 - 1)$$

$$C_0 = \int_S \left(1 + \left(\frac{vH_0}{G(\vec{r})} \right)^2 \right) |J| \left|\vec{D}_0\right| dl$$

$$C_1 = \int_S \left(\frac{vH_0}{G(\vec{r})} \right) |J| \left|\vec{D}_0\right| dl, \quad C_2 = \int_S \left(\frac{vH_0}{G(\vec{r})} \right)^2 |J| \left|\vec{D}_0\right| dl$$

Only one deterministic run needed to get $\left|\vec{D}_0\right|$

Single trace over ground plane



Self-capacitance (pF/m)

%change in H	Monte Carlo		FEM based Approach	
	Mean	Std deviation	Mean	Std deviation
10%	329.7429	11.5365	329.83	11.43
20%	331.0031	23.6053	331.48	23.31

Single trace over ground plane

Capacitance (pF/m)

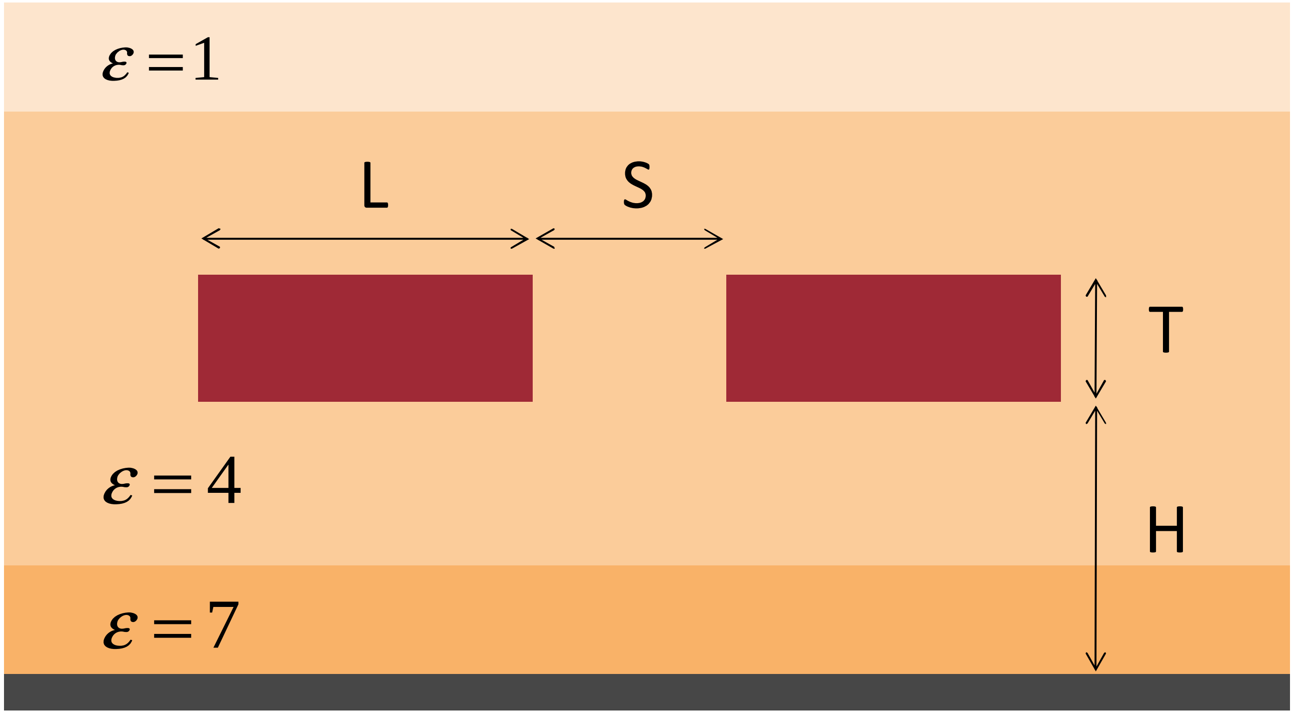
%change in L	Monte Carlo		FEM based Approach	
	Mean	Std deviation	Mean	Std deviation
10%	331.0028	7.26	329.26	7.48
20%	331.5143	14.92	329.26	14.96

Simulation time comparison

Time for 1 Capacitance extraction run ~1.2 s

- Monte Carlo : Time for 10000 runs ~ 12000 s
- Our approach ~ 2.0 s

Coupled symmetric microstrip

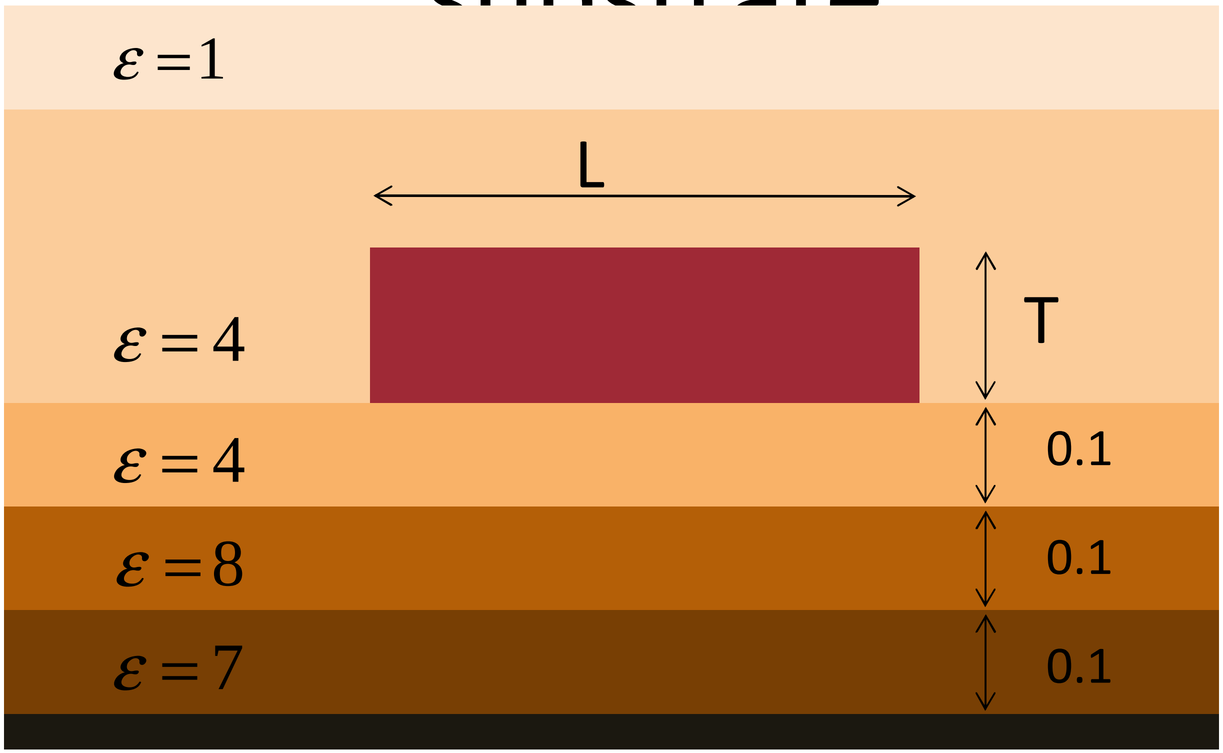


Mean Geometry dimensions : L=1 um, S = 0.15 um, H= 0.2 um

Self-capacitance (pF/m)

% change in H and S	Monte Carlo		FEM based Approach	
	Mean	Std deviation	Mean	Std deviation
10%	368.7324	13.04	368.85	14.05
20%	370.2421	26.83	370.83	28.65

Microstrip with multi-dielectric substrate



Self-capacitance (pF/m)

%change in each layer below Conductor	Monte Carlo (10000)		FEM based Approach	
	Mean	Std deviation	Mean	Std deviation
10%	268.94	6.76	268.01	6.26
20%	269.46	13.56	267.82	12.56

Remarks

- Expedient way for handling statistical variability in interconnect cross-sectional geometry
  - p.u.l. capacitance extraction 100x - 1000x faster than standard Monte Carlo
- Approach independent of the field solver used

Deterministic Model Order Reduction

$$(Y_{org} + sZ_{org} + s^2 Pe_{org})x = sB_{org} I$$

$$V = L_{org}^H x$$

Order: *N*

Model Order Reduction
e.g. Krylov subspace based methods

Projection matrix *F*

$$Y = F^H Y_{org} F$$

$$Z = F^H Z_{org} F$$

$$Pe = F^H Pe_{org} F$$

$$(Y + sZ + s^2 Pe)x = sBI$$

$$V = L^H x$$

Order n << N

Generalized Multiport Impedance Matrix using reduced model:

$$Z_G(s) = sL^H (Y + sZ + s^2 Pe)^{-1} B$$

Model order reduction under uncertainty

Deterministic Reduced Order Model

$$(Y + sZ + s^2 Pe)x = sBI$$
$$y = L^H x$$

Stochastic Reduced Order Model

$$(\tilde{Y} + s\tilde{Z} + s^2 \tilde{P}e)\tilde{x} = sBI$$
$$\tilde{V} = L^H \tilde{x}$$

$$\tilde{Y} = \tilde{F}^H \tilde{Y}_{org} \tilde{F}$$
$$\tilde{Z} = \tilde{F}^H \tilde{Z}_{org} \tilde{F}$$
$$\tilde{P}e = \tilde{F}^H \tilde{P}e_{org} \tilde{F}$$

Represent stochastic system matrices using polynomial chaos expansion:

$$\tilde{Y}_{org} = Y_0 + Y_1\xi_1 + Y_2\xi_2, \quad \tilde{Z}_{org} = Z_0 + Z_1\xi_1 + Z_2\xi_2$$
$$\tilde{P}e_{org} = P_0 + P_1\xi_1 + P_2\xi_2, \quad \tilde{F} = F_0 + F_1\xi_1 + F_2\xi_2$$

Augmented Stochastic Reduced Order Model

$$[(Y_0 + Y_1\xi_1 + Y_2\xi_2) + s(Z_0 + Z_1\xi_1 + Z_2\xi_2) + s^2(Pe_0 + Pe_1\xi_1 + Pe_2\xi_2)](x_0 + x_1\xi_1 + x_2\xi_2) = s(B_0 + B_1\xi_1 + B_2\xi_2)I$$

$$\begin{bmatrix} Y_0 & Y_1 & Y_2 \\ Y_1 & Y_0 & 0 \\ Y_2 & 0 & Y_0 \end{bmatrix}\begin{bmatrix} x_0 \\ x_1 \\ x_2 \end{bmatrix} + s\begin{bmatrix} Z_0 & Z_1 & Z_2 \\ Z_1 & Z_0 & 0 \\ Z_2 & 0 & Z_0 \end{bmatrix}\begin{bmatrix} x_0 \\ x_1 \\ x_2 \end{bmatrix} + s^2\begin{bmatrix} Pe_0 & Pe_1 & Pe_2 \\ Pe_1 & Pe_0 & 0 \\ Pe_2 & 0 & Pe_0 \end{bmatrix}\begin{bmatrix} x_0 \\ x_1 \\ x_2 \end{bmatrix} = s\begin{bmatrix} B_0 \\ B_1 \\ B_2 \end{bmatrix} I$$

Augmented reduced order model

$$(Y_{aug} + sZ_{aug} + s^2 Pe_{aug})x_{aug} = sB_{aug} I$$

$$V_{aug} = L_{aug}^H x_{aug}$$

$$Z_{aug} = sL_{aug}^H (Y_{aug} + sZ_{aug} + s^2 Pe_{aug})^{-1} B_{aug}$$

Order: *3n << N*

Computing polynomial chaos coefficients

Coefficient matrices in polynomial chaos expansion:

- Integrate over the random space and use orthogonality of the polynomials

$$\iint \tilde{Y}_{org}\rho_1(\xi_1)\rho_2(\xi_2)d\xi_1 d\xi_2 = \iint (Y_0 + Y_1\xi_1 + Y_2\xi_2)\rho_1(\xi_1)\rho_2(\xi_2)d\xi_1 d\xi_2$$

$$Y_0 = \iint \tilde{Y}_{org}\rho_1(\xi_1)\rho_2(\xi_2)d\xi_1 d\xi_2$$

$$\iint \tilde{Y}_{org}\xi_1\rho_1(\xi_1)\rho_2(\xi_2)d\xi_1 d\xi_2 = \iint (Y_0 + Y_1\xi_1 + Y_2\xi_2)\xi_1\rho_1(\xi_1)\rho_2(\xi_2)d\xi_1 d\xi_2$$

$$Y_1 = \iint \tilde{Y}_{org}\xi_1\rho_1(\xi_1)\rho_2(\xi_2)d\xi_1 d\xi_2$$

Smolyak Sparse Grid Integration

$$I(f) = \iint \tilde{Y}_{org}\rho_1(\xi_1)\rho_2(\xi_2)d\xi_1 d\xi_2 \approx \sum w_j f(u^j)$$

1-d integration rule: e.g. using chebyshev polynomial extrema

$$I(f) = \sum_{j=1}^{q} f(u^j)w^j \qquad u^j = -\frac{\cos \pi(j-1)}{q-1}, \; j = 1 \ldots q$$

The case of multiple random variables:

Deterministic Cartesian Product (DCP) rule:

$$I^Q[f] \equiv \left(I_i^{q_1} \otimes \ldots \otimes I_i^{q_N}\right)[f]$$

$$= \sum_{j_1=1}^{q_1} \ldots \sum_{j_N=1}^{q_N} f(u_1^{j_1}, \ldots, u_1^{j_N}).(w_1^{j_1} \otimes \ldots \otimes w_N^{j_N})$$

Number of calculations $\propto q^N$

Smolyak Sparse Grid Algorithm

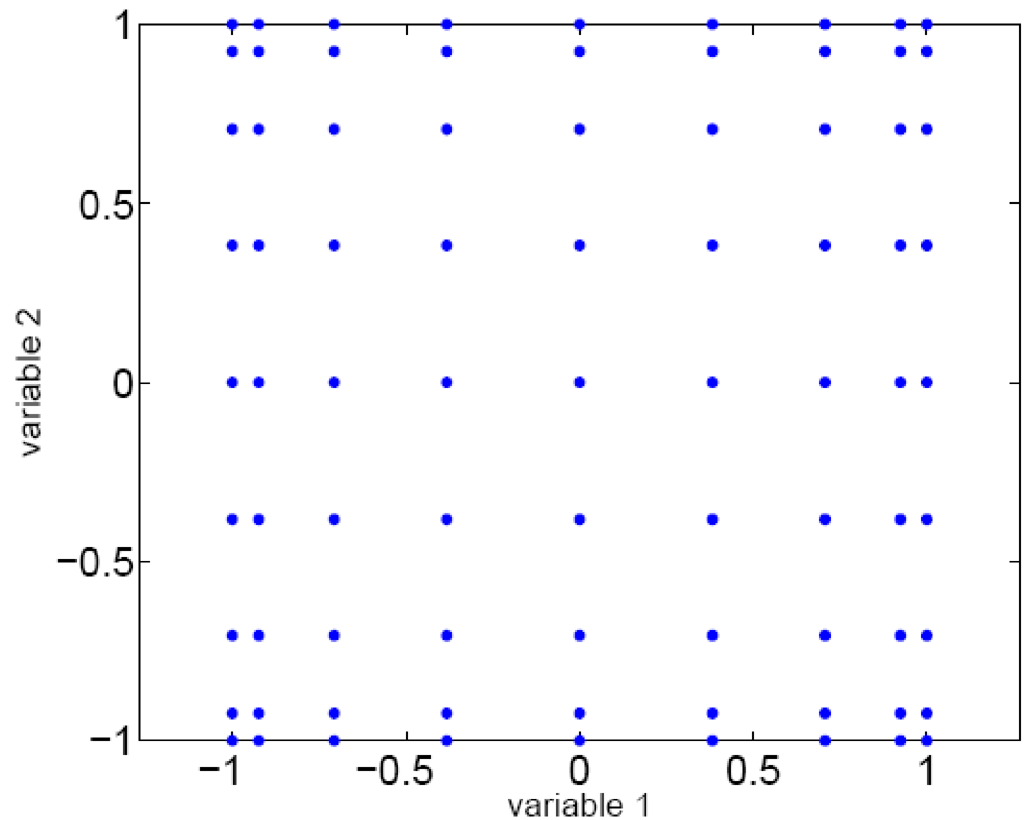
Idea: Not all points are equally important; hence, discard the least important ones

$$I^Q(f) \equiv A(J,N) =$$

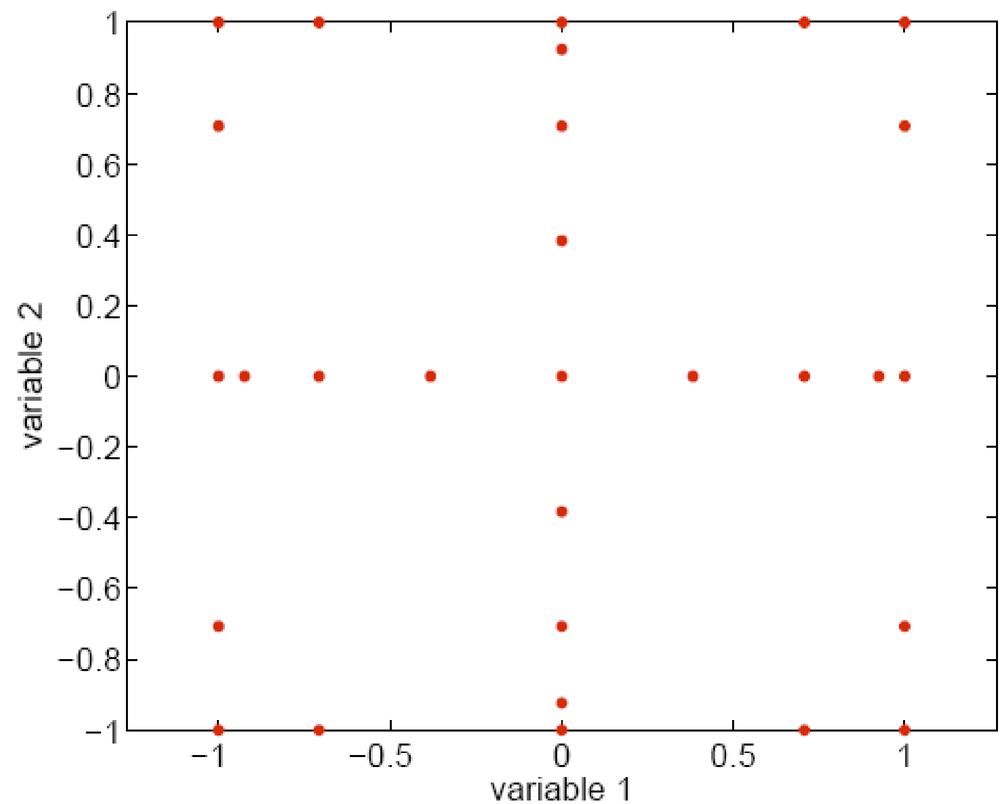
$$= \sum_{J-N+1 \le |i| \le J} (-1)^{J-|i|} . \begin{pmatrix} N-1 \\ J-|i| \end{pmatrix} .(I_{i_1} \otimes \ldots \otimes I_{i_N})$$

Number of calculations $\propto q(\log q)^{N-1}$

Comparison of grids generated using Tensor Product and Smolyak Algorithm



$$Q \propto q^N$$



Smolyak Sparse grid (29 points)

$$Q \propto N^p$$

q: number of points in 1-d rule
N: number of random dimensions
p: level in Smolyak algorithm

Algorithm for Stochastic MOR

- Represent uncertainty in the original system matrices through polynomial chaos expansion
- Generate sparse grid points and their corresponding weights using Smolyak Sparse grid algorithm
- Compute the transformation matrix through MOR of individual systems corresponding to Smolyak sparse grid points
- Compute the stochastic transform matrix
- Define the stochastic reduced order model

$$\tilde{Y}_{org} = Y_0 + Y_1\xi_1 + Y_2\xi_2$$

$$\theta_M = \{\xi_i\}, w_M = \{w_i\}$$

$$(Y_{org} + sZ_{org} + s^2 Pe_{org})x = sB_{org}I$$

$$x = F_i z$$

$$(Y + sZ + s^2 Pe)x = sBI$$

$$\tilde{F} = F_0 + F_1\xi_1 + F_2\xi_2$$

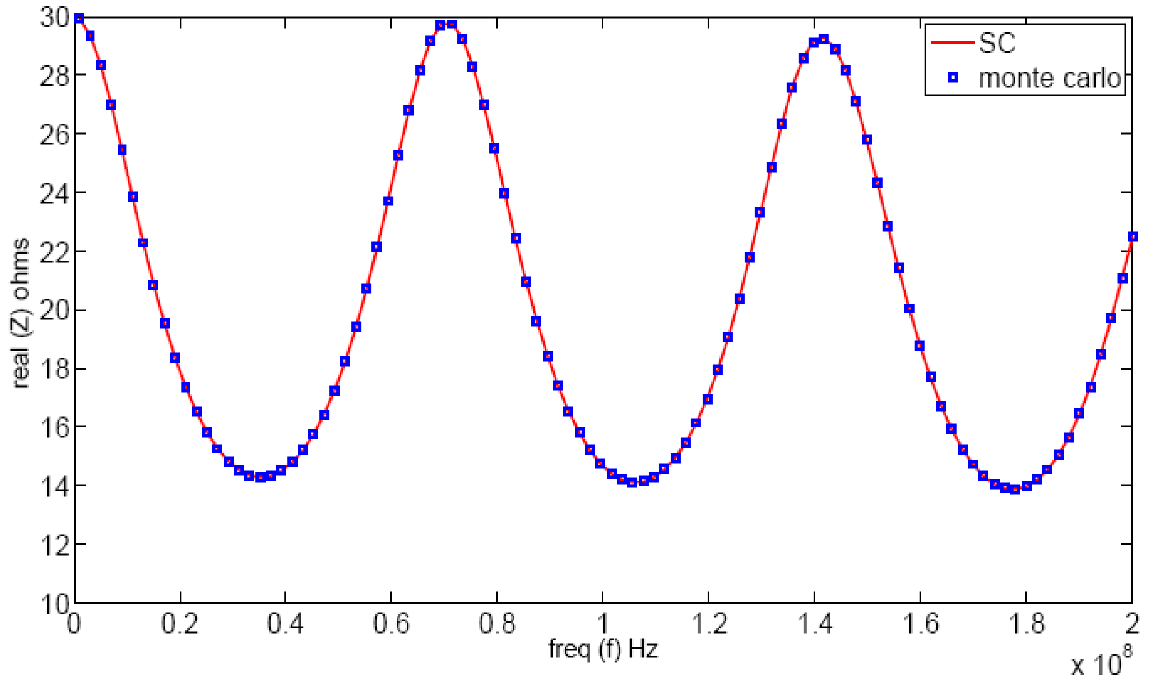
$$(Y_{aug} + sZ_{aug} + s^2 Pe_{aug})x_{aug} = sB_{aug}I$$

Example Study: Terminated coaxial cable

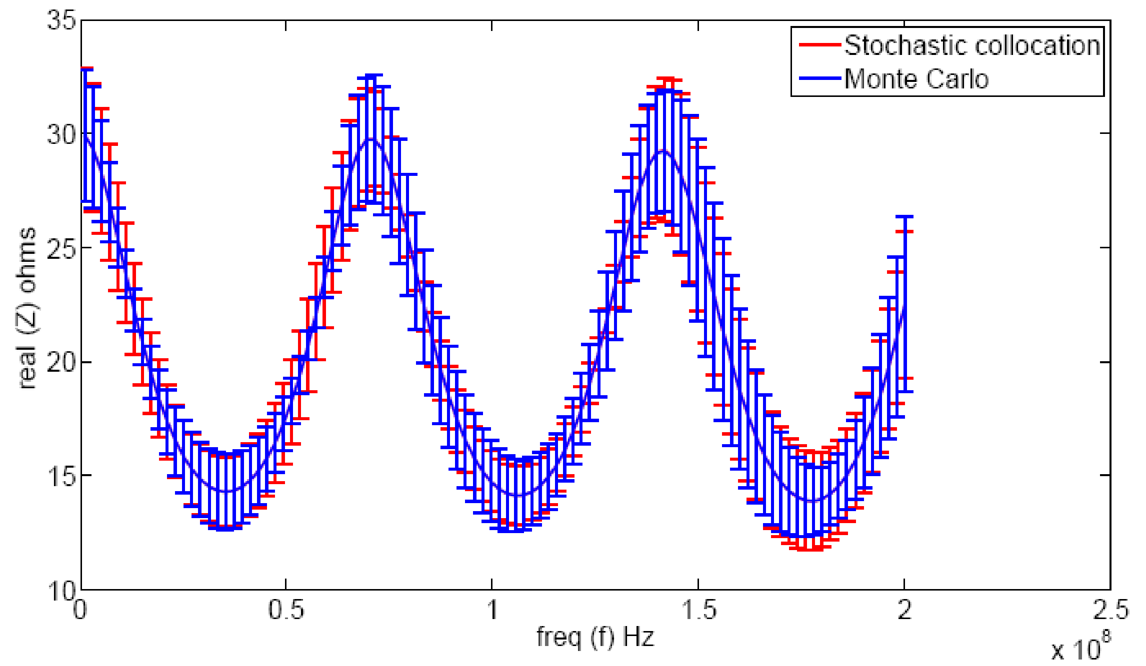
- Air-filled coaxial cable, terminated at a resistive load:
  - L=1m, inner radius=5mm, outer radius=10mm
  - FEM system (Y,Z,Pe) of order 36840
  - Reduced order system of order 20.
- Randomness in two inputs
  - Permittivity: uniform random variable in [3.4-4.4]
  - Load resistance: uniform random variable in [25-35] ohms
- Monte Carlo: 10201 simulations
- Smolyak: 29 points

Re{Zin(f)}

- Mean of real part of input impedance



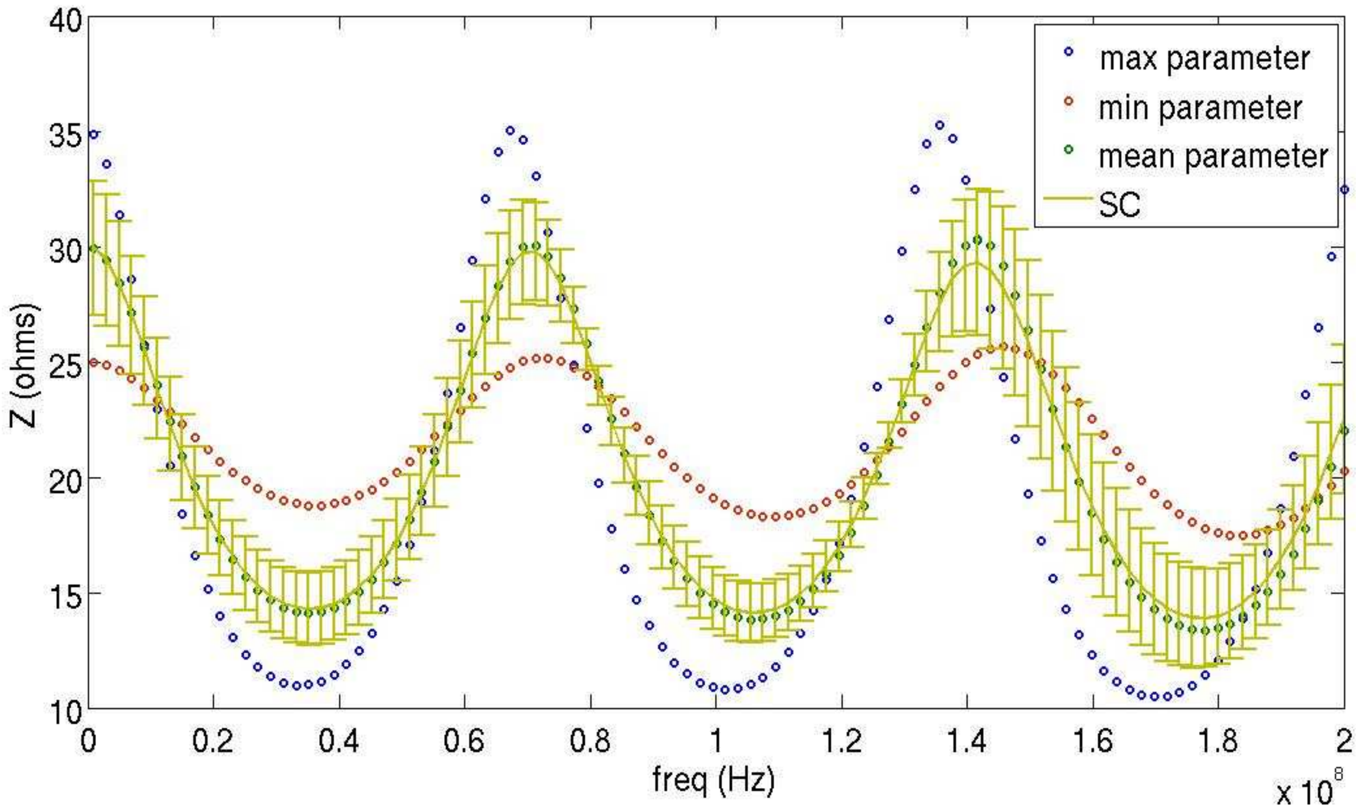
- Standard deviation of real part of input impedance



Corner simulations vs. stochastic simulations

- Standard practice to simulate corners for accounting for variability
- Corner simulations can be 'conservative'
- Coaxial cable example – consider corner values for random input parameters (ε, R_L)
  - (3.6,25)
  - (4.0,30)
  - (4.4,35)
- Compare with information generated using stochastic MOR

Corner vs. stochastic simulation



- Corner simulation appears very conservative
- Mean parameter solution is not accurate compared to the mean of stochastic simulation

Remarks

- Very good accuracy obtained with 100x to 1000x improvement in computation time compared to standard Monte Carlo.
  - Stochastic MOR model appropriate for time-domain simulations
- Stochastic MOR can have advantages over traditional corner based simulations
- Approach independent of the deterministic MOR method

- Variability and uncertainty is not a curse
  - It is an essential part of the dynamo of our evolution toward the next, more advanced state

"Chaos is the score upon which reality is written" – Henry Miller

- We should embrace uncertainty as an opportunity for tackling complexity
  - It will make our design tools more agile, more useful, and more conducive to complex system design flow
- It is critical for universities to pave the way down this path
  - Our future will not be built by deterministically-minded technologists and innovators