Set Point Control for Serial Manipulators Using Generalized Velocity Components Method

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Abstract **– In this paper PD control using a method given in [4] is presented. It is shown that by proper choosing of the Lyapunov function candidate a dynamic system with appropriate feedback is asymptotically globally stable in joint space. Presented control is new in the sense that it is derived in terms of generalized velocity components dynamics described by Loduha and Ravani. New control was tested on a model of manipulator with two degrees of freedom.**

Key Words. Robot control; quasi-velocities; gravitational forces, energy Lyapunov function, Lyapunov stability analysis.

I. INTRODUCTION

Dynamic equations of motion for serial manipulators are second order, nonlinear differential equations. In order to simplify the mass matrix which appear in these equations one can introduce quasi-velocities. The advantages of using quasi-velocities are as follows: 1) one can obtain natural splitting between momentum differential equations and kinematic differential equations, 2) decomposition of mass matrix which provides deeper insight into manipulator dynamics is possible, 3) mass matrix is diagonal which simplify its inversion. A set of new quasi-velocities was proposed Loduha and Ravani [4]. Similarly, as Jain and Rodriguez [1], they have introduced, instead of transformation in configuration space, a diagonalizing transformation in a quasi-velocity space. They have presented a diagonalized equation of motion for holonomic and nonholonomic systems using variables called the generalized velocity components. In this paper we consider two new controls in terms of these variables. This paper is organized as follows. In second Section diagonalized equation of motion are described. New control in quasi-velocity space is given in the third Section. Simulation results are presented in the fourth Section. The last section contains concluding remarks.

II. DYNAMIC EQUATIONS OF MOTION

Consider classical equations of motion for robot manipulators described as [5]:

$$M(\theta)\ddot{\theta} + C(\theta,\dot{\theta}) + G(\theta) = T \qquad (1)$$

where $M(\theta)$ stands for the system mass matrix,
$\theta, \dot{\theta}, \ddot{\theta}$ are vectors of joint variables, velocities, and accelerations, respectively,
$C(\theta,\dot{\theta})$ denotes Coriolis forces term,
$G(\theta)$ is a vector of gravitational forces,
T is a vector of joint moments.

In reference [4] authors consider systems described by Kane's equations. They introduce a transformation between joint velocities and generalized velocity components defined as follows:

$$\dot{\theta} = \Upsilon u \qquad (2)$$
$$u = \Upsilon^{-1}\dot{\theta} \qquad (3)$$

where Υ is a rate transformation matrix which depends on kinematical and dynamical parameters. Calculating a time derivative of (2) we obtain:

$$\ddot{\theta} = \dot{\Upsilon}u + \Upsilon\dot{u}. \qquad (4)$$

Inserting above equation into (1) and multiplying both sides by Υ^T one can write:

$$M(\theta)(\dot{\Upsilon}u + \Upsilon\dot{u}) + C(\theta,\dot{\theta}) + G(\theta) = T \qquad (5)$$
$$\Upsilon^T M(\theta)\Upsilon\dot{u} + \Upsilon^T[M(\theta)\dot{\Upsilon}u + +C(\theta,\dot{\theta})] + \Upsilon^T G(\theta) = \Upsilon^T T. \qquad (6)$$

Finally, Eq.(6) can be written as follows:

$$N\dot{u} + C(\theta,\dot{\theta},u) = \pi \qquad (7)$$

where

$$N = \Upsilon^T M(\theta)\Upsilon \qquad (8)$$
$$C(\theta,\dot{\theta},u) = \Upsilon^T[M(\theta)\dot{\Upsilon}u + C(\theta,\dot{\theta})] \qquad (9)$$
$$\pi = \Upsilon^T(T - G(\theta)). \qquad (10)$$

In equations (8)-(10) N is a diagonal matrix congruent to mass matrix of manipulator $M(\theta)$ (this matrix can be obtained using method described in [4]), $u, \dot{u}$ are vectors of generalized velocity components and its time derivative, respectively, $C(\theta,\dot{\theta},u)$ is a new Coriolis force vector and π is a vector of quasi-forces. In this paper we consider dynamic equations described by (7). Generalized velocity components are, in fact, a kind of quasi-velocities known from analitycal mechanics.

Recall Kane's equation of motion in terms of the generalized velocity components [4] for $\mathcal{N}$ rigid bodies:

$$\Upsilon^T M(\theta)\Upsilon\dot{u} + \sum_{i=1}^{\mathcal{N}}[m_i\Upsilon^T J_i^T \frac{d}{dt}(J_i\Upsilon)u + +\Upsilon^T\Omega_i^T I_i\frac{d}{dt}(\Omega_i\Upsilon)u + \Upsilon^T\Omega_i^T W_i I_i\omega_i - \Upsilon^T J_i^T f_i -\Upsilon^T\Omega_i^T\tau_i] = 0 \qquad (11)$$

where

$$M(\theta) = \sum_{i=1}^{\mathcal{N}} [m_i J_i^T J_i + \Omega_i^T I_i \Omega_i] \tag{12}$$

and Υ is the rate transformation matrix transforming joint velocities into generalized velocity components space,

m_i is the mass of i-th body,

J_i is the partial derivative of i-th body's mass center position with respect to the inertial reference frame,

Ω_i is the partial derivative of body i's angular velocity with respect to the time derivative of the generalized coordinates vector,

I_i is the central inertia matrix,

W_i is the angular velocity matrix associated with the i-th body, and written in terms of body i's natural frame,

ω_i is the angular velocity of i-th body,

f_i is the resultant active force acting at the mass center of the i-th body,

τ_i is the resultant moment.

For serial manipulators we can write (11) and (12) using (3) in the form of (6) where:

$$\begin{aligned} C(\theta,\dot{\theta}) &= \sum_{i=1}^{\mathcal{N}} [(m_i J_i^T \dot{J}_i + \Omega_i^T I_i \dot{\Omega}_i)\dot{\theta} + \\ &+ \Omega_i^T W_i I_i \omega_i] \quad (13) \\ G(\theta) &= -\sum_{i=1}^{\mathcal{N}} J_i^T f_i \quad (14) \\ T &= \sum_{i=1}^{\mathcal{N}} \Omega_i^T \tau_i. \quad (15) \end{aligned}$$

Physical interpretation of generalized velocity components (GVC). By standard numbering every component $u_k = \dot{\theta}_k + \delta_k$ is a sum of k-th relative joint velocity $\dot{\theta}_k$ and additional terms $\delta_k = \sum_{i=\mathcal{N}}^{k+1} w_{ki}\dot{\theta}_i$. These terms reflect the influence of all links of the manipulator. Every term is equal the relative joint velocity multiplying by a weight coefficient. These coefficients depend on link masses and also on geometrical and kinematical parameters in actual time instant and have no physical units (for manipulator with rotational joints). They change with the time during the motion. The generalized velocity component u_k is near the relative joint velocity then elements of mass matrix on the diagonal have big values and behind it - much smaller. Bigger value of the mass matrix element indicates that the appropriate relative joint velocity has a bigger contribution in component u_k. One can say that the generalized velocity component u_k represents the resultant velocity in a local instantaneous frame of reference.

III. PD CONTROL IN INTERNAL SPACE

Let a constant equilibrium point be assigned for the system as the vector of desired joint variables $\tilde{\theta}$ similarly as in [5]. Next we propose PD control in joint space.

Recall that [5] for standard equation of motion (1) PD-control in joint space is as follows:

$$T = -c_D\dot{\theta} + c_P\tilde{\theta} + G(\theta) \tag{16}$$

where $\tilde{\theta} = \theta_d - \theta$ is the joint error between the desired and actual posture.

PROPOSITION. The feedback control described as

$$\pi = -c_D u + \Upsilon^T c_P \tilde{\theta} \tag{17}$$

in which c_D is a positive definite control gain matrix renders the system stable in the sense of Lyapunov.

Proof. As a Lyapunov function candidate consider the following expression

$$\mathcal{L}(\theta, u) = \frac{1}{2} u^T N u + \tilde{\theta}^T c_P \tilde{\theta}. \tag{18}$$

The time derivative of N equals (by $M = M(\theta)$):

$$\dot{N} = \frac{d}{dt}(\Upsilon^T M \Upsilon) = \dot{\Upsilon}^T M \Upsilon + \Upsilon^T \dot{M} \Upsilon + \Upsilon^T M \dot{\Upsilon}. \tag{19}$$

Next we calculate the time derivative of the function (18) using equations (7)-(10) and the fact that $\frac{1}{2}\dot{M}(\theta)\dot{\theta} - C(\dot{\theta},\theta) = 0$ [5]. After transposition of (2) and subtracting $T = \Upsilon^{-T}\pi + G(\theta)$ (which arises from (10)) one can obtain:

$$\begin{aligned} \frac{d\mathcal{L}}{dt} &= u^T N \dot{u} + \frac{1}{2} u^T \dot{N} u - \dot{\theta}^T c_P \tilde{\theta} = \\ &= u^T[\pi - \Upsilon^T M \dot{\Upsilon} u - \Upsilon^T C(\theta,\dot{\theta}) + \frac{1}{2}(\dot{\Upsilon}^T M \Upsilon + \\ &+ \Upsilon^T \dot{M} \Upsilon + \Upsilon^T M \dot{\Upsilon}) u - \Upsilon^T c_P \tilde{\theta}] = \\ &= u^T[\pi + \Upsilon^T(\frac{1}{2}\dot{M}\dot{\theta} - C(\theta,\dot{\theta})) + \\ &+ \frac{1}{2}(\dot{\Upsilon}^T M \Upsilon - \Upsilon^T M \dot{\Upsilon}) u - \Upsilon^T c_P \tilde{\theta}] = \\ &= u^T(\pi - \Upsilon^T c_P \tilde{\theta}) = -u^T c_D u \leq 0. \end{aligned} \tag{20}$$

The function candidate decreases as long as $u \neq 0$ for all system trajectories. Notice that $\frac{d\mathcal{L}}{dt} \equiv 0$ only if $u \equiv 0$. The closed loop system is described as:

$$N\dot{u} + C(\theta, \dot{\theta}, u) = -c_D u + \Upsilon^T c_P \tilde{\theta}. \tag{21}$$

In classical description [5] the system reaches "equilibrium posture" with $\dot{\theta} \equiv 0$ and $\ddot{\theta} \equiv 0$. Time derivative of Eq. (3) is $\dot{u} = \dot{\Upsilon}^{-1}\dot{\theta} + \Upsilon^{-1}\ddot{\theta}$. Hence if $\dot{\theta} \equiv 0$ and $\ddot{\theta} \equiv 0$ then $u \equiv 0$ and $\dot{u} \equiv 0$. Next, from (7) and (13) we get $C(\theta, \dot{\theta}, u) = 0$. At the "equilibrium" we have:

$$\Upsilon^T c_P \tilde{\theta} = 0 \tag{22}$$

and also $\tilde{\theta} = \theta_d - \theta \equiv 0$ because Υ^T is invertible.

Jain and Rodriguez [1] have proposed equations of motion in terms of normalized and unnormalized quasi-velocities, respectively. These equations updated with gravitational forces can be written as follows:

$$\dot{\nu} + C(\theta, \nu) + G_\nu(\theta) = \epsilon \tag{23}$$

$$D\dot{\xi} + C(\theta, \xi) + G_\xi(\theta) = \kappa \tag{24}$$

where ν is a vector of normalized quasi-velocities, ξ - vector of unnormalized quasi-velocities, $C(\theta, \nu)$ - vector of Coriolis and centrifugal forces in diagonalized normalized equations of motion, $C(\theta, \xi)$ - vector of Coriolis and centrifugal forces in diagonalized unnormalized equations of motion,

D - articulated inertia about joint axes matrix, ϵ - vector of normalized quasi-moments, κ - vector of unnormalized quasi-moments, $G_\nu(\theta)$ and $G_\xi(\theta)$ - gravitational forces in normalized and unnormalized equations of motion, respectively.

In case of normalized and unnormalized quasi-velocities PD controls [2, 3] have the following form:

$$\epsilon = -c_D\nu + m^{-1}(\theta)c_P\tilde{\theta} + G_\nu(\theta) \quad (25)$$

$$\kappa = -c_D\xi + D^{\frac{1}{2}}m^{-1}(\theta)c_P\tilde{\theta} + G_\xi(\theta) \quad (26)$$

where c_D and c_P are positive diagonal control gain matrices, $\tilde{\theta} = \theta_d - \theta$ and $m(\theta)$ is a spatial operator - "square root" of mass matrix $M(\theta)$, namely $M(\theta) = m(\theta)m^T(\theta)$.

Interpretation of quasi-velocity PD control. Here we explain the difference between PD quasi-velocity control and standard PD control. Substitute k_D for standard case instead of c_D in (16). Notice that control (17) is globally asymptotically stable and the gain matrices c_D, c_P are positive definite. Control (17) provides a linear velocity feedback in the u formulation and also a nonlinear term consisting of matrix Υ^T, which depends on configuration of the manipulator. This term is a momentum like quantity. Observe, however that $\tilde{\theta}$ as a difference between desired and actual position tends to zero and then as the main term remains quasi-velocity control.

Generalized force vector using (10), (3) and (17) can be written as follows:

$$T = -(\Upsilon^{-1})^T c_D \Upsilon^{-1}\dot{\theta} + c_P\tilde{\theta} + G(\theta) = -k_D\dot{\theta} + c_P\tilde{\theta} + G(\theta) \quad (27)$$

where $k_D = (\Upsilon^{-1})^T c_D \Upsilon^{-1}$. Let us explain the significance of Eq.(27). Instead of using control of $\dot{\theta}$ vector as would be done usually in velocity feedback (proportional damping) a momentum like quantity is realized in this case because matrix Υ is configuration dependent. Assuming c_D as a constant scalar in Eq.(27), the term $(\Upsilon^{-1})^T\Upsilon^{-1}$ acts as a state dependent feedback gain matrix. We can realize a linear position control but not velocity control because of nonlinear configuration dependent matrix k_D. This momentum like quantity do not vanish because this term acts as a state dependent feedback gain matrix. This is shown in Fig.15.

IV. SIMULATION RESULTS

In this section we present simulation results for model of manipulator KARI-2 consisting of two degrees of freedom (double pendulum) using controls (16,17,25,26) described in previous section. The KARI-2 robot is characterized by the following set of parameters:

- links masses: $m_1 = 0.75kg, m_2 = 7.92kg$,
- link inertias: $J_1 = 0.359kgm^2, J_2 = 2.597kgm^2$,
- distance between the axis of rotation to the mass center: $p_1 = 0.28m, p_2 = 0.5235m$,
- length of links: $l_1 = 0.525m, l_2 = 0.595m$.

The aim is to achieve the set point control for desired trajectory in joint space described with spline parabolic-linear trajectory (SPLT) where $\theta_1(t) = \theta_2(t)$ change in $(0; \frac{\pi}{2})[rad]$ using

$$|\ddot{\theta}_1| = |\ddot{\theta}_2| = 2\pi[rad/s^2], \quad (28)$$

and with time duration $t_f = 3[s]$. Simulations were performed using the fourth-order Runge-Kutta formula and fixed step size $0.005[s]$ and control coefficients as follows:

$$c_D(1) = c_D(2) = 25, \quad c_P(1) = c_P(2) = 500. \quad (29)$$

Next we assume that joints are numbered from the tip to the base of the manipulator. The simulation results are presented in Figures 1 to 14. In Figure 1 the desired trajectories $\theta_1 = \theta_2$ in joints space are shown. Figure 2 shows desired velocity profile. Figure 3 illustrates a comparison between joint velocities and generalized velocity components (GVC) for KARI-2 manipulator. Joint errors for standard PD controller are shown in Figure 4 and for Loduha-Ravani PD controller in Figure 5. We can see that for standard case oscillations are present even after several seconds and have bigger amplitude as for LR case. For LR PD controller oscillations vanish after about 5.5 seconds and errors are rather smooth but these errors during work of manipulator have bigger values than for standard PD controller. For Jain-Rodriguez normalized case (JRN) (Figure 6) oscillations do not vanish so fast for the first joint but error value for this joint is comparable with standard one. Maximal values of errors during the work of manipulator are about $0.025, 0.06, 0.025[rad]$ for the first joint and ST, LR and JRN controls, respectively, and about $0.05, 0.11, 0.11[rad/s]$ for the second joint (we can decrease errors assuming larger values c_P or smaller values c_D). These errors tends to zero faster for LR PD control than for JRN or ST controls. In Figures 7, 8 and 9 we compared joint moments for standard, LR and JRN cases, respectively. Oscillations of moments vanish after about 5 second for LR PD control. But for standard case and for JRN PD control they are present even after 6 second. Maximal values of moments are bigger for standard case than for JRN or LR cases (for the second joint $20, 13$ and $12[Nm]$, respectively; for the first values are almost comparable). During simulations it was determined that PD joint space controllers give the same results in LR case and in JRU (unnormalized) case. In Figure 10 comparison between joint velocities and quasi-velocities for LR control and gravity is shown. The time history is similar as earlier because in controller we do not use gravitational term. In Figures 11 we can see joint errors $e1, e2$ for ST and LR cases with gravity. They are similar time history as earlier. From Figure 12, which compares joint errors after $5.5[s]$, arises that errors for LR PD case tends faster to zero than for ST PD controller. In Figures 13 and 14 we compared joint moments for standard (ST) and LR cases with gravity. Maximal values of moments are bigger for standard case than for LR case (for the second joint about $70[Nm]$ and $60[Nm]$, respectively; for the first values are almost comparable). Finally LR control oscillations vanish earlier. We can explain this fact that dissipation of energy is faster than in standard controller and that both controllers involve system variables in different configuration.

V. CONCLUSIONS

In this work we have introduced a new PD control scheme described in terms of Loduha-Ravani generalized velocity components. Simulations results indicate that one can realise effective PD control using quasi-velocities (called for LR case generalized velocity components). It was determined that PD joint space controllers give the same results in LR case and in JRU (unnormalized). Using PD controller (LR or JRU) we have oscillations with smaller amplitude then using standard PD controller and these oscillations vanish earlier for quasi-velocity case LR. But values of errors are at the beginning usually bigger for quasi-velocity controllers than for standard one. During regulation after some time errors for LR PD controller can faster tends to zero than for ST one. JRN control gives a little different results as other controls. For such controller coefficients have to be assumed with different values as for LR or JRU because these quasi-velocities involve direct dynamical parameters. PD control in terms of generalized velocity components takes place in an abstract space. Because of that we transform physical variables into quasi-velocity space and at the end we transform quasi-moments into physical moments which are input signals to controller.

Loduha-Ravani dynamic equations for serial manipulator are decoupled in the sense that we have diagonal mass matrix. Therefore we have to inverse only diagonal elements of this matrix. Using equations (11) and (2) we obtain natural splitting between dynamics and kinematics of a manipulator. We have given also an interpretation of GVC.

The relationships between standard equations of motion and new ones and also performances of controllers require further investigations.

VI. REFERENCES

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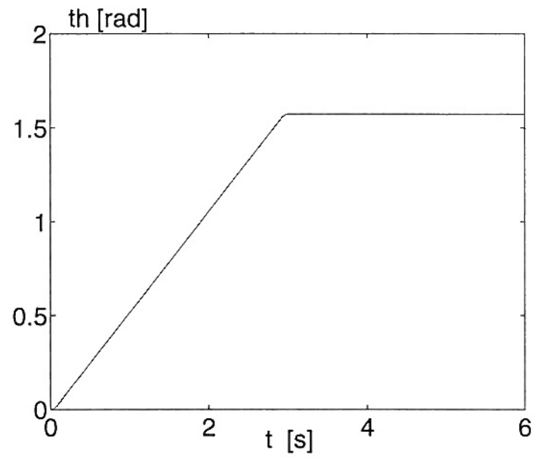

Fig. 1. Desired joint trajectory.

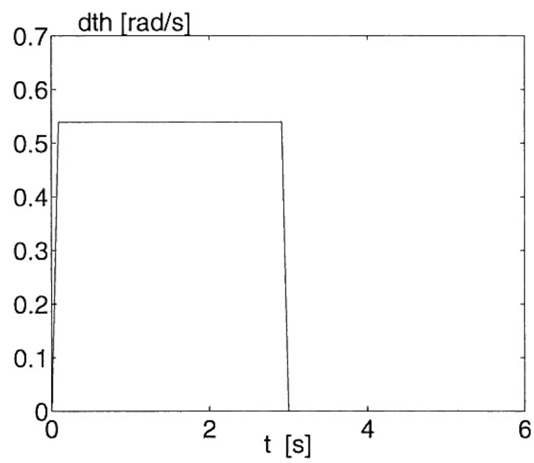

Fig. 2. Desired velocity profile.

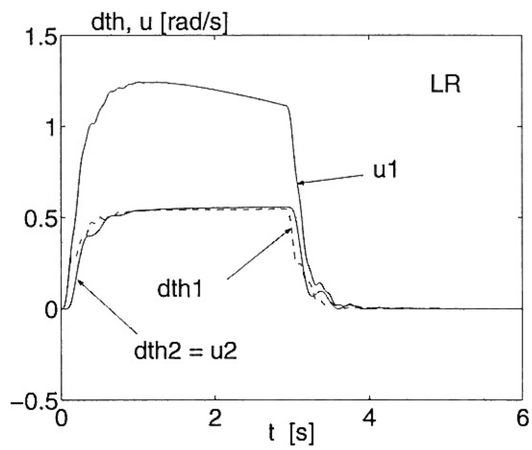

Fig. 3. Comparison between joint velocities $dth1, dth2$ and quasi-velocities $u1, u2$ for LR method.

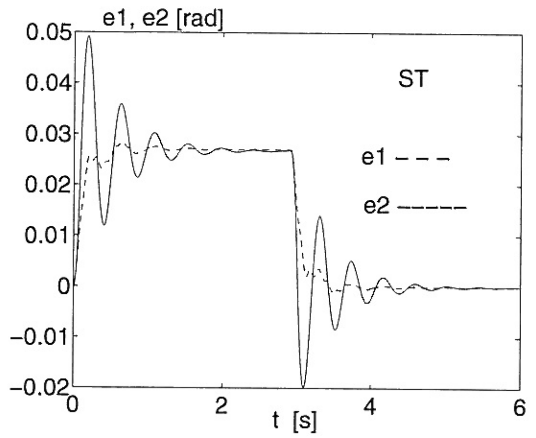

Fig. 4. Joint errors $e1, e2$ for standard case.

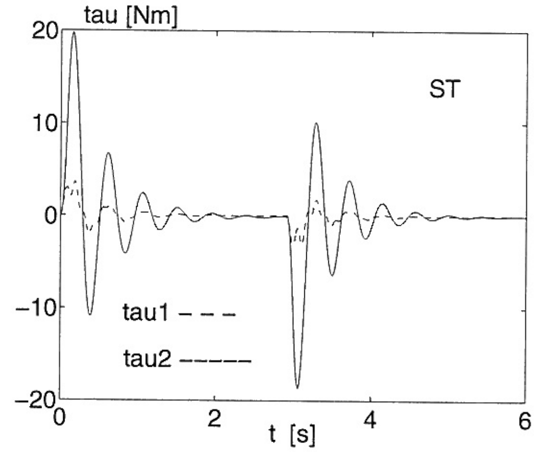

Fig. 7. Joint moments $tau1, tau2$ for standard case.

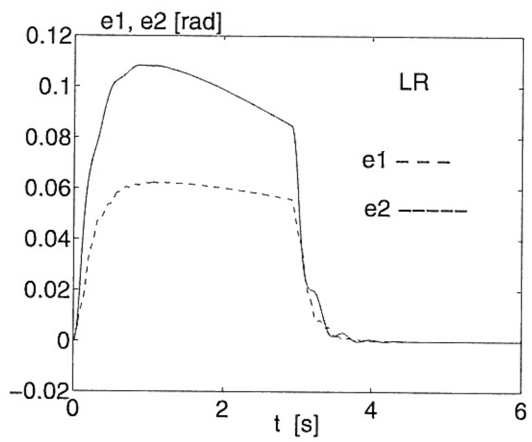

Fig. 5. Joint errors $e1, e2$ for LR case.

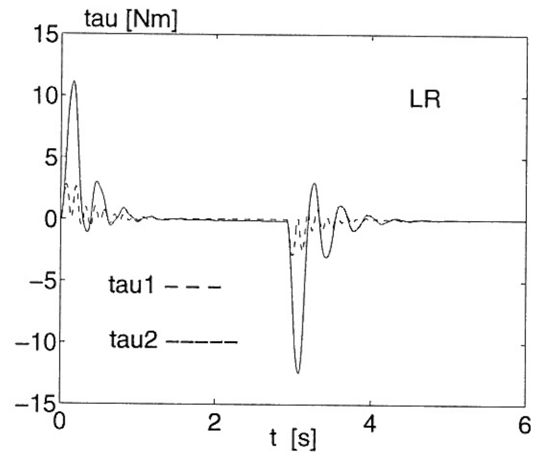

Fig. 8. Joint moments $tau1, tau2$ for LR case.

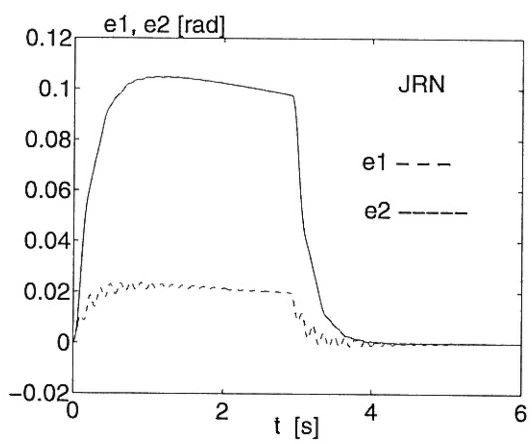

Fig. 6. Joint errors $e1, e2$ for JRN (normalized) case.

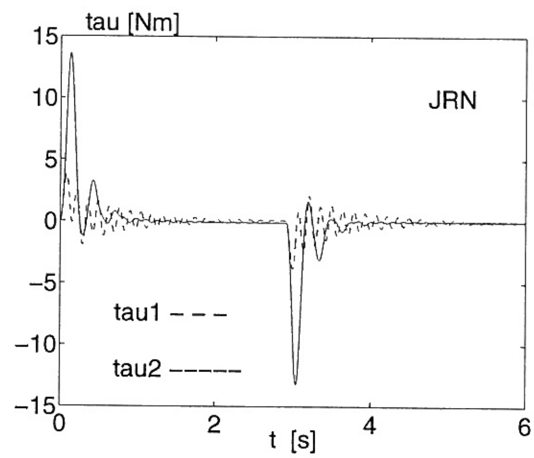

Fig. 9. Joint moments $tau1, tau2$ for JRN (normalized) case.

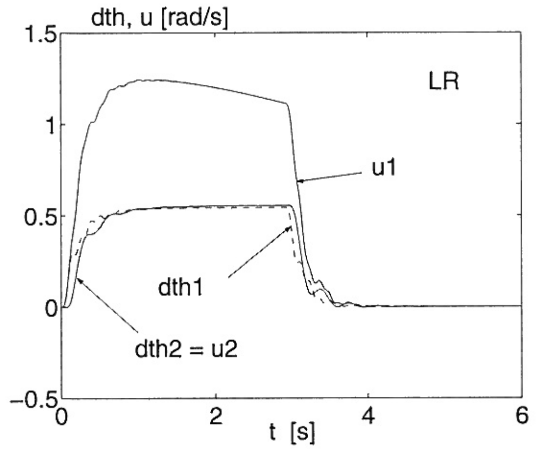

Fig. 10. Comparison between joint velocities $dth1, dth2$ and quasi-velocities $u1, u2$ for LR method with gravity.

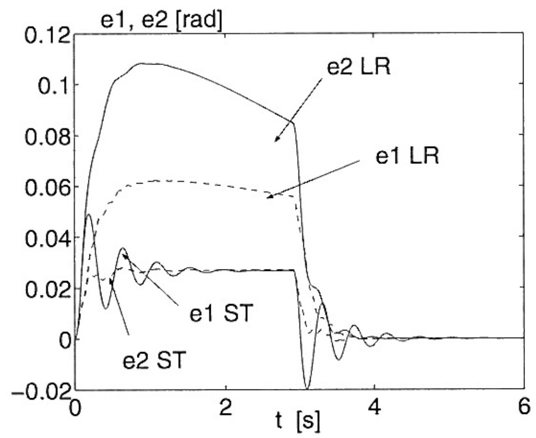

Fig. 11. Joint errors $e1, e2$ for standard case (ST) and LR, case with gravity.

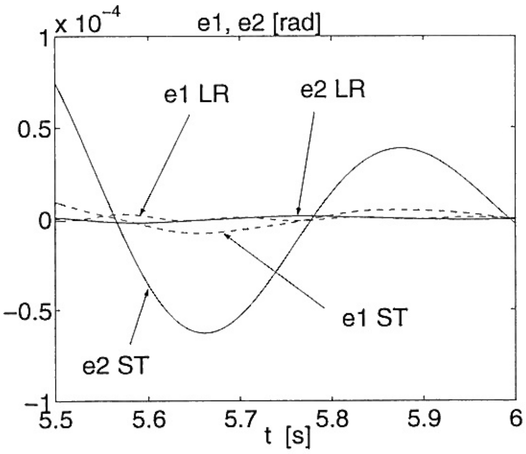

Fig. 12. Joint errors $e1$ after $5.5sec.$ for standard (ST) and LR, case with gravity.

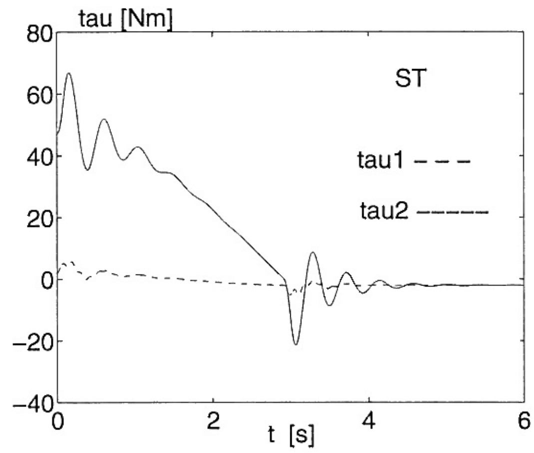

Fig. 13. Joint moments $tau1, tau2$ for standard, case with gravity.

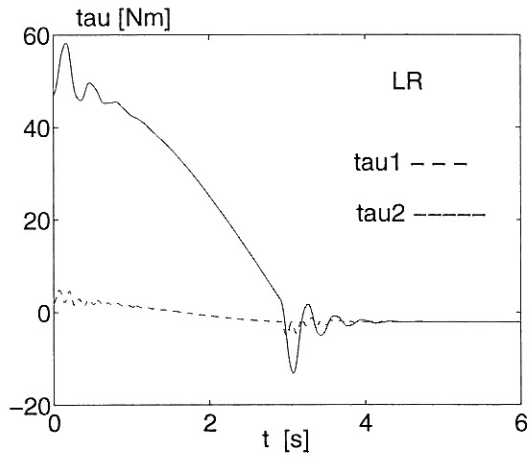

Fig. 14. Joint moments $tau1, tau2$ for LR, case with gravity.

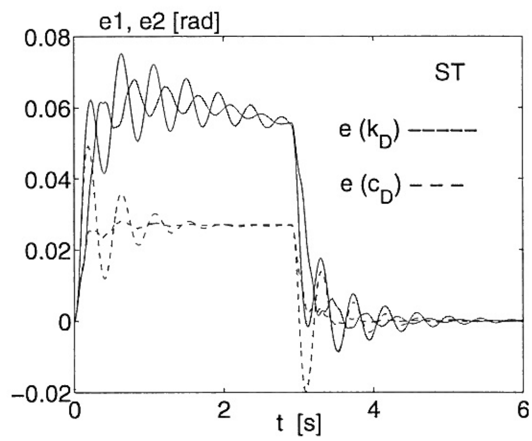

Fig. 15. Joint errors $e1, e2$ for standard PD (ST) control with matrices c_D and $k_D = \Upsilon^{-T} c_D \Upsilon^{-1}$.