

A Primer on Scattering Parameters, Part I: Definitions and Properties

Ivan A. Maio, Member, IEEE, maio@polito.it

Abstract—This primer offers a simple and comprehensive overview of the properties and usage of the scattering parameters of linear n-port elements. The primer is divided into two parts, addressing the basic theory and some key applications of the scattering equations, respectively. In this first part, the crucial role of the reference impedances defining the wave variables and scattering equations is thoroughly discussed. The normalization of the wave variables, the time-domain scattering equations and examples of the scattering equation for a transmission line segment are illustrated.

I. INTRODUCTION

The characterization of parts of electromagnetic systems as linear multiport elements is a fundamental tool for the analysis and simulation of EMC problems. In this context, the scattering equations have gained increasing importance. The scattering responses, in fact, are the primary quantities arising from high-frequency measurements [1] and the most suitable for full-wave analyses. As a consequence, the use of scattering data for the characterization of multiport elements as well as for the development of macromodels for system-level simulations has become a standard approach.

In this primer, the usage and the properties of the scattering equations of linear n-port elements are explained and illustrated by examples. The aim is to help readers not familiar with scattering equations to exploit them to characterize the linear n-port elements that occur in signal integrity and EMC problems. The presentation is devised to be as simple as possible and, at the same time, to highlight the important concepts involved in the use of the scattering equations.

II. PORT RELATIONS FOR LINEAR N-PORT ELEMENTS

In order to introduce the scattering equations of n-port elements, it is expedient to address the problem from a circuit theory point of view. Linear n-port elements are best characterized in the frequency domain, where the relations between the port variables take the form of linear algebraic equations. All possible set of equations can be easily defined by considering the characterization of a two-port element. For a two-port, the electrical status is defined by four port variables: the two port voltages and the two port currents. If either the two port currents or the two port voltages are selected as independent variables and the remaining port variables are expressed for the independent ones, then either the impedance or the admittance equations are obtained

$$\begin{aligned} \mathbf{V}(s) &= \mathbf{Z}(s)\mathbf{I}(s) \\ \mathbf{I}(s) &= \mathbf{Y}(s)\mathbf{V}(s) \end{aligned} \quad (1) \quad (2)$$

where $\mathbf{V}$ and $\mathbf{I}$ are the vectors of the Laplace transforms of port voltages and currents, respectively, and $\mathbf{Z}$ and $\mathbf{Y}$ are the impedance and admittance matrix of the characterized element, respectively. Throughout this paper, $s = \sigma + j\omega$ is used to indicate the complex frequency variable and boldface symbols to indicate matrices. Of course, the above equations hold for an arbitrary number of ports, as well.

Other set of equations can be defined by using as independent variables the current of one port and the voltage of the other port. In this case, hybrid equations arise, like the following one

$$\begin{bmatrix} I_1(s) \\ V_2(s) \end{bmatrix} = \begin{bmatrix} H_{11}(s) & H_{12}(s) \\ H_{21}(s) & H_{22}(s) \end{bmatrix} \begin{bmatrix} V_1(s) \\ I_2(s) \end{bmatrix} \quad (3)$$

where I_1 , I_2 , V_1 and V_2 are the port variables of the two-port element. Finally, if both variables of a port are selected as independent variables, then the chain parameter equations are obtained.

It is important to remind that the elements of the characteristic matrices are always network functions (*i.e.*, responses) of the circuit defined by the n-port with the ports closed on a specific set of loads. Such reference loads are the two terminal elements impressing the independent port variables of the equation that defines the characteristic matrix at hand. For equation (3), this concept is illustrated in Fig. 1, where the voltage and current sources apply the independent variables of the hybrid equation of the two-port element. According to this meaning, *for a given n-port a specific characteristic matrix exists provided the network defining the matrix has unique solution.*

From a formal point of view, the reference loads of a characteristic matrix do not limit its use in predicting the responses of the n-port element for arbitrary loads. In fact, the response of the n-port terminated by its actual loads is computed by means of the n-port equations and the load equations. The solution of this new set of equations leads to the sought responses.

III. THE SCATTERING MATRIX

Scattering equations relate auxiliary port variables that are defined by a linear transformation of port voltages and currents. The simplest auxiliary variables are named *voltage waves* and are defined by [1], [2]

$$\begin{aligned} A &= \frac{1}{2}(V + Z_r I) \\ B &= \frac{1}{2}(V - Z_r I) \end{aligned} \quad (4)$$

where V and I are the voltage and currents at the terminals of a port (defined by associate reference directions, see Fig. 2) and Z_r is the reference impedance defining the transformation. The

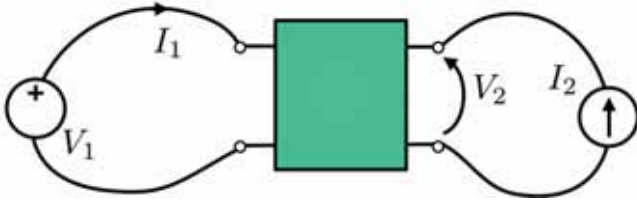


Fig. 1. Schematic of the network defining the hybrid characterization of (3).

port voltage and current can be obtained from the voltage waves as

$$\begin{aligned} V &= A + B \\ I &= Y_r (A - B) \end{aligned} \quad (5)$$

where $Y_r = 1/Z_r$ and either current and voltages or wave variables can be used to describe the electrical status of a port¹. Equations (4) can be readily generalized to n ports by replacing each variable with the corresponding vector of variables and Z_r with a diagonal matrix Z_r whose elements are the reference impedances of each port.

The scattering equation of an n -port element relates the vector of the B and A variables of the ports of the element

$$B(s) = S(s)A(s) \quad (6)$$

where S is the *scattering matrix* of the element for the reference impedance matrix Z_r defining the wave variables.

Now, as in the case of the equations relating port voltages and currents, the identification of the reference loads defining the scattering equations is in order. Since the reference loads of a matrix equation are the elements impressing the independent variables of the equations (e.g., see Fig. 1), the reference loads of (6) are those impressing the components of vector A at the ports of the n -port element. When the A variable of a port is given, from the first equation of (4) it follows that the voltage and the current of the port become related by

$$V = -Z_r I + 2A \quad (7)$$

This is the constitutive relation of a Thévenin element with an open circuit voltage $2A$ and equivalent impedance Z_r . For a two-port element, therefore, the entries of S are the responses of the network shown in Fig. 3. This demonstrates that, for the scattering matrix defined by the voltage waves, the reference load of the p -th port is the reference impedance of the voltage waves of that port with a series voltage source.

The entries of the scattering matrix are named *scattering parameters* or *scattering functions*. Every scattering function expresses one of the B s when all the A s are null, but one of them

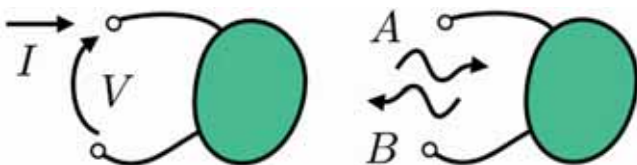


Fig. 2. Reference directions for the definition of the voltage waves of a port.

$$B_p = S_{pq}(s) A_q \quad \text{for} \quad A_k = 0 \quad \forall k \neq q \quad (8)$$

When the A variable of a port is null, its load is defined by $V = -Z_r I$ and the port is said to be *matched to its reference impedance*. Then S_{pq} yields the B_p due to A_q when all the ports of the element but port q are matched.

A. Example #1: Scattering Matrix of a Lumped Two-Port Element

This simple example illustrates the calculation of the scattering matrix on the two-port element of Fig. 4. The reference impedances are chosen equal to Z_r at both ports. For the network of Fig. 4, the Millman theorem yields

$$V_1 = V_2 = \frac{Y_r 2A_1 + Y_r 2A_2}{Y_r + sC + Y_r} \quad (9)$$

Since $B = V - A$, the following scattering matrix arises

$$S = \frac{1}{2Y_r + sC} \begin{bmatrix} -sC & 2Y_r \\ 2Y_r & -sC \end{bmatrix} \quad (10)$$

B. From Z to S

A classical problem in the characterization of n -port elements is the conversion of impedance and admittance equations into scattering equations. Figure 5 shows the network for the calculation of the scattering matrix of a two-port element characterized by its impedance matrix. The definitions of the voltage waves yield

$$\begin{aligned} 2A &= V + Z_r I \\ 2B &= V - Z_r I \end{aligned} \quad (11)$$

where $V = ZI$ and $Z_r = \text{diag}\{Z_{r1}, Z_{r2}\}$. The impedance equation allows to eliminate I from (11) as follows

$$\begin{aligned} (Z + Z_r)^{-1} 2A &= I \\ 2B &= (Z - Z_r)I \end{aligned} \quad (12)$$

leading to

$$S = (Z - Z_r)(Z + Z_r)^{-1} \quad (13)$$

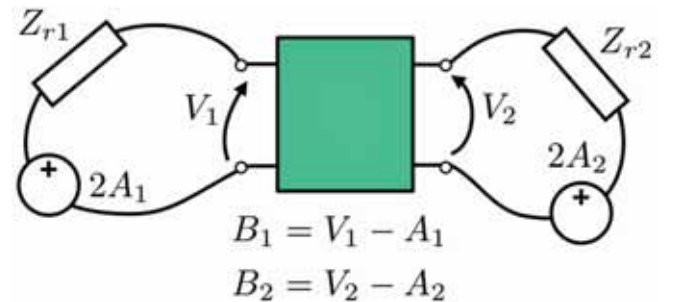


Fig. 3. Network defining the scattering responses for a two-port element. The port voltages of this network leads to the B voltage waves of the ports and hence to the scattering equations for the reference impedances Z_{r1} and Z_{r2} .

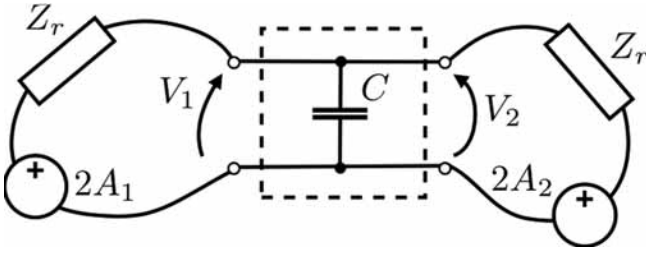


Fig. 4. The two-port element of Example #1 (dashed box) and the network defining its scattering matrix.

As an example, this equation can be used to compute (10) directly from the impedance matrix of the two-port element of Fig. 4

$$Z = \frac{1}{sC} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \quad (14)$$

Finally, it is worth noticing that (13) is the vector analogous of the relation defining the reflection coefficient of a scalar impedance Z with respect to the impedance Z_r . The reasons of this analogy are unfolded in Section V.

IV. THE REFERENCE IMPEDANCES

In contrast to equations involving port voltages and currents, scattering equations are defined by a set of (in principle) arbitrary reference loads, and this is of great help in many applications. In general, reference loads defined by finite impedances are termination conditions milder than the shorts and open circuits defining impedance, admittance, hybrid and chain equations. As a result, the conditions for the existence of the scattering equations (*i.e.*, for the existence of the solution of the network of Fig. 3) are weaker than the conditions for the existence of the equations involving port voltages and currents. In particular, *S* defined by a set of strictly positive reference resistances exists for any passive n -port [2]. Scattering equations, therefore, can handle a larger class of n -port elements.

Of course, the reference impedance of (4) can be also complex and frequency dependent, however this general case greatly complicates the discussion and is out of the scope of this primer. Hence, throughout this primer (except for Example #4), we restrict to real positive constant reference impedances (*i.e.*, to reference resistors).

The reference impedance values can be exploited to obtain scattering functions defined by reference loads closer to the operating load of the n -port element. This can be beneficial in measurement and modeling applications. In fact, even if, as pointed out in Sec. II, any characteristic matrix can be used to predict the responses of the n -port to its actual loads, there are practical limitations for matrices affected by errors. If the responses one is interested in are defined by loads very different from those defining the characteristic matrix that is available, then the limited precision of the matrix can lead to large errors. This problem can be best visualized by considering the poles of the characteristic matrix and those of the sought responses. If the two sets of poles are very different, the limited accuracy of the characteristic matrix will lead to large inaccuracies in the prediction of the new set of poles. The dependence of the poles of S on the reference impedances can be appreciated in the examples of Sec. VII.

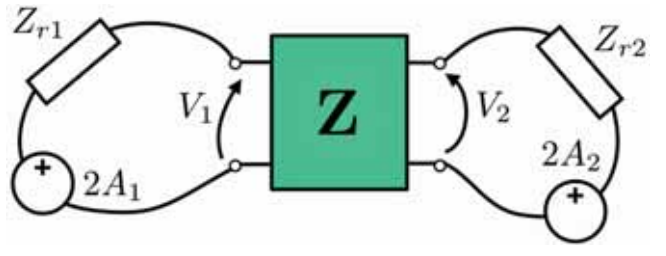


Fig. 5. Network for the calculation of the scattering matrix of a two-port element from its impedance matrix.

A. The Change of Reference Impedances

In order to cope with different sets of reference impedances, it is useful to compute the scattering matrix S' of a reference impedance matrix Z'_r from the matrix S that holds for Z_r .

Let

$$\begin{aligned} V &= A' + B' \\ I &= Y'_r(A' - B') \end{aligned} \quad (15)$$

be the expression of the port voltages and currents for the voltage waves defined by $Z'_r = (Y'_r)^{-1}$. Then the voltage waves defined by Z_r are given by

$$\begin{aligned} 2A &= V + Z_r I \\ &= A' + B' + Z_r Y'_r(A' - B') \\ 2B &= V - Z_r I \\ &= A' + B' - Z_r Y'_r(A' - B') \end{aligned} \quad (16)$$

These relations also write

$$\begin{aligned} 2A &= (1 + \zeta)A' + (1 - \zeta)B' \\ 2B &= (1 - \zeta)A' + (1 + \zeta)B' \end{aligned} \quad (17)$$

where ζ is the diagonal matrix $Z_r Y'_r$. When these expressions are replaced into the known scattering relation $B = SA$, S' is obtained

$$S' = [(1 + \zeta) - S(1 - \zeta)]^{-1}[S(1 + \zeta) - (1 - \zeta)] \quad (18)$$

If $Z_{rk} = Z_r$ and $Z'_{rk} = Z'_r \forall k$, then S' simplifies to

$$S' = (1 - PS)^{-1}(S - P1) \quad (19)$$

where $P = (Z'_r - Z_r)/(Z'_r + Z_r)$ is the reflection coefficient of Z'_r with respect to Z_r .

V. PHYSICAL MEANING OF SCATTERING RESPONSES

The circuit theory point of view is very useful to define the scattering equations and to compute S , yet it does not point out their physical meaning, which is covered in this Section. Let V and I be the voltage and current in a generic section of an ideal LC transmission line with characteristic impedance Z_0 (that is a real positive constant). Then they can be written as [3]

$$\begin{aligned} V &= V^+ + V^- \\ I &= Y_0(V^+ - V^-) \end{aligned} \quad (20)$$

where V^+ and V^- are the forward and backward voltages traveling on the transmission line, respectively. The comparison of the above equations to (5) shows that the voltage wave variables coincide with the forward and backward voltages of an LC line with characteristic impedance $Z_0 = Z_r$.

This interpretation of the wave variables highlights that S describes the scattering process of the traveling waves impinging on the n -port element. Figure 6 illustrates this idea. Let the n -port element be connected to n LC transmission lines with characteristic impedances Z_p , $p = 1, \dots, n$. Then the transmission lines carry to the ports the forward waves of A and the element reacts by exciting on the transmission lines the backward waves $B = SA$. If all transmission lines are matched, but the q -th one, then the q -th column of S describes the conversion of the wave impinging on the q -th port in backward waves excited at every port. In particular, the S_{qq} entry, describe the reflection at port q and S_{pq} the transmission from port q to port p .

The identification of wave variables with the waves propagating on a transmission line also explains why the wave variables are the primary quantities obtained from high-frequency measurements. For high-frequency fields, in fact, the size of the probing system becomes comparable to the wavelength and, from an experimental point of view, voltages and currents become ill defined quantities. The only quantities that remain well defined and measurable are the amplitudes of fields propagating in TEM waveguides of known geometry, namely the wave variables that are detected by *vector network analyzers* [1].

VI. POWER AND CURRENT WAVE VARIABLES

In addition to the voltage waves defined in Sec. III other definitions of the wave variables are possible. Since the wave variables are related to the waves on a transmission line, they are also related to the power flowing in the transmission line. The average power absorbed by a port during sinusoidal operation can be expressed for A and B as

$$\begin{aligned} P &= \operatorname{Re} \left\{ \frac{1}{2} V(j\omega) I^*(j\omega) \right\} \\ &= \operatorname{Re} \left\{ \frac{Y_r}{2} [|A|^2 - |B|^2 + 2(BA^* - B^*A)] \right\} \\ &= \frac{Y_r}{2} [|A|^2 - |B|^2] \end{aligned} \quad (21)$$

where Y_r is assumed real. In order to avoid the admittance factor in the expression of the average power, a different definition of the wave variables can be used

$$\begin{aligned} \check{A} &= \frac{1}{2\sqrt{Z_r}}(V + Z_r I) \\ \check{B} &= \frac{1}{2\sqrt{Z_r}}(V - Z_r I) \end{aligned} \quad (22)$$

that leads to

$$\begin{aligned} V &= \sqrt{Z_r}(\check{A} + \check{B}) \\ I &= (\check{A} - \check{B})/\sqrt{Z_r} \end{aligned} \quad (23)$$

The wave variables defined in this way are named *power waves* and are preferred in microwave applications. They allow to write the average power as

$$P = \frac{1}{2} [|\check{A}|^2 - |\check{B}|^2] \quad (24)$$

Finally, wave variables based on a current normalization are sometimes used. They are defined by

$$\begin{aligned} \bar{A} &= \frac{1}{2}(Y_r V + I) \\ \bar{B} &= \frac{1}{2}(Y_r V - I) \end{aligned} \quad (25)$$

The normalization used to define the wave variables, however, modifies the scattering functions only by a normalization factor, leaving their properties not affected. In fact, if K is a diagonal matrix collecting a set of arbitrary normalization coefficients, then the scattering matrix for the wave variables $\check{A} = KA$ and $\check{B} = KB$ is $S = KSK^{-1}$. In the rest of this paper, therefore, voltage waves are used.

VII. SCATTERING MATRIX OF TRANSMISSION LINES

Most applications of the scattering equations are in the area of high-frequency and microwave circuits, and involve n -port elements exhibiting propagation effects. This Section addresses the characterization of a segment of transmission line, that is the simplest example of distributed 2-port element.

A. Example #2: Scattering Matrix of an LC Line Segment for $Z_{r1} = Z_{r2} = Z_0$

The scattering matrix of an LC transmission line segment of length $\mathcal{L}$, characteristic impedance Z_0 and time delay $T = \mathcal{L}/v$ for $Z_{r1} = Z_{r2} = Z_r = Z_0$ is obtained from the responses of the network of Fig. 7. In every section of the transmission line, one can define the voltage waves associated to the current and voltage of the section. For $Z_r = Z_0$, such voltage waves coincide with the forward and backward voltages of the transmission lines, resulting in

$$\begin{aligned} A_1 &= V^+(0) \\ B_1 &= V^-(0) \\ A_2 &= V^-(\mathcal{L}) = e^{+sT} V^-(0) = e^{+sT} B_1 \\ B_2 &= V^+(\mathcal{L}) = e^{-sT} V^+(0) = e^{-sT} A_1 \end{aligned} \quad (26)$$

$$S = \begin{bmatrix} 0 & e^{-sT} \\ e^{-sT} & 0 \end{bmatrix} \quad (27)$$

Consistently, there is no reflection from the ports and the transmission function is an ideal delay. It is also worth noticing that the scattering functions are entire functions (no poles for finite s), as the network of Fig. 7 has no resonant behavior.

B. Example #3: Scattering Matrix of an LC Line Segment for $Z_{r1} = Z_{r2} = Z_r \neq Z_0$

The scattering matrix of the transmission line segment for arbitrary reference impedance can be easily computed by replacing the reference loads of Fig. 7 or by applying (19) to the scattering matrix (27). Here the latter way is demonstrated. This requires the following replacements in (19): the reference impedance for which S is known must be set to $Z_r = Z_0$; the reference impedance for which S' is desired must be set to $Z'_r = \bar{Z}_r$. Then $P = (\bar{Z}_r - Z_0)/(\bar{Z}_r + Z_0)$ and

$$S' = \begin{bmatrix} 1 & -Pe^{-sT} \\ -Pe^{-sT} & 1 \end{bmatrix}^{-1} \begin{bmatrix} -P & e^{-sT} \\ e^{-sT} & -P \end{bmatrix} \quad (28)$$

that leads to

$$\begin{aligned} S'_{11} = S'_{22} &= \frac{P(e^{-2sT} - 1)}{(1 - P^2 e^{-2sT})} \\ S'_{12} = S'_{21} &= \frac{(1 - P^2)e^{-sT}}{(1 - P^2 e^{-2sT})} \end{aligned} \quad (29)$$

In contrast to (27), the scattering functions of (29) do have poles, arising from the multiple reflection process that takes places between the ends of the transmission line segments, that are closed on $\bar{Z}_r \neq Z_0$. The poles of S' are located at

$$\begin{aligned} \sigma_k &= \frac{1}{2T} \log(|P|^2) \\ \omega_k &= k\pi/T, \quad \text{for } k = \dots -2, -1, 1, 2, \dots \end{aligned} \quad (30)$$

i.e., for a given line delay, they are at $(1/T) \log(|P|)$ from the imaginary axis, spaced of π/T . The larger is the mismatch of $\bar{Z}_r$ with respect to Z_0 , or the longer is the line segment, the closer are the poles to the imaginary axis. If $\bar{Z}_r$ were infinite or null, i.e., impedance or admittance equations were adopted, then the poles would be on the imaginary axis. As a consequence, the frequency responses associated to (29) feature periodic peaks of growing amplitude for growing $|P|$. This effect is demonstrated in Fig. 8, that shows the frequency responses of S'_{21} for different $\bar{Z}_r$ values. The simple transmission line segment, therefore, highlights one of the advantages of the scattering functions: for distributed structures that are prone to resonance, they can be much simpler and better behaved than impedance, admittance and transmission functions. Even if voltage and currents were measurable for high-frequency signals, the dynamic range of impedance and admittance functions would cause severe measurement problems.

C. Example #4: Scattering Matrix of an RLCG Line Segment for $Z_{r1} = Z_{r2} = Z_r = \sqrt{\ell/c}$

This example deals with the scattering functions of a lossy transmission line segment for $\bar{Z}_r$ coinciding with the characteristic impedance that the line would have if losses were removed. A possible application of this case is the measurement of the scattering responses of a low-loss 50 Ω TEM guiding structure

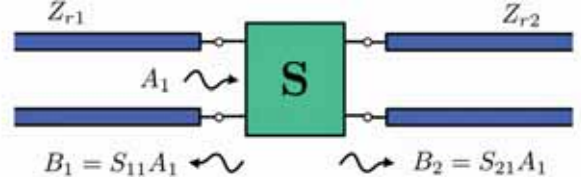


Fig. 6. Interpretation of the scattering functions of a two-port element as components of a scattering process. The ports of the element are connected to LC transmission lines with characteristic impedances Z_{r1} and Z_{r2} and the line on port #2 is matched.

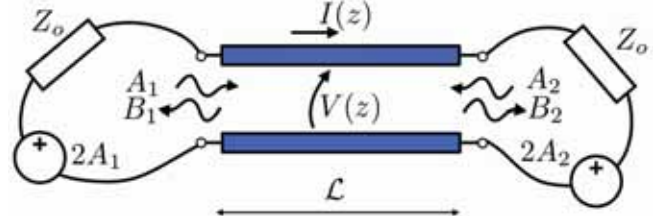


Fig. 7. Network for the calculation of the scattering matrix of an LC transmission line segment and the relevant variables.

by means of a network analyzer with 50 Ω reference impedance.

Let ℓ , c , $r(s)$ and $g(s)$ be the per-unit-length inductance, capacitance, resistance and conductance of the line, respectively and $T = \mathcal{L}\sqrt{\ell c}$ the time delay of the line segment. Again the scattering matrix is obtained by using (19) and the scattering matrix of the matched transmission line segment²

$$S = \begin{bmatrix} 0 & e^{-\gamma(s)\mathcal{L}} \\ e^{-\gamma(s)\mathcal{L}} & 0 \end{bmatrix} \quad (31)$$

where

$$\gamma\mathcal{L} = sT\sqrt{\left(1 + \frac{r}{s\ell}\right)\left(1 + \frac{g}{sc}\right)} \quad (32)$$

The resulting scattering functions are

$$\begin{aligned} S'_{11} = S'_{22} &= \frac{P(s)(e^{-2\gamma(s)\mathcal{L}} - 1)}{1 - P(s)^2 e^{-2\gamma(s)\mathcal{L}}} \\ S'_{12} = S'_{21} &= \frac{(1 - P(s)^2)e^{-\gamma(s)\mathcal{L}}}{1 - P(s)^2 e^{-2\gamma(s)\mathcal{L}}} \end{aligned} \quad (33)$$

where

$$\begin{aligned} P &= (1 - \zeta)/(1 + \zeta) \\ \zeta &= \sqrt{1 + \frac{r}{s\ell}} / \sqrt{1 + \frac{g}{sc}} \end{aligned} \quad (34)$$

The magnitudes of the scattering functions (33) are plotted versus frequency in Fig. 9 for a PCB trace. The geometry of the trace is assumed of stripline type and the following typical parameter values are used: $T = 1$ ns, $\sqrt{\ell/c} = 50$ Ω , trace width and thickness 150 μm and 35 μm , respectively. The per-unit-length conductance of the structure is neglected, whereas the per-unit-length resistance is described by means of the Holt's model

$$r(s) = r_{dc} + r_{dc}\sqrt{s/\pi f_s} \quad (35)$$

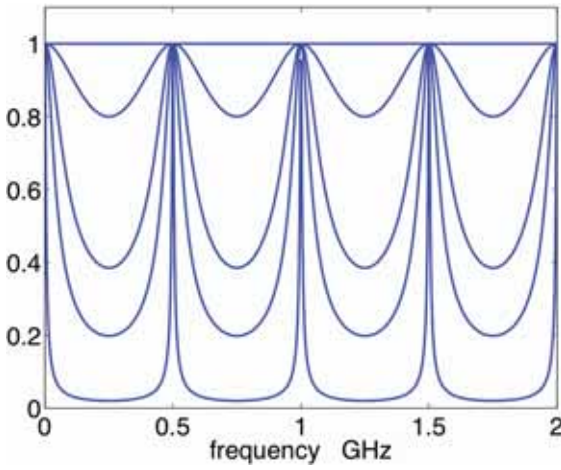


Fig. 8. Magnitude of S'_{21} versus frequency for an LC transmission line with $T = 1$ ns and $Z_0 = 50 \Omega$, and for $Z_r = 50, 100, 250, 500, 5000 \Omega$. The frequency responses is flat for $Z_r = 50 \Omega$ and increasingly resonant for growing Z_r .

where the estimated values of r_{dc} and f_s are $3.4 \Omega/\text{m}$ and 14.8MHz , respectively.

The frequency responses of Fig. 9 can be explained by observing that, for large s values, $S'_{11} \rightarrow 0$ and $S'_{21} \rightarrow \exp(-\gamma(s)\mathcal{L})$, namely S' tends to the scattering matrix (31) of the matched RLC line segment. Besides, the high-frequency decay of the transmission function is due to the attenuation introduced by skin losses, i.e., for $\omega \rightarrow \infty$, $\exp(-\gamma(j\omega)\mathcal{L}) \rightarrow \exp(-j\omega T) \exp(-R_s\sqrt{\omega})$, with $R_s = r_{dc}\mathcal{L}/2\sqrt{2\pi f_s}\sqrt{\ell/c}$. The mismatch introduced by losses has a weak effect on the behavior of the scattering functions, that remain well behaved over a wide bandwidth and are well suited for measurement purposes.

D. Time Domain Considerations

In signal integrity and EMC problems involving digital circuits, the time-domain analysis based on the transient characterization of n-port elements is particularly useful. For this application, the scattering equations have important advantages. The transient scattering equation of a n-port is defined by the inverse transform of (6)

$$\mathbf{b}(t) = \mathbf{s}(t) * \mathbf{a}(t) \quad (36)$$

where $\mathbf{s}$ is the inverse transform of $\mathbf{S}$ and $*$ denotes the convolution integral. The entries of $\mathbf{s}(t)$ are the scattering impulse responses, i.e., the responses of the reference circuit defining the scattering functions to ideal impulse sources.

Regardless of the way the convolution integral of (36) is computed³, the cost of computing the scattered wave variables grows with the complexity and duration of the scattering impulse responses. For distributed n-ports, scattering impulse responses defined by matched reference impedances are well behaved and short lasting, thereby leading to efficient and accurate time-domain characterizations. In contrast, the scattering impulse responses defined by strongly mismatched reference impedances (and therefore also the impulse responses of impedance, admittance and hybrid equations) are comb functions describing the multiple reflection process in the time domain, and are scarcely useful for the solution of transient problems.

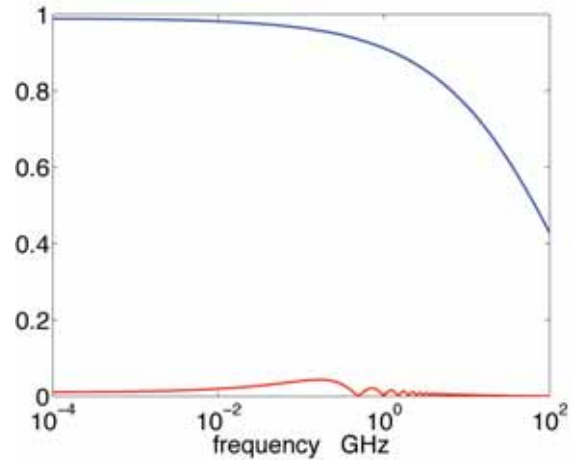


Fig. 9. Magnitude of S'_{11} (red line) and S'_{21} (blue line) versus frequency (log scale) for the RLC transmission line segment of Example #4.

The behavior of the scattering impulse responses defined by matched reference impedances can be appreciated from Example #2 and #4 above. In fact, in example #2, $s_{21}(t) = \delta(t - T)$, with $\delta(t)$ ideal pulse, is the ideal delay operator. Similarly, in Example #4, the transmission scattering function of the perfectly matched transmission line segment is

$$S_{21}(s) = e^{-\gamma(s)\mathcal{L}} = e^{-sT\sqrt{1+\frac{\mathcal{L}}{\mathcal{L}_0}}} = e^{-sT}H(s) \quad (37)$$

and $s_{21}(t) = h(t - T)$, where $h(t)$ is a single-sided pulse, taking into account the distortion and attenuation effects introduced by losses. The $h(t)$ function of the RLC case of Example #4 is shown in Fig. 10 by means of a logarithmic time scale in order to highlight the short-time distortion caused by the high-frequency losses of the resistance model (35). Finally, the comb structure of the impulse responses defined by mismatched reference impedances can be verified in Example #3.

VIII. Conclusion

This first part of the primer illustrates the features and properties of the scattering equations both from a circuit and a transmission line theory point of view. The crucial role of the reference impedances defining the scattering functions and the other network characteristic functions is pointed out and thoroughly discussed. The normalization of the wave variables, the time-domain scattering equations and examples of the scattering equation for a transmission line segment are also illustrated.

The advantages of the scattering equations stems from the nature of the variables their relate and from the role of their reference impedances. The wave variables are the only ones that can be measured up to microwave frequencies and the simplest variables for problems involving propagation effects. On the other hand, owing to the finite values of the reference impedances, the scattering equation are defined for a wide class of n-port elements. The reference impedances can be set by the user to obtain scattering functions simple and well behaved both in frequency and in time domain, facilitating the analysis and modeling of circuits and electromagnetic problems.

The second part of this primer will discuss the important relations between the passivity of an n-port and its scattering matrix. Besides, the use of scattering functions for the characterization of cascaded n-ports, the chain scattering parameters, and the development of macromodels from scattering data will be illustrated.

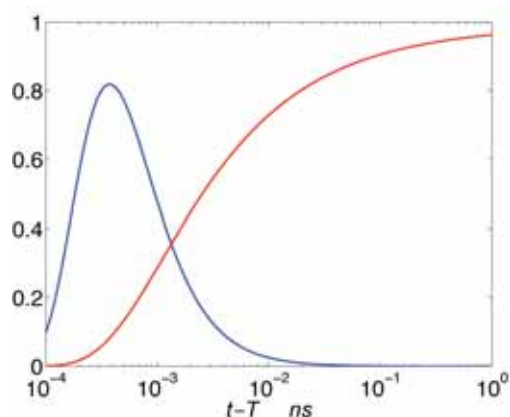


Fig. 10. Transmission impulse (blue line) and step (red line) responses of the RLC line segment of Example #4 for matched reference impedances. For representation purposes, the impulse response is scaled by a factor 5×10^{-11} s. A logarithmic time scale is used to highlight the short-time distortion produced by the high-frequency losses of the resistance model. The step response shows how the edge of and ideal step would be smoothed by the skin losses.

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¹The equivalence between electrical and wave variables holds if and only if Z_r is finite.

²In this case, the reference impedance that matches the line ends is $Z_0 = (\sqrt{\ell/c})\sqrt{1 + \frac{\tau}{sc}}/\sqrt{1 + \frac{q}{sc}}$ and is complex and frequency dependent. For complex reference impedances the energy meaning of the wave variables and the scattering matrix changes, yet their ability to characterize the n-port is maintained.

³The time-domain scattering equations are usually cast as a set of ordinary differential equations via real rational approximation of the scattering functions.

Biography



Ivan A. Maio received both the Laurea degree and the Ph.D. degree in electronic engineering from the Politecnico di Torino, Italy. Currently he is a Professor of Circuit Theory with the Department of Electronics of Politecnico di Torino. His research interests are in Nonlinear and Distributed Circuits and in Electromagnetic Compatibility. In particular, he works on modeling of lossy and non-uniform interconnects, on circuit equivalents via model order reduction, and on behavioral modeling of digital devices. He is the author of several papers in international journals and conference proceedings; reviewer for various international journals and conferences; Program Co-Chair of the 5th, 6th, 7th and 11th Workshop on Signal Propagation on Interconnects; and he served as Associate Editor of IEEE Transactions on Electromagnetic Compatibility for the PCB technical area.



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