The Fourth Most Referenced Transactions Paper of the EMC Society

Daniel D. Hoolihan, History Committee Chair

INTRODUCTION

As part of the 50th Anniversary celebration of the EMC Society of the IEEE (1957-2007), a review and comparison of past papers in the *IEEE Transactions on Electromagnetic Compatibility* was undertaken. In the three previous Newsletters, we have published the first, second and third most referenced papers, which are, respectively:

1. "Transient Response of Multiconductor Transmission Lines Excited by a Nonuniform Electromagnetic Field;" EMC-22, No. 2, May – 1980, Page 119 by A. K. Agrawal, H. J. Price, and S. H. Gurbaxani.
2. "Absorbing Boundary Conditions for the Finite-Difference Approximation of the Time-Domain Electromagnetic Field Equations;" EMC-23, No. 4, November – 1981, Page 377 by Gerrit Mur.
3. "Generation of Standard Electromagnetic Fields Using TEM Transmission Cells;" EMC-16, No. 4, November – 1974, Pages 189 -195 by Myron (Mike) L. Crawford.

In this issue, we are publishing the fourth most-referenced *IEEE Transactions on EMC* paper of the first 50 years of the EMC Society and it is by a well-known EMC technical person, Clayton Paul.

The title of the paper is "Frequency Response of Multiconductor Transmission Lines Illuminated by an Electromagnetic Field." It can be found in EMC-18, No. 4, November – 1976 starting on Page 183.

We hope you read and appreciate the significance of this historical article.

Frequency Response of Multiconductor Transmission Lines Illuminated by an Electromagnetic Field

CLAYTON R. PAUL, MEMBER, IEEE

***Abstract*—A well-known result [1], [2] for the response of a two-wire transmission line illuminated by a nonuniform electromagnetic field is extended to multiconductor lines. A simple matrix equation for the currents induced in arbitrary termination networks is obtained.**

I. INTRODUCTION

THE BASIC problem considered in this paper is a determination of the currents induced in termination networks associated with the $(n + 1)$-conductor uniform transmission line shown in Fig. 1 by spectral components of an incident, nonuniform electromagnetic field. The line is considered to be uniform in the sense that the $(n + 1)$ conductors are parallel to each other and the x direction and are circular wires having no cross sectional variation along their axes. One of the conductors, the zeroth conductor, is designated as the reference conductor for the line voltages and has a radius r_{w0}. The remaining n conductors have radii r_{wi} with $i = 1,\cdots,n$. The various separations between conductor pairs are d_{i0}, d_{j0}, d_{ij} with $i,j = 1,\cdots,n$.

The solution for the special case of a two-conductor line $(n = 1)$ was obtained by Taylor, Satterwhite, and Harrison in [1] and later in a more convenient form by Smith in [2]. These formulations were used in [3] to predict the responses

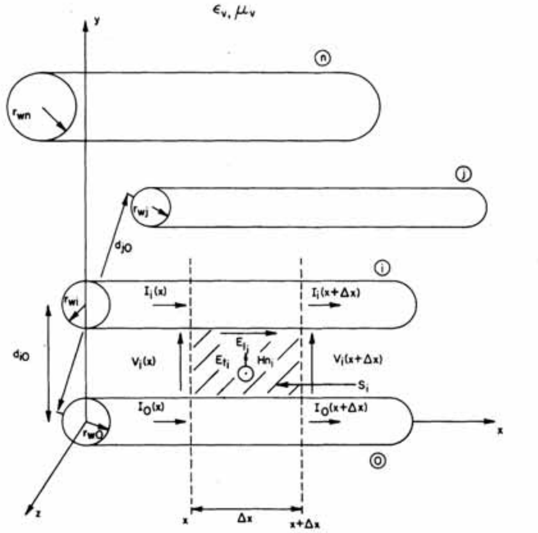

Fig. 1.

of multiconductor lines. The application of this result to multiconductor lines as in [3] is, of course, an approximation since the result in [1], [2] is, strictly speaking, not applicable to lines consisting of more than two conductors. Characteristic impedances of each set of isolated conductor pairs were employed in [3]. This also is an approximation since scalar characteristic impedances do not exist for multi-

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conductor lines and a characteristic impedance matrix is the proper quantity to be employed [5], [6], [9]. The case of a uniform plane wave incident on a three-conductor line in the transverse direction (perpendicular to the system's longitudinal (x) axis) with the electric field intensity vector polarized parallel to the line axis was obtained in [4]. Procedures for extending this result to multiconductor lines were indicated.

The distributed parameter transmission line equations for multiconductor lines with incident field illumination can be derived and are similar (with matrix notation employed) to the familiar equations for two-conductor lines [5], [6], [9]. Assuming sinusoidal excitation at a radian frequency $\omega = 2\pi f$, the electric field intensity vector, $\vec{\mathscr{E}}(x,y,z,t)$, and the magnetic field intensity vector, $\vec{\mathscr{H}}(x,y,z,t)$, are written as $\vec{\mathscr{E}}(x,y,z,t) = \vec{E}(x,y,z)e^{j\omega t}$ and $\vec{\mathscr{H}}(x,y,z,t) = \vec{H}(x,y,z)e^{j\omega t}$. The complex vectors $\vec{E}(x,y,z)$ and $\vec{H}(x,y,z)$ are the phasor quantities. Line voltages, $\mathscr{V}_i(x,t) = V_i(x)e^{j\omega t}$, of the ith conductor with respect to the zeroth conductor (the reference conductor) are defined as the line integral of $\vec{\mathscr{E}}$ between the two conductors along a path in the y,z plane. $V_i(x)$ is the complex phasor voltage. Line currents, $\mathscr{I}_i(x,t) = I_i(x)e^{j\omega t}$ associated with the ith conductor and directed in the x direction are defined as the line integral of $\vec{\mathscr{H}}$ along a closed contour in the y,z plane encircling only the ith conductor and $I_i(x)$ is the complex phasor current. The current in the reference conductor, $\mathscr{I}_0(x,t) = I_0e^{j\omega t}$, satisfies $I_0 = \sum_{i=1}^{n}(-I_i(x))$.

It is convenient to consider the effects of the spectral components of the incident field as per unit length distributed sources along the line [5], [9]. The sources appear as series voltage sources and shunt current sources as indicated in Fig. 2 for an "electrically small" Δx section of the line. The multiconductor transmission line equations may then be derived for the Δx subsection in Fig. 2 in the limit as $\Delta x \to 0$ as a set of $2n$ coupled, complex, ordinary differential equations [5], [9]

$$\dot{\boldsymbol{V}}(x) + j\omega \boldsymbol{L}\boldsymbol{I}(x) = \boldsymbol{V}_s(x) \tag{1a}$$

$$\dot{\boldsymbol{I}}(x) + j\omega \boldsymbol{C}\boldsymbol{V}(x) = \boldsymbol{I}_s(x). \tag{1b}$$

A matrix $\boldsymbol{M}$ with m rows and n columns is denoted as $m \times n$ and the element in the ith row and jth column is denoted by $[\boldsymbol{M}]_{ij}$. $\boldsymbol{V}(x)$ and $\boldsymbol{I}(x)$ are $n \times 1$ vectors of the line voltages and currents, respectively. The elements in the ith rows are $[\boldsymbol{V}(x)]_i = V_i(x)$ and $[\boldsymbol{I}(x)]_i = I_i(x)$ and $[\dot{\boldsymbol{V}}(x)]_i = (d/dx)V_i(x)$. The $n \times n$ real, symmetric, constant matrices $\boldsymbol{L}$ and $\boldsymbol{C}$ are the per-unit-length inductance and capacitance matrices, respectively. From Fig. 2, one can derive (1) and the entries in $\boldsymbol{L}$ and $\boldsymbol{C}$ become [9]

$$[\boldsymbol{L}]_{ii} = l_i + l_0 - 2m_{i0}$$
$$[\boldsymbol{L}]_{\substack{ij\\ i\neq j}} = l_0 + m_{ij} - m_{i0} - m_{j0} \tag{2}$$

and

$$[\boldsymbol{C}]_{ii} = c_{i0} + \sum_{\substack{j=1\\ i\neq j}}^{n} c_{ij} \qquad [\boldsymbol{C}]_{\substack{ij\\ i\neq j}} = -c_{ij}. \tag{3}$$

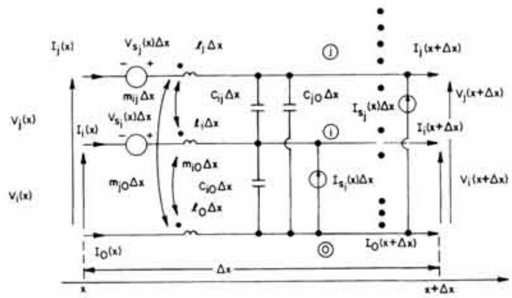


Fig. 2.

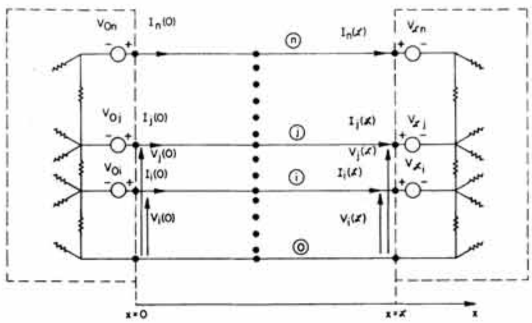


Fig. 3.

The entries in $\boldsymbol{V}_s(x)$ and $\boldsymbol{I}_s(x)$ are the per-unit-length distributed sources along the line induced by the incident field, i.e., $[\boldsymbol{V}_s(x)] = V_{s_i}(x)$ and $[\boldsymbol{I}_s(x)] = I_{s_i}(x)$, as shown in Fig. 2.

In order to consider general termination networks (and allowing independent sources in these networks) we choose to characterize these as generalized Thevenin equivalents [5], [6], [9]. For a line of total length $\mathscr{L}$, the equations for the termination networks at $x = 0$ and $x = \mathscr{L}$ are

$$\boldsymbol{V}(0) = \boldsymbol{V}_0 - \boldsymbol{Z}_0\boldsymbol{I}(0) \tag{4a}$$

$$\boldsymbol{V}(\mathscr{L}) = \boldsymbol{V}_{\mathscr{L}} + \boldsymbol{Z}_{\mathscr{L}}\boldsymbol{I}(\mathscr{L}) \tag{4b}$$

where $\boldsymbol{V}_0$ and $\boldsymbol{V}_{\mathscr{L}}$ are $n \times 1$ vectors of equivalent open circuit port excitation voltages, $[\boldsymbol{V}_0]_i = V_{0i}$ and $[\boldsymbol{V}_{\mathscr{L}}]_i = V_{\mathscr{L}i}$, and $\boldsymbol{Z}_0$ and $\boldsymbol{Z}_{\mathscr{L}}$ are $n \times n$ symmetric impedance matrices as shown in Fig. 3 [5], [6], [9]. This is, of course, a completely general and arbitrary characterization of these linear termination networks. The entries in these termination equations can be easily determined for a given network by considering $V_i(0)$ and $V_i(\mathscr{L})$ (the termination port voltages) as independent sources, and writing the loop current equations for each network where $I_i(0)$ and $I_i(\mathscr{L})$ are subsets of the loop currents in each network.

With the line immersed in a homogeneous medium (free space) with permittivity ε_v and permeability μ_v, the product of $\boldsymbol{L}$ and $\boldsymbol{C}$ becomes [5], [9]

$$\boldsymbol{LC} = \boldsymbol{CL} = \mu_v\varepsilon_v\boldsymbol{1}_n \tag{5}$$

where $\boldsymbol{1}_n$ is the $n \times n$ identity matrix with ones on the main diagonal and zeros elsewhere, i.e., $[\mathbf{1}_n]_{ii} = 1$, and $[\mathbf{1}_n]_{ij} = 0$, $i \neq j$.[1] For this case, the solution to (1) and (4) is in a simple form [5], [9]

$$[\cos(k\mathscr{L})\{\boldsymbol{Z}_0 + \boldsymbol{Z}_{\mathscr{L}}\} + j\sin(k\mathscr{L})\{\boldsymbol{Z}_C + \boldsymbol{Z}_{\mathscr{L}}\boldsymbol{Z}_C^{-1}\boldsymbol{Z}_0\}]\boldsymbol{I}(0) = -\boldsymbol{V}_{\mathscr{L}} + [j\sin(k\mathscr{L})\boldsymbol{Z}_{\mathscr{L}}\boldsymbol{Z}_C^{-1} + \cos(k\mathscr{L})\mathbf{1}_n]\boldsymbol{V}_0 + \hat{\boldsymbol{V}}_s(\mathscr{L}) - \boldsymbol{Z}_{\mathscr{L}}\hat{\boldsymbol{I}}_s(\mathscr{L}) \tag{6a}$$

$$\boldsymbol{I}(\mathscr{L}) = -j\sin(k\mathscr{L})\boldsymbol{Z}_C^{-1}\boldsymbol{V}_0 + [\cos(k\mathscr{L})\mathbf{1}_n + j\sin(k\mathscr{L})\boldsymbol{Z}_C^{-1}\boldsymbol{Z}_0]\boldsymbol{I}(0) + \hat{\boldsymbol{I}}_s(\mathscr{L}) \tag{6b}$$

where the wavenumber is $k = 2\pi/\lambda$, $\lambda = c/f$, $c = 1/\sqrt{\mu_v\varepsilon_v} \cong 3 \times 10^8$ m/s and the characteristic impedance matrix $\boldsymbol{Z}_C$ is [5], [6], [9]

$$\boldsymbol{Z}_C = c\boldsymbol{L}. \tag{7}$$

The inverse of an $n \times n$ matrix $\boldsymbol{M}$ is denoted by $\boldsymbol{M}^{-1}$ and $\hat{\boldsymbol{V}}_s(\mathscr{L})$ and $\hat{\boldsymbol{I}}_s(\mathscr{L})$ are given by [5], [9]

$$\hat{\boldsymbol{V}}_s(\mathscr{L}) = \int_0^{\mathscr{L}} \{\cos(k(\mathscr{L} - x))\boldsymbol{V}_s(x) - j\sin(k(\mathscr{L} - x))\boldsymbol{Z}_C\boldsymbol{I}_s(x)\}\,dx \tag{8a}$$

$$\hat{\boldsymbol{I}}_s(\mathscr{L}) = \int_0^{\mathscr{L}} \{\cos(k(\mathscr{L} - x))\boldsymbol{I}_s(x) - j\sin(k(\mathscr{L} - x))\boldsymbol{Z}_C^{-1}\boldsymbol{V}_s(x)\}\,dx. \tag{8b}$$

Solution of (6a) for the current vector, $\boldsymbol{I}(0)$, requires the solution of n complex equations in n unknowns ($I_i(0)$). Once (6a) is solved, (6b) yields the currents $\boldsymbol{I}(\mathscr{L})$ directly.

Although the equations may appear formidable, they are in a compact form and can be straightforwardly programmed on a digital computer. Furthermore, the form is not restricted to any particular value of n. The only difficulties are in determining $\boldsymbol{L}$ (or $\boldsymbol{C}$ through (5)), and determining $\hat{\boldsymbol{V}}_s$ and $\hat{\boldsymbol{I}}_s$ (which require that we determine $\boldsymbol{V}_s(x)$ and $\boldsymbol{I}_s(x)$). The determination of the equivalent sources $V_{s_i}(x)$ and $I_{s_i}(x)$ induced by the incident field will be the objective of the next section.

II. Determining the Equivalent Induced Sources

In order to determine the equivalent induced sources, $V_{s_i}(x)$ and $I_{s_i}(x)$, consider Fig. 1. The method used in [1] can be adapted here in a similar fashion. Faraday's law in integral form becomes

$$\oint_{C_i} \vec{E}\cdot d\vec{C}_i = -j\omega\mu_v \int_{S_i} \vec{H}\cdot\hat{n}\,dS_i \tag{9}$$

where S_i is a flat, rectangular surface in the x,y plane between wire i and wire 0 and between x and $x + \Delta x$ as shown in Fig. 1. The unit normal $\hat{n}$ is $\hat{n} = \hat{z}$ where $\hat{z}$ is the unit vector in the z direction, $dS_i = dx\,dy$ and C_i is a contour encircling S_i in the proper direction (counterclockwise according to the right-hand rule). Equation (9) becomes for the indicated integration[2]

$$\int_0^{d_{i0}} [E_{t_i}(y, x + \Delta x) - E_{t_i}(y,x)]\,dy - \int_x^{x+\Delta x} [E_{l_i}(d_{i0},x) - E_{l_i}(0,x)]\,dx = -j\omega\mu_v \int_x^{x+\Delta x}\int_0^{d_{i0}} H_{n_i}(y,x)\,dy\,dx \tag{10}$$

where E_{t_i} is the component of the total electric field (incident plus scattered) transverse to the line axis and lying along a straight line joining the two conductors, i.e., $E_{t_i} = E_y$; E_{l_i} is the component of the total electric field along the longitudinal axis of the line, i.e., $E_{l_i} = E_x$; and H_{n_i} is the component of the total magnetic field perpendicular to the plane formed by the two wires, i.e., $H_{n_i} = H_z$.

Defining the voltage between the two wires as

$$V_i(x) = -\int_0^{d_{i0}} E_{t_i}(y,x)\,dy \tag{11}$$

then

$$-\frac{dV_i(x)}{dx} = \lim_{\Delta x \to 0} \frac{1}{\Delta x}\int_0^{d_{i0}} [E_{t_i}(y, x + \Delta x) - E_{t_i}(y,x)]\,dy. \tag{12}$$

The total electric field along the wire surfaces is zero since we assume perfect conductors. (One can straightforwardly include finite conductivity conductors through a surface impedance as was done in [1]). Therefore (10) becomes in the limit as $\Delta x \to 0$

$$\frac{dV_i(x)}{dx} = j\omega\mu_v \int_0^{d_{i0}} H_{n_i}(y,x)\,dy. \tag{13}$$

The total magnetic field is the sum of an incident and a scattered field

$$H_{n_i}(y,x) = H_z(y,x) = \underbrace{\overset{(\mathrm{scat})}{H_z}(y,x)}_{\text{scattered}} + \underbrace{\overset{(\mathrm{inc})}{H_z}(y,x)}_{\text{incident}} \tag{14}$$

and the scattered field here is produced by the transmission line currents. The scattered flux passing between the two conductors per unit of line length is directly related to the scattered magnetic field and the per-unit-length inductance matrix, $\boldsymbol{L}$, as

$$\overset{(\mathrm{scat})}{\phi_i}(x) = -\int_0^{d_{i0}} \mu_v \overset{(\mathrm{scat})}{H_{n_i}}(y,x)\,dy = [l_{i1}, l_{i2}, \cdots, l_{in}]\begin{bmatrix} I_1(x) \\ I_2(x) \\ \vdots \\ I_n(x) \end{bmatrix} \tag{15}$$

[1] The property in (5) restricts the use of these results to bare wire lines in free space or other homogeneous media.

[2] In integrating from $y = 0$ to $y = d_{i0}$, we are implicitly assuming that r_{wi} and r_{w0} are much less than d_{i0}, i.e., the wires are sufficiently separated so that they may be replaced by filaments.

where $l_{ij} = [\boldsymbol{L}]_{ij}$. Substituting (15) and (14) into (13) and arranging for $i = 1,\cdots,n$ yields

$$\dot{\boldsymbol{V}}(x) + j\omega \boldsymbol{L}\boldsymbol{I}(x) = \begin{bmatrix} \vdots \\ j\omega\mu_v \int_0^{d_{i0}} \overset{(\text{inc})}{H_{n_i}}(y,x)\, dy \\ \vdots \end{bmatrix} \tag{16}$$

and the source vector $\boldsymbol{V}_s(x)$ in (1) is easily identified by comparing (16) and (1).

For transmission line theory to apply, the cross-sectional dimensions of the line (wire spacing, etc.) must be electrically small, i.e., $kd_{i0} \ll 1$ and $kd_{ij} \ll 1$. Thus the result indicates that the voltage, V_{s_i}, induced in the loop between the ith conductor and the zeroth conductor and between x and $x + \Delta x$ is equal to the rate of change of the incident flux penetrating this "electrically small" loop which, of course, makes sense.

Ampere's law yields

$$E_y = \frac{1}{j\omega\varepsilon_v}\left[\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}\right] \tag{17}$$

E_y will consist of scattered and incident field components and is written as

$$\begin{aligned} E_{t_i}(y,x) &= E_y(y,x) \\ &= \underbrace{\overset{(\text{scat})}{E_y}(y,x)}_{\text{scattered}} + \underbrace{\overset{(\text{inc})}{E_y}(y,x)}_{\text{incident}}. \end{aligned} \tag{18}$$

Substituting (17) into (11) we have

$$\begin{aligned} V_i(x) &= -\int_0^{d_{i0}} E_y(y,x)\, dy \\ &= \frac{1}{j\omega\varepsilon_v}\int_0^{d_{i0}} \left\{ \frac{\partial \overset{(\text{scat})}{H_z}(y,x)}{\partial x} + \frac{\partial \overset{(\text{inc})}{H_z}(y,x)}{\partial x} \right. \\ &\quad \left. - \frac{\partial \overset{(\text{scat})}{H_x}(y,x)}{\partial z} - \frac{\partial \overset{(\text{inc})}{H_x}(y,x)}{\partial z} \right\} dy. \end{aligned} \tag{19}$$

Utilizing (15) we obtain

$$\begin{aligned} V_i(x) = &-\frac{1}{j\omega\mu_v\varepsilon_v}\frac{d}{dx}\{[l_{i1},l_{i2},\cdots,l_{in}]\boldsymbol{I}(x)\} \\ &- \frac{1}{j\omega\varepsilon_v}\int_0^{d_{i0}} \frac{\partial \overset{(\text{scat})}{H_x}(y,x)}{\partial z}\, dy - \int_0^{d_{i0}} \overset{(\text{inc})}{E_{t_i}}(y,x)\, dy. \end{aligned} \tag{20}$$

If we assume that the currents on the wires are directed only in the x direction, i.e., (there are no transverse components of the currents on the wire surfaces), then $\overset{(\text{scat})}{H_x}(y,x) = 0$ and (20) becomes

$$\begin{aligned} V_i(x) = &-\frac{1}{j\omega\mu_v\varepsilon_v}\frac{d}{dx}\{[l_{i1},l_{i2},\cdots,l_{in}]\boldsymbol{I}(x)\} \\ &- \int_0^{d_{i0}} \overset{(\text{inc})}{E_{t_i}}(y,x)\, dy. \end{aligned} \tag{21}$$

Arranging these equations for $i = 1,\cdots,n$ we obtain the second transmission line equation

$$\begin{aligned} \dot{\boldsymbol{I}}(x) &+ j\omega\mu_v\varepsilon_v \boldsymbol{L}^{-1}\boldsymbol{V}(x) \\ &= -j\omega\mu_v\varepsilon_v\boldsymbol{L}^{-1}\begin{bmatrix} \vdots \\ \int_0^{d_{i0}} \overset{(\text{inc})}{E_{t_i}}(y,x)\, dy \\ \vdots \end{bmatrix}. \end{aligned} \tag{22}$$

Utilizing (5) in (22) ($\boldsymbol{C} = \mu_v\varepsilon_v\boldsymbol{L}^{-1}$) we obtain by comparing (16) and (22) to (1)

$$\boldsymbol{V}_s(x) = j\omega\mu_v \begin{bmatrix} \vdots \\ \int_0^{d_{i0}} \overset{(\text{inc})}{H_{n_i}}(y,x)\, dy \\ \vdots \end{bmatrix} \tag{23a}$$

$$\boldsymbol{I}_s(x) = -j\omega\boldsymbol{C}\begin{bmatrix} \vdots \\ \int_0^{d_{i0}} \overset{(\text{inc})}{E_{t_i}}(y,x)\, dy \\ \vdots \end{bmatrix}. \tag{23b}$$

The shunt current sources in $\boldsymbol{I}_s(x)$ are therefore a result of the line voltage induced by the incident electric field being applied across the per-unit-length line-to-line capacitances which, of course, satisfies our intuition.

III. Solution for $\hat{\boldsymbol{V}}_s(x)$ and $\hat{\boldsymbol{I}}_s(x)$

The final problem remaining is to obtain simplified versions of $\hat{\boldsymbol{V}}_s$ and $\hat{\boldsymbol{I}}_s$ in (8) to be directly used in (6). First consider the determination of $\hat{\boldsymbol{V}}_s(\mathscr{L})$. Substituting (23) into (8a) yields

$$\begin{aligned} \hat{\boldsymbol{V}}_s(\mathscr{L}) = j\omega\mu_v \int_0^{\mathscr{L}} &\left\{ \cos\left(k(\mathscr{L} - x)\right) \cdot \begin{bmatrix} \vdots \\ \int_0^{d_{i0}} \overset{(\text{inc})}{H_{n_i}}(y,x)\, dy \\ \vdots \end{bmatrix} \right\} dx \\ - k\int_0^{\mathscr{L}} &\left\{ \sin\left(k(\mathscr{L} - x)\right) \cdot \begin{bmatrix} \vdots \\ \int_0^{d_{i0}} \overset{(\text{inc})}{E_{t_i}}(y,x)\, dy \\ \vdots \end{bmatrix} \right\} dx. \end{aligned} \tag{24}$$

From Faraday's law we obtain

$$\overset{(\text{inc})}{H_{n_i}} = \frac{1}{j\omega\mu_v}\left[\frac{\partial \overset{(\text{inc})}{E_{l_i}}}{\partial y} - \frac{\partial \overset{(\text{inc})}{E_{t_i}}}{\partial x}\right]. \tag{25}$$

Substituting this into (24) yields

$$\begin{aligned}\hat{V}_s(\mathscr{L}) = &\int_0^{\mathscr{L}} \left\{ \cos(k(\mathscr{L}-x)) \begin{bmatrix} \vdots \\ \overset{(\mathrm{inc})}{E}_{l_i}(d_{i0},x) - \overset{(\mathrm{inc})}{E}_{l_i}(0,x) \\ \vdots \end{bmatrix} \right\} dx \\ &- \int_0^{\mathscr{L}} \left\{ \cos(k(\mathscr{L}-x)) \cdot \begin{bmatrix} \vdots \\ \int_0^{d_{i0}} \frac{\partial \overset{(\mathrm{inc})}{E}_{t_i}(y,x)}{\partial x}\, dy \\ \vdots \end{bmatrix} \right\} dx \\ &- k \int_0^{\mathscr{L}} \left\{ \sin(k(\mathscr{L}-x)) \cdot \begin{bmatrix} \vdots \\ \int_0^{d_{i0}} \overset{(\mathrm{inc})}{E}_{t_i}(y,x)\, dy \\ \vdots \end{bmatrix} \right\} dx. \end{aligned} \tag{26}$$

Utilizing Leibnitz's rule (see [7, p. 219]), (26) is equivalent to

$$\begin{aligned}\hat{V}_s(\mathscr{L}) = &\int_0^{\mathscr{L}} \left\{ \cos(k(\mathscr{L}-x)) \cdot \begin{bmatrix} \vdots \\ \overset{(\mathrm{inc})}{E}_{l_i}(d_{i0},x) - \overset{(\mathrm{inc})}{E}_{l_i}(0,x) \\ \vdots \end{bmatrix} \right\} dx \\ &- \int_0^{\mathscr{L}} \frac{\partial}{\partial x} \left\{ \cos(k(\mathscr{L}-x)) \cdot \begin{bmatrix} \vdots \\ \int_0^{d_{i0}} \overset{(\mathrm{inc})}{E}_{t_i}(y,x)\, dy \\ \vdots \end{bmatrix} \right\} dx \end{aligned} \tag{27}$$

and this may be written as

$$\begin{aligned}\hat{V}_s(\mathscr{L}) = &\int_0^{\mathscr{L}} \left\{ \cos(k(\mathscr{L}-x)) \cdot \begin{bmatrix} \vdots \\ \overset{(\mathrm{inc})}{E}_{l_i}(d_{i0},x) - \overset{(\mathrm{inc})}{E}_{l_i}(0,x) \\ \vdots \end{bmatrix} \right\} dx \\ &- \begin{bmatrix} \vdots \\ \int_0^{d_{i0}} \overset{(\mathrm{inc})}{E}_{t_i}(y,\mathscr{L})\, dy \\ \vdots \end{bmatrix} + \cos(k\mathscr{L}) \begin{bmatrix} \vdots \\ \int_0^{d_{i0}} \overset{(\mathrm{inc})}{E}_{t_i}(y,0) \\ \vdots \end{bmatrix} dy. \end{aligned} \tag{28}$$

Similarly $\hat{I}_s(\mathscr{L})$ may be obtained as

$$\begin{aligned}\hat{I}_s(\mathscr{L}) = &-j\boldsymbol{Z}_C^{-1} \int_0^{\mathscr{L}} \left\{ \sin(k(\mathscr{L}-x)) \cdot \begin{bmatrix} \vdots \\ \overset{(\mathrm{inc})}{E}_{l_i}(d_{i0},x) - \overset{(\mathrm{inc})}{E}_{l_i}(0,x) \\ \vdots \end{bmatrix} \right\} dx \\ &- j\boldsymbol{Z}_C^{-1} \left\{ \sin(k\mathscr{L}) \begin{bmatrix} \vdots \\ \int_0^{d_{i0}} \overset{(\mathrm{inc})}{E}_{t_i}(y,0)\, dy \\ \vdots \end{bmatrix} \right\}. \end{aligned} \tag{29}$$

The important quantity in (6a) is $\hat{V}_s(\mathscr{L}) - \boldsymbol{Z}_{\mathscr{L}}\hat{I}_s(\mathscr{L})$. Combining (28) and (29), this becomes

$$\begin{aligned}&\hat{V}_s(\mathscr{L}) - \boldsymbol{Z}_{\mathscr{L}}\hat{I}_s(\mathscr{L}) \\ &= \int_0^{\mathscr{L}} \left\{ [\cos(k(\mathscr{L}-x))\mathbf{1}_n + j\sin(k(\mathscr{L}-x))\boldsymbol{Z}_{\mathscr{L}}\boldsymbol{Z}_C^{-1}] \cdot \begin{bmatrix} \vdots \\ \overset{(\mathrm{inc})}{E}_{l_i}(d_{i0},x) - \overset{(\mathrm{inc})}{E}_{l_i}(0,x) \\ \vdots \end{bmatrix} \right\} dx \\ &\quad - \begin{bmatrix} \vdots \\ \int_0^{d_{i0}} \overset{(\mathrm{inc})}{E}_{t_i}(y,\mathscr{L}) \\ \vdots \end{bmatrix} dy \\ &\quad + [\cos(k\mathscr{L})\mathbf{1}_n + j\sin(k\mathscr{L})\boldsymbol{Z}_{\mathscr{L}}\boldsymbol{Z}_C^{-1}] \cdot \begin{bmatrix} \vdots \\ \int_0^{d_{i0}} \overset{(\mathrm{inc})}{E}_{t_i}(y,0) \\ \vdots \end{bmatrix} dy. \end{aligned} \tag{30}$$

Note that the equivalent forcing function on the right-hand side of (6a), $\hat{V}_s(\mathscr{L}) - \boldsymbol{Z}_{\mathscr{L}}\hat{I}_s(\mathscr{L})$, given in (30) is simply determined as a convolution of differences of the incident electric field vector along the wire axes, $\overset{(\mathrm{inc})}{E}_{l_i}(d_{i0},x) - \overset{(\mathrm{inc})}{E}_{l_i}(0,x)$, and a linear combination of integrals of components of the electric field vectors at the endpoints of the line which are transverse to the line, $\overset{(\mathrm{inc})}{E}_{t_i}(y,\mathscr{L})$ and $\overset{(\mathrm{inc})}{E}_{t_i}(y,0)$. This is, of course, precisely the result obtained by Smith [2] for two conductor lines. Substituting (30) into (6a) and setting $\boldsymbol{V}_{\mathscr{L}} = {}_n\mathbf{0}_1$, $\boldsymbol{V}_0 = {}_n\mathbf{0}_1$, i.e., no independent sources in the termination networks, one can verify that the result reduces for two conductor lines ($n = 1$) to the result given by Smith [2] since $\boldsymbol{Z}_C, \boldsymbol{Z}_{\mathscr{L}}, \boldsymbol{Z}_0$ become scalars for two conductor lines and (6a) becomes one equation in only one unknown $I(0)$. For uniform plane wave illumination of the line (which is usually the case of interest), (30) reduces to a much simpler form although the result allows for the more general case.

The final equations for the line currents then become (substituting (30) into (6))

$$[\cos(k\mathscr{L})\{Z_0 + Z_{\mathscr{L}}\} + j\sin(k\mathscr{L})\{Z_C + Z_{\mathscr{L}}Z_C^{-1}Z_0\}]I(0)$$
$$= -V_{\mathscr{L}} + [j\sin(k\mathscr{L})Z_{\mathscr{L}}Z_C^{-1} + \cos(k\mathscr{L})\mathbf{1}_n]V_0$$
$$+ \int_0^{\mathscr{L}} \{[\cos(k(\mathscr{L}-x))_n + j\sin(k(\mathscr{L}-x))Z_{\mathscr{L}}Z_C^{-1}]\overset{(\mathrm{inc})}{E_t}(x)\}\,dx - \overset{(\mathrm{inc})}{E_t}(\mathscr{L})$$
$$+ \{[\cos(k\mathscr{L})\mathbf{1}_n + j\sin(k\mathscr{L})Z_{\mathscr{L}}Z_C^{-1}]\overset{(\mathrm{inc})}{E_t}(0)\} \qquad (31a)$$

$$I(\mathscr{L}) = -j\sin(k\mathscr{L})Z_C^{-1}V_0 + [\cos(k\mathscr{L})\mathbf{1}_n + j\sin(k\mathscr{L})Z_C^{-1}Z_0]I(0)$$
$$- jZ_C^{-1}\int_0^{\mathscr{L}} \{\sin(k(\mathscr{L}-x))\overset{(\mathrm{inc})}{E_t}(x)\}\,dx$$
$$- jZ_C^{-1}\{\sin(k\mathscr{L})\overset{(\mathrm{inc})}{E_t}(0)\}. \qquad (31b)$$

where $\overset{(\mathrm{inc})}{E_t}(x)$, $\overset{(\mathrm{inc})}{E_t}(\mathscr{L})$, and $\overset{(\mathrm{inc})}{E_t}(0)$ are $n \times 1$ column vectors with the entries in the ith rows given by

$$[\overset{(\mathrm{inc})}{E_t}(x)]_i = \overset{(\mathrm{inc})}{E_{t_i}}(d_{i0},x) - \overset{(\mathrm{inc})}{E_{t_i}}(0,x) \qquad (32a)$$

$$[\overset{(\mathrm{inc})}{E_t}(\mathscr{L})]_i = \int_0^{d_{i0}} E_{t_i}(\rho_i,\mathscr{L})\,d\rho_i \qquad (32b)$$

$$[\overset{(\mathrm{inc})}{E_t}(0)]_i = \int_0^{d_{i0}} \overset{(\mathrm{inc})}{E_{t_i}}(\rho_i,0)\,d\rho_i \qquad (32c)$$

for $i = 1,\cdots,n$.

A word of caution in the interpretation of the notation is in order. Although it should be clear from the derivation, the reader should nevertheless be reminded that the integration path for the component $\overset{(\mathrm{inc})}{E_{t_i}}$ is in the y direction when the ith conductor is concerned. When other conductors are concerned, the integration path is a straight line in the y,z plane which joins the conductor and the zeroth conductor and is perpendicular to these two conductors. This is designated as ρ_i in (32) and replaces the y variable for the path associated with conductors i and 0. The notation may be cumbersome but the idea and implementation is quite simple.

IV. The Per Unit Length Inductance Matrix, *L*

One final calculation remains; the determination of the per unit length inductance matrix, L. Ordinarily this is a difficult calculation [8]. However, if we assume that the conductors are separated sufficiently such that the charge distribution around the periphery of each conductor is constant, then the conductors can be replaced by filamentary lines of charge. Typically, this will be quite accurate if the smallest ratio of conductor separation to wire radius is

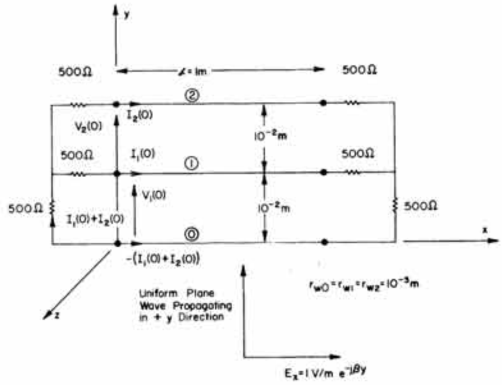


Fig. 4.

greater than 5 [8]. In this case, the entries in L are shown in the appendix to be

$$[L]_{ii} = \mu_v\varepsilon_v[C^{-1}]_{ii} \cong \frac{\mu_v}{2\pi}\ln\left(\frac{d_{i0}^2}{r_{wi}r_{w0}}\right) \qquad (33a)$$

$$[L]_{\substack{ij\\ i\neq j}} = \mu_v\varepsilon_v[C^{-1}]_{ij} \cong \frac{\mu_v}{2\pi}\ln\left(\frac{d_{i0}d_{j0}}{r_{w0}d_{ij}}\right). \qquad (33b)$$

For closer conductor spacings, proximity effect will alter the charge distributions from constant ones and numerical approximations must be employed to find C and L (through (5)) [8].

V. Computed Results

To show the simplicity of the result and indicate its equivalence to the result obtained by Harrison in [4], Example 1 considered in [4] will be computed by this method. Three wires all of radius 10^{-3} m lie in the x,y plane as shown in Fig. 4. A uniform plane wave with an electric field intensity magnitude of 1 V/m is propagating in the y direction and 500-Ω (purely resistive) loads connect each line to common nodes. The various distances in Fig. 1 are $d_{10} = 10^{-2}$ m, $d_{20} = 2 \times 10^{-2}$ m, and $d_{12} = 10^{-2}$ m. Z_0 and $Z_{\mathscr{L}}$ can be easily shown to be

$$Z_{\mathscr{L}} = Z_0 = \begin{bmatrix} 1000 & 500 \\ 500 & 1000 \end{bmatrix}.$$

The characteristic impedance matrix, using the values for the per unit length inductance matrix given in (33), becomes

$$Z_C = cL = 60\begin{bmatrix} \ln(100) & \ln(20) \\ \ln(20) & \ln(400) \end{bmatrix}$$

$\overset{(\mathrm{inc})}{E_{t_i}} = 0$ in (30) and the electric field intensity of the wave is

$$\overset{(\mathrm{inc})}{E_t}(y,x) = E_x e^{-jky}.$$

From this, one can determine

$$\overset{(\mathrm{inc})}{E_{t_1}}(d_{10},x) - \overset{(\mathrm{inc})}{E_{t_1}}(0,x) = e^{-jk10^{-2}} - 1$$
$$\overset{(\mathrm{inc})}{E_{t_2}}(d_{10},x) - \overset{(\mathrm{inc})}{E_{t_2}}(0,x) = e^{-j2k10^{-2}} - 1.$$

Two frequencies are considered in [4] in terms of $k\mathscr{L}$; $k\mathscr{L} = 1.5$, $k\mathscr{L} = 3.0$. Equations (31) and (32) with the above items were programmed on an IBM 370/165 computer in double precision arithmetic. The execution time (cpu time) was 0.01 s (1/100 s) and the results are

$$k\mathscr{L} = 1.5 \begin{cases} |I_0(0)| = 1.7662556E-5A & \underline{/I_0(0)} = 70.77^\circ \\ |I_1(0)| = 9.0756083E-8A & \underline{/I_1(0)} = -13.9^\circ \\ |I_2(0)| = 1.7671218E-5A & \underline{/I_2(0)} = -109.52^\circ \end{cases}$$

$$k\mathscr{L} = 3.0 \begin{cases} |I_0(0)| = 5.4543875E-5A & \underline{/I_0(0)} = 9.845^\circ \\ |I_1(0)| = 7.7363155E-7A & \underline{/I_1(0)} = -75.8^\circ \\ |I_2(0)| = 5.4608110E-5A & \underline{/I_2(0)} = -170.96^\circ. \end{cases}$$

The computed results obtained by Harrison's method and given in [4] are

$$k\mathscr{L} = 1.5 \begin{cases} |I_0(0)| = 1.766E-5A \\ |I_1(0)| = 9.076E-8A \\ |I_2(0)| = 1.767E-5A \end{cases}$$

$$k\mathscr{L} = 3.0 \begin{cases} |I_0(0)| = 5.454E-5A \\ |I_1(0)| = 7.736E-7A \\ |I_2(0)| = 5.461E-5A. \end{cases}$$

The results computed by this method are *exactly* those computed by Harrison's method in [4]. However, with this method only 2 simultaneous equations in the 2 unknowns, $I_1(0)$ and $I_2(0)$, are required to be solved ($I_0(0) = -I_1(0) - I_2(0)$). Harrison's method required the solution of 10 simultaneous equations in 10 unknowns. Furthermore, Harrison's method was restricted to uniform plane wave illumination of the line with the wave incident perpendicular to the line. Since $\boldsymbol{Z}_{\mathscr{L}} = \boldsymbol{Z}_0$ for this example and since the uniform plane wave are propagating broadside to the line, $\boldsymbol{I}(0) = \boldsymbol{I}(\mathscr{L})$.

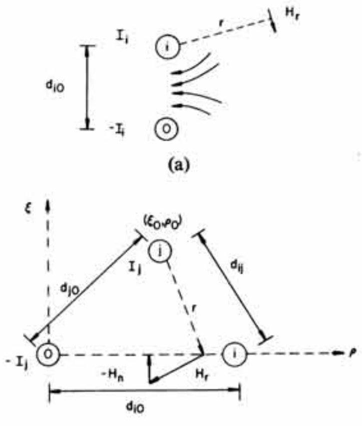
VI. Summary

A simple matrix equation for determining the frequency response of a multiconductor transmission line exposed to a nonuniform electromagnetic field has been presented. The formulation includes all mutual interactions and relates the response to the values of the electric field intensity vector transverse to the line at the line terminations and along the wire axes. The number of simultaneous equations to be solved for an $(n+1)$-conductor line is n and thus may be considered to be a minimum. The method can be easily extended to the case of n wires above an infinite ground plane where the ground plane is the reference conductor [9].

VII. Appendix

The purpose of this appendix is to show the relations in (33) for the entries in the per unit length inductance matrix, $\boldsymbol{L}$. The matrix $\boldsymbol{L}$ relates the scattered flux $\overset{(\text{scat})}{\boldsymbol{\phi}}$ passing between the wires to the wire currents as

$$\overset{(\text{scat})}{\boldsymbol{\phi}} = \begin{bmatrix} \overset{(\text{scat})}{\phi_1} \\ \vdots \\ \overset{(\text{scat})}{\phi_n} \end{bmatrix} \underbrace{\begin{bmatrix} l_{11} & \cdots & l_{1n} \\ \vdots & & \vdots \\ l_{n1} & \cdots & l_{nn} \end{bmatrix}}_{\boldsymbol{L}} \begin{bmatrix} I_1 \\ \vdots \\ I_n \end{bmatrix}. \quad \text{(A-1)}$$



(a)

(b)

Fig. 5.

The respective entries are determined as

$$l_{ii} = \left.\frac{\overset{(\text{scat})}{\phi_i}}{I_i}\right|_{I_1,\cdots,I_{i-1},I_{i+1},\cdots,I_n=0} \quad \text{(A-2a)}$$

$$\underset{i\neq j}{l_{ij}} = \left.\frac{\overset{(\text{scat})}{\phi_i}}{I_j}\right|_{I_1,\cdots,I_{j-1},I_{j+1},\cdots,I_n=0} \quad \text{(A-2b)}$$

and $l_{ij} = l_{ji}$. Large wire separations are assumed so that the wires may be replaced by filaments of current. When the wires are not widely separated, accurate values for $\boldsymbol{L}$ can be obtained by numerical methods [8].

Consider Fig. 5(a). The magnitude of the magnetic field intensity vector due to I_i on wire i at a distance $r > r_{wi}$ away from wire i is

$$H_r = \frac{I_i}{2\pi r} \quad \text{(A-3)}$$

and the total flux passing between wire i and wire 0 due to both currents is

$$\overset{(\text{scat})}{\phi_i} = \frac{\mu_v I_i}{2\pi}\left\{\int_{r_{wi}}^{d_{i0}} \frac{1}{r}\,dr + \int_{r_{w0}}^{d_{i0}} \frac{1}{r}\,dr\right\}$$

$$= \frac{\mu_v I_i}{2\pi} \ln\left(\frac{d_{i0}^{\,2}}{r_{wi} r_{w0}}\right). \quad \text{(A-4)}$$

Thus l_{ii} is easily identified as in (32a).

Consider Fig. 5(b). The portion of the flux $\overset{(\text{scat})}{\phi_i}$ passing between wire i and wire 0 due to $-I_j$ in the reference conductor is as above

$$\overset{(\text{scat})}{\phi_{i0}} = \frac{\mu_v I_j}{2\pi} \ln\left(\frac{d_{i0}}{r_{w0}}\right) \quad \text{(A-5)}$$

and the portion of the flux passing between wire i and wire 0 due to I_j in the jth conductor can be found to be

$$\overset{(\text{scat})}{\phi_{ij}} = -\mu_v \int_0^{d_{i0}} H_n \, d\rho$$

$$= -\frac{\mu_v}{2\pi} I_j \left\{ \int_{\rho=0}^{\rho=d_{i0}} \frac{(\rho-\rho_0)}{[\xi_0^2 + (\rho-\rho_0)^2]} \, d\rho \right\}$$

$$= \frac{\mu_v}{2\pi} I_j \left\{ \frac{1}{2} \ln \left[\frac{(\xi_0^2 + \rho_0^2)}{\xi_0^2 + (d_{i0}-\rho_0)^2} \right] \right\}. \qquad \text{(A-6)}$$

Combining (A-5) and (A-6) we obtain

$$\overset{(\text{scat})}{\phi_i} = \overset{(\text{scat})}{\phi_{i0}} + \overset{(\text{scat})}{\phi_{ij}} = \frac{\mu_v I_j}{2\pi} \ln \left(\frac{d_{j0} d_{i0}}{d_{ij} r_{w0}} \right) \qquad \text{(A-7)}$$

since

$$d_{ij}^2 = \xi_0^2 + (d_{i0} - \rho_0)^2 \qquad \text{(A-8a)}$$

$$d_{j0}^2 = \xi_0^2 + \rho_0^2 \qquad \text{(A-8b)}$$

and l_{ij} is easily identified as in (33b).

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HIGH INTENSITY RADIATED FIELDS (HIRF) COURSE

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Presenters

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