Adjoint dexterity inverse kinematics algorithm for mobile manipulators

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Abstract

We consider the inverse kinematic problem for mobile manipulators that may assume singular configurations. Referring to a formal analogy between the kinematics of stationary and mobile manipulators we adopt the adjoint manipulability matrix approach, and propose an inverse kinematics algorithm for mobile manipulators based on the adjoint dexterity matrix. The algorithm can solve both the regular as well as the singular inverse kinematic problem provided that a certain transversality condition holds. The performance of the new algorithm has been illustrated with computer simulations. They demonstrate that the algorithm is particularly efficient in combination with the Jacobian pseudoinverse (Newton) algorithm.

1 Introduction

A mobile manipulator is a robotic system composed of a nonholonomic mobile platform and a holonomic manipulator mounted atop of the platform. Due to a synergy of mobility properties of the platform with the manipulation capabilities of the manipulator mobile manipulators distinguish themselves by remarkably improved performance characteristics in comparison to their subsystems conceived distinctly [1, 2, 3, 4]. The mathematical modeling of mobile manipulators relies on control theory of nonlinear as well as linear, time-dependent systems. Employing the control theoretical perspective, we have developed in [5] a large portion of theory of mobile manipulators mimicking major concepts and results existing for stationary manipulators. In particular, we have presented inverse kinematics algorithms based on the pseudoinverse, the adjoint, and the singularity robust pseudoinverse of the analytic Jacobian of the mobile manipulator. Recently, this approach has been extended further towards defining a collection of extended Jacobian inverse kinematics algorithms [6].

The present paper continues an exploration of the analogy between stationary and mobile manipulators aimed at providing an inverse kinematics algorithm able to work at singular configurations of the mobile manipulator. As mentioned above, in [5] we have proposed a singularity robust pseudoinverse algorithm giving an approximate solution to the singular inverse kinematic problem. In this paper we are attempting at defining a singular inverse kinematics algorithm based on the adjoint dexterity matrix of the mobile manipulator. Such an algorithm resembles conceptually the adjoint manipulability or the Newton-Smale inverse kinematics algorithm for stationary manipulators introduced in [7], adapted to the path tracking problem in [8], and made a universal tool in [9]. Both the adjoint manipulability and the adjoint dexterity algorithms remain well defined at singular configurations provided that a transversality condition is satisfied. A detailed analysis of the relevance of the transversality condition in the context of singular path tracking for stationary manipulators has been accomplished in [10].

The adjoint dexterity inverse kinematics algorithm is subject to a thorough examination by computer simulations on an exemplary mobile manipulator composed of the unicycle-type platform equipped with the double pendulum manipulator. It turns out that the most efficient use of this algorithm is made in combination with the Jacobian pseudoinverse algorithm. The adjoint dexterity algorithm is then switched on around singular configurations, while the Jacobian pseudoinverse algorithm is used sufficiently far away from singularities. The switching point can be defined with regard to dexterity of the current configuration. We have shown that such a hybrid algorithm is capable of driving the mobile manipulator out of a singular configuration, and directing it toward a desirable position in the taskspace.

This paper is organized as follows. Basic concepts referring to the kinematics of mobile manipulators are introduced in section 2. In section 3 we resume main elements of the adjoint manipulability inverse kinematics algorithm for stationary manipulators. An analogous construction for mobile manipulators is accomplished in section 4. Computer simulations are described in section 5. Section 6 concludes the paper.

2 Basic concepts

We shall deal with mobile manipulators consisting of a nonholonomic mobile platform and a holonomic manipulator fixed to the platform. Assuming that $q \in R^n$, $x \in R^p$, $y \in R^r$ denote, respectively, generalized coordinates of the platform, joint positions of the manipulator, and task coordinates, we can represent the kinematics of the mobile manipulator as a driftless control system with outputs of the following form, [5],

$$\begin{cases} \dot{q} = G(q)u = \sum_{i=1}^{m} g_i(q)u_i \\ y = k(q,x). \end{cases} \tag{1}$$

The system (1) is equipped with $m(=n-l)+p$ inputs, where l is the number of independent nonholonomic constraints. The admissible control functions $u(\cdot)$ of (1) will be taken from the Hilbert space $L^2_m[0,T]$ of square integrable functions of time, defined on the interval $[0,T]$. The pairs $(u(\cdot),x)$ are called (endogenous) configurations of the mobile manipulator.

Given a configuration $(u(\cdot),x)$, we let $q(t) = \varphi_{q_0,t}(u(\cdot))$ denote the corresponding trajectory of the platform initialized at a posture q_0. We associate with the system (1) a variational system

$$\begin{cases} \dot{\xi} = A(t)\xi + B(t)v \\ \eta = C(t,x)\xi + D(t,x)w, \end{cases} \tag{2}$$

being a linear approximation to (1) along the triple $(u(t),x,q(t))$, i.e.

$$A(t) = \frac{\partial(G(q(t))u(t))}{\partial q}, \quad B(t) = G(q(t)),$$
$$C(t,x) = \frac{\partial k(q(t),x)}{\partial q}, \quad D(t,x) = \frac{\partial k(q(t),x)}{\partial x}. \tag{3}$$

The control system representation and the variational system defined above allow us to introduce the instantaneous kinematics of the mobile manipulator as the output reachability map of (1),

$$K_{q_0,T}(u(\cdot),x) = y(T) = k(\varphi_{q_0,T}(u(\cdot)),x), \tag{4}$$

and the analytic Jacobian,

$$J_{q_0,T}(u(\cdot),x)(v(\cdot),w) =$$
$$C(T,x)\int_0^T \Phi(T,s)B(s)v(s)\,\mathrm{d}s + D(T,x)w, \tag{5}$$

corresponding to the output reachability map of (2) initialized at $\xi_0 = 0$. The symbol $\Phi(t,s)$ denotes a fundamental matrix of (2) determined by the equations

$$\frac{\partial}{\partial t}\Phi(t,s) = A(t)\Phi(t,s), \quad \Phi(s,s) = I_n.$$

More details concerned with the definition of the kinematics and the analytic Jacobian can be found in [5, 11, 12].

Analogously as for stationary manipulators, we refer to a configuration $(u(\cdot),x)$ of the mobile manipulator as regular, if the Jacobian $J_{q_0,T}(u(\cdot),x)$ is surjective; otherwise the configuration is singular [5, 11]. The output reachability Gramian for the variational system

$$\mathcal{D}_{q_0,T}(u(\cdot),x) =$$
$$\left[C(T,x)\,D(T,x)\right]\begin{bmatrix}\int_0^T \Phi(T,s)B(s)B^T(s)\Phi^T(T,s)\,\mathrm{d}s & 0 \\ 0 & I_p\end{bmatrix}$$
$$\left[C(T,x)\,D(T,x)\right]^T \tag{6}$$

has been called the dexterity matrix of the mobile manipulator [5, 11].

3 Adjoint manipulability

Before presenting our inverse kinematics algorithm for mobile manipulators we have found instructive to reproduce the inverse kinematics algorithm for stationary manipulators [7], called the adjoint manipulability or the Newton-Smale inverse kinematics algorithm. The mathematics underlying this algorithm comes from [13]. Let $y = k(x)$, $x \in R^n$, $y \in R^r$ denote a coordinate representation of kinematics of a redundant stationary manipulator. Given a position and orientation y_d of the end effector, we need to compute a configuration x of the manipulator such that $k(x) = y_d$. To do so, first we choose any configuration $x \in R^n$. If $k(x) = y_d$, the problem is solved. Otherwise, we define a ray $\rho = \{\alpha(y_d - k(x)) \mid \alpha \in R\}$, joining the origin of the taskspace with $y_d - k(x)$. Let $J(x) = \frac{\partial k}{\partial x}(x)$ stand for the analytic Jacobian. Suppose that the kinematics are transverse to the ray, which means that

$$\operatorname{rank}\left[J(x),\ y_d - k(x)\right] = \operatorname{rank}\left[M(x),\ y_d - k(x)\right] = r,$$

where $M(x) = J(x)J^T(x)$ denotes the manipulability matrix of the manipulator [14]. If the transversality condition holds, the inverse image $k^{-1}(\rho)$ of the ray is an $(n-r+1)$-dimensional analytic submanifold of the jointspace. Clearly, the analytic Jacobian $J(x)$ maps tangent vectors to this submanifold into (tangent vectors to) the ray, i.e. for a certain $\alpha \in R$

$$J(x)\dot{x} = \alpha\,(y_d - k(x))\,. \tag{7}$$

A least squares solution of the above equation,

$$\dot{x} = -J^T(x)\lambda, \tag{8}$$

involves a vector of Lagrange multipliers satisfying

$$M(x)\lambda = \alpha\,(k(x) - y_d)\,. \tag{9}$$

To compute λ from (9) we shall use a matrix identity

$$A\ \mathrm{adj}A = I_r \det A, \tag{10}$$

where A is an $r \times r$ square matrix, adj$A$ denotes the adjoint matrix to A (the transpose matrix of algebraic complements of A). On assuming transversality, taking advantage of (10), and defining the function α as $\alpha(x) = \det M(x)$ (squared manipulability of configuration x), we obtain the following solution of (9)

$$\lambda = \mathrm{adj}M(x)\,(k(x) - y_d)\,. \tag{11}$$

Eventually, having substituted (11) into (8), we arrive at the adjoint manipulability (Newton-Smale) inverse kinematics algorithm ($\gamma > 0$)

$$\dot{x} = -\gamma J^T(x)\,\mathrm{adj}M(x)\,(k(x) - y_d)\,. \tag{12}$$

Suppose that $x(t)$ is a solution of (12) initialized at a certain configuration x_0, and define the taskspace error vector $e(t) = k(x(t)) - y_d$. It is easily seen that

$$\dot{e} = J(x)\dot{x} = -\gamma \det M(x)e,$$

therefore, every component e_i, $i = 1, 2, \ldots, r$ of the error evolves in time in accordance with the formula

$$e_i(t) = e_i(0)\exp\left(-\gamma\int_0^t \det M(x(s))\,\mathrm{d}s\right). \tag{13}$$

The formula (13) yields that whenever the integral $\int_0^t \det M(x(s))\,\mathrm{d}s$ tends with time to $+\infty$, the algorithm (12) solves the inverse kinematic problem for the stationary manipulator. The algorithm is able to drive the manipulator through singular configurations at which the transversality condition holds. Moreover, the end effector of the manipulator travels along a straight line in the taskspace.

4 Adjoint dexterity

Using the concepts of the instantaneous kinematics and the analytic Jacobian for mobile manipulators, introduced in section 2, we can construct the adjoint dexterity inverse kinematics algorithm for mobile manipulators, mimicking the ideas presented in section 3. To begin with, let us consider the inverse kinematic problem for the kinematics (4), so given a point $y_d \in R^r$ in taskspace we need to compute a configuration $(u(\cdot), x)$ of the mobile manipulator such that $K_{q_0,T}(u(\cdot), x) = y_d$. In order to solve this problem, we pick any configuration $(u(\cdot), x)$. If $K_{q_0,T}(u(\cdot), x) = y_d$, we are done. Otherwise, we define a ray $\rho = \{\alpha\,(y_d - K_{q_0,T}(u(\cdot), x)) \mid \alpha \in R\}$ in the taskspace.

Assume the transversality condition of the kinematics to the ray in the form

$$\dim\{\mathrm{Im}\,J_{q_0,T}(u(\cdot), x), y_d - K_{q_0,T}(u(\cdot), x)\} = r$$

or, equivalently,

$$\mathrm{rank}\big[\mathcal{D}_{q_0,T}(u(\cdot), x), y_d - K_{q_0,T}(u(\cdot), x)\big] = r. \tag{14}$$

Then, the inverse image $K^{-1}_{q_0,T}(u(\cdot), x)(\rho)$ of the ray forms a (Hilbert) submanifold, [15], of the configuration space, of codimension $r-1$. The analytic Jacobian transforms the tangent space to this submanifold into (the tangent space to) the ray, that leads to the following Jacobian equation

$$J_{q_0,T}(u(\cdot), x)(v(\cdot), w) = \alpha(y_d - K_{q_0,T}(u(\cdot), x))\,. \tag{15}$$

As in the case of stationary manipulators, in order to solve (15) for a pair $(v(\cdot), w)$ we apply the least squares method, i.e. we minimize the norm

$$\frac{1}{2}\left(\int_0^T v^T(s)v(s)\,\mathrm{d}s + w^T w\right)$$

with the equality constraint (15). A standard Lagrange multiplier technique, [16], requires the introduction of a Lagrangian

$$\mathcal{L}(v(\cdot), w, \lambda) = \frac{1}{2}\left(\int_0^T v^T(s)v(s)\,\mathrm{d}s + w^T w\right) +$$
$$\lambda^T(C(T, x)\int_0^T \Phi(T, s)B(s)v(s)\,\mathrm{d}s + D(T, x)w -$$
$$\alpha(y_d - K_{q_0,T}(u(\cdot), x))).$$

The necessary conditions for minimum yield

$$\begin{cases} v(t) = -B^T(t)\Phi^T(T, t)C^T(T, x)\lambda, \\ w = -D^T(T, x)\lambda, \end{cases} \tag{16}$$

where the vector $\lambda \in R^r$ of the Lagrange multipliers should be computed from the equation

$$\mathcal{D}_{q_0,T}(u(\cdot),x)\lambda = \alpha\left(K_{q_0,T}(u(\cdot),x) - y_d\right), \qquad (17)$$

containing the dexterity matrix (6). It follows from the transversality condition (14) that a solution

$$\begin{pmatrix}\lambda\\ \alpha\end{pmatrix} \in \operatorname{Ker}\left[\mathcal{D}_{q_0,T}(u(\cdot),x), K_{q_0,T}(u(\cdot),x) - y_d\right]$$

to the above equation is well defined, modulo the multiplication by a function of the configuration. Such a solution can be found by employing the identity (10) to the dexterity matrix. Consequently, we obtain

$$\lambda = \operatorname{adj}\mathcal{D}_{q_0,T}(u(\cdot),x)\left(K_{q_0,T}(u(\cdot),x) - y_d\right),$$
$$\alpha(u(\cdot),x) = \det\mathcal{D}_{q_0,T}(u(\cdot),x) = d^2_{q_0,T}(u(\cdot),x), \qquad (18)$$

where $d_{q_0,T}(u(\cdot),x)$ is the dexterity of the configuration $(u(\cdot),x)$ [5, 11]. As a matter of fact, the above solution provides us with the adjoint dexterity inverse kinematics algorithm for mobile manipulators, defined by the following formula

$$\frac{d}{d\theta}\begin{pmatrix}u_\theta(t)\\ x(\theta)\end{pmatrix} = -\gamma\begin{bmatrix}B_\theta^T(t)\Phi_\theta^T(T,t)C_\theta^T(T,x(\theta))\\ D_\theta^T(T,x(\theta))\end{bmatrix}$$
$$\operatorname{adj}\mathcal{D}_{q_0,T}(u_\theta(\cdot),x(\theta))\left(K_{q_0,T}(u_\theta(\cdot),x(\theta)) - y_d\right). \qquad (19)$$

In order to compute the left hand side of (19), we first take a control function $u_\theta(t)$, a joint position $x(\theta)$, and find a trajectory $q_\theta(t)$ of the platform as well as of the end effector $k(q_\theta(t),x(\theta))$. The variational system along the triple $(u_\theta(t),x(\theta),q_\theta(t))$ is defined by matrices $A_\theta(t)$, $B_\theta(t)$, $C_\theta(t,x(\theta))$, $D_\theta(t,x(\theta))$ in accordance with expression (3). The curve $(u_\theta(\cdot,x(\theta)))$ in the configuration space converges with $\theta \to +\infty$ to a configuration $((u(\cdot,x))$ such that $K_{q_0,T}(u(\cdot),x) = y_d$. The parameter $\gamma > 0$ is responsible for the speed of convergence of this algorithm.

Let us denote by $e(\theta) = K_{q_0,T}(u_\theta(\cdot),x(\theta)) - y_d$ the error in the taskspace corresponding to the configuration $(u_\theta(\cdot),x(\theta))$ of the mobile manipulator. Then, a straightforward computation involving expressions (5), (6), and the form (19) of the adjoint dexterity inverse kinematics algorithm allows us to deduce that

$$\frac{\mathrm{d}e(\theta)}{\mathrm{d}\theta} = -\gamma\det\mathcal{D}_{q_0,T}(u_\theta(\cdot),x(\theta))e(\theta),$$

and to establish the following equivalent of (13)

$$e_i(\theta) = e_i(0)\exp\left(-\gamma\int_0^\theta \det\mathcal{D}_{q_0,T}(u_s(\cdot),x(s)\,\mathrm{d}s\right),$$
$$i = 1,2,\ldots,r. \qquad (20)$$

Results of an examination of the algorithm (19) by computer simulations will be presented in the next section.

5 Simulations

For simulations we shall use the kinematics of a mobile manipulator presented in Fig. 1, composed of the unicycle-type mobile platform with a double pendulum manipulator mounted atop of the platform.

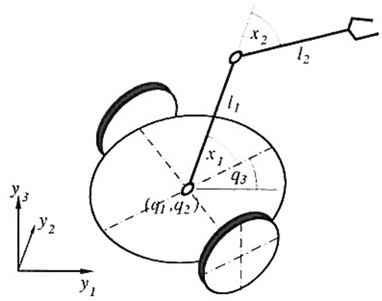

Figure 1. Double pendulum mounted on the unicycle.

The taskspace of this mobile manipulator is 3-dimensional and consists of the end effector positions in the Cartesian space. The control system representation (1) of the kinematics has the following form

$$\begin{cases}\dot q_1 = u_1\cos q_3\\ \dot q_2 = u_1\sin q_3\\ \dot q_3 = u_2,\end{cases}$$
$$y = k(q,x) = \begin{pmatrix}q_1 + (l_1\cos x_1 + l_2\cos x_{12})\cos q_3\\ q_2 + (l_1\cos x_1 + l_2\cos x_{12})\sin q_3\\ l_1\sin x_1 + l_2\sin x_{12}\end{pmatrix}, \qquad (21)$$

where we have substituted $x_{12} = x_1 + x_2$.

Singular configurations of this mobile manipulator are: $u_1(\cdot), u_2(\cdot)$ arbitrary, $x_1 = \pm\pi/2, x_2 = 0$ or π [11]. Now, a short computation shows that the dexterity matrix at these configurations assumes the form

$$\mathcal{D}_{q_0,T}(u(\cdot),x) = \begin{bmatrix}\mathcal{E}_{q_0,T}(u(\cdot),x) & 0\\ 0 & 0\end{bmatrix},$$

with $\mathcal{E}_{q_0,T}(u(\cdot),x)$ non-singular whenever $u_1(\cdot) \not\equiv 0$. This being so, as long as the platform is moving, any ray $\rho = \{\alpha(\rho_1,\rho_2,\rho_3)\,|\,\alpha \in R\}$ satisfying $\rho_3 \neq 0$ guarantees transversality.

In the representation (21) the controls have been taken in the form

$$u_i(t) = \beta_{i0} + \sum_{k=1}^{2}\alpha_{ik}\sin 2k\pi t + \beta_{ik}\cos 2k\pi t,$$

$i = 1,2$, that yields a 10-dimensional control space. Lengths of the manipulator arms amount to $l_1 = l_2 = 1$. The performance of the adjoint dexterity inverse kinematics algorithm has been tested on an exemplary inverse kinematic problem consisting in reaching by the end effector the desirable position $y_d = (3, 4, 0)$ from three singular initial configurations $u_{01}(t) = \beta_{10} = 0.0001$, $u_{02}(t) = \beta_{20} = 0.1, 0.01, 0.001$, $x(0) = (\pi/2, 0)$. It has been assumed that the initial posture of the platform is $q_0 = (0, 0, 0)$, and the control time horizon $T = 1$. Since the initial configurations are singular, an application of the Jacobian pseudoinverse (Newton) algorithm has failed. Therefore, in agreement with suggestions included in [7], in order to solve the inverse problems we have used a combination of the adjoint dexterity algorithm (19) and the Jacobian pseudoinverse algorithm

$$\frac{d}{d\theta}\begin{pmatrix} u_\theta(t) \\ x(\theta) \end{pmatrix} = -\gamma \begin{bmatrix} B_\theta^T(t)\Phi_\theta^T(T,t)C_\theta^T(T,x(\theta)) \\ \mathrm{D}_\theta^T(T,x(\theta)) \end{bmatrix} \mathcal{D}_{q_0,T}^{-1}(u(\cdot)_\theta, x(\theta))\,(K_{q_0,T}(u_\theta(\cdot), x(\theta)) - y_d)\,.$$

The task of the adjoint dexterity algorithm is to drive the mobile manipulator out of the singular configuration, and to enable the Jacobian pseudoinverse algorithm. The first algorithm is run with parameter $\gamma = 10^6$ until the squared dexterity $d^2_{q_0,T}(u_\theta(\cdot), x(\theta)) \geq 10^{-6}$. Then the second algorithm is switched on and run with $\gamma = 1$. The quality of solutions obtained in this way is illustrated in Figs 2-4. Additionally, Fig. 5 shows the speed of convergence of the algorithm. The initial flat parts of the plots in this figure correspond to the run of the adjoint dexterity algorithm. It may be checked that, respectively, 4, 8, and 18 iterations of the adjoint dexterity inverse kinematics algorithm suffice to enable the Jacobian pseudoinverse algorithm. The error plot in Fig 5 corresponding to the case $u_2 = 0.1$ shows a local increase of the error accompanying a switch to the Jacobian pseudoinverse algorithm.

6 Conclusion

Within the control theoretic paradigm we have developed a new inverse kinematics algorithm for mobile manipulators, called the adjoint dexterity inverse kinematics algorithm. Formally, the derivation of this algorithm is based on the results of Smale [13], and resembles a construction proposed for stationary manipulators in [7]. The adjoint dexterity algorithm is capable of solving the singular inverse kinematic problem for mobile manipulators at corank 1 singular configurations provided that the transversality condition (14) is satisfied. We have learnt form computer simulations that the algorithm works at the regular configurations as well as at singularities, but typically its convergence is much slower than of the Jacobian pseudoinverse algorithm (to some extent the speed of convergence might be controlled by tuning appropriately the parameter γ in (19)). For this reason, we recommend combining the adjoint dexterity algorithm

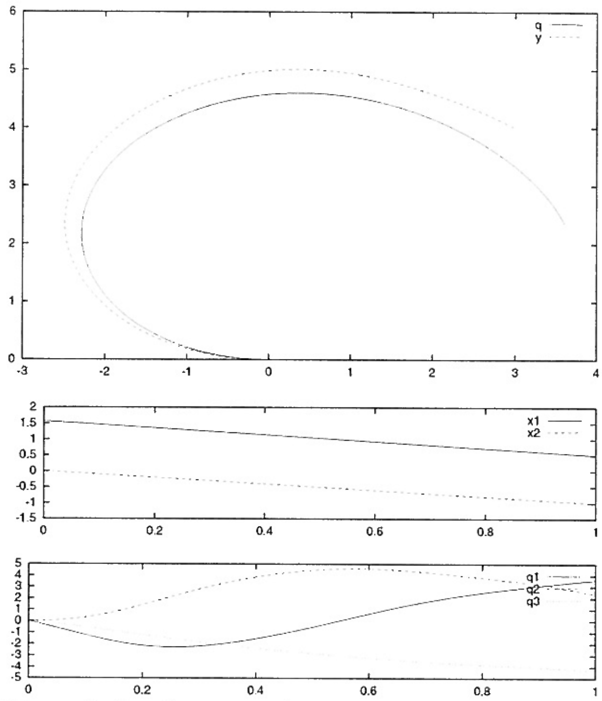

Figure 2. A solution to the inverse kinematic problem for $u_0(t) = (0.0001, 0.1)$, $x(0) = (\pi/2, 0)$.

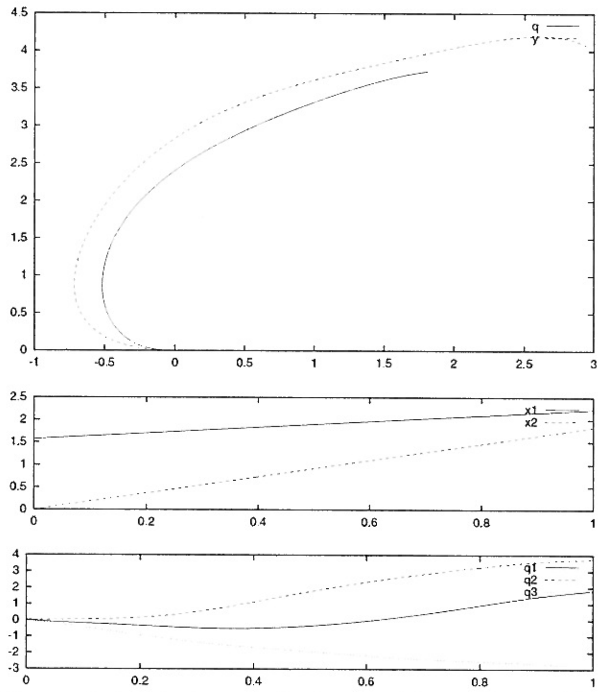

Figure 3. A solution to the inverse kinematic problem for $u_0(t) = (0.0001, 0.01)$, $x(0) = (\pi/2, 0)$.

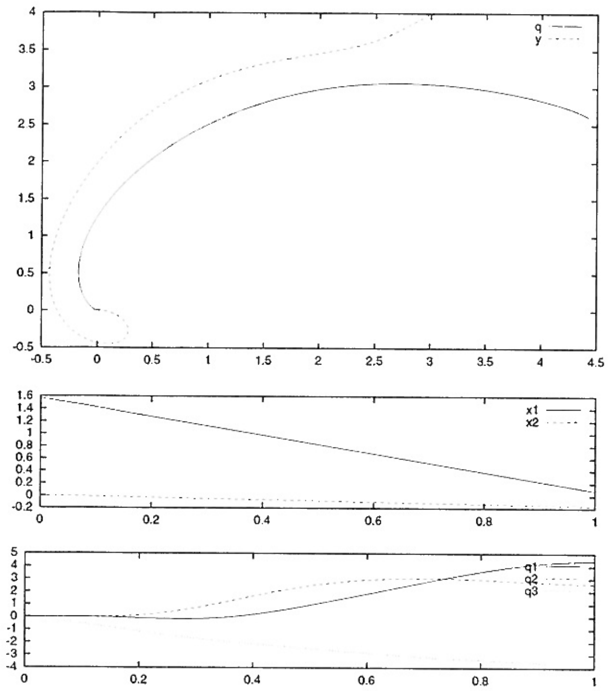

Figure 4. A solution to the inverse kinematic problem for $u_0(t) = (0.0001, 0.001)$, $x(0) = (\pi/2, 0)$.

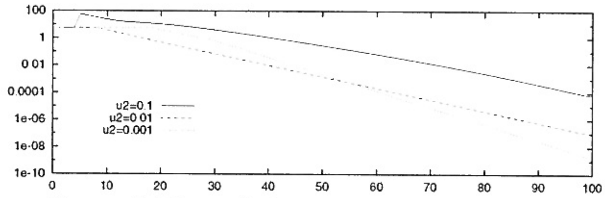

Figure 5. Plots of the error in the taskspace.

with the Jacobian pseudoinverse algorithm. The former should be used at and close to singularities, i.e. at configurations of poor dexterity, whereas the latter - when the mobile manipulator is enough far away from singularities. An efficient combination of these two algorithms have been observed in our computer experiments.

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