Motion Planning for Robots with Active Mechanical Compensation

Gustavo Arechavaleta, Felipe A. Machorro-Fernández and Vicente Parra-Vega

***Abstract*— Our interest is in steering robotic systems with active mechanical compensation from a given configuration to another. In particular, we consider a class of over-actuated robots with a controlled sliding mass at each link to compensate gravity effects. This active mechanical characteristic defines an equilibrium manifold ($\mathcal{EM}$) of the physical system. Although the configuration of the sliding masses induces a dynamic behavior, it does not alter the kinematic structure of the robot. It turns out that paths on $\mathcal{EM}$ require less control effort than paths on the configuration space ($\mathcal{CS}$). We propose to use sampling based motion planning (SMBP) strategies to capture the structure of $\mathcal{EM}$ that guarantee quasi-static equilibrium motions.**

I. INTRODUCTION

a) Motivation: Gravity compensation is desired in several robotic applications. For instance, in haptic schemes, physical human-robot interaction, rehabilitation interfaces and any application where apparent zero inertia is important. In this work we show that actuated sliding masses serve to compensate for gravity. We argue that: *a)* over-actuated robotic manipulators composed by revolute joints and sliding masses require less control effort to perform movements than classic manipulators; *b)* SBMP can be naturally applied for computing paths over $\mathcal{EM}$.

b) The main principle: The underlying idea comes from the Euler-Lagrange formalism where the equilibrium point, that represents a robot posture, nullifies the gradient of the potential energy (i.e. the gravity vector). Hence, it should be possible to induce a *modified* equilibrium point at some configurations of the robot such that the gravity effects become null.

c) Sketch of our strategy: We have introduced sliding masses as prismatic DoF *with their own control inputs* to modify the center of mass of each link (see Figure 1). Note that these new coordinates do not alter the kinematic structure of the robot but they induce some dynamic properties that must be considered to achieve static equilibrium at each joint [3] (see Sec. II). Since the gravity vector depends on joint and sliding coordinates, it is possible to define a set of algebraic constraints on the *extended* configuration space. Thus, the unconstrained subspace represents the so-called $\mathcal{EM}$. Because the allowable range of motion for each sliding mass is constrained by its link length, a set of disjoint equilibrium varieties may appear. Moreover, if obstacles come to play then $\mathcal{EM}$ may not be connected.

We propose to apply SBMP for capturing the topology of $\mathcal{EM}$ in a discrete data structure (see Sec. III). If two samples belong to the same equilibrium variety then a static equilibrium path exists to connect them. This means that along the path it is only necessary to activate the controls of sliding masses for executing the motion. However, if two samples belong to different components of $\mathcal{EM}$, then it is required to activate at least one of the joint controls together with the controls of sliding masses. The curve that connects these samples is called quasi-static equilibrium path.

Authors are with Robotics and Advanced Manufacturing Group, Centro de Investigación y de Estudios Avanzados del IPN, Saltillo, Coah. México. {garechav,vicente.parra}@cinvestav.edu.mx

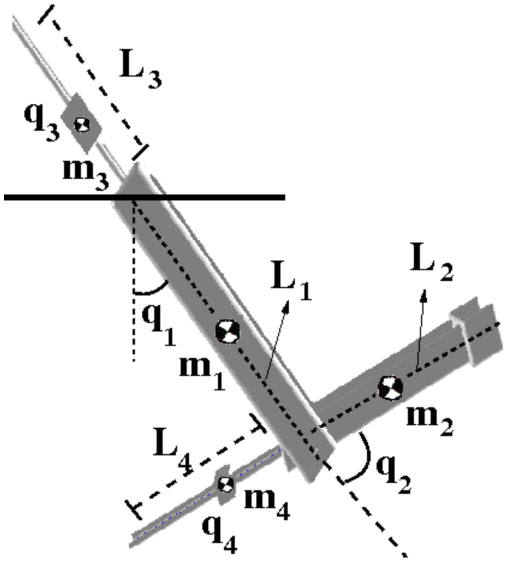

Fig. 1. Kinematics of a planar robotic arm with active mechanical compensation. In this case, $(q_1, q_2) \in \mathbb{S}^1 \times \mathbb{S}^1$ are the joint coordinates and $(q_3, q_4) \in \mathbb{R}^2$ represent the sliding coordinates.

II. MODELING

According to the Euler-Lagrange formalism, the class of rigid and fully revolute actuated robots satisfy

$$H(q)\ddot{q} + h(q, \dot{q}) + g(q) = \tau \quad (1)$$

where $q \in \mathcal{CS}$ defines the configuration of the kinematic structure and $n = \dim(\mathcal{CS})$ represents the dimension of its configuration space. The inertia matrix in $\mathbb{R}^{n\times n}$ is given by $H(q)$, $h(q, \dot{q}) = C(q, \dot{q})\dot{q} + B\dot{q}$ where Coriolis and Damping square matrices in $\mathbb{R}^{n\times n}$ are given by $C(q, \dot{q})$ and B matrices, respectively. The gravity $g(q)$ and generalized torques τ are in $\mathbb{R}^n$.

Let us define the *extended* coordinates as

$$q = \begin{pmatrix} q_j \\ q_s \end{pmatrix} \in \mathcal{CS} \times \mathbb{R}^m \quad (2)$$

where joint coordinates $q_j \in \mathcal{CS}$ define the postures of the kinematic chain. The sliding coordinates $q_s \in \mathbb{R}^m$ does not affect the kinematic chain, but they induce a dynamic behavior. The equations of motion of the *extended* system is represented by (1). It is possible to derive the following coupled equations of motion

$$H_j(q)\ddot{q}_j + T(q)\ddot{q}_s + h_j(q,\dot{q}) + g_j(q) = \tau_j \quad (3)$$
$$T(q)^T\ddot{q}_j + H_s(q)\ddot{q}_s + h_s(q,\dot{q}) + g_s(q) = \tau_s \quad (4)$$

where $T(q)$ is the triangular term of $H(q)$. Note that (3) stands for the dynamic contribution of the kinematic chain while (4) stands for the contribution coming from the sliding masses. Suppose that we design the control input $\tau = u + g(q)$ such that $u(t)$ assures no gravitational loads in the reachable space of singularity-free configurations. Then, this space is defined as

$$\mathcal{EM} = \{q = q_o | g(q_o) = 0\} \quad (5)$$

where the configuration q_o of the robot has no gravity effect.

III. Motion planning for gravity-free robots

The motion planning problem is formulated in terms of the *extended* coordinates in $\mathcal{CS} \times \mathbb{R}^m$. To satisfy gravity-free configurations, $g_j(q_j, q_s) = 0$, the following algebraic constraints are defined over $\mathcal{CS} \times \mathbb{R}^m$:

$$\begin{cases} q_{s_m} - f(q_{j_n}) = 0 \\ q_{s_{m-1}} - f(q_{j_{n-1}}, q_{j_n}) = 0 \\ \vdots \\ q_{s_1} - f(q_{j_i}, \ldots, q_{j_n}) = 0 \end{cases} \quad (6)$$

where q_{j_i} represents the first joint coordinate affected by $g(q)$ and $f(\cdot)$ gives the configuration of the corresponding sliding coordinate q_{s_k} where $g_j(q) = 0$ for q_{j_l}. Note that by manipulating the Schur complements of $H(q)$ the same constraints satisfy $g_j(q_j, q_s) = 0$ where $g_j(q_j, q_s) = g_j(q) - T(q)H_s(q)^{-1}g_s(q)$ (see [3]). The resulting $(n+m)-m$ unconstrained subspace of $\mathcal{CS} \times \mathbb{R}^m$ represents $\mathcal{EM}$.

We propose to use PRM-based methods to construct a discrete representation of $\mathcal{EM}$ (either compact or dense). The outcome is a *roadmap* where its nodes are collision-free equilibrium configurations connected by static and quasi-static equilibrium paths. Also, this can be achieved with RRT-based methods that are rather used for single-query [2].

Note that there exist in the literature specialized motion planners to cope with the equilibrium constraint of multi-limbed robots (see [1]). In this case, two types of constraints arise: 1) the set of contacts between the limbs and the operational surface; 2) the projection of the robot's center of mass remains within the support polygon which is defined on the operational space. In our case, however, there are not contact constraints and the static equilibrium is defined on the *extended* configuration space.

We modify the routine used in SBMP algorithms to discriminate invalid samples: feasible configurations are collision-free and satisfy (6). We also modify the local method to connect pairs of samples. In particular, we relax the gravity-free condition to connect disjoint equilibrium varieties by quasi-equilibrium paths. This means that at least one joint control must be activated in order to satisfy (6), i.e. more energy is required to perform the motion. Hence, the planner can automatically identify critical configurations over these equilibrium varieties. When pairs of samples are going to be connected a reward is assigned to static equilibrium paths while a penalty is associated to quasi-static equilibrium paths. This can be directly linked to an energy consumption criterion: $E = \int_0^1 (\dot{q}^T\tau)dt$.

IV. Case study

Consider a 2 DoF robot where its configurations q_j belong to $\mathbb{T}^2$ and 2 sliding coordinates q_s are in $\mathbb{R}^2$ (see Fig. 1). Thus, the *extended* configuration space is $\mathbb{T}^2 \times \mathbb{R}^2$. We define 2 constraints in terms of the link and sliding masses, link lengths and q to determine $\mathcal{EM}$:

$$\begin{aligned} q_{s_2} &= -m_2(L_2\cos(q_{j_1}+q_{j_2}))/(m_4\cos(q_{j_1}+q_{j_2})) \\ q_{s_1} &= -(1/m_3\cos(q_{j_1}))(m_1L_1\cos(q_{j_1}) + \\ &\quad m_2L_2\cos(q_{j_1}+q_{j_2}) + m_2L_1\cos(q_{j_1}) + \\ &\quad m_4q_{s_2}\cos(q_{j_1}+q_{j_2}) + m_4L_1\cos(q_{j_1})) \end{aligned} \quad (7)$$

The shape of $\mathcal{EM}$ is illustrated in Fig. 2 when $m_1 = 1.5, m_2 = 0.5, m_3 = 2.5, m_4 = 0.6$ and $L_1 = L_3 = 3, L_2 = L_4 = 2$. Gray regions correspond to configurations where $|q_{s_1}| > L_3$ (i.e. the first link is almost aligned to gravity).

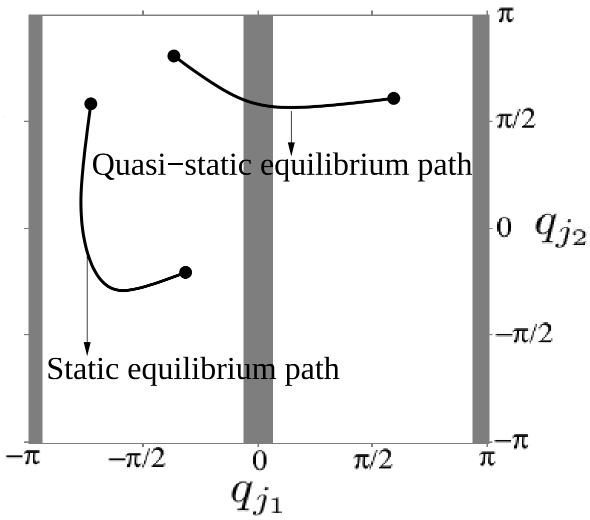

Fig. 2. The non-shaded regions correspond to $\mathcal{EM}$. Note that $q_1 = q_{j_1}, q_2 = q_{j_2}, q_3 = q_{s_1}$ and $q_4 = q_{s_2}$.

References

[1] T. Bretl. Motion Planning of Multi-Limbed Robots Subject to Equilibrium Constraints: The Free-Climbing Robot Problem. *The International Journal of Robotics Research*, vol. 25, no.4, pp. 317-342, April, 2006.

[2] S.M. LaValle. Motion Planning. *IEEE Robotics Automation Magazine*, vol. 18, no.1, pp. 79-89, March, 2011.

[3] F.A. Machorro-Fernández, V. Parra-Vega and E. Olguín-Díaz. Active Mechanical Compensation to Obtain Gravity-Free Robots: Modeling, Control, Design and Preliminary Experimental Results. *IEEE Int. Conf. on Robotics and Automation*, pp. 1061-1066, Kobe, Japan, May, 2009.