ARCHI: A redundant mechanism for machining with unlimited rotation capacities

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***Abstract** - This paper presents a 3 dof redundant parallel mechanism, ARCHI, dedicated to machining, as a sub-part of a 5-axis hybrid machine. We describe the redundant parallel mechanism design, its models, and discuss ways for its control.*

1 Introduction

The idea of parallel robots was first developed by Gough [1] and Stewart [2] in the 50's and 60's with the idea of "hexapods" (6 dofs) [3][4]. The 'revolution' introduced by Clavel and his Delta structure [5] in the late 80's, opened a new aera with machines able to reach extremely high accelerations and which are commonly used for pick-and-place; recently, we have used the Delta principle for machining applications and participated to the creation of UraneSx [6] (a 3-axis drilling-taping-boring machine-tool with linear drives, and a 3.5~5.0 g's acceleration capability). Considering the machining of complex shape objects, the solution often proposed by parallel mechanisms is dramatically different from the solution in use in industry; on one hand, most industrial machines (Figure 1) are based on a 5-dof serial chain whose weak point is the so-called 'head', *i.e.* the last two rotating joints (the key issues are the lack of stiffness, the limited speed, and the difficulty to create compact designs); on the other hand, most parallel-mechanism-based machines rely on 6-dof arrangement. Different practical designs have proven the efficiency of parallel mechanisms in this type of applications in terms of speed, accuracy and stiffness (see [7] and HexaM machine tool [8] proposed by Toyoda for example). Approaches using less actuators (that is, five actuators, for five machining axis) have been proposed recently to reduce cost and complexity (Figure 2), but they usually suffer from the same limitation that fully parallel chains: a limited tilting angle.
Hybrid serial / parallel or parallel / serial structures (Neos Tricept robot , Seoul University Eclipse Machine [9][10], DS Technologies Sprint) have to be mentioned as solutions for 5-axis machining: on one hand, the parallel / serial arrangement of Tricept offers a good dynamic behavior thanks to its parallel sub-part, but it still suffers from the 'natural' limitations of a serial 'head'; on the other hand, Sprint architecture guarantees a good behavior of the 'head' but is nevertheless limited in terms of tilting angle. We recently began to study alternate solutions based on the principle of Motion-Sharing where both the tool and the work-piece are moved, and where at least a part of the motion is due to parallel chains (P: Prismatic joint, U: Universal, S: Spherical, R: Revolute).

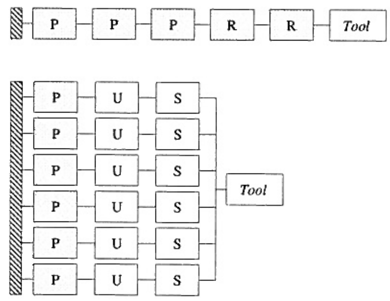

***Figure 1.** Classical (serial) 5-axis machine vs fully-parallel 6-dof machine (here, HexaM principle, Toyoda)*

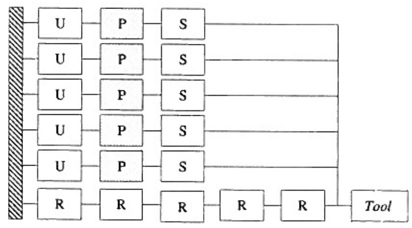

***Figure 2.** 5-axis, 5-dof with passive chain*

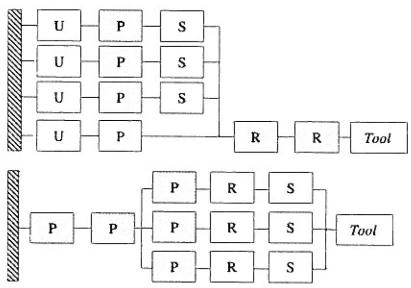

***Figure 3.** Hybrid Parallel-Serial (Tricept, Neos) or Serial-Parallel (Sprint, DS Technologies)*

We designed a new parallel redundant structure, ARCHI, as a part of a 5-axis equipment for machining. We describe

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here its basic design, kinematics and dynamics models, and discuss control strategies.

2 Motion-Sharing[2]: ARCHI basic design

We have already introduced one possible design, based on the so-called H4 structures ([11], [12], [13]), where 4 dof are dedicated to the tool motion, and 1 dof is at work-piece level. With the ARCHI project, we extend the idea of motion-sharing between tool and work-piece a step further, and we propose to give the tool a complete planar motion (2 translations, 1 rotation), and let the work-piece be moved along the remaining translation axis and about the remaining rotation axis.

Keeping in mind the needs for high stiffness in machining, we developed first ARCHI concept with linear drives and thus we focussed our efforts on planar parallel mechanisms driven by linear motors. Planar parallel mechanisms have been intensively studied (see [14]) and it turns out it is not possible to obtain a really large range of motion in rotation with classical arrangements. This is obviously because of the existence of singular positions, and among them, more specifically over-mobility singular positions where the machine stiffness is zero.

The machine we developed is designed to allow unlimited rotation capability thanks to a redundant parallel kinematic mechanism. Figure 4 is the ARCHI 'arrangement graph': four linear drives are fixed on the base, and then linked to the nacelle (carrying the spindle) thanks to *R-R* or *U-S* chains[3]; on the work-piece side, two joints are arranged in a serial way. Figure 5 shows a schematic view of such a mechanism, which can be seen as two (parallel) arms moving in a (x, y) plane and linked together *via* a rigid body and two pivots. Those 'two arms' collaborate to overcome singularities.

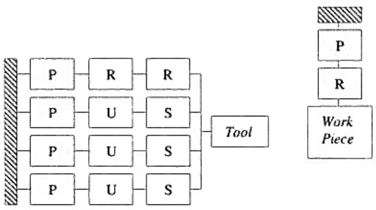


Figure 4. ARCHI basic design

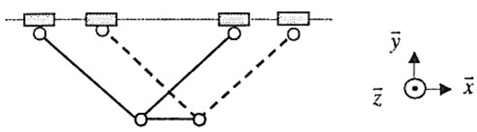


Figure 5. A possible configuration for ARCHI

Indeed, when the nacelle rotates from 0 to 2π, each chain can be in a singular position when the 'leg' is aligned with the nacelle: in this case, the singular chain cannot produce any torque about $\vec{z}$ axis. However, when a chain is singular, the three other chains are not singular and the complete mechanical structure stays controllable.

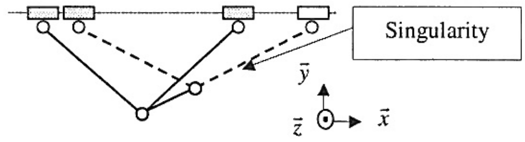


Figure 6. Actuation redundancy to overcome singularities

Such mechanisms offer in addition another advantage: they are extremely easy to build and assemble. We have designed our first ARCHI prototype as a simple test-bed (no focus an aesthetic) able to offer good performances: we use four LinearDrives motors, a FAEMAT spindle and few off-the-shelve mechanical components.

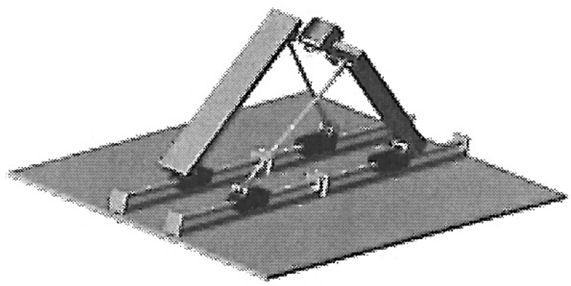


Figure 7. A CAD view of a practical design

Figure 8. A picture of the first ARCHI prototype[4]

3 Kinematic and dynamic modeling

Figure 9 describes the geometrical parameters.

[2] This concept already exists for some 'serial' machine-tools, and has been called the 'left-hand/right-hand' paradigm in the robotics community.

[3] The U-S chains can be replaced by S-S chains; they can also be replaced by R-R chains if a 'perfect' alignment of the joints can be obtained.

[4] For more photos, see: www.lirmm.fr/~marquet/ArchiConcept.html

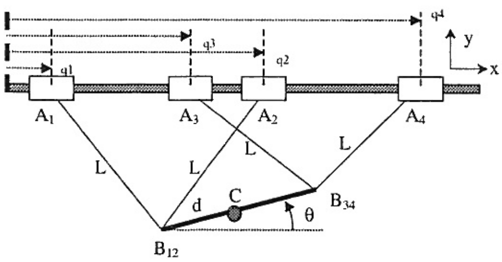


Figure 9. ARCHI parameters

3.1 Position relationship

Vector $q = [q_1 \quad q_2 \quad q_3 \quad q_4]$ denotes drives positions and $x = [x, y, \theta]$ is the nacelle configuration, described by the position of point C and the orientation of the nacelle. If coordinates of points B_{12} and B_{34} are respectively (x_{12}, y_{12}) and (x_{34},y_{34}), we have:

$$x_{12} = x - d.\cos\theta \qquad x_{34} = x + d.\cos\theta$$
$$y_{12} = y - d.\sin\theta \qquad y_{34} = y + d.\sin\theta.$$

- Inverse position relationship

$$q_1 = x_{12} - s_1 \quad q_2 = x_{12} + s_1 \quad q_3 = x_{34} - s_2 \quad q_4 = x_{34} + s_2$$

where: $s_1 = \sqrt{L^2 - {y_{12}}^2}$, $s_2 = \sqrt{L^2 - {y_{34}}^2}$

- Forward position relationship[5]

Depending on which set of 3 chains among the 4 is selected, different computations can be made, which are similar to:

$$x_{12} = \frac{1}{2}.(q_1 + q_3) \qquad y_{12} = -\sqrt{L^2 - \frac{1}{4}.(q_3 - q_1)^2}$$
$$x_{34} = \frac{1}{2}.(q_2 + q_4): \quad y_{34} = -\sqrt{L^2 - \frac{1}{4}.(q_4 - q_2)^2}$$

The position of C, and the nacelle angle, are given by:

$$x = \frac{x_{12} + x_{34}}{2}, \quad y = \frac{y_{12} + y_{34}}{2}, \quad \tan(\theta) = \frac{y_{34} - y_{12}}{x_{34} - x_{12}}$$

3.2 Velocity relationship

The relation between velocities of points (B_{12}, B_{34}) and C can be written in a matrix form:

$$\begin{bmatrix} \dot{x}_{12} \\ \dot{y}_{12} \\ \dot{x}_{34} \\ \dot{y}_{34} \end{bmatrix} = \begin{bmatrix} 1 & 0 & d.\sin\theta \\ 0 & 1 & -d.\cos\theta \\ 1 & 0 & -d.\sin\theta \\ 0 & 1 & d.\cos\theta \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix}$$

$$\boldsymbol{v} = \boldsymbol{G}\,\dot{\boldsymbol{x}}$$

The velocity relationship can be written thanks to the classical property:

$$V(B_{12}) \bullet A_k B_{12} = V(A_k) \bullet A_k B_{12}, \ \text{k=1,2}$$

$$V(B_{34}) \bullet A_k B_{34} = V(A_k) \bullet A_k B_{34}, \ \text{k=3,4.}$$

(• denotes the dot product)

These equations can be grouped in a matrix form such as:

$$\boldsymbol{J_v}\, \boldsymbol{v} = \boldsymbol{J_q}\, \dot{\boldsymbol{q}}$$

Where:

$$\boldsymbol{J_v} = \begin{bmatrix} x_{12} - q_1 & y_{12} & 0 & 0 \\ x_{12} - q_2 & y_{12} & 0 & 0 \\ 0 & 0 & x_{34} - q_3 & y_{34} \\ 0 & 0 & x_{34} - q_4 & y_{34} \end{bmatrix}$$

$$\boldsymbol{J_q} = \begin{bmatrix} x_{12} - q_1 & 0 & 0 & 0 \\ 0 & x_{12} - q_2 & 0 & 0 \\ 0 & 0 & x_{34} - q_3 & 0 \\ 0 & 0 & 0 & x_{34} - q_4 \end{bmatrix}$$

This can be expressed as:

$$\boldsymbol{J_v}\,\boldsymbol{G}\,\dot{\boldsymbol{x}} = \boldsymbol{J_q}\,\dot{\boldsymbol{q}} \ \text{ or } \ \boldsymbol{J_x}\,\dot{\boldsymbol{x}} = \boldsymbol{J_q}\,\dot{\boldsymbol{q}}$$

If $\boldsymbol{J_q}$ is not singular, this leads to:

$$\dot{\boldsymbol{q}} = \boldsymbol{J_m}\,\dot{\boldsymbol{x}}$$

where:

$$\boldsymbol{J_m} = \begin{bmatrix} 1 & \frac{y_{12}}{s_1} & d.(\sin\theta - \frac{y_{12}}{s_1}.\cos\theta) \\ 1 & -\frac{y_{12}}{s_1} & d.(\sin\theta + \frac{y_{12}}{s_1}.\cos\theta) \\ 1 & \frac{y_{34}}{s_2} & d.(-\sin\theta + \frac{y_{34}}{s_2}.\cos\theta) \\ 1 & -\frac{y_{34}}{s_2} & d.(-\sin\theta - \frac{y_{34}}{s_2}.\cos\theta) \end{bmatrix}$$

[5] Better results could be obtained by resorting to iterative numerical scheme methods, where the Cartesian position vector is estimated as:

$$\boldsymbol{x}_{n+1} = \boldsymbol{x}_n + \boldsymbol{J}(\boldsymbol{x}_n, \boldsymbol{q}_n).[\boldsymbol{q}_d - \boldsymbol{q}_n]$$

3.3 Singularity issue

The Jacobian conditioning index of a mechanism permits to evaluate its isotropy. In our case, we compared the Jacobian of the redundant machine to the Jacobians of the 4 partial mechanisms composed of only 3 arms (i.e. non-redundant mechanisms): $J_{124}, J_{134}, J_{123}, J_{234}$.

The first series of curves depicts the behavior of each set of chains: singularities occur clearly when the condition number tends to infinity.

On the contrary, the redundant complete machine offers good conditioning for both matrices, guaranteeing that no singularity occurs (of course, under-mobility singularity still exists for this mechanism, but they are easy to manage as for most parallel mechanisms).

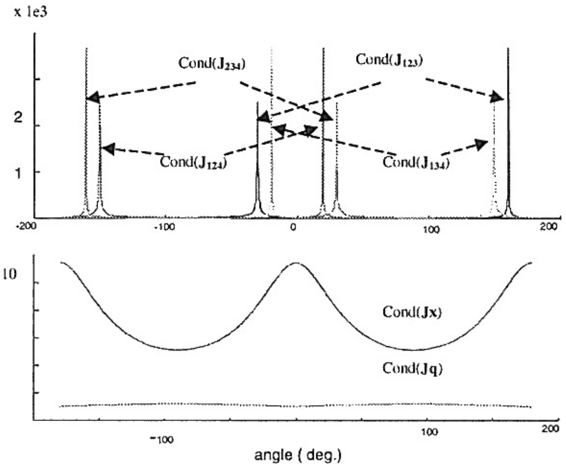


Figure 10. Jacobian conditioning of redundant and non-redundant mechanisms for $x = 1$ m, $y = -0.4$ m and $\theta \in [-200,200]$ degrees ($L = 1$ m, $d = 0.2$ m)

3.4 Dynamic relationship

In this part arms dynamics is neglected. Let's consider f_d and f_n as, respectively, drive's and nacelle's forces. The basic relation of dynamics can be written:

$$J_m^t.(f_d - k.\dot{q}) + M_n.g + f_n = J_m^t.M_d.\ddot{q} + M_n.\ddot{x}$$

with:

- $\mathbf{M_n}$: Matrix containing nacelle's weights
- $\mathbf{M_d}$: Matrix containing drives weights
- $\mathbf{k}$: drives friction coefficient
- $\mathbf{g}$: gravity vector.

Moreover, the actuators acceleration is:

$$\ddot{q} = J_m.\ddot{x} + \dot{J}_m.\dot{x}$$

and the model becomes:

$$[J_m^t.M_d.J_m + M_n].\ddot{x} + [J_m^t.M_d.\dot{J}_m + k.J_m].\dot{x}$$
$$+ f_n + M_n.g + J_m^t.f_d = 0$$

This relation can also be expressed on the classical form:

$$f_n = \tilde{A}(x).\ddot{x} + \tilde{H}(x, \dot{x})$$

4 Control strategies simulation results

Thanks to all the previous models, we have built a simulator of ARCHI including of course dynamic effects, and the possibility of introducing errors on different parameters. We present here simulation results for a typical motion combining a translation and a rotation from $\pi/4$ to $\pi/2$; during that motion, the chain No 4 crosses a singularity.

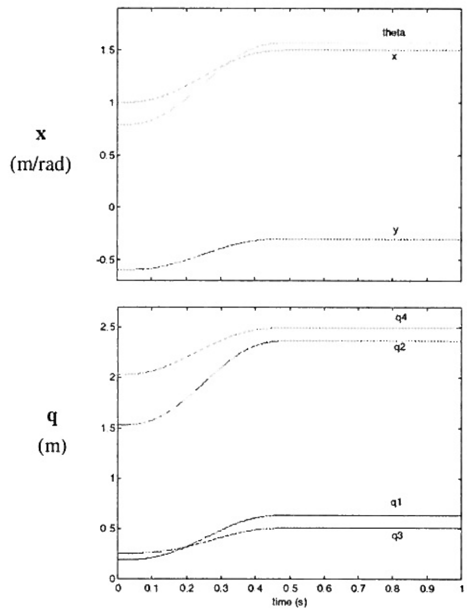


Figure 11. A typical test trajectory (Cartesian and joint motions)

4.1 Independent joint space control

Most (if not all) industrial machine-tools control systems are based on independent (linear) joint control loops; with actuation redundancy, any dimensional error in kinematic models leads to non-convergence problems as soon as an integral effect is included in the control loops. Figure 12 shows such a problem with a 2% error on one leg length. Our idea is then to study an alternate approach (dynamic Cartesian scheme) with different control strategies.

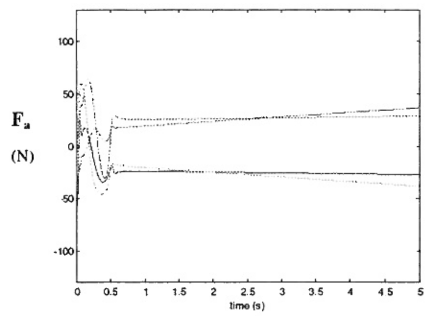


Figure 12. Motors forces do not converge in case of dimensional error

4.2 General approach: Dynamic Cartesian scheme

To overcome the problems due to dimensional errors, we propose to use only Cartesian control schemes; moreover, since parallel mechanisms are intended to offer high speeds and high accelerations, we consider that taking into account dynamics is mandatory for better performances.
However, redundancy issue has been considered too: the non-unicity of drives forces corresponding to a given external force may be addressed in various ways.

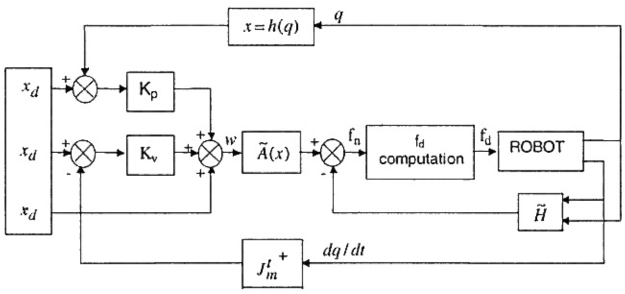


Figure 13. Dynamic Cartesian control structure

4.3 "2-arm robot like" control

We have explained in section 2 that ARCHI could be seen as a two-arm robot carrying a solid object. Thus appealing control strategies may be similar to those proposed for two-arm robots; for example, Dauchez [15] proposed to define 'internal forces' as forces acting inside the carried object. In a similar way, we can define control strategies where the internal force acting in the nacelle is considered.

4.3.1 Without gravity effect

The simplest way to consider such 'internal force' is to regard statics only: then the internal force ('inside' the nacelle) is defined as the difference between the static forces, f_1 and f_2, produced by each 'individual robot' (i.e. each 2-dof sub-part), projected on the nacelle direction (given by vector $\boldsymbol{n}$ in Figure 14). An appealing control strategy is then to set this 'internal force' to zero:

$$(f_1 - f_2) \bullet \boldsymbol{n} = 0$$

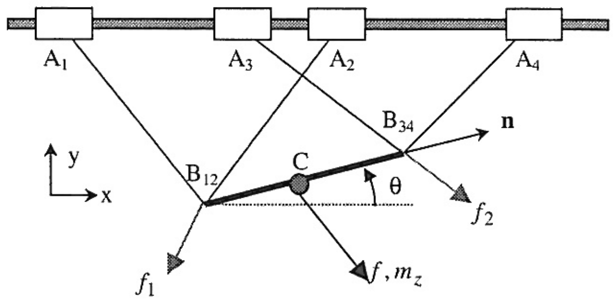


Figure 14. Forces on the mechanism

This condition leads to:

$$J_f \cdot \begin{bmatrix} f_{1x} & f_{1y} & f_{2x} & f_{2y} \end{bmatrix}^t = \begin{bmatrix} f_x & f_y & m_z & 0 \end{bmatrix}^t$$

with:

$$J_f = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ d.\sin\theta & -d.\cos\theta & -d.\sin\theta & d.\cos\theta \\ \cos\theta & \sin\theta & -\cos\theta & -\sin\theta \end{bmatrix}$$

Moreover,

$$f_d = [J_x^{-1}.J_q]^t . \begin{bmatrix} f_{1x} & f_{1y} & f_{2x} & f_{2y} \end{bmatrix}^t$$

Then drives forces are given by:

$$f_d = [J_x^{-1}.J_q]^t . J_f^{-1} . \begin{bmatrix} f_x & f_y & m_z & 0 \end{bmatrix}^t$$

4.3.2 With gravity effect

In this case, the supplementary condition is:

$$(f_1 - f_2).\boldsymbol{n} = M_n.G.\sin\theta$$

4.4 Minimizing a given criterion

As $dim(Ker(J_m)) = 1$, the equation $J_m^t.f_d = f$ has an infinity of solutions [16][17]:

$$f_d = J_m^{t\,+}.f + [I - J_m^{t\,+} J_m^t].z\,,\ z \in \Re^4$$

In fact, $[I - J_m^{t\,+} J_m^t].z$ belongs to the null-space [18] and represents internal forces in the mechanism:

$$[I - J_m^{t\,+} J_m^t].z \in Ker(J_m)$$

Then, drives forces computation can be realized by minimizing a given criterion [19][20][21] as the 2-norm or the infinite norm of drives forces.

4.4.1 Pseudo-inverse based control

The solution corresponding to $z=0$, i.e. the result of a pseudo-inversion, minimizes $\|f_d\|_2$ (if $z=0$, the internal forces are equal to zero).

4.4.2 Infinity-norm based control

Minimization of the infinite norm of f_d can be performed if we choose $z=-k.grad(\|f_d\|_\infty)$, $k \in \Re^+$. It corresponds to a minimization of the maximal drive force.

4.5 Simulation results

Figure 15 and Figure 16 describe respectively drives forces and the 2 norm of the internal force in the case of a 2-arm control. If we neglect gravity effects, the static internal force is quite important whereas if we don't neglect these effects the static internal force is very small. For the control using the pseudo-inverse (Figure 17), the result is better than the 2-arm control because drives forces component in the null-space is equal to zero. Figure 18 shows that the minimization of the infinite norm of drives forces is more efficient than the simple pseudo-inverse, but the curve representing drives forces presents undesirable "discontinuities". Perhaps a good solution could consist in the minimization of a p norm, p > 2 (compromise between the 2 norm and the infinite norm). If the two-arm control seems to be less efficient than the control using the minimization of a given criterion, however it is very simple to implement and it needs few calculations.

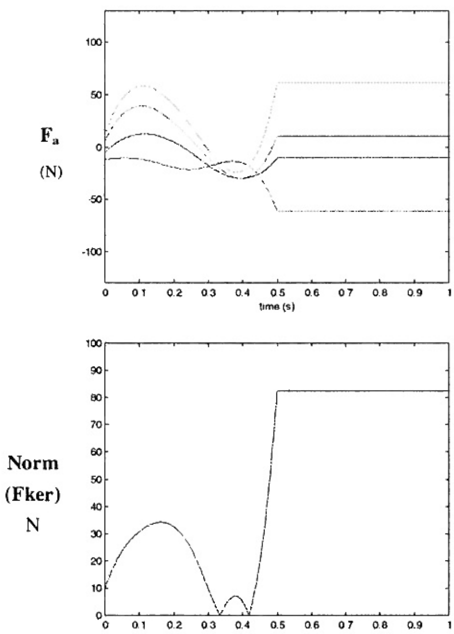


Figure 15. Motors forces and norm of the 'internal' forces with a 'two-arm like' control, without gravity

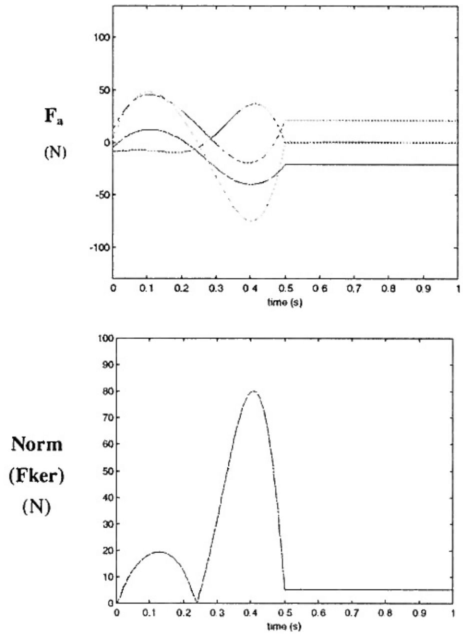


Figure 16. Forces with 'two-arm like' control, with gravity

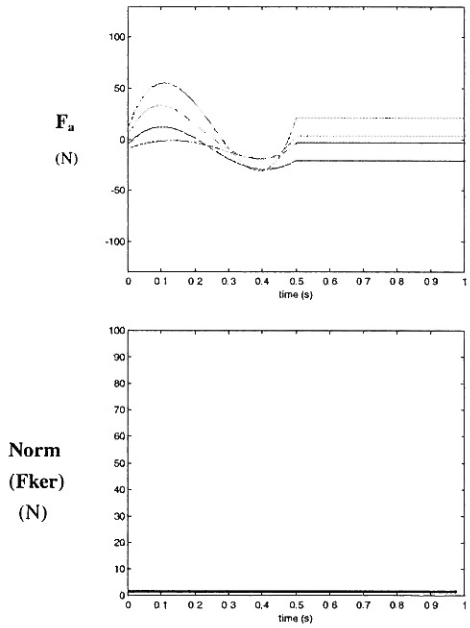
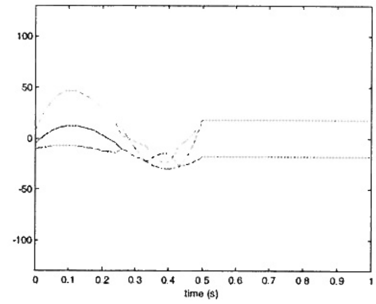


Figure 17. Pseudo-inverse based control



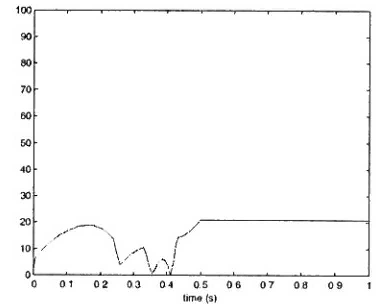


Figure 18. Infinity-norm based control.

5 Conclusion.

In this paper we described Archi, a new 3 dof redundant parallel robot actuated by four drives. After giving its kinematic and dynamic models, we demonstrated its ability to easily evolve in the whole workspace and to allow unlimited rotations. Simulations based upon different joint forces computations methods were compared, a prototype was built and the implementation of a control software is under way. Then, Archi robot will be included as a part of an hybrid robot dedicated to machining.

Acknowledgements:
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