Comparison of the Application of the Symplectic and the "Partially Stretched Orthogonal Transformations" in a New Branch of Adaptive Control for Mechanical Devices

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Abstract — In the last few years certain researchers exerted considerable effort to develop a special branch of soft computing aiming at the control of mechanical devices. This research started from considerations strictly restricted to the realm of classical mechanical systems from which the Symplectic Group describing their general inner symmetry was chosen as a source of uniform structures in this new branch, while tuning their free parameters as well as the possibilities of replacing this process by simple abstract algebraic operations of limited calculations were considered, too. Later on via considering the mathematical properties of these procedures more generally the potential application of several Lie groups as well as using the Symplectic Group in electric and electromechanical systems was outlined. Till now the "Generalized Lorentz Group", the "Stretched Orthogonal Group", and the special, so-called "Minimum Operation Symplectic Transformations" were considered for this purpose. Observing the fact that these methods apply rigid rotation of orthonormal set of vectors in an abstract space one or two times, the aim of the present paper is achieve further reduction of computational complexity by introducing the "Partially Stretched Orthogonal Matrices". As it is demonstrated via simulations made for 3 DOF SCARA arm this group eliminates the deficiencies of the originally used "Stretched Orthogonal Group".

I. INTRODUCTION

A new approach aiming at the adaptive control of mechanical systems of which we have insufficient and inaccurate knowledge regarding their dynamic properties and dynamic interaction with their environment was initiated in [1]. It was conform to the need for developing a universally useful controller making it unnecessary to include a particular model of the robot-work-piece interaction in the control software. Instead of this it aimed at a more intelligent control which is able to "learn" the main features specific to the technological operation under consideration. For this purpose the Symplectic Transformations describing the inner symmetry of the classical mechanical systems were used as "uniform models", a "deformation principle" as the basis of adaptivity, the idea of "partial system identification" [2] as well as the "Standard Symplectising Algorithm" described e.g. in [3], too, were utilized. The main idea of this approach was summarized in [4].

Following the first successes it cropped up soon that essential phenomenological problems, that is that the "momentum part" of the canonical coordinates cannot be directly measured, the Symplectic Group as well as the symplectising algorithm were temporarily dropped and the Orthogonal Group diagonalizing a model inertia matrix as well as rotational and stretch parameters tuned in the conventional causal manner as usual in several sub-fields of soft computing were considered [5-7]. This approach was found to be strongly akin to the traditional Soft Computing (SC) approaches.

Following the analysis of the "uncontrolled norms" occurring in the "Standard Symplectising Algorithm" the idea of the "Minimum Operation Symplectic Transformations" was invented by Tar in [8]. The "decent" behavior of this transformation made it possible to drop the simultaneous tuning of numerous parameters as it was done in the case of e.g. [5-7], and to search further possibilities for realizing learning as "partial system identification" carried out via explicit algebraic procedures [9].

The present state of this approach seems to correspond to the creation of a new branch of Soft Computing for particular problem classes possibly wider than that of the control of mechanical systems. On this basis the author of the present paper has the opinion that it would be expedient to contribute to its potential success by further decreasing its computational complexity via using a group of transformations far simpler than the till investigated ones.

Formal analysis of the mathematical properties of the

"Minimum Operation Symplectic Transformations" opened the way to the potential application of other Lie groups than the Symplectic Group. New ones as the Generalized Lorentz Group and the "Stretched Orthogonal Group" were investigated for this purpose [10]. A study of these groups revealed that each of the transformations till considered can be built up of rigid rotations of orthonormal vectors and of certain stretch and shrink operations. The difference between them rather consists of the number and dimensions of the vectors considered while the basic idea of the partial system-identification essentially remains the same.

The paper is organized as follows: Part II briefly summarizes the idea of "partial and temporal system identification" and the role of the possible Lie groups in it. In Part III analysis of the transformations related to the "Minimum Operation Symplectic Transformations", the "Generalized Lorentz Group" and the "Stretched Orthogonal Group" are given. Following that the "Partially Stretched Orthogonal Transformations" are defined. In Part IV simulations results are analyzed, while Part V contains the concluding remarks.

II. THE IDEA OF PARTIAL AND TEMPORAL SYSTEM-IDENTIFICATION

Formally any adaptive control problem can be formulated as follows: there is given some *imperfect model of the system to be controlled,* on the basis of which some *command signal* is calculated for a desired input $\mathbf{i}^d$ as $\mathbf{e}=\varphi(\mathbf{i}^d)$. If the system's *response to this command (inverse dynamics)* is described by the *unknown function* $\mathbf{i}^r=\psi(\varphi(\mathbf{i}^d))\equiv f(\mathbf{i}^d)$, normally the realized response $\mathbf{i}^r$ differs from the desired one $\mathbf{i}^d$. Normally one can obtain information via observation only on the "net" function $f()$, and that this function considerably may vary in time for instance due to the external environmental influences. Furthermore, in general the only practical tool to "manipulate" the nature of this function is only the *deformation* of its actual input $\mathbf{i}^{d*}$ in comparison with the *desired one*. The aim is to achieve and maintain the $\mathbf{i}^d=f(\mathbf{i}^{d*}):=\mathbf{g}(\mathbf{i}^d)$ state, that is the problem of adaptive control can also be formulated as a fixed-point problem. [Only the nature of the *model function* $\varphi()$ can be directly manipulated.]

On the basis of the idea of the renormalization transformation applied in Chaos Theory, in [11] the so called "Modified Renormalization Algorithm" was suggested for finding the proper deformation factor for a SISO systems in the form of a series as

$$s_{n+1}f\left(s_n s_{n-1}\ldots s_1 x^d\right)=x^d \qquad (1)$$

If the situation of $s_n\rightarrow 1,\ldots s_n s_{n-1}\ldots s_1\rightarrow s$ occurs sx^d just corresponds to the desired deformation of the input according to the above program (several convergence issues also were investigated in [11]).

To For MIMO systems similar series of nonsingular transformations can be invented but extra restrictions must be made to eliminate the inherent ambiguity as $\mathbf{s}_{n+1}\mathbf{f}\left(\mathbf{s}_n\mathbf{s}_{n-1}\mathbf{s}_{n-2}\ldots\mathbf{s}_1\mathbf{x}^d\right)=\mathbf{x}^d,\ \{\mathbf{s}_{n+1},\mathbf{s}_n,\mathbf{s}_{n-1},\ldots,\mathbf{s}_1\}$. Appropriate convergence is required not only in the norm of the vectors but in their direction, too. For avoiding **0** to **0** or **0** to **finite**-type singularities we may introduce at least one or more "formal", that is physically not interpreted parameters in **f** and in $\mathbf{x}^d$, too. The idea is as follows: via putting appropriate number of almost arbitrary column vectors near **f** and $\mathbf{x}^d$, two quadratic matrices [**f**] and [$\mathbf{x}^d$] can be obtained. Let us so utilize ambiguity to obtain non-singular, easily invertible matrices also forming some Lie group. In this case the $[\mathbf{s}_{n+1}]\,[\mathbf{f}]=[\mathbf{x}^d]$ matrix equation can simply be solved as $[\mathbf{s}_{n+1}]=[\mathbf{x}^d]\,[\mathbf{f}]^{-1}$, and the solution so obtained will also belong to the same Lie group.

III. THE PARTICULAR TRANSFORMATIONS HERE CONSIDERED

For instance any Lie group defined by a basic quadratic expression defined with a constant **G** as $\mathrm{M}^T\mathbf{G}\mathrm{M}=\mathbf{G}, \det\mathbf{G}\neq\mathbf{0}, \det\mathrm{M}=1$ is appropriate since $\mathrm{M}^{-1}=\mathbf{G}^{-1}\mathrm{M}^T\mathbf{G}$. The cases in which $\mathbf{G}=\mathbf{I}, \mathbf{g}=\langle 1,\ldots,1,-c\rangle, \Im=\begin{bmatrix}\mathbf{0} & \mathbf{I}\\ -\mathbf{I} & \mathbf{0}\end{bmatrix}$ (*c* is the velocity of light) correspond to the Orthogonal Group, the Generalized Lorentz Group, and the Symplectic Group.

In the case of the Orthogonal Group it is easy to create a rotation that rotates a given vector to be parallel with an other one and leaves their orthogonal sub-space unchanged. This means that only the minimum of the necessary transformations is executed, so in this sense the given transformation is as close to the identity operator as possible.

$$\begin{aligned}
\mathbf{u}^{(1)}&=\frac{1}{\mathbf{f}^{(1)T}\mathbf{f}^{(1)}}\left[\mathbf{f}^{(1)}-\frac{\mathbf{f}^{(2)T}\mathbf{f}^{(1)}}{\mathbf{f}^{(2)T}\mathbf{f}^{(2)}}\mathbf{f}^{(2)}\right]\\
\mathbf{u}^{(2)}&=\frac{1}{\mathbf{f}^{(2)T}\mathbf{f}^{(2)}}\left[\mathbf{f}^{(2)}-\frac{\mathbf{f}^{(1)T}\mathbf{f}^{(2)}}{\mathbf{f}^{(1)T}\mathbf{f}^{(1)}}\mathbf{f}^{(1)}\right]\\
\tilde{\mathbf{f}}^{(1)}&=\frac{\mathbf{f}^{(1)}}{\mathbf{f}^{(1)T}\mathbf{f}^{(1)}+\mathbf{f}^{(2)T}\mathbf{f}^{(2)}}\\
\tilde{\mathbf{f}}^{(2)}&=\frac{\mathbf{f}^{(2)}}{\mathbf{f}^{(1)T}\mathbf{f}^{(1)}+\mathbf{f}^{(2)T}\mathbf{f}^{(2)}}\\
\tilde{\mathbf{u}}^{(1)}&=\frac{\mathbf{u}^{(1)}}{\mathbf{u}^{(1)T}\mathbf{u}^{(1)}+\mathbf{u}^{(2)T}\mathbf{u}^{(2)}}\\
\tilde{\mathbf{u}}^{(2)}&=\frac{\mathbf{u}^{(2)}}{\mathbf{u}^{(1)T}\mathbf{u}^{(1)}+\mathbf{u}^{(2)T}\mathbf{u}^{(2)}}
\end{aligned} \qquad (2)$$

$$\mathbf{S} = \left[\begin{array}{cccc|cccc} \mathbf{f}^{(1)} & \mathbf{u}^{(2)} & \mathbf{e}^{(3)} & \ldots & -\tilde{\mathbf{f}}^{(2)} & -\tilde{\mathbf{u}}^{(1)} & \mathbf{0} & \ldots \\ \hline \mathbf{f}^{(2)} & \mathbf{u}^{(1)} & \mathbf{0} & \ldots & \tilde{\mathbf{f}}^{(1)} & \tilde{\mathbf{u}}^{(2)} & \mathbf{e}^{(3)} & \ldots \end{array}\right]$$

In similar way the "Minimum Operation Symplectic Group" and the "Generalized Lorentz Group" **S** were invented to obtain such "minimum transformations" as given above. In them $\mathbf{f}^{(1)}$, $\mathbf{f}^{(2)}$ must be linearly independent non-zero vectors as the upper and the lower components of the vector **f**. The unit vectors $\{\mathbf{e}^{(i)}: i=3,\ldots,2n\}$ are pairwisely orthogonal to each other and to the sub-space stretched by $\mathbf{f}^{(1)}$, and $\mathbf{f}^{(2)}$. They can simply be obtained via rigidly rotating a whole original orthonormal set.

The situation is very similar in the case of the Generalized Lorentz Group. If **f** is a physically interpreted vector, $f=|\mathbf{f}|$, $c=1$, and one additional dimension must be added to it and a Lorentzian matrix is obtained as

$$\left[\begin{array}{c|c|c|c|c} \mathbf{e}^{(f)}\sqrt{f^2/c^2+1} & \mathbf{e}^{(2)} & \ldots & \mathbf{e}^{(DOF)} & \mathbf{f} \\ \hline f/c^2 & \mathbf{0} & \ldots & \mathbf{0} & \sqrt{f^2/c^2+1} \end{array}\right] \quad (3)$$

Here $\mathbf{e}^{(f)}$ is the unit vector parallel with **f**, $c=1$, the other $\mathbf{e}^{(i)}$s are pairwisely orthogonal unit vectors, each of them is orthogonal to $\mathbf{e}^{(f)}$, too. These vectors can again be obtained by rigidly rotating an initial orthonormal set in the $\mathbf{e}^{(1orig)}$, **f** plain, too.

The "Stretched Orthogonal Group" is the group of matrices having the form of $\mathbf{T}=s\mathbf{W}$, in which **W** is orthogonal matrix, and $s>0$ is a positive parameter. These **T** matrices trivially form a Lie group and they can be easily constructed for a pair of non-zero vectors **a** and **b** as **a=Tb**. In this operation simply $s=|\mathbf{a}|/|\mathbf{b}|$, and **W** makes a rotation concerning only the two-dimensional sub-space stretched by **a** and **b**.

It is clear that while the calculation of the "Stretched Orthogonal Transformations" is very simple, it immediately yields the solution sought for, its stretch factor s concerns any sub-spaces. This is in contrast to the Lorentz matrices or the Minimum Operation Symplectic Matrices, which apply stretch/shrink only in the case of well-defined sub-spaces, while the other sub-spaces are rotated only. This may explain that the application of the "Stretched Orthogonal Group" normally gave considerably weaker results than the Generalized Lorentzian Matrices or the Minimum Operation Symplectic Matrices gave.

It also is plausible that a relatively simple remedy can be applied, namely replacing $\mathbf{T}=s\mathbf{W}$ with

$$\mathbf{J} = \mathbf{I} + (s-1)\frac{\mathbf{b}\mathbf{b}^T}{\mathbf{b}^T\mathbf{b}}, \quad \mathbf{T} = \mathbf{W}\mathbf{J} \quad (4)$$

It is evident that if **a** is orthogonal to **b**, then **Ja=a**. If $\mathbf{c}=\beta\mathbf{b}$, then $\mathbf{J}\mathbf{c}=s\beta\mathbf{b}$, that is these transformations apply stretch/shrink only the desired direction, and leaves the orthogonal sub-spaces unchanged. Following this direction-selective shrink/stretch, rotation comes as a gentle operation. It is worth noting that in spite to the case of the matrices of "Stretched Orthogonal Group", these "partially stretched matrices" does not form a special group. They are simply the elements of the group of the non-singular quadratic matrices of positive determinant.

The realization of such transformations in INRIA's SCILAB 2.5 code is very simple:

```
// function [s,W,Fi]=rotshr2(a,b)
// shrinks and rotates b to move it ino a
// the output is the appropriate //
// shrinking factor, the apprpropriate
// matrix, and the angle of rotation
function [s,W,Fi]=rotshr2(a,b);
[DIM,col]=size(a);
small=1e-12; // small factor to avoid
// division by 0
small10=10*small;
W=eye(DIM,DIM);
na=norm(a,'fro');
nb=norm(b,'fro');
s=na/(nb+kicsi)+small;
be=b/(nb+small);
ae=a/(na+small);
Fi=acos(ae'*be); // the proper angle of
// rotation
sFi=sin(Fi);
cFi=cos(Fi);
c=ae-(be'*ae)*be; // the part of ae
// orthogonal to be
cnorm=norm(c,'fro');
if (cnorm>small)&(nb>small10) // in this
// case rotation is needed otherwise the
// two vectors were parallel to each
// other
c=c/cnorm;
W1=eye(DIM,DIM)-be*be'-
c*c'+cFi*(c*c'+be*be')+sFi*(-
be*c'+c*be'); // rotates b into a
end; // if cnorm>small
J=eye(DIM,DIM)+(s-1)*be*be';
// stretches in the direction of b
// selectively
W=W1*J; //the output matrix
```

Apart from its last two lines this algorithm is the same as the code of the "Stretched Orthogonal Transformations".

IV. SIMULATION RESULTS

In the given simulation example a standard 3 DOF SCARA arm of one telescopic, and two rotary joints was considered. The calculations were made by using INRIA's Scilab 2.5. The end-point of the arm was fixed to a damped dashpot of viscous coefficient *Vis=100 Ns/m* and spring-constant *Sp=1000 N/m* to represent unmodelled environmental interaction. In the space of the joint coordinates a PID type control was prescribed. It could be exactly realized only in the possession of the perfect dynamic model of the system and the environmental interactions. It was supposed that the electric drives of this system are "perfect" ones, that is they immediately deliver the commanded torque without any delay or internal dynamics. To avoid instabilities due to very big transformations in the beginning transient process, instead of a "full correction" a "regulated correction" was applied with linear interpolation in the deformation as

$$\xi = \frac{\left\| \mathbf{x}^d - \mathbf{f} \right\|}{1 + \max\left(\left\| \mathbf{x}^d \right\|, \left\| \mathbf{f} \right\| \right)}, \qquad \lambda = 1 + \varepsilon_1 + \left(\varepsilon_2 - 1 - \varepsilon_1 \right) \frac{\xi}{1+\xi}, \; \hat{\mathbf{x}}^d = \mathbf{f} + \lambda\left(\hat{\mathbf{x}}^d - \mathbf{f} \right) \tag{5}$$

For large relative difference $\chi \to \varepsilon_2$, a small value moderating the norm of the transformation matrices to be applied, while for small difference it approaches $(1+\varepsilon_1)$. A small positive ε_1 means a kind of slight extrapolation of the tendency, while small positive ε_2 means a kind of "regulation" of the too considerable differences. In the simulations $\varepsilon_2 = 10^{-14}$ and $\varepsilon_1 = 0.2$ was applied.

In the simulations a "simple" and a "more intricate" nominal trajectory was prescribed. Fig. 1 displays the results for the simple non-adaptive PID control, and the adaptive versions based on the "symplectic", the "stretched orthogonal", and the "partially stretched orthogonal" matrices. In Fig. 2 the appropriate trajectory reproduction errors are given for the simulated motion with respect to the workshop system of coordinates in m units. Fig. 3 reveals that the new approach has considerable success for the fast motion and that the step-by-step rotations and stretches/shrinks applied by it are very close to the unit operator.

V. CONLUSIONS

The figures make it evident that the "partially stretched orthogonal" quadratic of (DOF+1) dimension seriously improve the quality of control, their result in a trajectory reproduction of almost the same precision as the much more complicated --also quadratic-- symplectic transformations of the dimension of 2(DOF+1). The weak quality resulted by the simple "stretched orthogonal group" emphasizes the significance of the sub-spaces of which no sufficient information is available for the controller. It is expedient to make further investigations for the application of the "partially stretched orthogonal matrices".

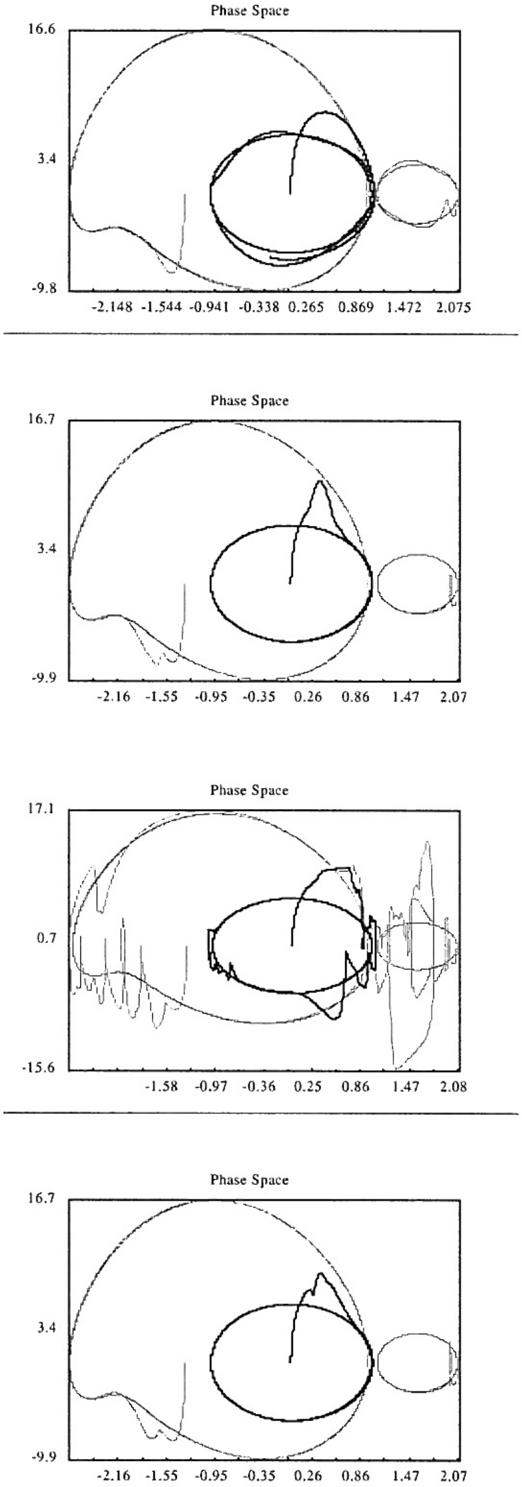

Fig. 1: The nominal and simulated phase space for the "non-adaptive", the "symplectic", the "stretched orthogonal", and the "partially stretched orthogonal" cases; q_1 [rad] solid, q_2 [rad] dashed, q_3 [m] dotted.

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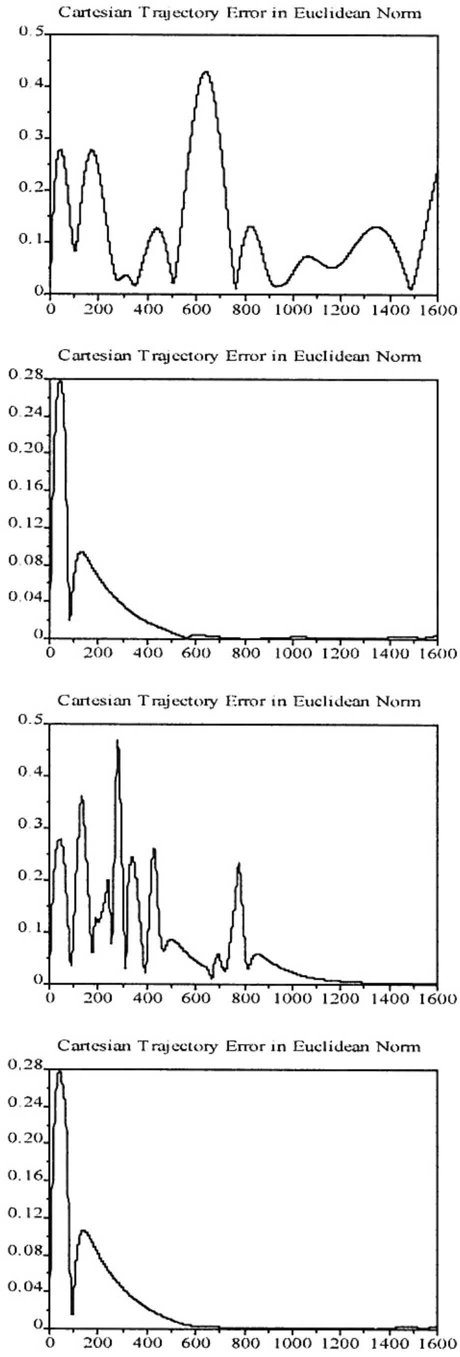

Fig. 2: The simulated trajectory reproduction error for the "non-adaptive", the "symplectic", the "stretched orthogonal", and the "partially stretched orthogonal" cases [in *m* units]

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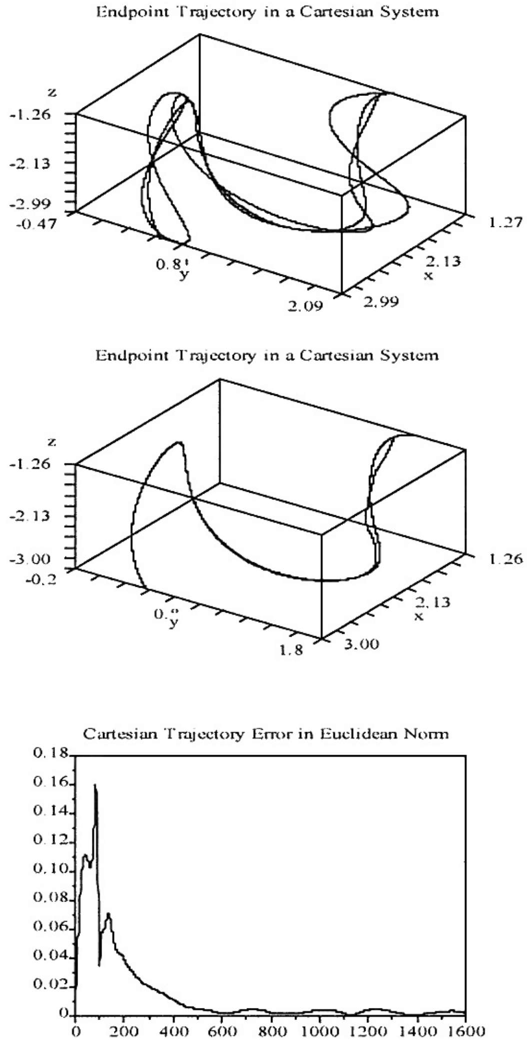

Fig. 3: The simulated trajectory reproduction for the fast motion for the "non-adaptive", and the "partially stretched orthogonal" cases, and its error for the latter case [in *m* units]

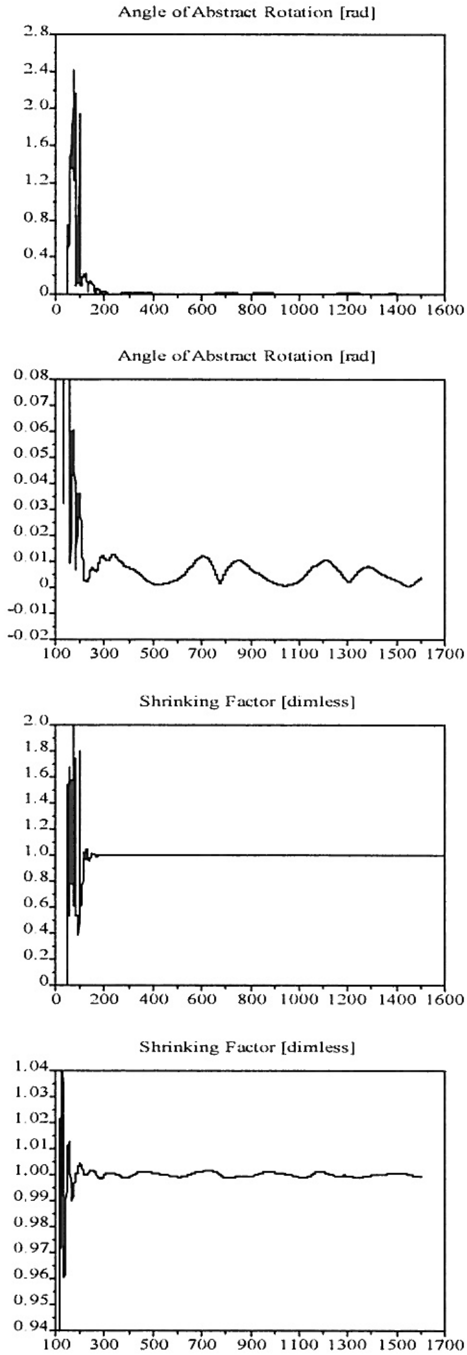

Fig. 4: The angle of abstract rotation [*rad*] and shrinking/stretching correcton factors [dimless]"partially stretched orthogonal" case with exaggerated excerpts

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