The Fifth Most Referenced Transactions Paper of the EMC Society

Dan Hoolihan, History Committee Chair

INTRODUCTION

As part of the 50th Anniversary Celebration of the EMC Society of the IEEE (1957-2007), a review and comparison of past papers in the *IEEE Transactions on Electromagnetic Compatibility* was undertaken. In the four previous Newsletters, we have published the first, second, third, and fourth most referenced papers, which are, respectively:

1. "Transient Response of Multiconductor Transmission Lines Excited by a Nonuniform Electromagnetic Field;" EMC-22, No. 2, May – 1980, Page 119 by A. K. Agrawal, H. J. Price, and S. H. Gurbaxani.
2. "Absorbing Boundary Conditions for the Finite-Difference Approximation of the Time-Domain Electromagnetic Field Equations;" EMC-23, No. 4, November – 1981, Page 377 by Gerrit Mur.
3. "Generation of Standard Electromagnetic Fields Using TEM Transmission Cells;" EMC-16, No. 4, November – 1974, Pages 189 -195 by Myron (Mike) L. Crawford.
4. "Frequency Response of Multiconductor Transmission Lines Illuminated by an Electromagnetic Field," EMC-18, No. 4, November – 1976, Pages 183-190 by Clayton R. Paul.

In this issue, we are publishing the fifth Most-Referenced EMCS Transactions paper of the first 50 years of the EMC Society and it is written by two gentlemen; J. G. Costas and B. Boverie who were both with Sandia National Laboratories in Albuquerque, New Mexico when the paper was written.

The title of the paper is "Statistical Model for a Mode-Stirred Chamber" and it was first published in the *IEEE Transactions on EMC* in Volume 33, No. 4. in November of 1991.

Again, we hope you take the time to read and appreciate the significance of this historical article.

[3] J. V. Evans, "Theory and practice of ionosphere study by Thomson scatter radar," *Proc. IEEE*, vol. 57, pp. 496–530, Apr. 1969.
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Statistical Model for a Mode-Stirred Chamber

Joseph G. Kostas and Bill Boverie

Abstract—**The probability density functions for one- and three-dimensional fields in a mode-stirred chamber are derived and verified with chi-square goodness-of-fit tests on experimental data. Maximum-likelihood estimators of the functions' parameters are derived, and their accuracy is determined as a function of the amount of data. These results are applied to estimating chamber Q. The amount of data required for a given accuracy is determined.**

I. Introduction

A mode-stirred chamber (MSC) is a test facility that makes it possible to generate large electromagnetic fields (~ 100 V/m) with modest-sized sources, but the price paid is that the fields are no longer deterministic. The large fields are obtained by using a metal room as a high Q overmoded cavity. Q's as high as 150 000 have been reported in the literature [1]. The EM energy is injected via an antenna inside the room, and the field strength builds up until the loss in the room equals the input power. This also results in a standing wave pattern and, hence, nonuniform illumination of the test object. This, in turn, requires use of statistics to interpret the results. Good statistics require using a wavelength (λ) that is much smaller than the room dimensions and providing some means, usually a metal paddle wheel, for "stirring" the modes. The small wavelength (relative to room dimensions) results in a highly overmoded condition, the number of modes being approximately [2].

$$n \cong abc/\lambda^3$$

where a, b, and c are the room dimensions. The mode structure is further complicated by the test object, antenna, and paddle wheel, making a deterministic field model impractical.

The field in an MSC is, therefore, considered to be a random process. For a given paddle-wheel angle, the field varies with position in the chamber in a random manner. Likewise the field at a given point in the chamber varies randomly with paddle-wheel angle. Tests indicate that the ratio of peak to average values remains approximately the same throughout the chamber, provided that the measurement point is at least a third of a wavelength from the walls or the paddle wheel [1].

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The authors are with Sandia National Laboratories, Albuquerque, NM 87185.

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Tests are performed using either average or peak values. For example, a transfer function for a shielded cable can be defined as the ratio of the voltage or current at the cable end to the incident E-field, as a function of frequency. If the test object is measured in an anechoic chamber, all these parameters are deterministic. However, if it is measured in an MSC, either average or peak values of the parameters are used, for which comparable results are reported [1]. MSC and anechoic chamber tests give comparable results, except that any "gain" possessed by the test object is lost in the MSC [3]. Normally, this difference is only a few dB.

Statistical modeling of MSC fields beyond use of averages or peak values has received limited attention in the literature. Tests indicated that the power received by an antenna (where the power received is considered to be a random variable) in the chamber is exponentially distributed [4]. The field incident on the antenna is proportional to the square root of the received power and should, therefore, be Rayleigh distributed. Plots of experimentally determined field probability density functions (pdf's) appear to be Rayleigh distributed [5]. A theoretical model indicates that the power density is chi-square distributed with two degrees of freedom (dof) [6], which is the same as an exponential distribution. Thus, these three references come to the same basic conclusion. However, as noted below, this conclusion applies to only one component of the field; the resultant of the three components has a different distribution.

The literature has nothing to say about uses for the pdf. One use is derivation of optimal estimators for test quantities of interest. Another is the derivation of the size of errors in these estimators due to statistical sampling. Since the field is a random process, samples are random variables, and any function of random variables is itself a random variable. The estimators thus vary from the true value, though the size of the error decreases with the amount of data. Both of these uses for the pdf are considered in this paper.

This paper derives a model for the pdf of the MSC field for both one- and three-dimensional fields. The model is verified with chi-square goodness-of-fit tests on experimental data taken with a point sensor (instead of the usual horn antenna). The model is then used to derive maximum-likelihood estimators of the pdf parameters, and the accuracy of these estimators is determined as a function of the amount of data used to make the estimate. The result is then applied to estimating chamber Q.

II. Theoretical Model

The only case of interest is that in which a large number of modes are present in the MSC. In this case, the field at a point is the vector sum of the contributions of all these modes. Moving the paddle wheel changes the way the energy in the field is distributed among the modes. The energy in a given mode can be thought of as a random variable that depends on paddle-wheel angle.

Since the field at a point has a phase as well as an amplitude, six parameters are required to fully describe the field. The six parameters are in-phase and quadrature components in each of three orthogonal directions. Each of these six components is the sum of a large number of random variables (the mode amplitudes) and by the central limit theorem, should be normally distributed. Furthermore, if the measurement point is well away from any wall, it is reasonable to assume that all six components are independent and identically distributed. Finally, the means can all be assumed to be zero if there is not a significant direct-path signal from the antenna in the chamber to the measurement point. This is a good assumption if the antenna is near and pointed into a corner. (If there is a significant

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direct-path signal, the in-phase components will have nonzero means.)

The magnitude of the resultant (vector sum of the field in the three dimensions) is then the square root of the sum of the squares of six identically distributed, zero mean, normal random variables and is, therefore, chi distributed with six degrees of freedom, or

$$f(E) = \frac{E^5}{8\sigma^6} e^{-E^2/2\sigma^2}.$$

However, most MSC measurements only respond to one dimension of the field. This appears to be the case in [4], [5], and [6] and implies, though the references did not so state, that the antennas used had linear polarization. In that case, the magnitude of the received voltage (or current) is proportional to just one dimension of the field. Each dimension of the field has in-phase and quadrature components and is therefore chi distributed with two dof, which is the same as a Rayleigh distribution, or

$$f(E) = \frac{E}{\sigma^2} e^{-E^2/2\sigma^2}.$$

The chamber power density (in each dimension), and hence the received power, is proportional to the square of the field, and is, therefore, exponentially distributed, or

$$f(P) = \frac{1}{2\sigma^2} e^{-P/2\sigma^2}.$$

Recall that, in all three probability distributions, σ^2 is the variance of each of the six field components.

III. Experimental Data

Field measurements were made in an existing shield room, converted to use as an MSC. The chamber was a metal shield room made of clamped galvanized-steel panels. Chamber dimensions were approximately 10 × 20 × 8 ft. Histograms were made at ten locations in the chamber for three frequencies (4, 6, and 8 GHz). Point measurements were made using a small E-field probe with linear polarization. The probe is a square-law device with an output voltage proportional to the square of the E-field [7]. The probe voltage would then be expected to be exponentially distributed. A schematic diagram of the chamber and instrumentation system is shown in Fig. 1.

For these tests, the tuner was moved in discrete steps of 3°. For each tuner position, the electric field was measured and stored for later processing. The net input power to the chamber was 0.25 m Watt.

A representative histogram is shown in Figs. 2(a) and 2(b). The histogram is an aid in displaying the distribution of sample values. For example, 49 electric field values lie between 0 and 0.1 V^2/m^2, 18 samples lie between 0.1 and 0.2 V^2/m^2, etc.

The histogram is characteristic of any probe orientation within the chamber. These data were taken near the center of the chamber at 4 GHz. The histogram clearly has an exponential shape, except for a few "outliers," i.e., values higher than those predicted by the exponential distribution. This general result, exponential distribution with outliers, was found for all the histograms taken.

The usual check for fit between data and a distribution function is the chi-square goodness-of-fit test. This test is based on the random variable,

$$\chi^2 = \sum_{i=1}^{m} \frac{(k_i - n_i)^2}{n_i}$$

where m is the number of data intervals, k_i is the number of samples in the ith interval, and n_i is the expected number of samples in the interval if the hypothesized distribution is correct. The χ^2 variable will be chi-square distributed (if the amount of data is large) with $m - p - 1$ degrees of freedom, p being the number of parameters in the assumed distribution that are calculated from the data. Obviously, the smaller χ^2, the more likely the distribution is correct. The value of χ^2 calculated from the data is compared to a threshold value determined by the confidence level selected for the test. If χ^2 is less than the threshold, the hypothesized distribution is selected as being correct. The test data supported the exponential distribution at the 95% confidence level, even with the outliers, in all but one case. Trimming the outliers results in the exponential distribution being accepted in all cases.

The reason for the outliers is unclear. They could be an artifact of the relatively low Q of the chamber (see below). They may also be a result of the measurement being a point measurement. Other experimenters do not report outliers [4], [5], but they used an antenna instead of a point sensor. An antenna will average the field over its aperture and might "average out" the outliers.

IV. Estimators

One of the main advantages of having a pdf for the chamber field is that estimators and their accuracy can be calculated. The maximum-likelihood estimator (MLE) will be used here because it has several useful properties. First, it generally leads to intuitively reasonable estimators. Second, it is always asymptotically unbiased, i.e., its mean is the true value if the amount of data is large. Third, there is a useful theorem for determining the accuracy of MLE estimators [8].

The MLE derivation involves writing the joint pdf for the data samples, assuming the samples are independent. The pdf parameters are then selected to maximize the value of the joint pdf. In this case it is easier to maximize the natural log of the pdf, and since log is a monotonic function, this is the same as maximizing the pdf. Taking a derivative with respect to σ^2, the only parameter in the above distributions, and setting it equal to zero yields

$$\hat{\sigma}^2 = \frac{1}{6n} \sum_{i=1}^{n} E_i^2 \triangleq \frac{\overline{E^2}}{6}$$

for the three-dimensional (chi-6) pdf and

$$\hat{\sigma}^2 = \frac{1}{2n} \sum_{i=1}^{n} E_i^2 \triangleq \frac{\overline{E^2}}{2}$$

for the one-dimensional (Rayleigh) pdf, where n is the number of data samples. Thus the estimator is just the average of the square of the E-field divided by the number of degrees of freedom.

V. Estimator Accuracy

For large sample sizes, σ^2 is approximately normally distributed, and in this case its variance can be shown to be [8, p. 212]

$$\text{var}\left[\hat{\sigma}^2\right] = \left\{-nE\left[\frac{\partial^2 \ln f}{\partial(\sigma^2)^2}\right]\right\}^{-1}$$

where E denotes the expected value and f is the pdf. Applying this equation gives

$$\text{var}\left[\hat{\sigma}^2\right] = \frac{\sigma^4}{3n}$$

for the three-dimensional (chi-6) pdf and

$$\text{var}\left[\hat{\sigma}^2\right] = \frac{\sigma^4}{n}$$

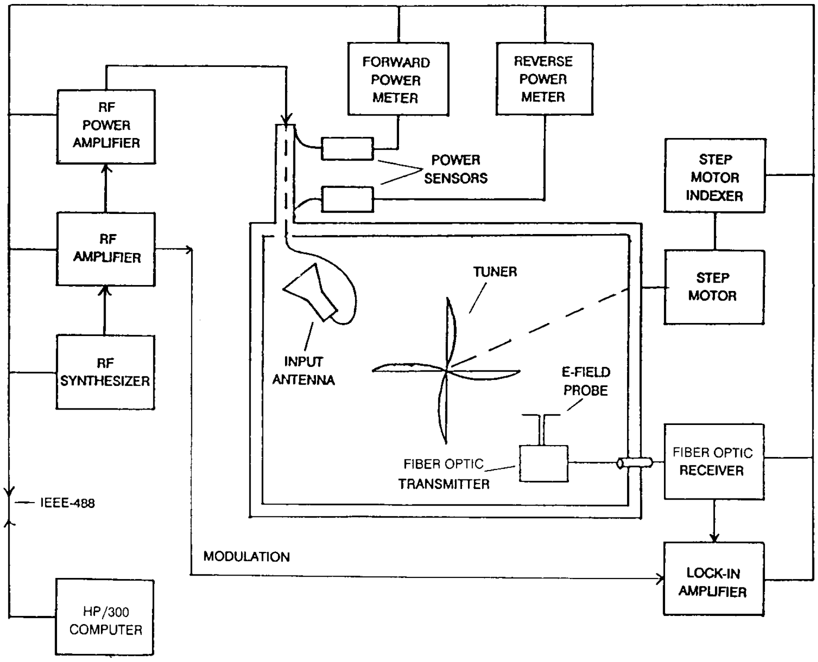


Fig. 1. MSC schematic diagram.

for the one-dimensional (Rayleigh) pdf. The normalized (divided by σ^2) accuracy (standard deviation) of $\hat{\sigma}^2$ is thus $1/\sqrt{n}$ and $1/\sqrt{3n}$ for one- and three-dimensional data, respectively.

The normalized accuracy of $\hat{\sigma}^2$ makes it possible to determine how much data is needed for a desired accuracy. Since $\hat{\sigma}^2$ is normally distributed and unbiased, the probability (confidence) that it is within k standard deviations(s) of its mean (σ^2) is

$$\text{prob}\left[\sigma^2 - ks < \hat{\sigma}^2 < \sigma^2 + ks\right] = 2\,\text{erf}\,(k)$$

where "erf" is the error function

$$\text{erf}\,(k) = \frac{1}{\sqrt{2\pi}} \int_0^k e^{-y^2/2}\,dy.$$

Requiring $\hat{\sigma}^2$ to be within a confidence interval of d dB implies

$$d\,\text{dB} = 10 \log \frac{1 + k/\sqrt{zn}}{1 - k/\sqrt{zn}}$$

where k determines the confidence, and z is the number of dimensions of the field data (1 or 3). Solving for n gives

$$n = \frac{k^2}{z}\left(\frac{10^{d/10} + 1}{10^{d/10} - 1}\right)^2.$$

For example, if d is 1 dB and the desired confidence is 90% ($k = 1.65$), then n is 69 or 207 for $z = 3$ dimensions or 1 dimension, respectively.

VI. Chamber Q

The estimate of σ^2 is directly related to the chamber Q. Chamber Q is a useful quantity because it allows prediction of the mean field strength from the input power. The highly overmoded chamber makes Q calculations based on resonant bandwidths difficult, if not meaningless. A better approach is to use the basic definition of Q, which is 2π times the ratio of the energy stored per cycle to the energy dissipated per cycle. This leads to

$$Q = \frac{\omega\epsilon\overline{E^2}V}{2P} = \frac{\omega\overline{B^2}V}{2\mu P}$$

where V is the chamber volume and P is the net input power. Note that it is the square of the E- or B-field that is averaged and that it is the three-dimensional field that is of interest. If a one-dimensional field is used, then Q must be multiplied by three to include the energy stored in the other two dimensions. Q can also be expressed in terms of $\hat{\sigma}^2$, or, for E-fields,

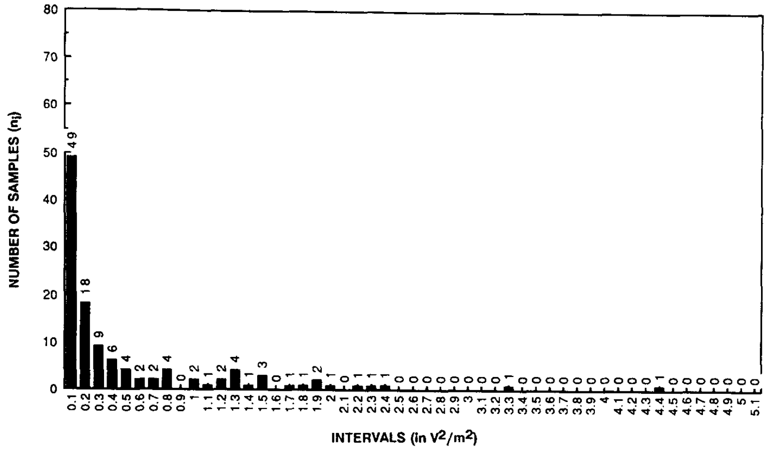
$$Q = \frac{3\omega\epsilon\hat{\sigma}^2 V}{P}.$$

The advantage of this expression is that the normalized accuracy for Q can be seen to be the same as that for $\hat{\sigma}^2$, or $1/\sqrt{n}$ and $1/\sqrt{3n}$ for one- and three-dimensional data, respectively. The above equation for n, the amount of data required for a given accuracy, therefore also applies to Q.

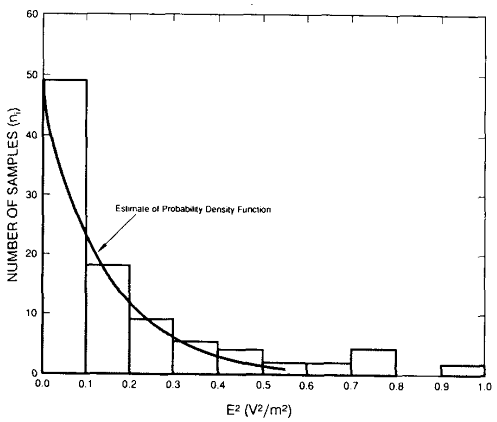
Chamber measurements were used to determine Q values from 1 to 10 GHz. The Q is approximately 20 000 from 2 to 10 GHz, and drops off rapidly below 2 GHz (See Fig. 3). The prediction for Q based only on wall losses is [9]

$$Q = 1.5\left[\frac{V}{\mu S\delta}\right]\left[1 + \frac{3\pi}{8k}\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)\right]^{-1}$$

where S is the total surface area, δ is the skin depth, and k is the wave number. Since the second bracket is essentially one, Q should vary as the square root of frequency (because of $1/\delta$). This equation predicts a much higher Q than was measured. The low Q and its independence of frequency indicate that loss mechanisms other than Joule heating are important. At the present time, a good physical explanation for the low Q is not forthcoming. However, an effort has been made to eliminate electrical conductivity from considera-



(a)



(b)

Fig. 2. (a) Histogram of square of electric field (one component) at a position close to the center of the chamber and at a frequency of 4 GHz. (b) An expanded version of Fig. 2(a) with an estimated probability density function superimposed.

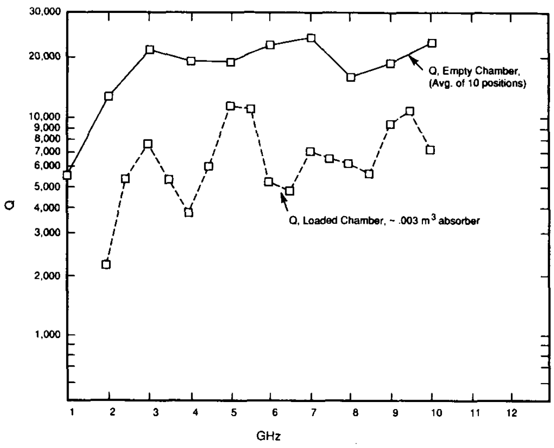


Fig. 3. Q-measurement summary.

tion. A sample of the chamber wall was measured for electrical conductivity by the Microwave Standards Division of Sandia National Laboratories. The conductivity of the wall coating was determined to be $1.26 \cdot 10^7$ S/m, which is close enough to published values of the conductivity of zinc ($1.74 \cdot 10^7$ S/m, [10]) to assume the walls can be taken to be zinc (the steel is not coming into play). RF leakage is another possibility for the low Q. The steel panel construction of this chamber contains approximately 250 linear ft of intermediate overlapping flat stock to hold the panels together. The chamber has been checked for leakage, with no detectable leakage. However, the leakage is diffuse and would be difficult to detect for the relatively low power input levels.

The chamber was loaded with approximately 0.003 m^3 of rf absorber placed 25 in off the floor, and the Q was again measured from 2 to 10 GHz. Fig. 2 gives the loaded Q, along with the previously obtained unloaded Q. Q drops by a factor of 2 to 6, depending on frequency. The loaded Q was measured in only one position in the chamber. The rather large reduction in Q by a small amount of material is impressive.

VII. Conclusions

Each of the three components of the field in the chamber is Rayleigh distributed, which is the same as a chi distribution with two degrees of freedom, and the resultant is thus chi distributed with six degrees of freedom. Each component of the power density is then exponentially distributed. Experimental data confirm these distributions, though unexpected high values, or outliers, were consistently found.

The maximum likelihood estimator for σ^2, the only parameter in these distributions, is just the average of the square of the field divided by the number of degrees of freedom (2 or 6). The normalized accuracy of these estimates is $1/\sqrt{n}$ and $1/\sqrt{3n}$ for one- and three-dimensional data, respectively. Chamber Q is proportional to σ^2 and has the same normalized accuracy.

Acknowledgment

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