An adaptive control law for mobile manipulators with parametric uncertainty in the dynamics

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Abstract

In this paper a new adaptive control algorithm for mobile manipulator (a rigid manipulator mounted on a wheeled mobile nonholonomic platform) has been presented. For proper cooperation between manipulator and mobile platform it is necessary to control the orientation of the platform. To control the behaviour of the whole composed system the backstepping procedure has been applied. First, we use some control algorithm for the kinematics of the mobile manipulator to preserve the so-called posture tracking (it means the position and orientation tracking) for wheeled mobile platform. Next, we take under consideration the dynamics and the kinematics of whole mobile manipulator. The presented control algorithm maybe considered as a dynamic version of the classical adaptive Slotine & Li [8] controller. Theoretical considerations have been completed with simulation study.

Keywords— **nonholonomic constraint, wheeled mobile platform, universal control.**

1 Introduction.

There exist numerous desired trajectory tracking algorithms for rigid manipulators based on different levels of knowledge of the robot dynamics. In other words, a solution of the trajectory tracking problem for rigid manipulators is well known and can be found in the literature. On the other hand, the trajectory tracking problem for wheeled mobile robots in presence of nonholonomic constraint has been solved. In this paper we consider a rigid holonomic manipulator mounted on a top of a wheeled mobile platform. Such a combined system is able to perform manipulation tasks in a much larger workspace than a fixed-base manipulator but this approach introduces a few of new issues that are not present in the each subsystem considered separately. First, the dynamics of the combined system are much more complicated because they include dynamic interactions between mobile platform and manipulator [11], [5]. Such dynamics can be partially or completely unknown. Second, a particular point in the workspace can be reached either by moving the manipulator or moving the mobile platform or by a composed motion of both [10] (a cooperation). Third, some problems with control of such composed system occur [1].

In further considerations we will discuss a dynamic model of the mobile manipulator and we will propose some solution of the trajectory tracking problem designed to full dynamic equations of the mobile manipulator. Our goal in this paper is to find an adaptive control algorithm which preserves the proper cooperation between two subsystems: the nonholonomic subsystem (the mobile platform) and the holonomic one (the rigid manipulator) in a case when the dynamics have parametric uncertainty in a model. The wheeled mobile platform has been chosen to enlarge the workspace of the manipulator. The manipulator has to follow the own desired trajectory which defines a task of this subsystem. The task of the platform is to follow the desired trajectory without slip of wheels. It means that the desired trajectory for the mobile platform must satisfy the nonholonomic constraint (1) and the desired trajectory must be planned for the whole system simultaneously [4].

For such a system we will introduce a new adaptive control algorithm which is in fact the dynamic version of the Slotine & Li trajectory tracking algorithm. In many papers we can find a solution to the trajectory tracking problem but only for the kinematics of the nonholonomic system (for instance mobile platform) without any information how to control the whole system. Many authors assume that the dynamics can be fully linearized but in a case of any uncertainty in the model it is not obvious and should be considered separately. The control algorithm proposed in this paper preserves an asymptotically stable trajectory tracking for the whole mobile manipulator with parametric uncertainty in the model.

2 Mathematical model of a mobile manipulator.

We will analyse mechanical system which consists of two subsystems: a wheeled mobile platform (often called in the literature 'mobile robot') and a rigid manipulator. As a vector of generalized coordinates we take $q = (q_m^T, q_r^T)^T$, where a symbol q_m denotes the vector of generalized coordinates describing the behaviour of the mobile platform

$$q_m = (x, y, \theta, \phi_1, \beta_1, \ldots, \phi_k, \beta_k)^T, \quad q_m \in R^n,$$

where x, y denote Cartesian position of the mass centre of the platform, θ denotes orientation of the platform, ϕ_i is an angular position of ith wheel and β_i denotes an angle of the steering wheel. A symbol q_r denotes the vector of joint coordinates of the manipulator

$$q_r = (\theta_1, \ldots, \theta_p)^T, \qquad q_r \in R^p.$$

The wheeled mobile platform should move without slip of wheels. It is equivalent to the assumption that the velocity at the contact point between each wheel and a surface is equal to zero (in other words, the mobile manipulator motion is frictionless). It establishes the following relationship between generalized coordinates q_m and generalized velocities $\dot{q}_m$:

$$A(q_m)\dot{q}_m = 0, \tag{1}$$

where:

$A(q_m)$ - is full rank $(m \times n)$ matrix.

The expression (1) defining nonholonomic constraint implies the existence of the vector of auxiliary velocities, which satisfy

$$\dot{q}_m = G(q_m)\eta, \tag{2}$$

where:

$G(q_m)$ - full rank $n \times m$ matrix for which holds

$$A(q_m)G(q_m) = 0, \tag{3}$$

η - m-dimensional vector of auxiliary velocities.

In further considerations we will assume that the mobile platform will be moved on the horizontal plane. It means that the potential energy of the platform is constant and does not influence the behaviour of the whole system. A Lagrange function for the whole system, which consists of nonholonomic mobile platform and the manipulator mounted on it, is as follows

$$L(q, \dot{q}) = K_m(q_m, \dot{q}_m) + K_r(q, \dot{q}) - V_r(q) \tag{4}$$

where

$K_m(q_m, \dot{q}_m) = \frac{1}{2}\dot{q}_m^T Q_m(q_m)\dot{q}_m$ - kinetic energy of the platform,

$K_r(q, \dot{q}) = \frac{1}{2}\dot{q}^T Q(q)\dot{q}$ - kinetic energy of the manipulator,

$V_r(q)$ - potential energy of the manipulator,

$q = \begin{pmatrix} q_m \\ q_r \end{pmatrix} \in R^N$, $N = n + p$, - coordinates of the mobile manipulator.

After substituting the Lagrange function into the d'Alembert Principle, we will get the following equations of the mobile manipulator dynamics

$$\begin{aligned} Q(q)\ddot{q} + Q_m(q_m)\ddot{q}_m + C(q, \dot{q})\dot{q} &+ \\ C_m(q_m, \dot{q}_m)\dot{q}_m + D(q_r) &= A^T\lambda + Bu \end{aligned} \tag{5}$$

or more detailed

$$\begin{bmatrix} Q_{11} + Q_m & Q_{12} \\ Q_{21} & Q_{22} \end{bmatrix} \begin{bmatrix} \ddot{q}_m \\ \ddot{q}_r \end{bmatrix} + \begin{bmatrix} C_{11} + C_m & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \begin{bmatrix} \dot{q}_m \\ \dot{q}_r \end{bmatrix}$$

$$+ \begin{bmatrix} 0 \\ D \end{bmatrix} = \begin{bmatrix} A^T\lambda \\ 0 \end{bmatrix} + \begin{bmatrix} Bu_m \\ u_r \end{bmatrix}.$$

An input vector u_m denotes forces and torques applied to some wheels of the mobile platform and a vector u_r denotes torques applied to joints of the rigid manipulator. From the fact that for the mobile platform with nonholonomic constraint always exists the vector of the auxiliary variables expressed as (2), it is profitable to rewrite the dynamics separately for each subsystem and eliminate the vector of the Lagrange multipliers λ from the first matrix equations as follows

$$\begin{bmatrix} G^T(Q_{11} + Q_m)G & G^T Q_{12} \\ Q_{21}G & Q_{22} \end{bmatrix} \begin{bmatrix} \dot{\eta} \\ \ddot{q}_r \end{bmatrix} +$$

$$\begin{bmatrix} G^T(C_{11} + C_m)G + G^T(Q_{11} + Q_m)\dot{G} & G^T C_{12} \\ C_{21}G & C_{22} \end{bmatrix} \begin{bmatrix} \eta \\ \dot{q}_r \end{bmatrix}$$

$$+ \begin{bmatrix} 0 \\ D \end{bmatrix} = \begin{bmatrix} G^T B & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} u_m \\ u_r \end{bmatrix}.$$

To simplify the notation, we will describe the above equations in the following way

$$Q^*(q)\dot{z} + C^*(q, z)z + D^*(q) = B^*(q)u, \tag{6}$$

$$z = \begin{bmatrix} \eta \\ \dot{q}_r \end{bmatrix}.$$

The above matrix equation (6) we will call the dynamics of the mobile manipulator in coordinates $z^T = (\eta, q_r)^T$ and the equation

$$\dot{q}_m = G(q)\eta \tag{7}$$

will be called the kinematics of the mobile manipulator.

2.1 Properties of the model of a mobile manipulator.

In this section we must consider basic properties of the model given by (6). As for the mentioned equation, we have

Property 1: Real inertia matrix Q^* is always symmetric and positive definite.
Above property is simply to show. The second property has been shown in [2]. It states that if we get a matrix of Coriolis and centrifugal forces as a Christoffel's symbol namely

$$C^*_{ij}(q) = \sum_{k=1}^{m+p} c^i_{kj}(q)\dot{q}_k,$$

where coefficients are equal to

$$c^i_{kj}(q) = \frac{1}{2}\left(\frac{\partial Q^*_{ij}(q)}{\partial q_k} + \frac{\partial Q^*_{ik}(q)}{\partial q_j} - \frac{\partial Q^*_{jk}(q)}{\partial q_i}\right),$$

then a skew-symmetry between the inertia matrix and Coriolis matrix does not hold anymore. In other words,

Property 2: For many mobile manipulators exists some matrix $C_k \neq 0$, such that

$$\frac{d}{dt}Q^* = (C^* + C_k) + (C^* + C_k)^T.$$

It is obvious that Property 2 must be checked before starting the control process.

Property 3: Input matrix $B^*(q)$ has always full rank.

2.2 Linear parametrization of the model of the mobile manipulator.

From theory of adaptive systems for rigid manipulators (e.g.[7]) we know that the dynamics of such robot can be expressed as a linear function of some parameters, which depend on mass and inertia moments of robot joints as follows

$$Q(q,a)\ddot{q} + C(q,\dot{q},a)\dot{q} + D(q,a) = Y(\ddot{q},\dot{q},\dot{q},q)a,$$

where a is a vector of real (constant) parameters. Now we want to generalize such approach for a model of mobile manipulator with correction matrix C_k as

$$Q^*(q,a)\dot{z} + C^*(q,z,a)z + D^*(q,a) - C_k(q,z,a)e_z =$$
$$= Y_k(\dot{z}, z, e_z, z, q)a,$$

with variables defined as follows

$$z_d = \begin{bmatrix} \eta_r \\ \dot{q}_{ref} \end{bmatrix}, \qquad e_z = z - z_d = \begin{bmatrix} \eta - \eta_r \\ \dot{q}_r - \dot{q}_{ref} \end{bmatrix},$$

where the first argument of Y_k matrix is related to acceleration, which the inertia matrix is multiplied by, the second one describes velocity, which the Coriolis matrix is multiplied by, the third is a vector multiplied by the correction matrix C_k and next arguments are arguments of all elements of the above model. It is easy to show, that the correction matrix C_k, which is a linear combination of matrices $\dot{Q}^*$ and C^* (see Property 2), can be expressed as a linear function of the same parameters as the two mentioned matrices.

3 Control problem statement.

In the introduction we have written that in the paper we will find a control law preserving the proper cooperation between the mobile platform and the rigid manipulator mounted on top of it. The manipulator has to follow the desired trajectory $q_{rd}(t)$ which defines a task of this subsystem. The task of the platform is to follow the desired trajectory $(x_d(t), y_d(t), \theta_d(t))$ (it is so-called 'posture tracking') without skip of wheels-it means that the desired trajectory for the mobile platform must satisfy the nonholonomic constraint (1) [4].

Our purpose in this paper will be to address the following control problem for mobile manipulators:

> Find a control law $u^T = (u_m^T, u_r^T)$ such that a mobile manipulator with parametric uncertainty in the dynamic model follows a desired trajectory $(x_d(t), y_d(t), \theta_d(t), q_{rd}^T(t))$ without skip and surge of platform wheels and tracking errors converge against zero.

We treat the solution to the control problem proposed in this paper as a backstepping procedure [4]:

1. Find the vector of velocities η_r for equation (7) which guaranties the convergence the real trajectory of the platform to the desired trajectory $(x_d(t), y_d(t), \theta_d(t))$ in a presence of nonholonomic constraint. This problem is nontrivial because a dimension of the vector η is essentially smaller than a dimension of the vector of states for each class of mobile platforms but for every mobile platform exists a control algorithm (due to Brockett's condition [9] - depending on time or not smooth) preserving asymptotic tracking of desired trajectory for whole vector of states. In simulations we have taken into consideration a new dynamic control algorithm introduced by Lee, Lee and Teng for mobile platform of (2,0) class.
2. If we know the velocities $\eta_r(t)$ which satisfy the above condition, define the control algorithm preserving the convergence of the trajectory of the whole system (the mobile manipulator) to the desired trajectory $(x_d, y_d, \theta_d, q_{rd}^T)$ in a case of parametric uncertainty in the dynamic model.

4 Control algorithm for mobile manipulator.

In this section we present main result, namely the mathematical solution to the problem considered in this paper.

Theorem 1: [6]
Consider a model of a mobile manipulator given by (6) and (7). Let desired trajectories satisfy the nonholonomic constraint and $\eta_r(t)$ be a solution to the posture tracking problem for wheeled mobile platform. We propose following control law u for the mobile manipulators

$$u(t) = (B^*(q))^{-1}\{Q^*(q,\hat{a})\dot{z}_d + C^*(q,z,\hat{a})z_d + D^*(q,\hat{a}) - C_k(q,z,\hat{a})e_z - K(t)e_z\} =$$

$$= (B^*(q))^{-1}\{Y_k(\dot{z}_d, z_d, e_z, z, q)\hat{a} - K(t)e_z,\} \qquad (8)$$

where

$$e_z = \begin{bmatrix} \eta - \eta_r \\ \dot{q}_r - \dot{q}_{ref} \end{bmatrix} = \begin{bmatrix} e_v \\ s \end{bmatrix} = \begin{bmatrix} e_v \\ \dot{e} + \Lambda e \end{bmatrix},$$

$$K(t) = \begin{bmatrix} K_m(t) & 0 \\ 0 & K_d(t) \end{bmatrix}, \qquad (9)$$

$$e = q_r - q_{rd}, \qquad \Lambda = \Lambda^T > 0,$$

and $\hat{a}(t)$ is a vector of the parameter estimates. $K(t)$ is a matrix of the dynamic gains of the universal adaptive controllers expressed as

$$K(t) = diag\{K_i(t)\} \quad i = 1,..,m+p, \qquad (10)$$

where $K_i(0)$ should be greater than 0 and where $K_i(t)$ is the gain of the local universal adaptive controller connected to the i-th coordinate. Each of the local universal adaptive controllers has a form

$$\dot{K}_{mi}(t) = | e_{vi} |^2, \quad i = 1,..,m, \qquad (11)$$

$$\dot{K}_{dj}(t) = | s_j |^2, \quad j = 1,..,p. \qquad (12)$$

Let the adaptation law for parameters be as follows

$$\dot{\hat{a}}(t) = \dot{\tilde{a}}(t) = -\Gamma^{-1}Y_k^T(\dot{z}_d, z_d, e_z, z, q)e_z, \qquad (13)$$

where $\hat{a}(t) - a = \tilde{a}(t)$ is an estimation error for unknown parameters. Then for every $q(0)$ a solution $(e_z(t), K(t))$ of the closed-loop system has the following properties:

$$\lim_{t\to\infty} e_z(t) \to 0,$$

$$\lim_{t\to\infty} K_i(t) \text{ exists and is finite,} \quad i = 1,..,m+p.$$

It is easy to observe that the presented control algorithm is a modification and generalization of well known Slotine & Li adaptive control algorithm. Variable s is defined similar to classical Slotine & Li algorithm and it is called a sliding mode.

5 Proof of the main theorem.

To prove the Theorem 1 we have to get the equation of the closed-loop system. After some calculation we get

$$Q^*\dot{e}_z + C^*e_z + C_k e_z = Y_k(\dot{z}_d, z_d, e_z, z, q)\tilde{a} - K(t)e_z. \qquad (14)$$

5.1 Proof of the asymptotic convergence of tracking errors to zero.

As a Lyapunov like function we take

$$V(e_v, s, \tilde{a}, t) = \frac{1}{2}e_z^T Q^* e_z + \frac{1}{2}\tilde{a}^T\Gamma\tilde{a}, \qquad \Gamma = \Gamma^T > 0.$$

The time derivative of V along the trajectories of the closed-loop system (14) with the adaptation law (13) is equal to

$$\begin{aligned}\dot{V} &= e_z^T Q^*\dot{e}_z + \frac{1}{2}e_z^T\dot{Q}^*e_z + \tilde{a}^T\Gamma\dot{\tilde{a}} \\ &= e_z^T\{-(C^* + C_k)e_z + Y_k(\dot{z}_d, z_d, e_z, z, q)\tilde{a} - K(t)e_z\} \\ &+ \frac{1}{2}e_z^T\dot{Q}^*e_z + \tilde{a}^T\Gamma\left\{-\Gamma^{-1}Y_k^T(\dot{z}_d, z_d, e_z, z, q)e_z\right\}.\end{aligned}$$

From Property 2 we have

$$\begin{aligned}\frac{1}{2}e_z^T\dot{Q}^*e_z &= \frac{1}{2}e_z^T(C^* + C_k)e_z + \frac{1}{2}e_z^T(C^* + C_k)^T e_z \\ &= e_z^T(C^* + C_k)e_z,\end{aligned}$$

therefore

$$\dot{V} = -e_z^T K(t)e_z = -e_v^T K_m(t)e_v - s^T K_d(t)s \leq 0.$$

From (11) and (12) we conclude that the gains in matrices $K_m(t)$ and $K_d(t)$ can only increase with time and this fact we use to evaluate $\dot{V}$ in the following way

$$\dot{V} \leq -e_v^T K_m(0)e_v - s^T K_d(0)s \leq 0. \qquad (15)$$

According to Yoshizawa-La Salle Theorem [3] (it is a version of La Salle Invariance Principle for autonomous systems) we conclude that e_v and s converge against zero asymptotically fast.

5.2 Proof of the convergence of the matrix gains $K(t)$ to the limited number.

From (15) and from properties of square forms we have the following evaluation of $\dot{V}(t)$

$$\begin{aligned}\dot{V} &\leq -e_v^T K_m(0)e_v - s^T K_d(0)s \\ &\leq - \| e_v \|^2 \underline{\lambda}(K_m(0)) - \| s \|^2 \underline{\lambda}(K_d(0)) \leq 0\end{aligned}$$

where $\underline{\lambda}(K_m(0))$ is the smallest eigenvalue of the matrix K_m for $t = 0$ and $\underline{\lambda}(K_d(0))$ is the smallest eigenvalue of the matrix K_d for $t = 0$. We can rewrite the above inequality as follows

$$\dot{V}(t) \leq -\alpha \| e_v \|^2 \leq 0, \qquad \alpha = \underline{\lambda}(K_m(0)) > 0.$$

After integration we get

$$V(t) \leq V(0) - \alpha \int_0^t \| e_v(\tau) \|^2 \, d\tau. \qquad (16)$$

On the other hand, from definition of the Lyapunov like function we know that

$$\forall \quad t \geq 0 \qquad\qquad V(t) \geq 0. \qquad (17)$$

Using (16) and (17) we have

$$0 \leq V(0) - \alpha \int_0^t \| e_v(\tau) \|^2 \, d\tau, \qquad \forall \ t \geq 0. \qquad (18)$$

From (18) and after some manipulation we obtain the following expression

$$\int_0^t \| e_v(\tau) \|^2 \, d\tau \leq \frac{V(0)}{\alpha} < \infty,$$

also

$$\begin{aligned} K_{mi}(\infty) &= \int_0^\infty | e_{vi}(\tau) |^2 \, d\tau + K_{mi}(0) \\ &\leq \int_0^\infty \| e_v(\tau) \|^2 \, d\tau + K_{mi}(0) \\ &\leq \frac{V(0)}{\alpha} + K_{mi}(0) < \infty. \qquad (19) \end{aligned}$$

Similarly, we can estimate the boundary of $K_{di}(t)$ as follows

$$K_{di}(\infty) \leq \frac{V(0)}{\beta} + K_{di}(0) < \infty, \qquad \beta = \underline{\lambda}(K_d(0)) > 0.$$

This completes the proof.

6 Simulation study.

We consider in this paper a wheeled mobile manipulator which consists of a two wheel mobile platform belonging to the class (2,0) and a rigid RTR manipulator. The object of our simulation is presented in Fig.1. The motion of the mobile manipulator is fully described by the following vector of original generalized coordinates:

$$q = \begin{bmatrix} x & y & \theta & \phi_1 & \phi_2 & \theta_1 & \theta_2 & \theta_3 \end{bmatrix}^T,$$

where:

x, y	–	Cartesian position of the platform,
θ	–	orientation of the mobile platform,
ϕ_i	–	angular position of the fixed wheels,
θ_i	–	joint variables of the manipulator.

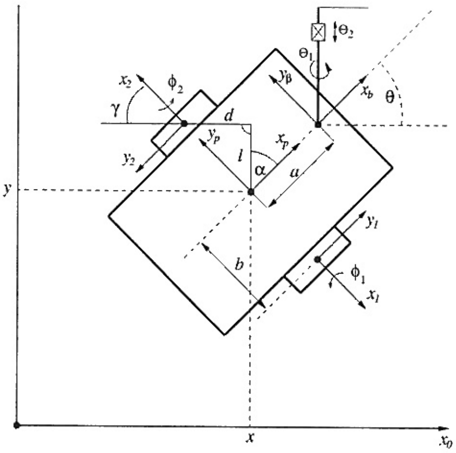

Figure 1. Mobile manipulator which consists of RTR manipulator mounted on a top of wheeled mobile platform of the (2,0) class.

6.1 Control algorithm for the kinematics - Lee, Lee & Teng control algorithm .

We consider the kinematics for a wheeled mobile platform of the (2,0) class

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} \cos\theta \cdot v \\ \sin\theta \cdot v \\ w \end{pmatrix}, \quad \begin{pmatrix} v \\ w \end{pmatrix} = \begin{pmatrix} \phi_1 + \phi_2 \\ \phi_1 - \phi_2 \end{pmatrix}.$$

A task of the mobile platform is to track some trajectory. The desired trajectory has to meet the kinematics which are consequence of existence of the nonholonomic constraint

$$\begin{pmatrix} \dot{x}_d \\ \dot{y}_d \\ \dot{\theta}_d \end{pmatrix} = \begin{pmatrix} \cos\theta_d \cdot v_d \\ \sin\theta_d \cdot v_d \\ w_d \end{pmatrix}. \qquad (20)$$

Velocities v_d and w_d are these velocities which a system has to have to execute a motion exactly along the desired trajectory (x_d, y_d, θ_d). To present control algorithm it is necessary to express tracking errors in so-called reference frame

$$\begin{pmatrix} x_e \\ y_e \\ \theta_e \end{pmatrix} = Rot(Z, -\theta) \begin{pmatrix} e_x \\ e_y \\ e_t \end{pmatrix} =$$

$$\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} x_d - x \\ y_d - y \\ \theta_d - \theta \end{pmatrix}.$$

Lee, Lee & Teng control algorithm [4] ensures asymptotic convergence of the reference tracking errors (x_e, y_e, θ_e) to zero. For the sake of non-singularity of the $Rot(Z, -\theta)$ matrix we see that asymptotic convergence of reference tracking errors implies asymptotic convergence of simple tracking errors (e_x, e_y, e_t)

to zero. Control law v_r, w_r preserving the convergence of the reference tracking errors to zero is as follows

$$
\begin{aligned}
v_r &= k_0 x_e + v_d \cos\theta_e, \\
w_r &= 1 + \frac{\beta}{\alpha} + k_1(\theta_e + \frac{\varepsilon y_e}{1+\Lambda}), \\
\beta &= \varepsilon \left\{ \frac{\dot{h} y_e - h w_d x_e + h v_d \sin\theta_e}{1+\Lambda} + \right. \\
&- \left. \frac{h y_e}{(1+\Lambda)^2 \Lambda}(y_e v_d \sin\theta_e - k_0 x_e^2) \right\}, \\
\alpha &= 1 + \frac{\varepsilon h x_e}{1+\Lambda} - \frac{\varepsilon h \theta_e y_e}{(1+\Lambda)^2 \Lambda}, \\
\Lambda &= \sqrt{\theta_e^2 + x_e^2 + y_e^2}, \qquad (21)
\end{aligned}
$$

where $k_0, k_1 > 0$, $h(t) = 1 + \gamma\cos(t - t_0)$, $0 < \gamma < 1$ and $0 < \varepsilon < \frac{1}{2(1+\gamma)}$. The most important thing is to define the appropriate $h(t)$ function and γ and ε parameters. A form of $h(t)$ function and a choice of the parameters depends on a choice of the desired trajectory. In further considerations we will test a behaviour of our object during tracking of the following desired trajectory

$$(x_d, y_d, \theta_d, \theta_{1d}, \theta_{2d}, \theta_{3d}) = \left(10\sin t, -10\cos t, t, \frac{\pi}{2}, 1, \frac{\pi}{2}\right).$$

For such desired trajectory the function h should be chosen as $h(t) = 1$. As the object of simulations we will choose a model of the wheeled mobile manipulator presented in Fig.1. The dynamics of the considered mobile manipulator have a very complicated form and therefore will be omitted in this paper (but they are presented in [5]). The goal of simulations is to examine the behaviour of the mobile manipulator in few situations: with only kinematic control (Lee, Lee & Teng control algorithm without the dynamics), with kinematic control and exact linearization of the dynamics, with the control algorithm introduced in this paper. We want also to show the asymptotic trajectory tracking for joint coordinates of the rigid manipulator. The simulations have been made with the MATLAB package and the SIMULINK toolbox [1].

7 Concluding remarks.

In the paper the new adaptive control algorithm for mobile manipulator has been introduced. This control law is in fact a backstepping procedure. First, for the control of the kinematics every control algorithm preserving so-called 'posture tracking' (trajectory tracking for position and orientation coordinates) can be

[1]MATLAB package and the SIMULINK toolbox were available thanks to Wroclaw Centre of Networking and Supercomputing.

a) b) c)

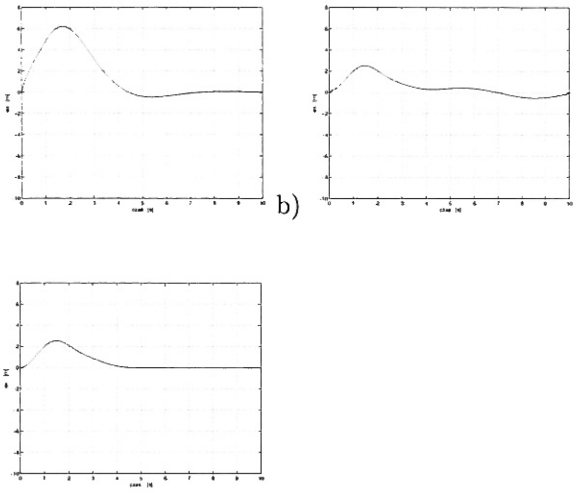

Figure 2. Tracking error e_x for mobile platform (2,0) for different control algorithms: a) new adaptive control, b) exact linearization, c) Lee, Lee & Teng without the dynamics.

used. We prefer the dynamic algorithms which are smooth because the function describing the control for the kinematics (that is $\eta_r(t)$) and its first derivative ($\dot{\eta}_r(t)$) have to exist. The new adaptive control algorithm we treat as a second step in the backstepping procedure. This control law makes possible to control motion of the whole system, e.g. the dynamics and the kinematics with parameter uncertainty. Theoretical considerations have been completed with simulations which have shown that the influence of the dynamics with known parameters as well as the dynamics with unknown parameters on the behaviour of the mobile manipulator is noticeable.

8 Acknowledgements

This paper contributes to the research project *Modelling, motion planning and control of mobile manipulators* sponsored by the Polish State Committee of Scientific Research.

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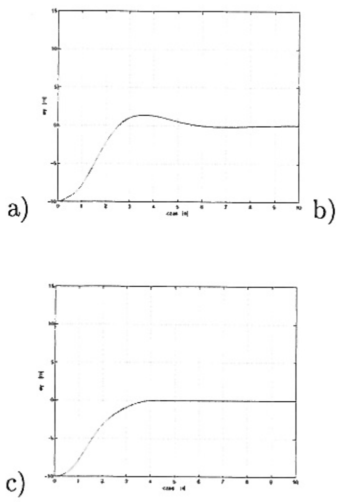

Figure 3. Tracking error e_y for mobile platform (2,0) for different control algorithms: a) new adaptive control, b) exact linearization, c) Lee, Lee & Teng without the dynamics.

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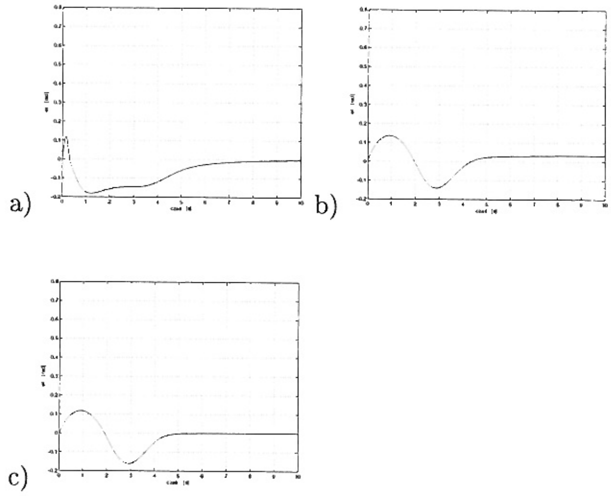

Figure 4. Tracking error e_t for mobile platform (2,0) for different control algorithms: a) new adaptive control, b) exact linearization, c) Lee, Lee & Teng without the dynamics.

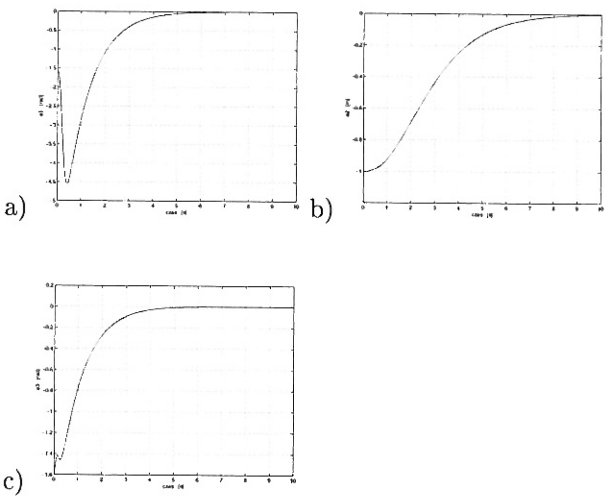

Figure 5. Hypothetical tracking errors for joints of the rigid RTR manipulator: a) e_1, b) e_2, c) e_3.