ASYMPTOTIC PRESERVING IMPLICIT MAXWELL SOLVERS

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n this work we present an extension of the Boundary Integral Treecode (BIT), a grid free electrostatic *O(N log N)* field solver, to an implicit electromagnetic field solver. We start by formulating Maxwell's equations in vector potential form and make use of the Lorenz gage to arrive at,

$$\frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2} - \Delta\psi = \frac{\rho}{\varepsilon}, \qquad \frac{1}{c^2}\frac{\partial^2 A}{\partial t^2} - \Delta A = \mu J\,.$$

Focusing on the second equation, a method of lines transpose methodology is applied to the system, i.e., we discretize the time operator and construct an integral solution for the resulting modified Helmholtz equation,

$$\Delta A^{n+1} - \frac{1}{c^2\Delta t^2}A^{n+1} = -\left(\frac{2A^n - A^{n-1}}{c^2\Delta t^2}\right) - \mu J.$$

Using the free space Greens function, we arrive at,

$$A^{n+1} = \int_\Omega \left(\frac{2A^n - A^{n-1}}{c^2\Delta t^2} - \mu J\right) G(x \mid y)\, d\Omega$$
$$+ \oint_{\partial\Omega} \left(A^{n+1}\nabla G(x \mid y) - G(x \mid y)\nabla A^{n+1}\right)\cdot n\, dS$$

As in BIT, the volumetric integral is the particular solution, the boundary integral term is used to construct a correction to the particular solution. After discretizing the volumetric integral, the resulting sum over the volume is efficiently evaluated using fast summation. The new method computes a magnetic field that is by construction divergence free in the computational domain. Further, the implicit Maxwell solver is Asymptotic Preserving, i.e., it recovers the Darwin approximation of Maxwell's equations in the long time limit. As a proof of concept, we apply the new method to simulate the wave equation. A potential criticism for the vector potential form of Maxwell under the Lorenz gauge, is that the method is not charge conserving. To explore this issue, we compute solutions to Maxwell's equations for specified J and ρ, and leverage these test problems to numerically explore the issue of charge conservation, using the Lorenz gauge as a residual. Further, we explore using the residual to construct an iterative algorithm, based on defect correction, in which each update improves change conservation. Time permitting, we will explore some basic particle based EM solutions for the Vlasov-Maxwell system.

1. A. Christlieb, R. Krasny, J. Verboncoeur, J. Emhoff and I. Boyd, "Grid-Free Plasma Simulation techniques", IEEE Trans on Plasma Sci. (34), No. 2, pp 149-165.
2. A. Christlieb, R. Krasny and J. Verboncoeur, "Efficient Particle Simulation of a Virtual Cathode using a Grid-Free Treecode Poisson Solver", IEEE Trans on Plasma Sci. (32), No. 2, pp 384-389.

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