

An Improved Algorithm for Computing Roadmaps of Algebraic Sets

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We consider the problem of computing a roadmap of any given a real algebraic variety in R^k defined by a polynomial of degree bounded by d . Single exponential algorithm for this problem has been known for a while starting from the work of Canny, Grigoriev and Vorobjov, Gournay and Risler, Heintz, Roy and Solerno, and Basu, Pollack and Roy. The best complexity bound for a (deterministic) algorithm for computing such a roadmap was due to Basu, Pollack and Roy (JAMS 1999), and this algorithm had a complexity bound of $d^{O(k^2)}$. The $O(k^2)$ term in the exponent seemed essential because of a certain paradigm of making recursive calls, which was common to all known single exponential algorithms till date. As such it had remained a challenging problem for over a decade to try to improve this exponent. Based on some prior work by Mohab Safey El Din and Eric Schost, together with the semi-algebraic techniques developed in the work of Basu, Pollack and Roy, we have very recently succeeded in improving this exponent.

More precisely, we prove the following. Let R be a real closed field and $(D \subset R)$ a subring. We give an algorithm that takes as input a polynomial $(Q \subset D[X_1, \dots, X_k])$, and computes a description of a roadmap of the set of zeros, $Z(Q, R^k)$, of Q in R^k . The complexity of the algorithm measured by the number of arithmetic operations in the ring D , is bounded by $d^{O(k\sqrt{k})}$ where $d = \deg(Q) \geq 2$. In this talk I will try to explain the main ideas underlying the above result, and explain its importance.

Note: Joint work with Marie-Francoise Roy, Mohab Safey El Din, and Eric Schost.