A new adaptive terminal sliding mode control scheme for robotic manipulators designed following an energy-based approach

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Abstract

In this paper it is presented a terminal sliding-mode adaptive control scheme for robotic manipulators designed following an energy-based approach. The control comprises two basic terms: a composite adaptive term which implements a feedback law for compensating the modelled dynamics and a non-linear sliding-mode term for overcoming the unmodelled dynamics and perturbations. The resulting closed-loop system is proved to be stable and it is also shown that the trajectory-tracking error converges to zero in finite time. Moreover, an upper bound of this error convergence time is calculated. Finally, the design is evaluated by means of simulations.

1 Introduction

For more than a decade, the design of adaptive control schemes for robot manipulators has been an active field of research (Craig 1988, Barambones and Etxebarria 1999). Because of the inherent nonlinear and time-varying nature of robotic manipulators, adaptive controllers have been found appropriate to achieve consistent performance in the presence of payload and configuration variations (Ortega and Spong 1989). From a theoretical viewpoint the existence of convergent adaptive control laws has been established, although in most cases the convergence is asymptotic, which implies in principle an infinite convergence time.

On the other hand, variable structure control has been proved to be a solid framework to achieve robust performance against uncertainties and external disturbances, and as a result, sliding control has been successfully used in robotic applications (Slotine and Li 1991). A well reported drawback of conventional sliding control is the rather high gains usually involved in the controllers, which seriously limits their practical implementation. Moreover, as it happens in most nonlinear control schemes, often analyzed using Lyapunov-like methods, conventional sliding techniques only guarantee the asymptotic error convergence, which means that the tracking error does not converge to zero in a finite time.

In this paper adaptive and sliding control methods are combined to be applied to the control of robotic manipulators (Slotine and Li 1991, Barambones and Etxebarria 2000) but with a new energy-based design that guarantees that the tracking error is eliminated in finite time. As a first ingredient of the design a composite adaptive scheme which drives the parameter adaptation using both the tracking error and the prediction error, is used to get faster parameter convergence and smaller tracking errors. This results in a decrease of the model dynamical uncertainties which allows to use lower sliding gains, thus resolving one of the implementation problems addressed above. Moreover, a nonlinear filtered error is introduced in the switching control law, similar to the so-called terminal sliding mode control (Zhihong and O'Day 1999), which leads to a tracking error converging to zero in finite time. This property contrasts with the theoretical infinite convergence time associated to the asymptotic behavior of conventional adaptive and sliding control schemes.

2 Problem formulation and control design

The vector equations of motion of a n-link robot manipulator can be written as:

$$\mathcal{T} = M(\Theta)\ddot{\Theta}+C(\Theta,\dot{\Theta})\dot{\Theta}+G(\Theta)+F(\Theta,\dot{\Theta})+D(t) \quad (1)$$

where $\mathcal{T}$ is a $n \times 1$ vector of joint torques; $\Theta, \dot{\Theta}$ and $\ddot{\Theta}$ are the $n \times 1$ vectors of joints positions, speed and accelerations, respectively; $M(\Theta)$ is the $n \times n$ mass matrix of the manipulator; $C(\Theta, \dot{\Theta})$ is an $n \times n$ vector of centrifugal and Coriolis terms which is chosen so that the matrix $\dot{M} - 2C$ is skew-symmetric; $G(\Theta)$ is an $n \times 1$ vector of gravitational terms, $F(\Theta, \dot{\Theta})$ is an $n \times 1$ vector of friction terms, and $D(t)$ is an $n \times 1$ vector whose elements represent the dynamic uncertainties caused by unmodelled dynamics and noise. It is assumed that this uncertainty vector is bounded (i.e $\|D(t)\| \leq \rho \quad \forall t$).

The equation of motion (1) form a set of coupled nonlinear ordinary differential equations which are quite complex, even for simple manipulators. However, as it is well known this equations have some useful properties:

(i) $\underline{m}I \leq M(\Theta) = M^T(\Theta) \leq \overline{m}I, \quad 0 < \underline{m} \leq \overline{m}$.

(ii) Given a proper definition of the unknown parameter vector, it is possible to obtain the following linear dependence:

$$M(\Theta)\ddot{\Theta} + C(\Theta,\dot{\Theta})\dot{\Theta} + G(\Theta) + F(\Theta,\dot{\Theta}) = Y(\Theta,\dot{\Theta},\ddot{\Theta})A$$

where A is an q-dimensional vector containing the system dynamical parameters and $Y(\Theta, \dot{\Theta}, \ddot{\Theta})$ is an $n \times q$ matrix often refereed to as regressor matrix, whose elements are nonlinear known functions.

(iii) There are a parameter vector B of dimension $v \leq r$, whose elements are a subset of the parameter vector A introduced in (ii), and a vector $H(\dot{\Theta}, \Theta)$ of dimension v, whose elements are a nonlinear functions, so that the manipulator energy can be written in the next form:

$$E(t) = H(\dot{\Theta},\Theta)^T B + d(t)$$

where $d(t)$ is the term which take into account the uncertainties.

(iv) From the energy conservation principle it is deduced that the change of this is equal to the power supplied by the motors minus the power consumed due to the frictions.

$$\left[\mathcal{T} - F(\Theta,\dot{\Theta})\right]^T \dot{\Theta} = \frac{dE}{dt}$$

From the properties **(iii)** and **(iv)** and taken into account that the friction term can be written as a product of a unknown parameter vector χ (of dimension $r - v$) for a regressor Φ, this is:

$$F(\Theta,\dot{\Theta})^T\dot{\Theta} = \Phi(\Theta,\dot{\Theta})^T\chi$$

it is concluded that:

$$\mathcal{T}^T\dot{\Theta} - \Phi(\Theta,\dot{\Theta})^T\chi = \frac{dE}{dt} = \frac{dH(\dot{\Theta},\Theta)}{dt}B + \dot{d}(t) \quad (2)$$

Filtering the two members of equation (2 by a first order low pass filter it is obtained:

$$\frac{1}{s+\lambda_f}[\mathcal{T}^T\dot{\Theta}] = \left[H(\dot{\Theta},\Theta) - \frac{\lambda_f}{s+\lambda_f}H(\dot{\Theta},\Theta)\right]B + \frac{1}{s+\lambda_f}\left[\Phi(\Theta,\dot{\Theta})^T\chi\right]$$

where s is the *Laplace* transform variable and λ_f is the cut frequency of the filter, which is adequately chosen to eliminate the uncertainties term.

The previous equation can be written in the next linear in the parameters form:

$$y = W(\dot{\Theta},\Theta)^T A \quad (3)$$

where y is the filtered power:

$$y = \frac{1}{s+\lambda_f}[\mathcal{T}^T\dot{\Theta}]$$

W is a vector formed by non-linear known functions:

$$W(\dot{\Theta},\Theta) = \begin{bmatrix} H(\dot{\Theta},\Theta) - \dfrac{\lambda_f H(\dot{\Theta},\Theta)}{s+\lambda_f} \\ \dfrac{1}{s+\lambda_f}\Phi(\Theta,\dot{\Theta}) \end{bmatrix} \quad (4)$$

and A is the full parameter vector defined in the property **(ii)**, which is composed by the dynamic coefficients of the robot plus the friction term coefficients.

$$A = \begin{bmatrix} B \\ \chi \end{bmatrix}$$

The control problem may be formulated as follows: Let $\Theta_d(t)$ be a given twice differentiable desired trajectory, and define the tracking error as $E(t) = \Theta(t) - \Theta_d(t)$. The control objective is to ensure the tracking error to converge to zero, while maintaining bounded all signals in the system.

Assuming the knowledge of a bound on the uncertainty vector, ρ, let us define the vector control input to be of the form:

$$\mathcal{T} = \hat{\mathcal{T}} - KS^r - \mathcal{P}\,\mathrm{sgn}(S) \quad (5)$$

where $K = diag(k_1,\ldots,k_n) > 0$, $\mathcal{P} = \mathrm{diag}(\rho)$, $S^r = [s_1^r,\ldots,s_n^r]^T$, $\mathrm{sgn}(S) = [\mathrm{sgn}(s_1),\ldots,\mathrm{sgn}(s_n)]^T$, and S is the sliding variable defined as $S = \dot{E} + \Lambda E^p$ with $\Lambda = \mathrm{diag}(\lambda_1,\ldots,\lambda_n) > 0$ and $E^p = [e_1^p,\ldots,e_n^p]^T$, and the numbers p and r are defined so as to satisfy also the following condition:

$$p, r = \frac{z_1}{z_2}, \quad z_1, z_2 \in \mathbf{Z}_+, \quad z_1 < z_2, \quad z_1, z_2 \text{ odd}, \quad \text{and } p > \tfrac{1}{2} \quad (6)$$

Also $\hat{T}$ is the predicted torque, defined as:

$$\begin{aligned}\hat{T} &= \hat{M}(\Theta)\ddot{\Theta}_r + \hat{C}(\Theta,\dot{\Theta})\dot{\Theta}_r + \hat{G}(\Theta) + \hat{F}(\Theta,\dot{\Theta}) \\ &= Y(\Theta,\dot{\Theta},\dot{\Theta}_r,\ddot{\Theta}_r)\hat{A} \qquad (7)\end{aligned}$$

where $\hat{M}$, $\hat{C}$, $\hat{G}$, $\hat{F}$, and $\hat{A}$ denote the estimates of M, C, G, F and A respectively, $\dot{\Theta}_r = \dot{\Theta}_d - \Lambda E^p = \dot{\Theta} - S$ and $\ddot{\Theta}_r = \ddot{\Theta}_d - p\Lambda\,\mathrm{diag}(e_1^{p-1},\ldots,e_n^{p-1})\dot{E}$.

The dynamical parameters of the system are updated according to the following composite adaptive law:

$$\dot{\hat{A}} = -\Gamma\left[Y^T(\Theta,\dot{\Theta},\dot{\Theta}_r,\ddot{\Theta}_r)\,S + \gamma W^T(\Theta,\dot{\Theta})e_f^r\right] \quad (8)$$

where Γ is a positive definite matrix, γ is a positive number, and

$$e_f = \hat{y} - y = W(\dot{\Theta},\Theta)\hat{A} - W(\dot{\Theta},\Theta)A = W(\dot{\Theta},\Theta)\tilde{A}$$

is the filtered prediction error.

Figure 1 illustrates the block diagram of this sliding adaptive control scheme.

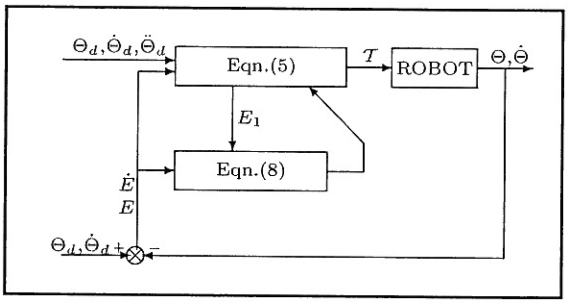

Figure 1: Proposed robust composite adaptive control scheme

Remark 1. It should be noted that in the terms $(\cdot)^r$ and $(\cdot)^p$, the power and root operations are odd, and that only the real roots are considered. Therefore this operations maintain the sign of their arguments.

3 Stability analysis

The following results on differential inequalities (Hale 1969) will be used for the subsequent stability analysis.

Definition . If $f(V,t)$ is a scalar function of scalars $V(t), t$ in some open connected set $\mathbf{D}$, then a function $V(t)$, $t_0 \le t < t_1$, $t_1 > t_0$ is a solution of the differential inequality

$$\dot{V}(t) \le f(V(t),t) \quad (9)$$

on $[t_0,t_1)$ if $V(t)$ is continuous on $[t_0,t_1)$ and its derivative on $[t_0,t_1)$ satisfies (9).

Lemma 1 . Let $f(y(t),t)$ be continuous on an open connected set $\mathbf{D} \in \mathbf{R}^2$ and assume that the initial value problem for the scalar equation,

$$\dot{y}(t) = f(y(t),t), \quad y(t_0) = y_0 \quad (10)$$

has a unique solution. If $y(t)$ is a solution of (10) on $t_0 \le t < t_1$ and $V(t)$ is a solution of (9) on $t_0 \le t < t_1$ with $V(t_0) \le y(t_0)$, then $V(t) \le y(t)$ for $t_0 \le t < t_1$.

Lemma 2 . Assume that a continuous positive-definite function $V(t)$ satisfies the following differential inequality:

$$\dot{V}(t) \le -\alpha V^\eta, \quad \forall t \ge t_0,\ V(t_0) \ge 0, \quad (11)$$

where $\alpha > 0$, $0 < \eta < 1$ are constants. Then, for any given t_0, $V(t)$ satisfies the following inequality:

$$\begin{aligned}V^{1-\eta}(t) \le V^{1-\eta}(t_0) - \alpha(1-\eta)(t-t_0) \\ t_0 \le t \le t_1 \qquad (12)\end{aligned}$$

and $\quad V(t) = 0, \quad \forall t \ge t_1$

where t_1 is given by:

$$t_1 = t_0 + \frac{V^{1-\eta}(t_0)}{\alpha(1-\eta)} \quad (13)$$

Before presenting the main stability results, the following assumption is stated:

$\mathcal{A}1$. The regressor matrix satisfies the following condition:

$$\int_t^{t+T} Y^T Y dt \ge \alpha I \quad \alpha, T > 0 \quad (14)$$

This condition is verified if the regressor matrix Y satisfies the so called "persistent excitation" condition which always can be achieved by choosing a sufficiently rich-in-frequency reference signal.

Now the following result can be stated:

Theorem 1 *Consider the robotic maniputator given by (1). Then, if assumption ($\mathcal{A}1$) is verified, the control law (5) with the adaptation law (8) lead the closed loop outputs Θ and their derivatives $\dot{\Theta}$ to track asymptotically the desired trajectories Θ_d and their derivatives $\dot{\Theta}_d$. Moreover, the tracking error $E = \Theta - \Theta_d = 0$ for $t \ge t_T$ with $t_T < \infty$.*

Proof: Define the following Lyapunov function candidate:

$$V = V_1 + V_2 = \frac{1}{2}S^T M S + \frac{1}{2}\tilde{A}^T\Gamma^{-1}\tilde{A} \quad (15)$$

whose time-derivative is:

$$\begin{aligned}
\dot{V} &= S^T M\dot{S} + \frac{1}{2}S^T\dot{M}S + \tilde{A}^T\Gamma^{-1}\dot{\tilde{A}} \\
&= S^T M\ddot{\Theta} - S^T M\ddot{\Theta}_r + \frac{1}{2}S^T\dot{M}S + \tilde{A}^T\Gamma^{-1}\dot{\tilde{A}} \\
&= S^T\left[\mathcal{T} - C(S+\dot{\Theta}_r) - G - F - D\right] - \\
&\quad - S^T M\ddot{\Theta}_r + \frac{1}{2}S^T\dot{M}S + \tilde{A}^T\Gamma^{-1}\dot{\tilde{A}} \\
&= S^T\left[\mathcal{T} - M\ddot{\Theta}_r - C\dot{\Theta}_r - G - F - D\right] + \\
&\quad + \frac{1}{2}S^T\left[\dot{M} - 2C\right]S + \tilde{A}^T\Gamma^{-1}\dot{\tilde{A}} \\
&= S^T\left[\mathcal{T} - Y(\Theta,\dot{\Theta},\dot{\Theta}_r,\ddot{\Theta}_r)A - D\right] + \tilde{A}^T\Gamma^{-1}\dot{\tilde{A}} \\
&= S^T\left[Y\hat{A} - KS^r - \mathcal{P}\,\mathrm{sgn}(S) - YA - D\right] + \\
&\quad + \tilde{A}^T\Gamma^{-1}\dot{\tilde{A}} \\
&= S^T Y\tilde{A} - S^T K S^r - \rho^T|S| - D^T S - \\
&\quad - \tilde{A}^T Y^T S - \gamma\tilde{A}^T W^T e_f^r \\
&\leq -S^T K S^r - \gamma e_f\, e_f^r \qquad (16)
\end{aligned}$$

The first term on the right-hand of $\dot{V}$ satisfies:

$$S^T K S^r = \sum_{i=1}^{n} k_i|s_i|^{r+1} \geq \alpha_1\left[\sum_{i=1}^{n}\frac{1}{2}\overline{m}s_i^2\right]^\eta \geq \alpha_1 V_1^\eta \qquad (17)$$

where $\eta = \dfrac{1+r}{2} < 1$, $\alpha_1 = k_{min}\cdot\left(\dfrac{2}{\overline{m}}\right)^\eta$ with $k_{min} = min\{k_i\}$.

It should be noted that from the conditions over the number r (see eqn.6) the term $S^T S^r$ is a positive value.

The second term of V, V_2 satisfies:

$$V_2 = \frac{1}{2}\tilde{A}^T\Gamma^{-1}\tilde{A} \leq \frac{1}{\alpha_2}\tilde{A}^T W^T W\tilde{A} = \frac{1}{\alpha_2}e_f\, e_f \qquad (18)$$

with $\alpha_2 = \dfrac{2\lambda_{min}(W^T W)}{\lambda_{max}(\Gamma^{-1})}$ where $\lambda_{max}(\cdot)$ and $\lambda_{max}(\cdot)$ denotes the maximum and minimum eigenvalues of $(\cdot)$, respectively.

Finally, the second-right term of $\dot{V}$ satisfies:

$$\gamma e_f^T e_f^r = \gamma|e_f|^{r+1} \geq \gamma\alpha_2^\eta\left[\frac{1}{\alpha_2}e_f^2\right]^\eta \geq \gamma\alpha_2^\eta V_2^\eta \qquad (19)$$

Then, from the previous results, one can obtain that the V derivative verifies:

$$\dot{V} \leq -\alpha_1 V_1^\eta - \gamma\alpha_2^\eta V_2^\eta \leq -\alpha_m\left(V_1+V_2\right)^\eta = -\alpha_m V^\eta \qquad (20)$$

where $\alpha_m = min\{\alpha_1, \gamma\alpha_2^\eta\}$.

Since V is clearly a positive definite function and $\dot{V}$ is negative definite, the closed loop system is asymptotically stable. Moreover, from Lemma 2, it follows that the time t_r for reaching the sliding mode ($V^{1-\eta}(t_r) = 0$) is:

$$t_r = \frac{V^{1-\eta}(0)}{\alpha_m(1-\eta)} \qquad (21)$$

On the sliding mode, for $t \geq t_r$, it is verified:

$$S = \dot{E} + \Lambda E^p = 0 \qquad (22)$$

Now, it is defined:

$$V_s = \frac{1}{2}E^T E \qquad (23)$$

The time derivative of V_s on the sliding mode is:

$$\begin{aligned}
\dot{V}_s &= E^T\dot{E} \\
&= -E^T\Lambda E^p \\
&= -\sum_{i=1}^{n}\lambda_i(e_i^2)^{\eta_s} \\
&\leq -\lambda_{min}2^{\eta_s}\left[\frac{1}{2}\sum_{i=1}^{n}e_i^2\right]^{\eta_s} \\
&= -2^{\eta_s}\lambda_{min}V_s^{\eta_s} \qquad \forall t \geq t_r \qquad (24)
\end{aligned}$$

where $\eta_s = \frac{1+p}{2} < 1$, and $\lambda_{min} = \min(\lambda_i)$, $i = 1,\cdots n$. Therefore, from Lemma 2, it follows that $E(t) = 0$ for $t \geq t_T$ with:

$$t_T = t_r + \frac{V_s^{1-\eta_s}(t_r)}{2^{\eta_s}\lambda_{min}(1-\eta_s)} \qquad (25)$$

It should be noted that from the conditions over the number p (see eqn.6) the term $E^T\Lambda E^p$ is a positive value.

Remark 2. On the terminal sliding mode ($S = 0$), it follows from (22) that $\ddot{\Theta}_r$ can be expressed as:

$$\begin{aligned}
\ddot{\Theta}_r &= \ddot{\Theta}_d - p\,\Lambda\,\mathrm{diag}(e_1^{p-1},\ldots,e_n^{p-1})\dot{E} \\
&= \ddot{\Theta}_d + p\,\Lambda\,\mathrm{diag}(e_1^{p-1},\ldots,e_n^{p-1})\,\Lambda E^p \\
&= \ddot{\Theta}_d + p\,\Lambda^2 E^{2p-1} \qquad (26)
\end{aligned}$$

Therefore, p, as it was required in equation (6), must be chosen such that $p > \frac{1}{2}$ to ensure the boundedness of $\ddot{\Theta}_r$ as the tracking error E goes to zero.

4 Simulation results

In this section we will consider the control of the simple planar manipulator with two revolute joints. Let us fix the notation as follows: For each link i (i=1,2) θ_i denotes the joint angle; m_i denotes the mass; l_i denotes the length; l_{c_i} denotes the distance from the previous joint $(i-1)$ to the center of mass of link i; and I_i denotes the moment of inertia of link i about an axis perpendicular to the plane, passing through the center mass of link i.

Using the well-known Lagrangian equations in classical dynamics, one can show that the dynamic equations of the robot are:

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} + \begin{bmatrix} g_1 \\ g_2 \end{bmatrix} + \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} + \begin{bmatrix} c\dot{\theta}_2 & c\dot{\theta}_1 + c\dot{\theta}_2 \\ -c\dot{\theta}_1 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} \quad (27)$$

where the coefficients d_{ij}, g_i, f_i and c are:

$$\begin{aligned} d_{11} &= m_1 l_{c_1}^2 + I_1 + m_2[l_1^2 + l_{c_2}^2 + 2 l_1 l_{c_2} \cos\theta_2] + I_2 \\ d_{22} &= m_2 l_{c_2}^2 + I_2 \\ d_{12} &= d_{21} = m_2 l_1 l_{c_2} \cos\theta_2 + m_2 l_{c_2}^2 + I_2 \\ c &= -m_2 l_1 l_{c_2} \sin\theta_2 \\ g_1 &= m_1 l_{c_1} g \cos\theta_1 + m_2 g[l_{c_2}\cos(\theta_1+\theta_2) + l_1\cos\theta_1] \\ g_2 &= m_2 l_{c_2} g \cos(\theta_1+\theta_2) \\ f_1 &= v_1\dot{\theta}_1 \\ f_2 &= v_2\dot{\theta}_2 \end{aligned}$$

where g is the gravity acceleration, and v_i are the viscous friction coefficients.

Using a proper parametrization, the dynamic equation of the robot can be put in linear dependence. It is defined the following (2×7) regressor matrix $Y(\Theta, \dot{\Theta}, \dot{\Theta}_r, \ddot{\Theta}_r)$ whose elements are:

$$\begin{aligned} &y_{11} = \ddot{\theta}_{r1} && y_{21} = 0 \\ &y_{12} = \ddot{\theta}_{r2} && y_{22} = \ddot{\theta}_{r1} + \ddot{\theta}_{r2} \\ &y_{13} = (2\ddot{\theta}_{r1} + \ddot{\theta}_{r2})\cos\theta_2 - && y_{23} = \cos\theta_2 \ddot{\theta}_{r1} + \\ &-[\dot{\theta}_2\dot{\theta}_{r1} + (\dot{\theta}_1 + \dot{\theta}_2)\dot{\theta}_{r2}]\sin\theta_2 && +\sin\theta_2 \dot{\theta}_1 \dot{\theta}_{r1} \\ &y_{14} = g\cos\theta_1 && y_{24} = 0 \\ &y_{15} = g\cos(\theta_1+\theta_2) && y_{25} = g\cos(\theta_1+\theta_2) \\ &y_{16} = \dot{\theta}_1 && y_{26} = 0 \\ &y_{17} = 0 && y_{27} = \dot{\theta}_2 \end{aligned}$$

and the unknown dynamical parameters:

$$\begin{aligned} &a_1 = I_1 + m_1 l_{c_1}^2 + m_2 l_1^2 + I_2 + m_2 l_{c_2}^2 \\ &a_2 = m_2 l_{c_2}^2 + I_2 && a_3 = m_2 l_1 l_{c_2} \\ &a_4 = m_1 l_{c_1} + m_2 l_1 && a_5 = m_2 l_{c_2} \\ &a_6 = v_1 && a_7 = v_2 \end{aligned} \quad (28)$$

As it was indicated in the section 1, the robot energy can be put in the form $E = H(\Theta, \dot{\Theta})B$. For the robot of this example, the H elements are:

$$\begin{aligned} &h_1 = \dot{\theta}_1^2 && h_2 = \left(\dot{\theta}_2 + 2\dot{\theta}_1\right)\dot{\theta}_2 \\ &h_3 = 2\dot{\theta}_1\left(\dot{\theta}_1 + \dot{\theta}_2\right)\cos(\theta_2) \\ &h_4 = g\sin(\theta_1) && h_5 = g\sin(\theta_1+\theta_2) \end{aligned}$$

and the terms of B are:

$$B = [a_1 \; a_2 \; a_3 \; a_4 \; a_5]^T$$

Finally, the parametrization of the friction terms is made using the function matrix:

$$\Phi = \begin{bmatrix} \dot{\theta}_1 & 0 \\ 0 & \dot{\theta}_2 \end{bmatrix} \quad (29)$$

and the parameter vector:

$$\chi = \begin{bmatrix} a_6 \\ a_7 \end{bmatrix} \quad (30)$$

As can be observed, the parameter vector A is formed by the elements of the vector B plus the elements of the vector χ

In all the example the following values for the robot's parameters are assumed (SI units):

$$\begin{matrix} m_1 = 2 & m_2 = 1.2 & I_1 = 0.25 & I_2 = 0.15 & v_1 = 0.1 \\ l_1 = 1 & l_{c_1} = 0.4 & l_2 = 0.7 & l_{c_2} = 0.3 & v_2 = 0.1 \end{matrix}$$

Using this values, the real (unknown) dynamical parameters a_i are:

$$A = [2.00 \;\; 0.25 \;\; 0.38 \;\; 2.1 \;\; 0.33 \;\; 0.1 \;\; 0.1]^T \quad (31)$$

The following values have been chosen for the controller parameters:

$$\begin{gathered} \lambda = \mathrm{diag}([10\ 10]) \quad \Gamma = \mathrm{diag}([0.5\ 0.5\ 0.5\ 0.5\ 0.5]) \\ K = [1.5\ 1.5] \quad \mathcal{P} = \mathrm{diag}([0.1\ 0.1]) \\ r = \frac{9}{11} \quad p = \frac{13}{15} \quad \gamma = 0.1 \quad \lambda_f = 10 \end{gathered}$$

and it is assumed that there is an uncertainty around 50% in the dynamical parameters of the robot, and therefore these are incorrectly initialized with:

$$\hat{A}(0) = [1.5 \;\; 0.15 \;\; 0.25 \;\; 1.5 \;\; 0.25 \;\; 0 \;\; 0]^T \quad (32)$$

In the example, the robot start at position $\Theta = [0 \;\; 0]^T$ and the control objective is to follow the desired trajectory:

$$\Theta_d = \begin{bmatrix} 0.3\sin\left(0.7t - \frac{\pi}{2}\right) + 0.3\sin\left(0.1t - \frac{\pi}{2}\right) + 0.61 \\ 0.5\sin\left(0.9t - \frac{\pi}{2}\right) + 0.5\sin\left(0.1t - \frac{\pi}{2}\right) + 1.1 \end{bmatrix}$$

Figure 2 shows the tracking errors for the joints. As it can be observed, after a small time, both tracking errors tend to zero. Figure 3 shows the control signals for the joints.

The control signals are smooth because the dynamical composite parameter estimation allows to use a small sliding gain, since the sliding control only needs to compensate for the unmodelled dynamics and not for the parameter uncertainties. As a result, in the figures it can be observed that the chattering phenomenon is greatly reduced with respect to conventional sliding mode control schemes. Finally in Figure 4 it is presented the time evolution of the estimated parameters which, as it is observed, tend to the real parameters.

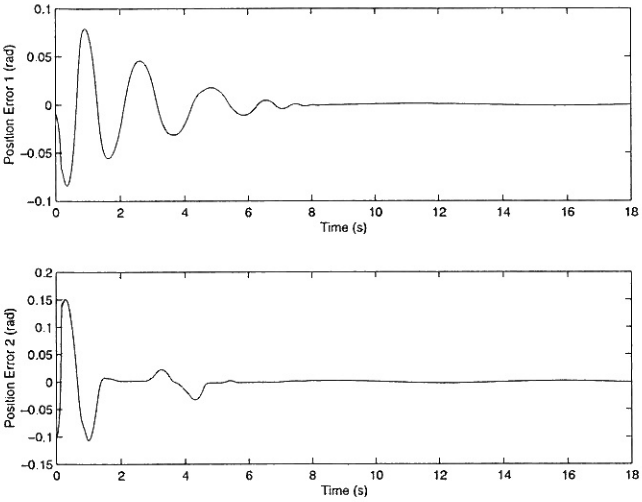

Figure 2: Error signal for the joints

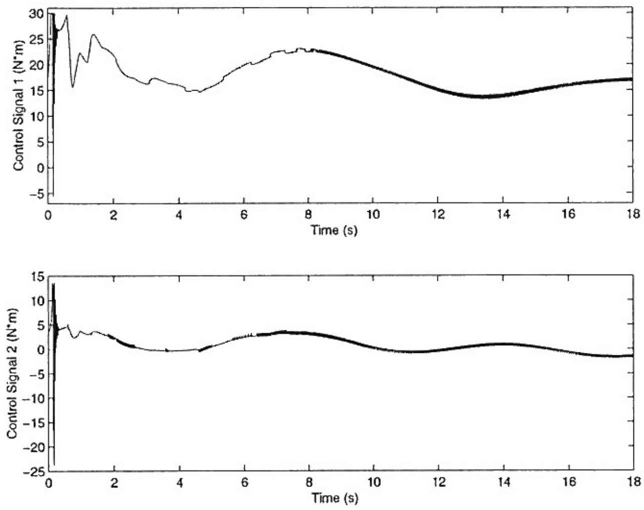

Figure 3: Control signal for the joints

5 Conclusions

In this paper an energy-based approach to the design of a robust adaptive control scheme for robotic manipulators has been presented. A combined adaptive-sliding control strategy has been employed. The adaptive part has been used to compensate the assumed dynamics of the model, and the sliding-mode part has been included in the design to overcome the unmodelled dynamics and perturbations. It has been shown that the use of a composite adaptive control law allows to use lower sliding gains, which leads to a more easily implementable design. Also it has been proved that the closed loop system remains stable and that the tracking errors are eliminated in finite time, and an upper bound of this time has been calculated. Finally, a set of simulations has been presented to illustrate the performance of the proposed design.

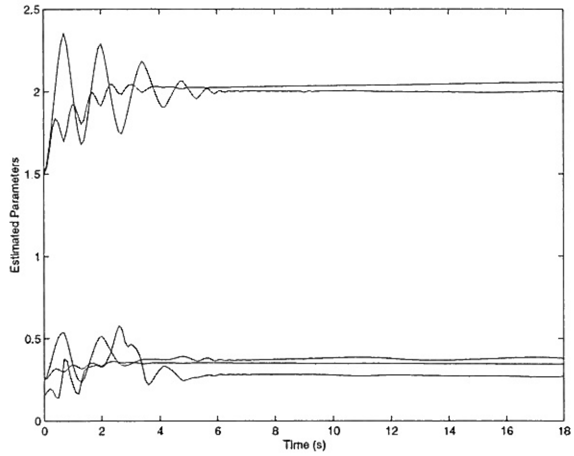

Figure 4: Estimated parameters of the system

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