Practical Papers, Articles and Application Notes

Flavio Canavero, Technical Editor

In this issue, I propose to the readers of this column two diverse, but very interesting papers.

The first article is entitled "Ground Based Air-Conditioning System Interfered Aircraft Communication Channel" by Norbert Kohns. In this paper, the author reports a case of electromagnetic interference experienced by an aircraft VHF radio. He describes in detail how he identified the external source that was neither previously evident nor easy to find. He found that the source of disturbance was the electronic circuitry controlling an electric motor inside an airport hangar. The author discusses also how he was able to suppress the spurious oscillation of the control electronics by placing a capacitor on the circuit board. This paper offers a systematic analysis of the cause of interference and provides a very good example of troubleshooting work that I'm sure readers will enjoy.

The second paper is by Clayton R. Paul and discusses loop inductance. This is the first of a two-part series on "What Do We Mean by "Inductance?" The second part, dedicated to partial inductance will be published in the next issue. This is a very enlightening contribution on the concept of inductance of which several misconceptions and confusion exists. Professor Paul, with his plain and neat style that we all know from his many books, presents the basics of electromagnetic induction in a very accessible (though absolutely rigorous) manner and reveals the subtleties related to the interpretation and calculation of this electrical parameter.

With this series of two papers by Professor Paul, I would like to start an "Education Corner" in this column. I hope to continue with a sequence of enlightening contributions clarifying fundamental ideas, thus helping EMC Society members to better perform in their profession.

In conclusion, I encourage (as always) all readers to actively participate in this column, either by submitting manuscripts they deem appropriate, or by nominating other authors having something exciting to share with the community. I will follow all suggestions and with the help of independent reviewers, it is my sincere hope to be able to provide a great variety of enjoyable and instructive papers. Please communicate with me, preferably by email at canavero@ieee.org.

Ground Based Air-Conditioning System Interfered Aircraft Communication Channel

Norbert Kohns, Member, IEEE

***Abstract*—This paper describes a very unusual cause of VHF band interference and the technique for how the source of radiation was determined. An electronic circuit that controls a motor driven air intake flap of an air-conditioner heat exchanger, "mutated" into a broadband VHF transmitter, jamming a large segment of the VHF band.**

I. INTRODUCTION

Pilots on aircraft NATO 1 (N1) reported multiple squelch breaks on radio VHF 5. This specific feature occurred during taxi, takeoff and landing with the radio tuned to the main operating base Geilenkirchen tower frequency of 140.075 MHz. When the VHF radio squelch opened, a very loud buzzing was heard on the headset. Furthermore, this fault was reported only intermittently by the pilots, as it did not occur on some days. Because the phenomenon could not be isolated and eliminated in an adequate time frame, aircraft commanders refused to fly N1 until the problem was solved.

II. FACT-FINDING

A spectrum analyzer (SA) connected to the dual band antenna VHF 5 of N1 (parked at spot 10) showed a broad band of spectral lines cluttering above and below the tower frequency of 140.075 MHz (see Fig. 1).

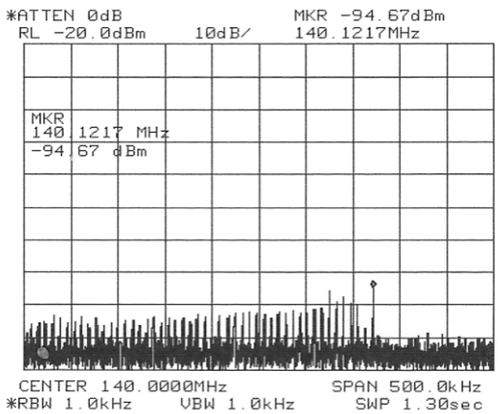

Fig. 1. Spectral lines cluttering the VHF air band. Each line represents a VHF radio frequency carrier. The highest frequency is at 140.1217 MHz, which is a few kHz above the tower frequency. The power level of the signals is well within the squelch breaking level of radio VHF 5 (-110 dBm).

A similar measurement was done on the legacy aircraft 444, which was parked on spot 9 (see Fig. 2).

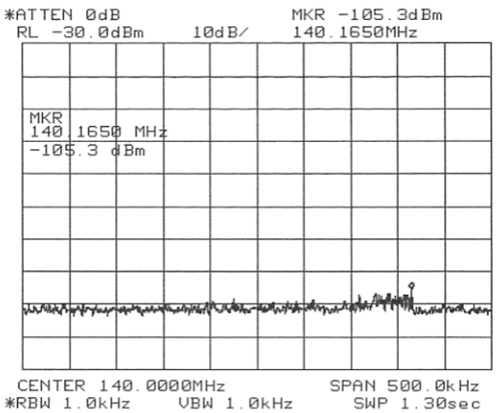

Fig. 2. The RF-power from the VHF antenna to the VHF radio receiver input is about 10 dB less than on N1. Therefore, only the highest signal levels can be resolved with the same SA SPAN and resolution bandwidth (RBW). The highest frequency is at 140.1650 MHz. The squelch did not automatically open with this low signal level on the legacy aircraft 444. This indication explains why N1 was the only aircraft to exhibit this defect. However, the same buzzing tone was heard on the headset when the squelch was manually opened on the legacy aircraft.

III. LOCATING THE SOURCE OF INTERFERENCE

In order to avoid a possible ground loop with the SA, the external power source and the ground potential of the aircraft, a battery operated DC to AC converter was used to apply power to the SA. All aircraft power was shut down and the aircraft power cable was disconnected. The test result was virtually the same as shown in Figure 1 above. It was now clear that the defect was an externally generated VHF band-jamming signal. The signal source was pinpointed to the area inside Hangar 1 by use of a handheld VHF band radio scanner. The buzzing signal was heard at all locations inside and in front of Hangar 1 and towards the runway. Hangar 1 is located 300 meters away from the center of the runway. However the buzzing signal could be received at both ends of the runway. The length of the runway is more than 3000 meters. With the SA, a plot was taken inside Hangar 1 (see Fig. 3).

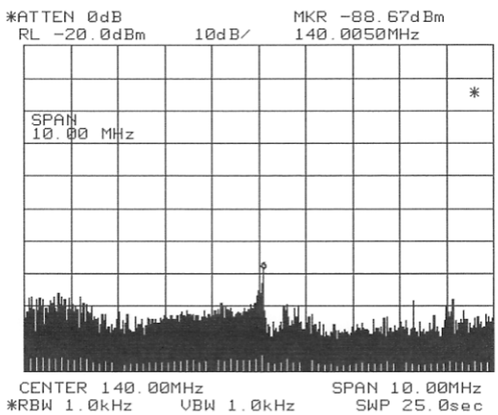

Fig. 3. With a SPAN setting of 10 MHz the VHF jamming source peaks clearly at 140.0050 MHz out of the frequency spectrum. The plot was taken inside Hangar 1.

The audio output of the handheld scanner was measured with an oscilloscope (see Fig. 4 and 5) in order to determine the modulation type of the interfering signal.

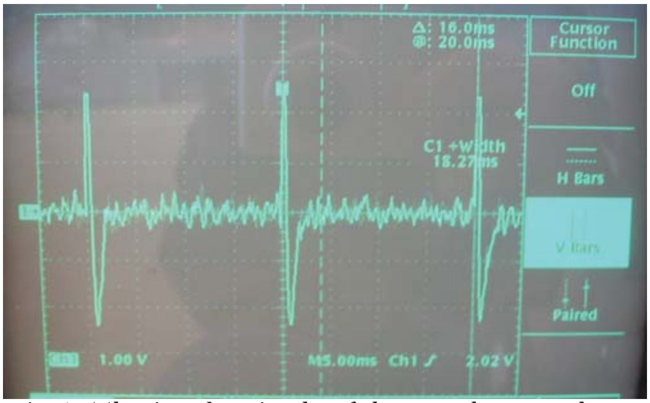

Fig. 4. The time domain plot of the scope shows a pulse signal with a repetition time of 20 ms, which equates to a frequency of 50 Hz. This explains the buzzing noise that is heard on the headset.

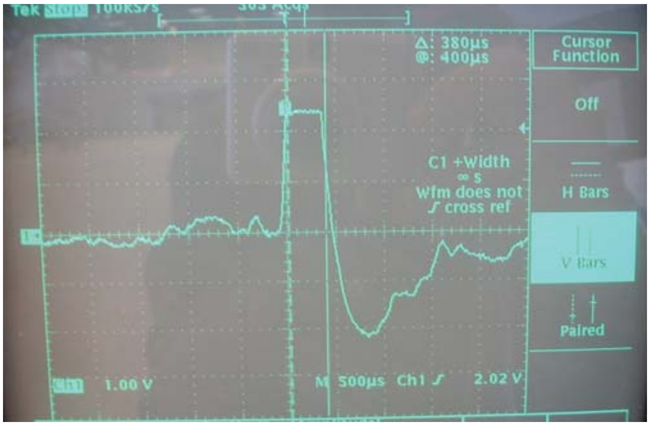

Fig. 5. The pulse width is approximately 380 µs.

IV. FINDING THE SOURCE OF INTERFERENCE

In order to confirm that the signal source was located somewhere inside Hangar 1 it was decided to completely shut down the mains power from the adjacent Building 217 and Hangar 1 on the next day. The test result is shown in Fig. 6 and 7.

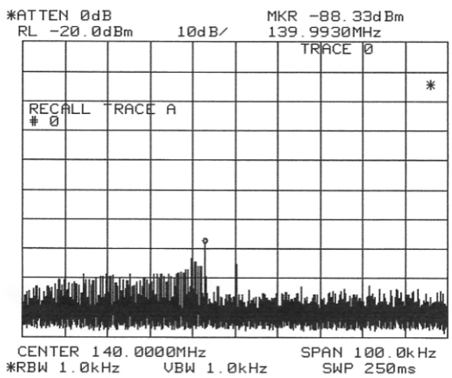

Fig. 6. On the following day this plot was taken inside Hangar 1 five minutes before the power was completely switched off. It should be noted that the highest frequency has shifted to the left and the peak interference frequency is now at 139.9930 MHz. The tower frequency is cleared at this moment. This observation explains why the interference was intermittent and not present every day.

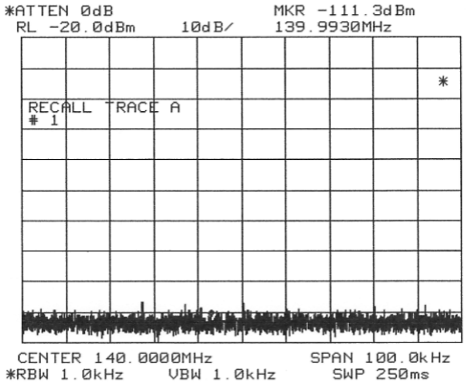

Fig. 7. This plot was taken during the power off time inside Hangar 1. All interference signals diminished to less than -110 dBm. This proved that the source of the VHF squelch breaks on aircraft N1 was located inside Hangar 1.

During the power shut down time of Hangar 1, a plot was also taken at the VHF 5 antenna on aircraft N1. The distance between the aircraft N1 and Hangar 1 is about 200 meters (see Fig. 8).

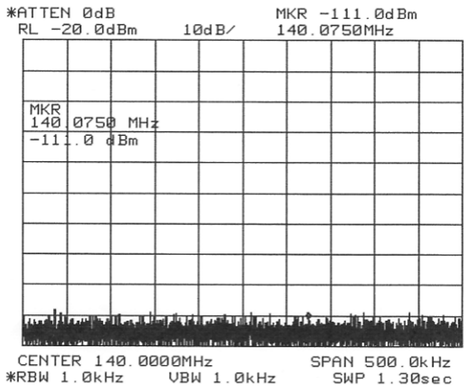

Fig. 8. This plot was taken at the VHF 5 antenna on aircraft N1 during the power off time of Hangar 1. All interference signals diminished to less than -110 dBm. This confirms the measurements inside Hangar 1.

V. TRIANGULATE THE INTERFERENCE

The challenge was now to find the actual source that generated this kind of interference. With the assistance of the StOV-German Garrison Administration-electricians and air conditioning specialists, Hangar 1 was powered down again. Power was reapplied to the Hangar in discrete sections in an effort to localize the source. The interference returned when power was applied to the air conditioning system. The air conditioning specialists attempted to isolate the subsystem that was causing the interference. While the handheld scanner was monitored, various functions were switched off and back on. The interference coincided with power being removed and restored to the motor control circuit that moves a flap inside of one of the heat exchangers mounted on the roof of Hangar 1. Heat exchanger 24 was identified as the originator and was inspected on the roof of the building (see Fig. 9)

The motor driven air intake flaps control the airflow inside each of the heat exchangers.

The suspected motor assembly was removed for further investigation (see Fig. 10).

Fig. 9. View of the roof of Hangar 1.

Fig. 10. Motor assembly.

All heat exchangers mounted on the roof were inspected with the handheld scanner. The results revealed a second assembly showing similar symptoms. It was also removed for further investigation

The bench test for the above-mentioned flap controller confirmed that the electronic motor control circuitry had "mutated" to become a broad band VHF air band transmitter when the flap reached the mechanical limit at the flap open position (see Fig. 11 and 12).

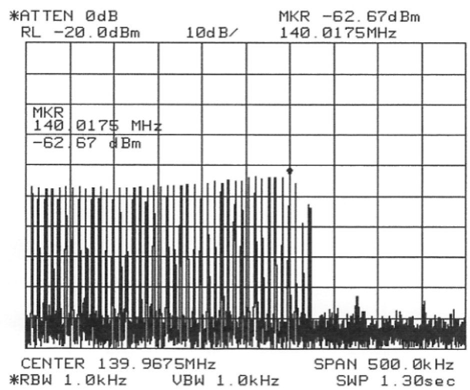

Fig. 11. This plot was taken during the bench test with a monopole antenna at a distance of two meters from the flap controller.

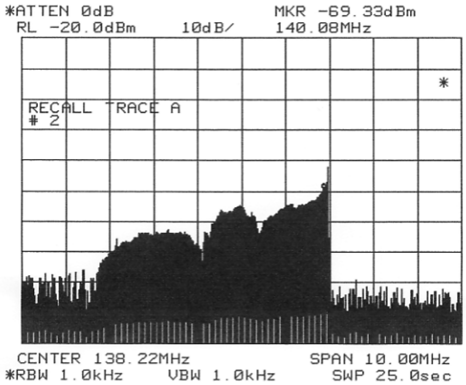

Fig. 12. Same test condition as in Fig. 11 except with a wider SPAN setting. The jamming signal is almost 5 MHz wide and has an envelope like a pulse signal.

VI. TROUBLESHOOTING THE MOTOR CONTROL UNIT

It was necessary to reengineer the schematic diagram of the circuit board in order to fully understand the circuit function of the motor control unit and to isolate the failing mechanism (see Fig. 13 and 16).

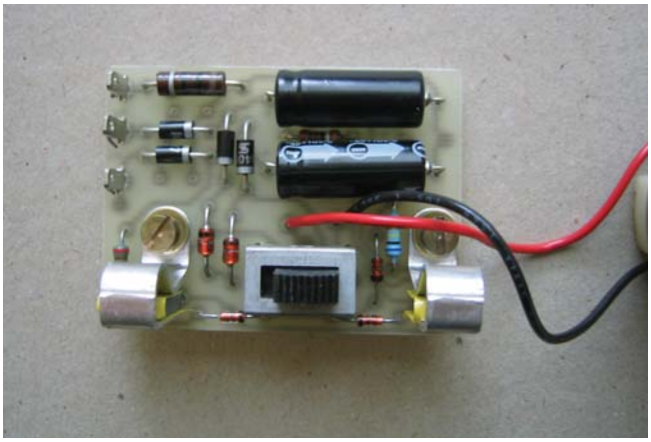

Fig. 13. Motor control unit circuit board.

VII. OBSERVATIONS

The circuit operates with 24 VAC under normal conditions. When the circuit is operated with 24 VDC instead and the clockwise motor rotation is stopped, the circuit begins to oscillate at a stable frequency of 133.5 MHz (see Fig. 14).

VIII. FUNCTIONAL DESCRIPTION OF MOTOR CONTROL CIRCUIT AND TROUBLESHOOTING

The circuit consists of a constant current source, which supplies 50 mA of current to the connected DC motor. Connecting 24 VAC between KL2 pin 1 and 2 supplies 17 VDC across the motor terminals KL1 pin 1 and 2. The current is regulated to 50 mA and causes the motor to turn in a clockwise direction. The flap moves to the upper mechanical limit, which forces the motor to stop. The resistance across the motor decreases and, due to the current regulation of the voltage across the motor, drops to 7 VDC to prevent the motor from overloading. At that moment the circuit starts to oscillate at VHF frequencies. A 50 Hz (20ms) AC ripple is riding on the bias

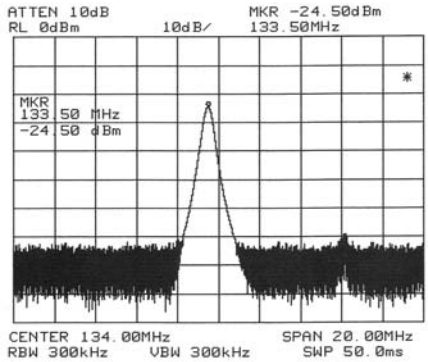

Fig. 14. When the circuit is supplied with 24 VAC similar to normal operating conditions, the carrier frequency shown above is pulse modulated with a 50 Hz signal (see Fig. 11 and 12). The pulse modulation generates a broad spectrum of side band emissions that cover a large band of frequencies within the VHF air band.

current to the base of transistor T1, which produces a combination of pulse and amplitude modulation. This modulation generates multiple radio frequency side bands (see Fig. 11 and 12). Connecting 24 VAC between KL2 pin 1 and 3 supplies 17 VDC with reversed polarity across the motor terminals KL1 pin 1 and 2. The motor turns counterclockwise and the flap stops at the lower mechanical limit. No oscillation occurs at this point. Troubleshooting the circuit was difficult because probing the circuit with an oscilloscope stopped the oscillation at almost any test point. For example, oscillation stopped when the emitter of T1 was measured. C1 and C2 were therefore removed for the measuring of their capacitance. Both values were well within limits. Reinstalling the capacitors in reverse positions stopped the oscillation at both mechanical flap stops. Measuring the equivalent series resistance (ESR) values indicated the location of the problem. C1 had a twice as high ESR as C2. In addition, the circuit board layout adds some instability due to the design of the trace between T1 and R4 (see Fig. 15).

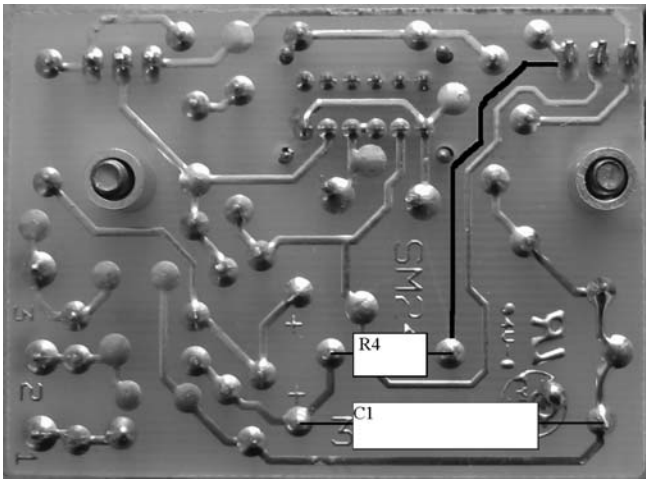

Fig. 15. The black highlighted circuit board trace between the emitter of T1 via R4 to C1 acts as an inductor. Oscillation will stop when a small capacitor (100nF) is placed at the emitter of T1 to ground.

IX. RF-COUPLING PATH

The electrical control central that provides power to all motor control circuits is located in a separate room inside Hangar 1. It supplies 24 VAC motor control signals and ground to the motor control circuits on the roof of Hangar 1 via unshielded cables (30 to 50 meters long vertically mounted). Measurements were taken with an RF-current probe at the cable that was connected to the defective motor control circuit reviled, that the cable was the radiating element. This explained the large transmitting range of the oscillating circuitry.

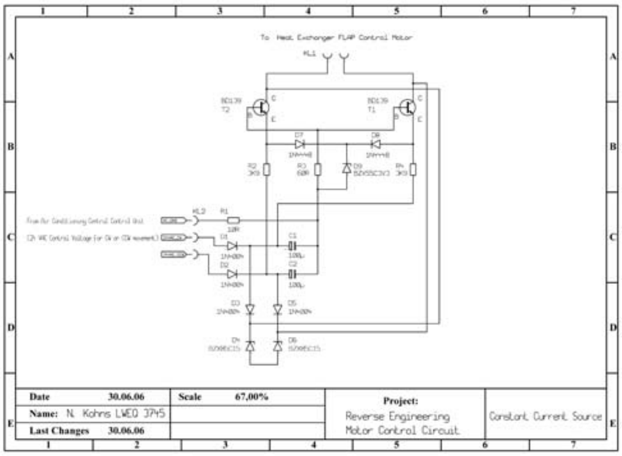

Fig. 16. Control circuit Schematic Diagram

X. CONCLUSION

Because of flight safety considerations the motor control units in all heat exchangers were replaced with a newer model.

BIOGRAPHY

Norbert Kohns *was born in Weißenthurm, Germany, in 1949. For 25 years, he has been employed as a NATO civilian. Currently he serves as a Principal Technician and Maintenance Instructor of the NATO AWACS Electronic Support Measures shop, located at the NATO Airbase in Geilenkirchen, Germany. In addition, he is the focal point of EMI/EMC related issues of the NATO E-3A fleet.*

What Do We Mean By "Inductance"? Part I: Loop Inductance

Clayton R. Paul, Life Fellow, IEEE

***Abstract*—This is the first part of a two-part article in which the concept and calculation of the inductance of a current-carrying loop are discussed. These are obtained using Faraday's law. In Part II, the ability to uniquely attribute portions of the inductance of a current loop to segments of that current loop using the concept of partial inductances is discussed. The intent of these articles is to provide meaningful and unambiguous formulations of inductance and partial inductance that are based on solid scientific principles in order to dispel the various misconceptions and erroneous conclusions that have arisen about these important concepts.**

***Index Terms*—inductance, current loops, magnetic flux, Faraday's law**

The concepts of resistance, capacitance and inductance are fundamental to the analysis of lumped electric circuits [1, 2]. We all understand the meaning and calculation of resistance. If we pass a current I through a block of material and measure the resulting voltage drop V across it, the resistance of the block of material is the ratio $R = V/I$. The concept and calculation of capacitance is also easily understood. If we have two bodies (conductive or not as in the case of ESD), we can calculate the capacitance of the structure by placing charge on them with $+Q$ on one body and an equal amount of charge but opposite in sign $-Q$ on the other body. A voltage V will be induced by this charge between the two bodies. The capacitance of this structure is the ratio $C = Q/V$. The units of capacitance are Farads which is named for Michael Faraday who, interestingly, had more to do with inductance than capacitance. We visualize electric field lines $\vec{E}$ produced by the charge that are directed from the positively-charged body to the negatively-charged body. The voltage between the two bodies is obtained as $V = -\int_{-}^{+} \vec{E} \cdot \vec{dl}$ [3, 4]. This is called the *line integral* along a path from the negatively-charged body to the positively-charged body. The integral of the *dot product* $\vec{E} \cdot \vec{dl}$ means that we sum (with an integral) the products of the *components* of the electric field lines that are *tangent to the path* and the differential lengths of this path dl. This is a sensible definition since the electric field vector has two components: one along the path and one perpendicular to the path. The component perpendicular to the path should not contribute to the result. The magnitude of the electric field and hence the magnitude of the resulting voltage is directly proportional to the magnitude of the charges Q. Hence the capacitance of this structure is dependent only on the geometrical shapes of the two bodies, their physical orientation with respect to each other, e.g., their separation, and the properties of the material they are immersed in, e.g., air, Teflon, etc. Alternatively, suppose we apply a voltage source of value V between the two bodies. Charge $Q = CV$ will be deposited from the source onto these bodies with the amount of charge Q that the bodies can store on them depending on the capacitance of the structure and the voltage applied between them. Hence capacitance represents the ability of the structure to store charge.

The concept and calculation of inductance seem to be less well understood thereby promoting numerous misconceptions and inappropriate terminology being applied to it. This leads to considerable misunderstanding about inductance and errors in its calculation that we intend to rectify in this article. While capacitance results from the separation of charge, inductance results from the movement of charge: an electric current. In this article we will examine the concept and calculation of inductance using fundamental scientific laws (Faraday's law). This understanding of the basic meaning of inductance will allow the unambiguous calculation of inductances for various configurations of *closed loops of current*. In Part II we will also examine how to *uniquely* apportion parts of the inductance of a closed loop to portions of that loop leading to the important concept of *partial inductance* which allows the calculation of *ground bounce* and *power rail collapse* that are critically important in the design of an electronic system for signal integrity [5, 6].

We routinely model electronic circuits with a *lumped-circuit model* which is a particular interconnection of the lumped-circuit elements of resistance, capacitance and inductance [1, 2]. We then solve these lumped-circuit models for the resulting voltages and currents of those elements using Kirchhoff's voltage law (KVL) and current law (KCL). *These lumped-circuit models and the voltages and currents obtained from them are only valid so long as the largest physical dimension of the circuit is electrically small, i.e., much less than a wavelength at the frequency of excitation, f, of that circuit* [3, 4]. A wavelength is $\lambda = v/f$ where v denotes the velocity of propagation of, for example, the currents along the connection leads attached to the elements. If the surrounding medium is free space (for all practical purposes air) then the velocity of propagation is approximately $v = 3 \times 10^8$m/s. If a sinusoidal source excites the circuit and has a frequency of 300 MHz, a wavelength is 1 meter, and if the excitation frequency of the source is 3 GHz a wavelength is 10 cm or approximately 4 inches. In the case of a printed circuit board (PCB) the velocities of propagation of the signals carried by the lands on the board are about 60% of that of free space which is due to the interaction of the electromagnetic fields produced by those signals with the board substrate and hence the wavelengths are smaller than in air. In lumped circuits we can ignore the effects of the connection leads attached to the lumped elements because their lengths must be electrically small, i.e., $\ll \lambda$, in order for the model to be valid. If a connection lead that is attached to an element is electrically long, currents at the two endpoints of this lead *will not be the same* but will have a phase difference between them [3, 4, 6]. If the length of the connection lead is one-half wavelength, these currents at the endpoints of the lead will be 180° out of phase with each other. If the length of the connection lead is only $\lambda/100$, the phase difference between the two currents at the endpoints is an inconsequential 3.6° and can be ignored. This phase difference translates in the time domain to a time delay; the current at one end of the connection lead and the current at the other end will have a time delay between them. For a connection lead of length L this time delay (in seconds) can be written as $T_D = L/v = (L/\lambda)(1/f) = (L/\lambda)P$ where $P = 1/f$ is the period of the sinusoidal waveforms. Hence the sinusoidal waveforms at the two ends of the connection lead will be shifted in time relative to each other by a fraction of their period, L/λ. If the connection lead is electrically short, $L \ll \lambda$, then the two waveforms will be almost coincident in time and the time delay can be ignored. Otherwise the time delay will be significant. The conventional lumped-circuit analyses that we use ignore the effects of the connection leads which is valid *only if* the largest dimension of the circuit is electrically small, say, less than $1/10\,\lambda$. This important restriction allows us to make some important simplifying assumptions in the calculation of the lumped elements and should be kept in mind throughout both articles.

I. FARADAY'S LAW AND INDUCTANCE

Maxwell's equations govern *all* that we, as electrical engineers, do. However, we routinely make simplifying approximations of a problem such as symmetry, electrically small dimensions (as in the case of lumped circuit models), etc. in order to be able to solve Maxwell's equations approximately for a specific problem. It is often claimed that "We aren't using Maxwell's equations" in solving everyday problems when in fact we are (always) using an approximation of them. For example, lumped-circuit models as well as the formulations of resistance, capacitance and inductance in those models represent approximations of Maxwell's equations [3]. Maxwell's equations are very simple to understand, but calculations from them are usually quite difficult to achieve except for some very ideal configurations. Maxwell's equations are commonly stated as four laws: Faraday's law, Ampere's law, Gauss' law for the electric field (which provides that electric field lines that begin on positive charge must terminate on negative charge), and Gauss' law for the magnetic field (which provides that all magnetic field lines must form closed loops, i.e., there are no isolated sources or sinks of the magnetic field unlike the electric field) [3, 4]. In addition to these four laws, the law of Conservation of Charge (which was apparently first postulated by Benjamin Franklin) is implied as a fifth law. It can be shown that the two laws of Gauss can be derived from Faraday's and Ampere's laws and the law of Conservation of Charge so that actually there are only three unique equations [3, 4].

The following important principle must be kept in mind:

All notions about inductance and calculations of it result from Faraday's law.

In order to state Faraday's law in unambiguous mathematical terms, consider Figure 1 which shows an open surface s that has a contour or path *c* surrounding it. With reference to Figure 1,

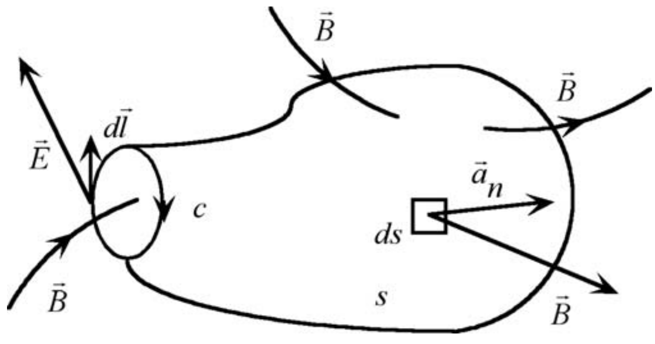

Fig. 1.

Faraday's law can be stated in mathematical form as [3, 4]

$$emf = -\frac{d\psi}{dt} \tag{1}$$

where the *electromotive force* around the *closed loop c* is

$$emf = \oint_c \vec{E} \cdot d\vec{l} \tag{2}$$

and the *magnetic flux* that passes through the *open surface s* is

$$\psi = \int_s \vec{B} \cdot d\vec{s} \tag{3}$$

The contour *c* of the *closed loop* can be thought of as either a conducting material (as in the case of a wire) or an imaginary contour of non-conducting material (as in the case of free space) and $\vec{E}$ is the *electric field intensity vector* with units of volts/meter along that contour. The *dot product* in the integrand of the *emf* in (2), $\vec{E} \cdot d\vec{l}$, means that we take the product of the electric field lines that are *tangent to the contour* and the differential lengths of this contour dl. We then sum these products (with an integral) to obtain the *emf* around that closed path. Again, $\vec{E}$ has a component parallel or tangent to this path and a component perpendicular to this path, and the components that are perpendicular to this path do not contribute to the sum. Observe that the electromotive force in (2) has the units of volts and acts like a voltage. However the minus sign that was present in the previous definition of voltage is absent here so that instead of being a voltage *produced by charge*, the *emf* represents a form of *voltage source inserted in the loop*. If the electrical dimensions (in wavelengths) of the closed loop are electrically small ($\ll \lambda$) we may treat this *emf* as a lumped voltage source and place it anywhere in the loop.

The right-hand side of Faraday's law in (1) is the rate of *decrease* (the negative sign is referred to as Lenz's law) of the magnetic flux ψ given in (3) that passes through the surface *s* that the closed loop *c* encloses, and $\vec{B}$ is the *magnetic flux density* vector with units of Webers/m^2 or Tesla. The result of the *surface integral* in (3), ψ, gives the *net magnetic flux passing through the surface that is enclosed by the contour c*. The units of that flux are Webers. A vector differential surface of that surface is $d\vec{s} = ds\,\vec{a}_n$ where $\vec{a}_n$ is the unit normal to the surface. The dot product $\vec{B} \cdot d\vec{s}$ in the *surface integral* in (3) means that we take the product of the components of $\vec{B}$ that are *perpendicular to the surface* and the differential surfaces ds. Then we add (with an integral) these products to give the *net magnetic flux* ψ *leaving* *(or passing through) the open surface s*. This is again sensible since $\vec{B}$ has two components: one perpendicular to the surface and one that is tangent to the surface. The component of $\vec{B}$ that is tangent to the surface does not (and should not) contribute to the net flux passing *through the surface*.

So we may interpret Faraday's law as providing that:

A time-varying magnetic field passing through an open surface s will induce (produce) an electric field around the contour c that encircles the surface.

This is the process behind some particle accelerators that accelerate charged particles to enormous speeds and smash them into other particles in order to break those particles into their constituent pieces. A large, time-varying magnetic field creates an electric field that exerts a force on electric charge. The path here into which the electric field is induced is an imaginary contour in space. Faraday's law also makes possible electric transformers and electric motors and generators among an enormous number of other applications that are absolutely essential to our daily lives and commerce. In an electric generator, coils of wire rotate around a shaft and pass through a magnetic field thereby causing a time-varying magnetic field in those coils of wire. Hence, voltages are induced in those coils of wire by Faraday's law thereby producing "electricity". Without Faraday's law you would be reading this article by candlelight! This article would have been written by hand because computers would also not be possible without Faraday's law.

The contour or path c in the general statement of Faraday's law can be thought of as the mouth of a "balloon" which can be inflated to give different surfaces s as illustrated in Figure 1. All these surfaces give the same result so long as the contour c remains the same. Magnetic flux lines that enter and leave the surface and do not pass through the mouth of the balloon do not contribute to the net flux through the surface and hence do not contribute to the induced electric field. Only those magnetic field lines that pass through the mouth of the balloon contribute to the net flux exiting the balloon surface. The direction of the contour c and the direction of "out of" the open surface s are related by the *right-hand rule*. Placing the fingers of our right hand in the direction of the contour c, our thumb will point in the direction of "out of the open surface".

To simplify the discussion we have chosen a flat surface and a circular contour enclosing that surface as shown in Figure 2. Again, the components of the magnetic flux density that penetrate or pass through this surface are those that are normal (perpendicular) to the surface, $\vec{B} \cdot \vec{ds}$ and $\vec{ds} = ds\ \vec{a}_n$ where $\vec{a}_n$ is a unit normal to the surface. Again, this is sensible because the components of $\vec{B}$ that are tangent (parallel) to the surface do not "exit" the surface. Faraday's law provides that we may replace the *effect* of the magnetic flux density vector passing through the surface by inserting an equivalent voltage source whose value is

$$V = \frac{d\psi}{dt} \tag{4}$$

into the contour of the loop that encloses the surface. In order to talk about an "inductance" of this loop, we will assume that the physical dimensions of this loop are electrically small ($\ll \lambda$). Furthermore, we will consider the loop contour to be constructed of a conducting material such as a wire (a conductor having a circular, cylindrical cross section). We can lump these effects of the time-changing magnetic field through the loop into a lumped voltage source whose value is given in (4) and place it anywhere in the loop contour because we assume that the loop dimensions are electrically small.

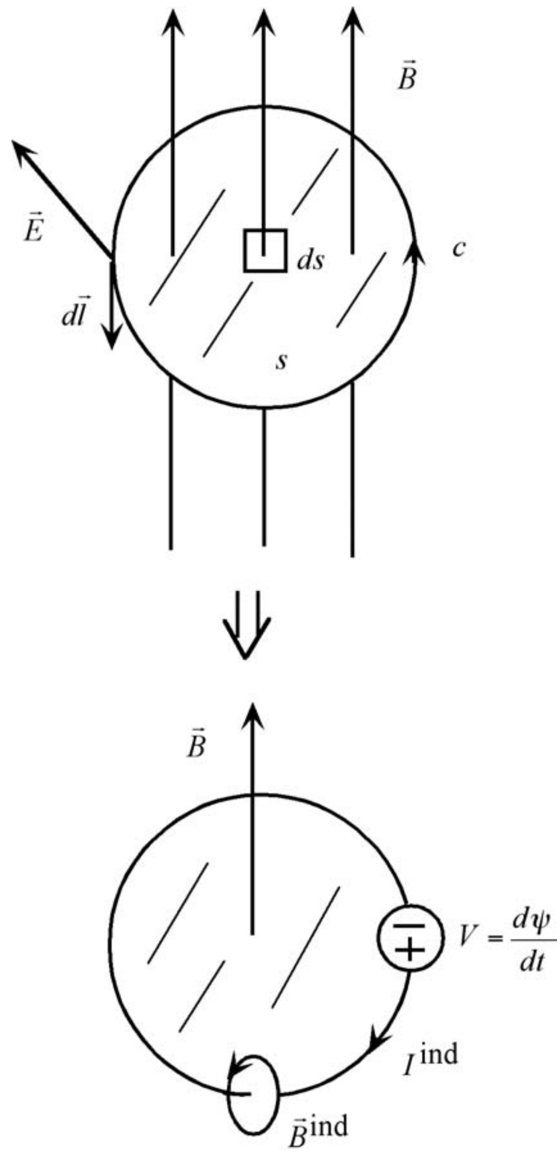

Fig. 2.

Getting the correct polarity of this source is critical. Faraday's law essentially provides that the voltage source representing the induced *emf* has a polarity such that it *opposes (Lenz's law) the rate of change of the magnetic flux through the loop*. A foolproof way of getting the source polarity right is the following. The source should tend to induce or "push" a current I^{ind} around this conducting loop in a direction such that this induced current produces another induced magnetic flux $\vec{B}^{\text{ind}}$ that *opposes* any change in the original magnetic field $\vec{B}$. This is a very sensible result because if the magnetic field induced by

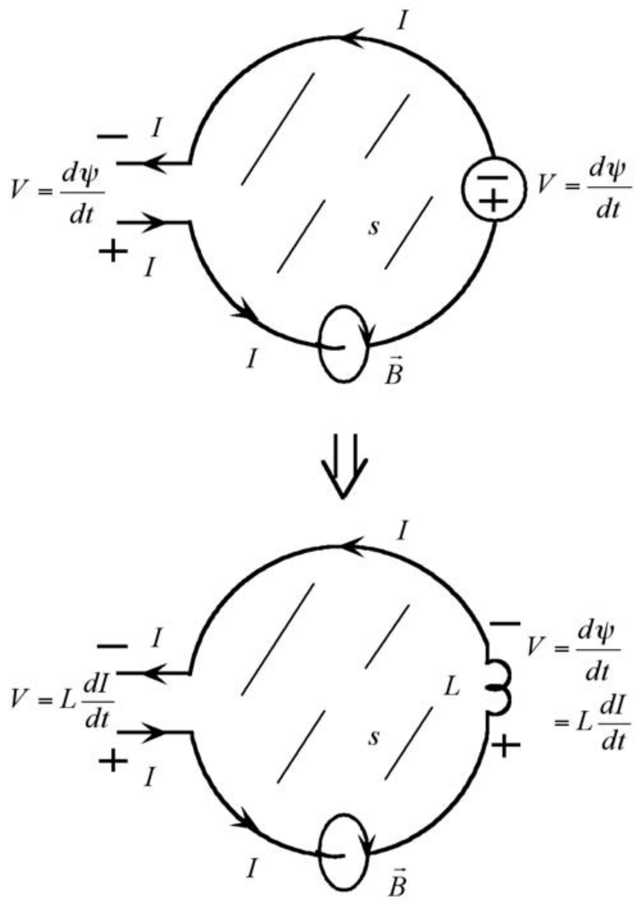

Fig. 3.

the source did not *oppose the original magnetic field*, an induced current would produce an induced magnetic flux that would increase the net magnetic flux through the loop thereby inducing a larger induced voltage and a larger induced magnetic field, and so on. As we will show in Section II, a current in a wire produces a magnetic field whose direction can be obtained with the *right-hand rule*. That is, if we place the thumb of our right hand in the direction of the current, the fingers will give the direction of the induced magnetic field about the wire. This is shown in Figure 2. If the original magnetic flux through the surface enclosed by the loop is directed upward as shown, the source should have a polarity such that it tends to push a current out of its positive terminal that circulates clockwise thereby producing (by the right-hand rule) an induced magnetic field that is directed downward *through the loop surface* such that this induced magnetic field opposes the original magnetic field.

Observe that the value of the induced voltage source V in (4) depends on the *time rate-of-change* of the magnetic flux. Hence either a large B field that is slowly varying with time (such as a 60 Hz power frequency current) or a small B field that is rapidly varying with time (such as a 2 GHz current in a cell phone) will have a similar effect. This explains why these concepts are becoming more and more important as the clock speeds in digital circuits increase thereby making our jobs as EMC engineers more difficult.

Now let us define the *inductance of the loop*. Suppose we open the loop at some point and inject a current I around that loop, circulating in the counterclockwise direction as shown in Figure 3. This will produce a magnetic field $\vec{B}$ and magnetic flux ψ that, by the right-hand rule, will be directed upward through the surface of the loop that is enclosed by the current. Hence we have the original problem shown in Figure 2 and the induced source in (4) is inserted as shown. The calculation of the inductance of a loop is directly analogous to the calculation of the capacitance of a structure. In order to calculate the capacitance between two bodies, we place equal and opposite charges on the two bodies and then calculate the voltage induced between the two bodies by that charge. The capacitance is the ratio of the charge and the voltage produced by it. In a similar fashion we define the inductance of a current loop:

The inductance of a current-carrying loop is defined as the ratio of the total magnetic flux penetrating the surface of the loop and the current of the loop that produced it:

$$L = \frac{\psi}{I} \qquad \text{Henrys} \tag{5}$$

The units of inductance are Henrys honoring Joseph Henry of Albany, New York who essentially discovered Faraday's law at about the time as Faraday. The magnetic flux ψ through the loop is directly proportional to the current I that produced it and hence the inductance of the loop L is only a function of the loop shape and material properties of the surrounding medium. According to Faraday's law, the voltage source that is induced in this loop has a value that is the rate-of-change of the total magnetic flux penetrating the loop so we obtain, substituting (5) into (4),

$$\begin{aligned} V &= \frac{d\psi}{dt} \\ &= L\frac{dI}{dt} \end{aligned} \tag{6}$$

which is the usual result for the voltage across the terminals of an inductor. Figure 3 shows that we can replace this induced source with the usual inductor symbol, and the voltage induced across this inductance is given in (6). This voltage appears at the terminals of the opened loop like a Thevenin equivalent open-circuit voltage.

II. INDUCTANCES OF IMPORTANT GEOMETRIES

We next consider the use of these results to calculate the inductance of some important geometries. It is important to keep in mind that *there are relatively few geometries for which inductance can be calculated exactly*. Once again, this reiterates the important point that the meaning and interpretation of Maxwell's equations are very simple but their use in making calculations is usually very difficult unless some approximations are made. All of the geometries that we will consider will be composed of

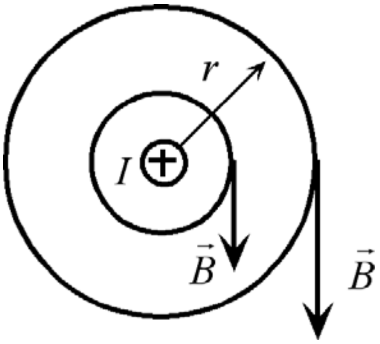

Fig. 4.

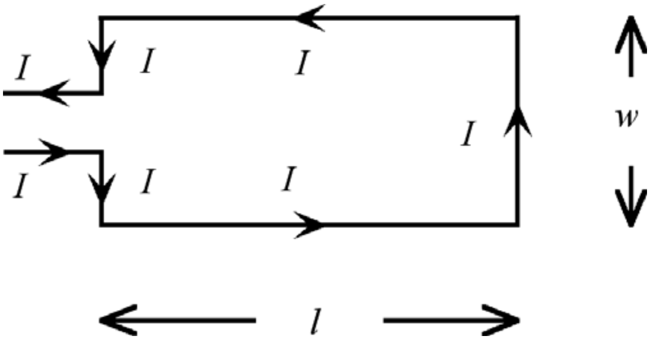

Fig. 5.

loops of current which can be thought of as wires. In order to do this we need to obtain a fundamental result for the magnetic field that is produced by a current-carrying wire.

Consider an infinitely long wire carrying a dc current I as shown in Figure 4. The wire is assumed to be infinitely long in order to avoid having to consider "fringing" of the magnetic fields at the endpoints of a finite-length wire. Ampere's law is, for dc, [3, 4]

$$\oint_c \vec{H} \cdot d\vec{l} = I_{\text{enclosed}} \tag{7}$$

The vector $\vec{H}$ is the *magnetic field intensity* vector with units of A/m. Ampere's law provides that if we sum the products of the components of $\vec{H}$ that are *tangent* to a *closed* path enclosing a current and the differential path lengths dl of that path we will obtain the total current passing through the surface enclosed by this path. By symmetry, the magnetic field about the wire will form concentric circles. Since $\vec{H}$ is directed circumferentially and is therefore tangent to circular contours about the wire, the dot product can be removed from (7). Similarly, by symmetry, the magnitude of $\vec{H}$ is the same at all points of a contour of radius r. Thus H can be removed from the integrand in (7) and we obtain a simple equation for the magnetic field about a current-carrying wire at a distance r from the wire [3, 4]:

$$\begin{aligned} B &= \mu_0 H \\ &= \frac{\mu_0 I}{2\pi r} \end{aligned} \tag{8}$$

where $\mu_0 = 4\pi \times 10^{-7}$ is the permeability of free space (assuming there are no ferromagnetic materials in the space surrounding the wire). Again this shows that the direction of the magnetic field can be determined with the *right-hand rule*: placing the thumb of our right hand in the direction of the current, the fingers will give the direction of the resulting magnetic field which forms concentric circles about the wire as shown in Figure 4.

This is the basis of the current probe that is used extensively in EMC as a noninvasive way to measure a current in a wire [6]. The current produces a magnetic field that circulates in the ferromagnetic core of the current probe that is placed around the current. Turns of wire are wound on that core such that the (time-varying) magnetic flux circulating in the core induces, by Faraday's law, a voltage at the terminals of this coil of wire that can be measured. The "transfer impedance" of the current probe is then the ratio of the induced voltage and the current enclosed by the probe. Hence the current probe relies on both Ampere's law and Faraday's law [4].

Consider a rectangular loop consisting of sides of lengths l and w as shown in Figure 5. We will use the above result for the magnetic field about an infinitely long wire to compute the total flux through the loop by ignoring the fringing of the fields at the ends of these sides that are of finite length. We superimpose the fluxes through the loop due to each of the currents in the four sides. This gives the loop inductance to be calculated as

$$\begin{aligned} L &= \frac{\psi}{I} \\ &\cong \underbrace{2\frac{\int_{l=0}^{l}\int_{r=r_w}^{w}\frac{\mu_0 I}{2\pi r}\,dr\,dl}{I}}_{\text{top and bottom}} + \underbrace{2\frac{\int_{w=0}^{w}\int_{r=r_w}^{l}\frac{\mu_0 I}{2\pi r}\,dr\,dw}{I}}_{\text{sides}} \\ &= \mu_0\frac{l}{\pi}\ln\left(\frac{w}{r_w}\right) + \mu_0\frac{w}{\pi}\ln\left(\frac{l}{r_w}\right) \end{aligned} \tag{9}$$

Calculation of the inductance of circular loops is somewhat more difficult but some approximate results can again be obtained using the basic definition of inductance given in (5) [3, 4]. The inductance of a circular loop of radius r that is constructed with a thin wire of radius r_w is approximately $L \cong \mu_0 r[\ln(\frac{8r}{r_w}) - 2]$ for $r_w \ll r$. The inductance of a long solenoid of length l and radius a consisting of N turns of wire (that are closely spaced) along its surface can be approximately obtained by assuming that the magnetic flux has the same value at all points of the interior of the solenoid as $L \cong \mu_r \mu_0 N^2 \pi\, a^2 l$ where μ_r is the relative permeability of the (ferromagnetic) core material of the solenoid. The inductance of a toroid consisting of N turns of closely-spaced wire on a toroidal core of rectangular cross section having an inner radius a, an outer radius b and a cross-sectional thickness t is approximately $L \cong \mu_r \mu_0 \frac{N^2 t}{2\pi}\ln(\frac{b}{a})$ where μ_r is the relative permeability of the (ferromagnetic) core material. This toroidal geometry is typical of *common-mode chokes* used in EMC to block common-mode currents [6]. For cores having a thickness that is much less than the radius of the core, $(b - a) \ll a$, this result simplifies to [4] $L \cong \mu_r \mu_0 \frac{N^2 A}{2\pi}$ where A is the cross-sectional area of the core. Doubling the number of turns on a solenoid or a

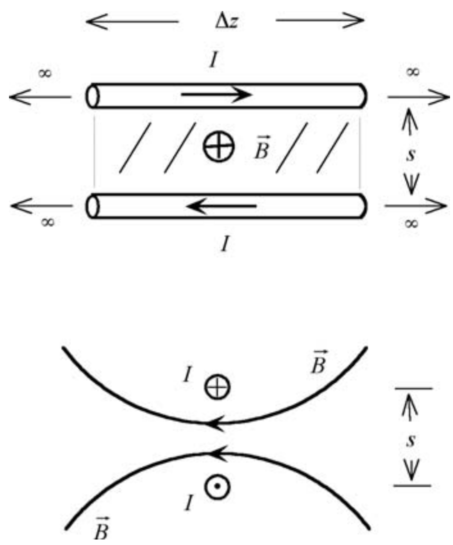

Fig. 6.

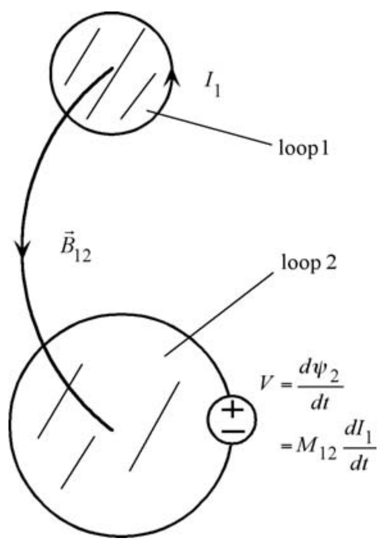

Fig. 7.

toroidal core quadruples the inductance.

The last structure we will consider is the parallel-wire transmission line consisting of two wires of infinite length and radii r_w that are separated by distance s shown in Figure 6. Again we will use the basic result for the magnetic field about an infinitely long wire given in (8) and superimpose the results for the two wires to obtain the magnetic flux through the loop of a portion of the line enclosed by the two wires of length Δz and width s. The structure has infinite length so we cannot talk about an inductance of it but instead must use the concept of a *per-unit-length inductance* [5, 6]. Superimposing the magnetic fluxes due to both wires we obtain the inductance of a section of the line of length Δz as

$$\begin{aligned} L &= \frac{\psi}{I} \\ &= 2\frac{\int\limits_{z=0}^{\Delta z}\int\limits_{r=r_w}^{s} \frac{\mu_0 I}{2\pi r}\, dr dz}{I} \\ &= \frac{\mu_0}{\pi}\Delta z \ln\left(\frac{s}{r_w}\right) \qquad \text{Henrys} \qquad (10) \end{aligned}$$

The *per-unit-length* inductance of the line is

$$\begin{aligned} l &= \frac{L}{\Delta z} \\ &= \frac{\mu_0}{\pi} \ln\left(\frac{s}{r_w}\right) \qquad \text{Henrys/meter} \qquad (11) \end{aligned}$$

The per-unit-length inductances of a coaxial cable and one wire above an infinite ground plane are the two remaining simple inductance calculations and are similarly derived in [3, 4].

III. MUTUAL INDUCTANCE

Mutual inductance is similarly defined between two distinct loops as the ratio of the magnetic flux penetrating the surface enclosed by the second loop and the current in the first loop which produced the flux in the second loop as illustrated in Figure 7:

$$M_{12} = \frac{\psi_2}{I_1} \qquad \text{Henrys} \qquad (12)$$

Again, the magnetic flux through the second loop is proportional to the current of the first loop that produced it so that the mutual inductance between the two loops is dependent only on their shape, their relative orientations and the material properties of the surrounding medium. Observe that the voltage source representing the induced *emf* in loop 2 has a polarity, again according to Faraday's law, such that it tends to induce a current in the second loop that produces a magnetic flux that opposes the original magnetic flux that passes through its surface which is due to the current in the first loop. There really are no differences between self and mutual inductance except in the calculation of the flux passing through the second loop.

IV. SUMMARY

The purpose of this article is to reconsider the concept of inductance and to give precise meaning to it based on scientific laws. Inductance and capacitance are among the most fundamental concepts in electrical engineering and in the daily practice of EMC. It is imperative that we have a sound understanding of their meaning based on scientific laws so that we can avoid becoming trapped in "word games" and "intuition" that often get us into trouble. We need to think critically about these extremely important and fundamental concepts if we are to accomplish our goal of being proficient and successful EMC engineers.

There are many examples of where these concepts have become corrupted and then propagated as "fact" thereby confusing and misleading other EMC engineers. As an example, a recent article in the July 2007 issue of *Printed Circuit Design & Manufacture* magazine is entitled "What is Inductance?". In that article the author states that "We use the terms "loop" and "partial" to describe the amount of the loop about which we are *counting the rings of field lines* (emphasis added)…" He also shows our Figure 4 wherein he states that "…the energy in the field is about *the total number of rings of magnetic field lines around the conductor* (emphasis added)…" These statements make no sense. We showed in (8) that the value of the magnetic field about a current I that is associated with a specific radius r about that current is $B = \mu_0 I/2\pi r$. Hence *the number of magnetic field rings about a current-carrying wire is infinite*! Simply re-compute the *value* of B for another radius and draw the concentric ring representing that value and continue doing this indefinitely for the infinite number of other possible values of r! This is similar to plotting the collector-emitter characteristic of a bipolar junction transistor. We plot the collector-emitter voltage versus the collector current for *selected values of the base current*. If we plotted this for *all possible values of base current*, the plot would be covered in pencil lead! Others have offered the similarly erroneous assertion that inductance is related to *the number of* magnetic flux lines "wrapping around" the current. All of this confusion and the resulting erroneous conclusions could have been avoided if we simply go back to basic principles and base our discussion and calculation of inductance on Faraday's law which is what it was derived from in the first place. Very few concepts need to be "newly invented". The old concepts have stood the test of time.

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BIOGRAPHY

Clayton R. Paul *(S'61-M'70-SM'79-F'87-LF'06) received the B.S. degree from The Citadel, Charleston, SC in 1963, the M.S. degree from Georgia Institute of Technology, Atlanta, GA in 1964, and the Ph.D. degree from Purdue University, Lafayette, IN in 1970, all in electrical engineering.*

He is Emeritus Professor at the University of Kentucky where he was a member of the faculty in the department of electrical engineering for 27 years. He is currently the Sam Nunn Eminent Professor of Aerospace Systems Engineering and Professor of Electrical and Computer Engineering at Mercer University, Macon, GA. He is the author of numerous textbooks on electrical engineering subjects, and has published numerous technical papers, the majority of which are in his primary research area of electromagnetic compatibility (EMC) of electronic systems. From 1970 to 1984, he conducted extensive research for the US Air Force in modeling crosstalk in multiconductor transmission lines and printed circuit boards. From 1984 to 1990 he served as a consultant to the IBM Corporation in the area of product EMC design.

Dr. Paul is an Honorary Life Member of the IEEE EMC Society and is the recipient of the 2005 IEEE Electromagnetics Award and the 2007 IEEE Undergraduate Teaching Award. **EMC**