ENTROPIC LATTICE BOLTZMANN MHD ALGORITHMS USING SCALAR DISTRIBUTION FUNCTIONS

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Lattice Boltzmann (LB) representations are mesoscopic algorithms that exploit a simple collide-stream lattice scheme [1] to recover the dissipative incompressible MHD equations. With the linear collisional relaxation being purely local and the streaming requiring communication only between neighboring lattice sites, LB algorithms have shown ideal parallelization to all cores available.

These algorithms are based on a scalar distribution function for the density/velocity field and a vector distribution function for the magnetic field. Another excellent feature of these LB-MHD algorithms is that div **B** = 0 can be enforced to machine accuracy since the magnetic field is recovered as the zeroth moment of the vector distribution function. The asymmetric magnetic stress term (in **u – B** interchange) is then determined from the 1st moment. From this it can be shown that div **B** is determined by the trace of an antisymmetric tensor [1,2].

However, the standard vector-scalar distribution approach is prone to numerical instabilities for high Reynolds and magnetic Reynolds numbers. For Navier-Stokes turbulence, entropic LB algorithms have been developed in which the detailed balance arguments permits a stable algorithm for arbitrary high Reynolds number. The use of a vector distribution function precludes the introduction of a discrete H-theorem. Here, we investigate the use of a scalar distribution to model the magnetic field. While a discrete Htheorem can now be enforced with the positive-definiteness of both distribution functions, the asymmetric magnetic stress term is recovered by the introduction of appropriate forcing terms. 2D MHD turbulence is first investigated. Another advantage of the scalar representation is the reduction in computational memory requirements as well as simpler implementation of boundary conditions.

The Orszag-Tang vortex will be examined as well as some LES closure schemes using Elsasser variables.

[1] P. J. Dellar, J. Computat. Phys.**179**, 95 (2002)
[2] G. Vahala, M. Soe, J. Yepez, L. Vahala, J. Carter and S. Ziegeler, Comm. In Comput. Phys. **4**, 624 (2008)

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