Medial Zones in Motion Planing Applications

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***Abstract*— The popularity of medial axis in shape modeling and analysis comes from several of its well known fundamental properties. For example, medial axis captures the connectivity of the domain, has a lower dimension than the space itself, and is closely related to the distance function constructed over the same domain.**

We formally define the new concept of a *medial zone* of an *n*-dimensional semi-analytic domain Ω that subsumes the medial axis $\mathcal{MA}(\Omega)$ of the same domain as a special case, and can be thought of as a 'thick' skeleton having the same dimension as that of Ω. We show that the medial zone $\mathcal{MZ}(\Omega)$ of Ω converges to either $\mathcal{MA}(\Omega)$ or Ω itself, and is homeomorphic to the domain. Our formulation of medial zones, and hence of medial axes of semi-analytic domains reveals their attractive theoretical and computational properties, including intriguing computational paradigms for 2- or 3-dimensional semi-analytic sets with rigid or evolving boundaries. Due to the fact that the medial zones fuse some of the critical geometric and topological properties of both the domain itself and of its medial axis, re-formulating problems in terms of medial zones affords the 'best of both worlds' in applications such as robotic and autonomous navigation, and design automation.

I. MOTIVATION

The *medial axis* introduced by Blum [1] for biological shape measurement and description has become an important tool in geometric modeling and computing due to its compact representation of the essential topologic and geometric properties of a shape. For example, an important result described by Lieutier [4] states that a shape and its medial axis are connected the same way regardless of the dimension of the space (i.e., they are homotopy equivalent), which is exploited in many applications, including those that focus on shape similarity.

Medial axes have gained popularity in the context of motion planning due to their 'equidistance' to the boundary of the domain. This property of medial axes becomes particularly important in motion planning when navigating along narrow passages, a situation in which other practical algorithms, including probabilistic roadmap (PRM) planners, do not fare well. Since the medial axis contains, by definition, points that are equally spaced to the boundary of the domain, planning a path along the medial axis becomes a natural choice for domains with narrow passages. On the other hand, medial-axis based planners tend to produce non-optimal paths in the regions that are away from the narrow passages, because the medial axis continues to remain in the 'middle' of the domain as shown in Figure 1(a).

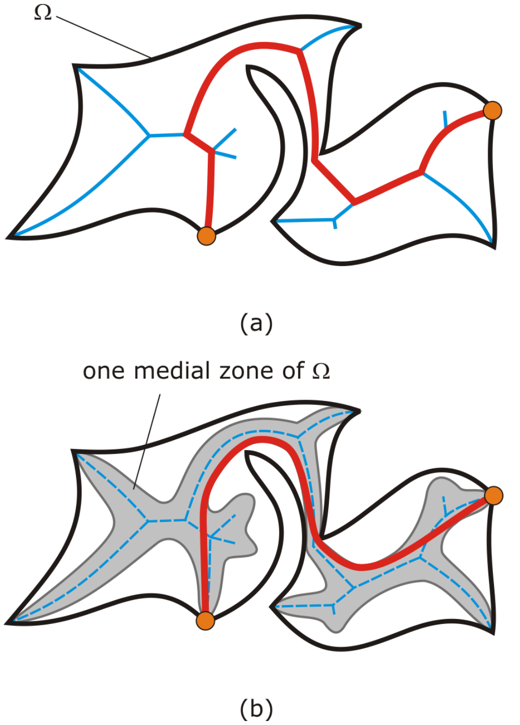

Fig. 1. Path planning along medial axis (a) and medial zone (b) of the same domain. The path in figure (b) is 13.34% shorter and has a visibly more uniform curvature than the path shown in (a). The example shows a planar domain only for illustration purposes.

We formally define the new concept of a *medial zone* of an *n*-dimensional semi-analytic domain Ω that subsumes the medial axis $\mathcal{MA}(\Omega)$ of the same domain as a special case, and can be thought of as a 'thick' skeleton having the same dimension as that of Ω. We show that the medial zone $\mathcal{MZ}(\Omega)$ converges to either $\mathcal{MA}(\Omega)$ or Ω itself, is homeomorphic to the domain, and has attractive theoretical and computational properties.

The formulation of medial zones [3] builds on our earlier work on computing medial axes of planar non-rigid domains [2] . We show that, by constructing the exact distance functions with R-functions that operate on the real valued halfspaces bounding the domain as logic operators, we infer the information needed to localize

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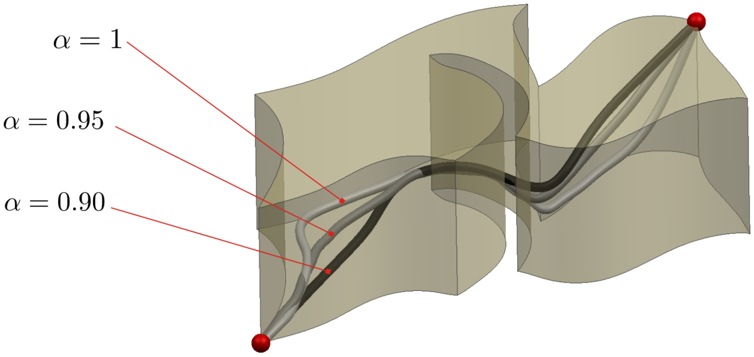

Fig. 2. Path planning in 3D with medial zones.

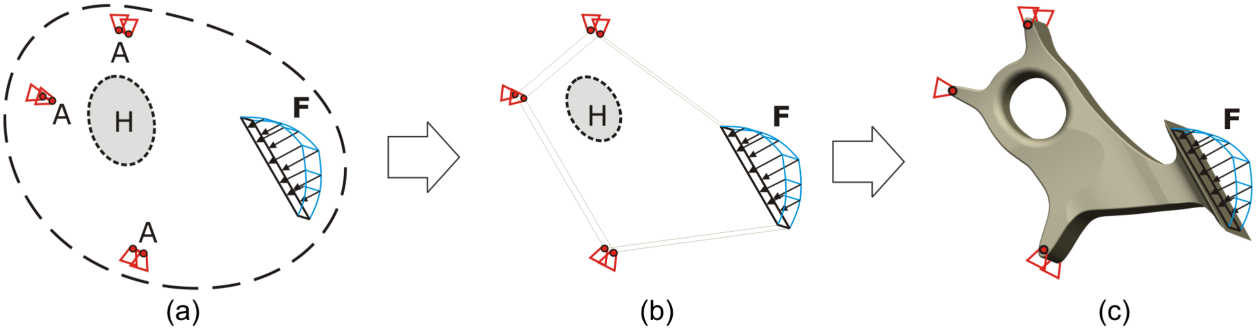

Fig. 3. Automating shape synthesis: the medial zone provides a topologically correct initial guess whose size can be adjusted by the degrees of freedom provided by the medial zones. The initial guess shown in (c) is a subset of the medial zone.

the computation of medial zones and medial axes even as the geometry and topology of the domain evolve.

R-functions have been invented in the 60's by V.L. Rvachev [5], [6], who called these functions "logically charged functions". They provide the means to construct a C^n function over a domain defined by primitive halfspaces. The main contribution of the theory of R-functions to the topic of this work is to replace these logical operations by real-valued functions, which generates an implicit representation for any semi-analytic set Ω.

The 3D paths planned along the medial axis and inside two medial zones are illustrated in Figure 2. We note that the medial axis contains sharp features (vertices and edges), which should be reflected in the path planned along the medial axis. However, in order to improve the clarity of the illustration, we offset all three paths and show them in Figure 2 as generalized cylinders. In turn, this visual artifact is hiding the sharp features that would otherwise be seen in path planned along the medial axis in Figure 2(d).

In practical terms, we demonstrate that the use of medial zones in path planning applications retains the properties of medial axes in the neighborhood of narrow passages, while resulting in globally shorter paths. Furthermore, given the topological guarantees of the medial zones, they can provide a powerful new paradigm to automate the shape synthesis of mechanical artifacts as illustrated in Figure 3.

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