Optimal Motion Planning for Robust Sensorless Part Orientation

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***Abstract*—Part orientation is a necessary first step for many machining and manipulation tasks. We investigate a sensorless method for steering a sphere in its 5D state space despite model perturbations that scale the sphere diameter by an unknown but bounded constant. The controllers we investigate consist of motion paths. We demonstrate solutions to this problem under two actuators, then show how this method can be extended to control schemes requiring no actuators.**

I. Introduction

In this paper we consider the problem of steering a sphere to a desired configuration despite model perturbations that scale the sphere diameter by an unknown but bounded constant. We focus on the sphere because it is a ubiquitous elementary component in manufacturing. Additionally, sphere manipulation through rolling is a canonical example of a non-flat nonholonomic mechanism.

A. One Sphere

Consider the sphere on a plane that rolls without slipping. Such a sphere has an x, y location and an orientation in $SO(3)$. We describe its configuration by $g = (x, y, R)$ and its configuration space $\mathcal{G} = \mathbb{R}^2 \times SO(3)$. The differential system is

$$\frac{d\,x(t)}{dt} = u, \quad \frac{d\,y(t)}{dt} = v,$$
$$\frac{d\,R(t)}{dt} = R(t) \begin{bmatrix} 0 & 0 & -u/r \\ 0 & 0 & v/r \\ u/r & -v/r & 0 \end{bmatrix} \tag{1}$$

Here $R(t)$ is the rotation matrix in $SO(3)$, u, v are the control functions and r is the sphere radius. The problem is to minimize $\frac{1}{2}\int_0^T \left(u^2 + v^2\right) dt$ over all possible solution curves of (1) satisfying the boundary constraints $g_{start} = (x(0), y(0), R(0))$ and $g_{goal} = (x(T), y(T), R(T))$, $g_{start}, g_{goal} \in \mathcal{G}$. This formulation uses the velocity of the center of the ball, $[u_1, u_2]$ as control inputs. Depending on the nature of the problem, the inputs might be subject to minimum turning radius and the constraint $g(t) \in \mathcal{G}_{free}$ to consider collision avoidance.

B. Ensemble Control of Spheres

We will solve this motion planning problem, but under a model perturbation that scales the sphere diameter by some unknown, bounded constant, i.e. $r_{actual} = r\epsilon$, $\epsilon \in [1-\delta, 1+\delta]$. However, rather than try to steer a single sphere governed by the perturbed kinematic model, our approach is to steer an uncountably infinite collection of spheres parameterized by ϵ, each governed by the exact kinematic model

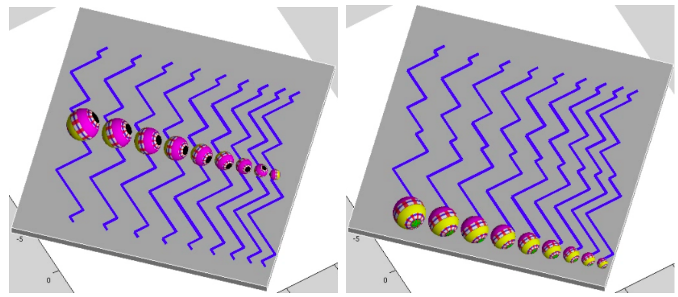

Fig. 1. Nine spheres, with radius $[0.5, \ldots, 1.5]$ roll down identical grooves, shown in blue. All spheres finish with a net rotation of $\approx \pi$ about their y-axis.

$$\frac{d\,x(t,\epsilon)}{dt} = u, \quad \frac{d\,y(t,\epsilon)}{dt} = v,$$
$$\frac{d\,R(t,\epsilon)}{dt} = R(t,\epsilon) \begin{bmatrix} 0 & 0 & -u/(r\epsilon) \\ 0 & 0 & v/(r\epsilon) \\ u/(r\epsilon) & -v/(r\epsilon) & 0 \end{bmatrix} \tag{2}$$

Following the terminology introduced by recent work in control theory [1], [11]–[15], we call this fictitious collection of spheres an *ensemble* and call the model (2) an *ensemble control system*. The idea is that if we can find open-loop inputs $u(t)$ and $v(t)$ that result in $g(0, \epsilon) = g_{start}$ and $\|g(T, \epsilon) - g_{goal}\| \leq \mu$ for all $\epsilon \in [1-\delta, 1+\delta]$, then we can certainly guarantee that the actual sphere, which corresponds to one particular value of ϵ, will be steered from the start to the goal.

One optimization problem then is to minimize

$$\frac{1}{2}\int_0^T \left(u^2 + v^2\right) dt \tag{3}$$

over all possible solution curves of (2) under the constraint that $\|g(T, \epsilon) - g_{goal}\| \leq \mu$, $\forall \epsilon \in [1-\delta, 1+\delta]$ for some $\mu > 0$.

If we want the sphere rolling to be driven by gravity, it would be convenient to constrain $[u, v] \in \mathbb{R}^+$.

A solution to (3) is not included in this note. A proof of controllability and an algorithm for steering an ensemble toward a goal orientation as a function of ϵ in $SO(3)$, $\mathbf{R}(\epsilon)_{goal}$ is given in section III-A of [16]. Interestingly, this algorithm is in-place such that $\Delta x = \Delta y = 0$. Therefore, to get to a goal $[x, y, \mathbf{R}_{goal}] \in \mathbb{R}^2 \times SO(3)$ requires only first rolling about the world y-axis to the desired x position, followed by a roll about the world x-axis to the desired y position. These two movements generate some rotation $\mathbf{R}(\epsilon)_{x,y}$. Since the algorithm in III-A of [16] is in-place, we can apply it to generate the rotation $(\mathbf{R}(\epsilon)_{x,y})^{-1}\,\mathbf{R}(\epsilon)_{goal}$ and we have the full solution. Note that a different solution based on Fourier coefficients is given by Pryor in [21], [22]. Neither of these feasible solutions is proven to be optimal.

II. Related Work

A. part orientation

We are motivated by progress in sensorless part manipulation, particularly the work of [8] and [4] showing that simple actuators are often sufficient to robustly orient a wide array of planar objects without using sensors. These works employed a tray that could be tilted in two axis [8] and parallel-jaw grippers [4] . These methods exploit differences in part geometry. Robustly orienting the rounded surface of a sphere offers special challenges due to its inherent symmetry.

B. sphere manipulation

Manipulation of spherical objects by rolling has been investigated in depth by members of the math, control, and robotic manipulation community.

This research can be traced to Brocket and Dai who analyzed an approximation of the problem and determined the optimal controller for this approximation [3]. Jurdjevic determined the optimal shortest length paths, showing that the optimal solution curve minimizes the integral of the geodesic curvature and that these curves are solutions to Euler's elastica problem [10]. Li provided a symbolic algorithm for steering the system [17], Marigo gave a numeric algorithm [18], and Oriolo and Vendittelli presented an iterative approach for stabilizing the ball-plate system [20]. Robotic ball-plate systems solutions have been implemented [2], [18]. Choudhury and Lynch showed that a single degree of freedom manipulator was sufficient for orienting the sphere, and designed a successful experiment consisting of an elliptical bowl mounted on top of a linear motor with the bowl primary axis oriented 45 degrees from the linear motor orientation [6]. This problem has produced several practical stabilizing controllers [5], [7], [19].

Svinin and Hosoe extended the problem for ball-plate systems with limited contact area [24], [25]. This enables manipulations of objects with spherical portions.

C. ensemble control

We are motivated by the work on *ensemble control* in [11]–[15]. These works studied the controllability properties of the *Bloch equations*, a unit vector in $\mathbb{R}^3$. The sphere, which moves in $\mathbb{R}\times SO(3)$ adds both position and the full rotation matrix to the problem.

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