Adaptive Control of an Electrical Actuator

R. Merzouki, J. C. Cadiou and N. K M'Sirdi
Laboratoire de Robotique de Paris
10-12, avenue de l'Europe
78140 Vélizy
e-mail: merzouki@robot.uvsq.fr

Abstract— **A compensation method for friction and backlash effects in control system is presented. This method entails the use of a non linear observers to estimate respectively the friction force which is modeled as Tustin's model, and the transmitted torque inside the dead zone. The friction observer model is based on the pressure of contact between surfaces. These observers are chosen to ensure a global stability of the system.**

Simulations results verify the theory and show that the method can improve the performance of a control system which it is used. Although, experimental results improve more this performance which are taken from a test bench representing an electrical actuator.

Keywords : ***backlash, dead zone, transmitted torque, friction force, electrical actuator.***

I. Introduction

Friction appears in the surface of a body to prevent slipping on another body (static friction) or that dissipates the mechanical energy in case of slip (dynamic friction). This friction produces a tangential force to the surface of contact, between rigid bodies which are tighten one against the other. In a first time, the rigid bodies remain in almost relative immobility; beyond a certain intensity of the applied force, they slip. The slip dissipates the energy and worn the surfaces. Except for that phenomenon, the presence of a dead zone due to the backlash effects introduces an hysteretic behaviour. It causes a non stability to the system. Backlash is necessary in mechanical motions, but when it increases due to the wear, it disturbs the performances of the system. These latter make difficult in practice, the position control of the mechanical system with a big precision. In a closed loop case, the non stability of the system is due principally to the apparition of dynamic disruptions, cycles limits and the static errors, because of the presence of friction and backlash. Among research dealing with the friction effects: Friedland in [5] developed a new algorithm in order to estimate the constant of the dynamic system. This algorithm is a reduced order observer containing two non linear functions, one is the Jacobean of the second. A good choice of the non linear function in the observer allows an asymptotic stability of the error. Then, Amin in [2] has developed two types of observers: The first one estimates the friction force as a constant of the time, the second one is used to estimate the relative velocity of the motion between surfaces in contact. Canudas in [4] proposed a model of friction that includes different effects like: Hysteretic behaviour and stiction.effect It is from this model that we have developed our friction observer. Research is also develloping in control of systems with backlash. We can cite: Brandenbourg & Schaffer[2] have studied the influence and partial compensation of simultaneously acting backlash and coulomb friction in a speed and position controller elastic two-mass system. Recker & al [8] and Tao & Kokotovic [9] have worked on the adaptive control of system with backlash. On this subject, different mathematical models are proposed: Tao & Kokotovic [9] have modelised a backlash inverse model based on hysterisis cycle. Cadiou & M'Sirdi [3] have developed a differentiable model basing on the dead zone characteristic.

In this paper, we use an adaptative control of an electrical actuator based on an a new approach in estimation of a friction force, based on the pressure of contact between surfaces. The transmitted torque acted in the dead zone interval is estimated in order to compensate the backlash effect. The use of two non linear observers [5] and [6] for the last estimation increases the performence of our control. An asymptotic convergence of the estimation error of friction and transmitted torque with the system in a closed loop has been proofed. Simulation and experimental results show the improvement of the system performences.

II. MODELS

A. System model

Our system Figure (1-left) is described by the following equation:

$$U + w = I.\ddot{\theta} + F \qquad (1)$$

where:

U : Control signal, w : Transmitted torque $(N.m)$, I : Inertia of the system $(kg.m^2)$, F : Friction force $(N.m)$.

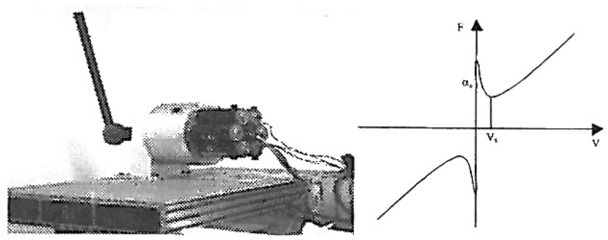

Fig. 1. Left: Electrical Actuator - Right: Tustin's Friction Model

B. Friction reference model

Friction force is represented by Tustin's model Figure(1-right). This model is composed on three intervals: the first one is static where Coulomb friction is acting and modeled as a constant times the sign of the velocity. The second interval describes Stribeck effect by an exponential decrease of the friction force from Stiction value until the end of the transient period. The third interval is corresponding to the dynamic state, linearly depends on the relative velocity.

Our experimental actuator given by figure (1-left) is dividing in two parts: motoring part describes by six DC motors, and reducer part which presenting frictions and backlash imperfections.

The friction force is expressed as follows:

$$F = (\alpha_0 + \alpha_1.e^{-\beta\left|\dot{\theta}\right|} + \alpha_2.\left|\dot{\theta}\right|).sign(\dot{\theta}) \qquad (2)$$

where: F is the Friction force $(N.m)$, α_0 is the Coulomb friction coefficient $(N.m)$, α_1 is the Stiction friction coefficient $(N.m)$, α_2 is the Viscous friction coefficient $(N.m.s/rad)$, $\dot{\theta}$ represents the relative velocity (rad/s) and β a graphical parameter.

C. *Backlash reference model*

Figure(2-right) describes a backlash spring system which is characterized by its stiffness K and backlash magnitude of $2j_0$. When we apply on the system a force F_2 we will find that the evolution of F_1 will depend on the displacement of point A as follows:

if $-j_0 < X_A < j_0$ then $F_1 = 0$

if $|X_A| > j_0$ then $F_1 = K.(X_A - j_0.sign(X_A))$.

Our experimental actuator Figure (1-left) is divided in two parts: motoring part which describes six DC motors, and reducer part which represents the frictions and the backlash imperfections.

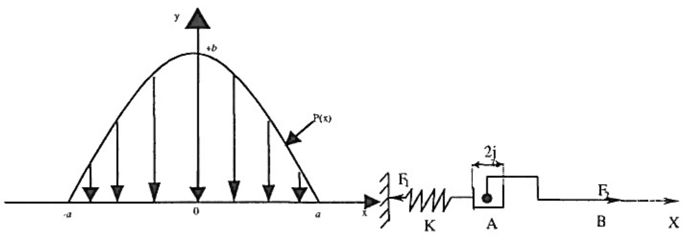

Fig. 2. Left: Pressure Distribution Right: Backlash spring system

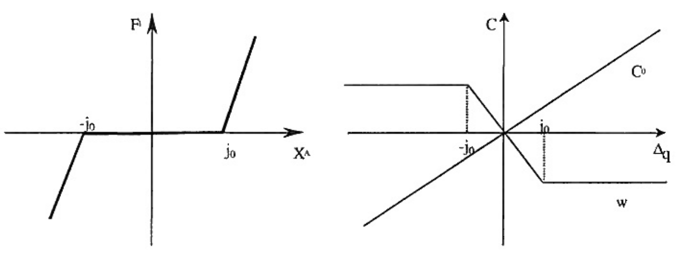

Fig. 3. Left: The backlash model - Right: The backlash model splited in two parts

We can give another representation of the torque C transmitted by the reducer to the load, by splitting the last representation Figure(3-left) in a sum of two functions Figure(3-right):

$$C = C_0 + w. \qquad (3)$$

Where $C_0 = K.\Delta\theta$. depends on $\Delta\theta = \theta_s - N.\theta_e$ position between the input and output reducer's positions, with N represents the reducer coefficient

And:

$$w = -K.j_0.\frac{1 - e^{-\gamma.\Delta\theta}}{1 + e^{-\gamma.\Delta\theta}} \qquad (4)$$

j_0 is the maximum magnitude of the backlash.

III. Estimation

A. *Estimation of Friction force*

A.1 Our Friction observer

A.1.a Quasi-static mode. We consider that during a mechanical motion, transmitted through surfaces in contact, the pressure distribution $P(x)$ is given by Figure (2-left).

In this case, we have selected a distribution which is represented by a half of an ellipse and is limited by $-a$ and $+a$. Where a is a positive value which defines the maximum of deflection [9], and it's calculated as follows:

$$a = \frac{\alpha_0}{\sigma_0} \qquad (5)$$

where: σ_0 is the stiffness in a surface contact (N.m/rad). The simplest representation of this last distribution is:

$$P(x) = b.\sqrt{1 - \frac{x^2}{a^2}} \qquad (6)$$

Referring to the physical theory, the Friction force in a quasi-static period is depending on the normal force N, so that:

$$F_1 = \mu.N \qquad (7)$$

where: μ is a dynamic coefficient of the Friction force F_1.

Knowing that the relation between the normal force N and the pressure $P(x)$, then the Friction force can be expressed as:

$$F_1 = \int_{-a}^{+a} a.\mu.P(x).dx \qquad (8)$$

by replacing (6) in (8), we will obtain:

$$F_1 = \int_{-a}^{+a} a.\mu.b.\sqrt{1 - \frac{x^2}{a^2}}.dx \qquad (9)$$

We put: $x = R.\sin\alpha$ then: $dx = a.\cos\alpha.d\alpha$ with: $\alpha \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, then:

$$F_1 = b.\mu.\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} a^2.\cos^2\alpha.d\alpha \qquad (10)$$

$$F_1 = b.a^2.\mu.\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} (1-\sin^2\alpha).d\alpha \tag{11}$$

$$F_1 = \frac{\pi.b.a^2.\mu}{2} \tag{12}$$

We can notice that in this mode, and from (12), Friction force depends linearly on the dynamic coefficient μ.

A.1.b Dynamic mode. When $P(x=\pm a)=0$, the evolution of the Friction force will depend on the relative velocity $\dot{\theta}$ between surfaces in contact:

$$F_2 = \alpha_2.\,\dot{\theta} \tag{13}$$

where: α_2 is the viscous Friction coefficient $(N.s/m)$.

So, the friction force F will be expressed in the last both modes as follows:

$$\begin{aligned} F &= F_1 + F_2 \\ F &= \frac{\pi.b.a^2.\mu}{2} + \alpha_2.\,\dot{\theta} \end{aligned} \tag{14}$$

Now, the estimation of F depends on the estimation of a dynamic friction coefficient μ as follows:

$$\hat{\mu} = \frac{1}{a^3}.(\alpha_1 + \frac{\alpha_0-\alpha_1}{(1+\frac{\dot{\theta}}{\dot{\theta}_s})^2}).sign(\dot{\theta}) \tag{15}$$

where: α_1 is the stiction coefficient (N) and $\dot{\theta}_s$is the stribeck velocity (m/s).

We notice that $\hat{\mu}$ is defined in $\left]-\dot{\theta}_{\max},0\right[\cup\left]0,+\dot{\theta}_{\max}\right[$, so it takes its values in $\left]-\frac{\alpha_1}{a^3},-\frac{\alpha_O}{a^3}\right[\cup\left]\frac{\alpha_1}{a^3},\frac{\alpha_0}{a^3}\right[$ domain. Thus, the function $\hat{\mu}(\dot{\theta})$ decreases exponentially from the initial condition. For this latter, we try to represent its variation by using an exponential function in its definition domain. We obtain:

$$\hat{\mu} = \frac{1}{a^3}.(\alpha_1 + (\alpha_0-\alpha_1).e^{-\frac{|\dot{\theta}|}{|\dot{\theta}_s|}}).sign(\dot{\theta}) \tag{16}$$

If we know a priori that the output low velocity representation is approximately propotional to the time. Otherwise, $\hat{\mu}$ is defined in a low velocities interval, thus $\hat{\mu}$ could be expressed as follow:

$$\hat{\mu} = \frac{1}{a^3}.(\alpha_1 + (\alpha_0-\alpha_1).e^{-\frac{\lambda}{|\dot{\theta}_s|}.t}).sign(\dot{\theta}) \tag{17}$$

λ has a positive value and depend on the gravity.

In order to show more the negative representation of $\hat{\mu}$ (i.e $\dot{\theta}<0$), we put $\hat{\mu}$ as a function of $t.sign(\dot{\theta})$, then equation (17) will be defined by:

$$\hat{\mu} = \frac{1}{a^3}.(\alpha_1 + (\alpha_0-\alpha_1).e^{-\frac{\lambda}{|\dot{\theta}_s|}.\left|t.sign(\dot{\theta})\right|}).sign(\dot{\theta}) \tag{18}$$

Now we give another description of equation (17) by introducing a differential representation as follow:

$$\dot{\hat{\mu}} = -\frac{\lambda}{\left|\dot{\theta}_s\right|}.\hat{\mu} + \frac{\alpha_1.\lambda}{a^3.\left|\dot{\theta}_s\right|}.sign(\dot{\theta}) \tag{19}$$

with: $\hat{\mu}(0) = \frac{\alpha_O}{a^3}.sign(\dot{\theta})$.

Then, the friction force estimation will be written as:

$$\widehat{F} = \frac{\pi.b.a^2.\hat{\mu}}{2} + \alpha_2.\,\dot{\theta} \tag{20}$$

B. Estimation of Backlash

B.1 The dead zone magnitude observer

Let us consider the following representation of the backlash effects, defined by the relative motion of part 1 according to the part 2 Figure(4). Where:

j_0: represents the dead zone magnitude (rad), I: Inertia of part 1 $(kg.m^2)$, θ_0: Initial position (rad).

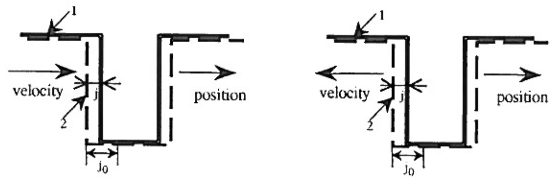

Fig. 4. Left & Right: Backlash mechanism

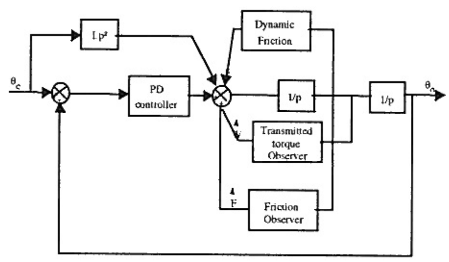

Fig. 5. Bloc Diagram

We can represent the dead zone as follow:

$$\frac{dj}{dt} = \dot{\theta}_{init}\,.e^{-\frac{\alpha_2.|\theta_0|.t}{I}} \tag{21}$$

This model defines the variation of the backlash magnitude inside the dead zone during the motion. According to the material characteristics of the system, this magnitude variation is proportional to the inertia of the part 1, and to the relative velocity between the two parts. But it's inversely proportional to the viscous friction coefficient and initial position simultaneously.

In order to make easier the control and the parametric estimation of the model (21), we consider the following formulation:

$$\frac{d^2j}{dt^2} = -\frac{\alpha_2.\,|\theta_0|}{I}.\frac{dj}{dt} \tag{22}$$

where: $\frac{dj}{dt}(0) = \dot{\theta}_{init}$, the initial condition on bcklash magnitude variation.

The last formulation can be represented by a state space representation in order to study the observer stability. Then, the estimpation of j is given by:

$$\ddot{\hat{j}} = -\frac{\alpha_2.|\theta_0|}{I}.\dot{\hat{j}} + k.e$$

e is the position error, $\hat{j}$ is the estimated backlash magnitude and k a positive gain.

B.2 Estimation of transmitted torque

We have seen in section (2) that the transmitted torque C can be represented by the add of two torques: C_0 linear and w nonlinear of $\Delta\theta$. This latter is acting in the dead zone Figure (6-right). So it can be modelized into:

$$w = -K.j_0.\frac{1 - e^{-\gamma.\Delta\theta}}{1 + e^{-\gamma.\Delta\theta}} \quad (23)$$

where γ represents a graphical positive parameter with high value,

$\Delta\theta = \theta_s - N.\theta_e$ is the position between the input and the output reducer positions, K is the system stiffness, and j_0 represents the dead zone magnitude. Then, we can approximate w to:

$$w = -K.j_0.sign(\Delta\theta) \quad (24)$$

The variation of $\Delta\theta$ is defined by: $\Delta\,\dot{\theta} = \dot{\theta}_s - N.\,\dot{\theta}_e$

B.2.a Case of direct motion. In this case, $\Delta\theta$ and $\Delta\,\dot{\theta}$ have the same sign, that's mean part 1 follows part 2 in the positive sense Figure(4-left).

We represent the torque estimation Figure(6-right) by the following equation:

$$\widehat{w} = K.j_0.(e^{-\gamma|\Delta\theta|} - 1).sign(\Delta\theta) \quad (25)$$

which approach the curve described in Figure (6-right).

The torque estimation error is defined by:

$$\widetilde{w} = w - \widehat{w} \quad (26)$$

when $\Delta\theta \neq 0$ and γ is higher, then w will approach to a signum function.

The variation of this error will be equal to :

$$\dot{\widetilde{w}} = -\,\dot{\widehat{w}} \quad (27)$$

using equation (24), and the approximation of the signum function with an important value of γ:

$$sign(\Delta\theta) \simeq \frac{1 - e^{-\gamma.\Delta\theta}}{1 + e^{-\gamma.\Delta\theta}} \quad (28)$$

then $\dot{\widetilde{w}}$ will be calculated as follow:

$$\begin{aligned} \dot{\widetilde{w}} &= -\gamma.K.j_0.\Delta\,\dot{\theta}\,.(-e^{-\gamma.|\Delta\theta|}).sign^2(\Delta\theta) \\ &\quad + K.j_0.(e^{-\gamma.|\Delta\theta|} - 1).\frac{2.\gamma.\Delta\,\dot{\theta}\,.e^{-\gamma.\Delta\theta}}{(1 + e^{-\gamma.\Delta\theta})^2} \end{aligned} \quad (29)$$

expression $\frac{2.\gamma.\Delta\dot{\theta}.e^{-\gamma.\Delta\theta}}{(1+e^{-\gamma.\Delta\theta})^2}$ goes to zero when $\Delta\theta \neq 0$, and goes to $\frac{1}{2}.\Delta\,\dot{\theta}$ when $\Delta\theta = 0$ where γ is a high and a positive value. We know also that when $\Delta\theta$ is around *zero value*, the transmitted torque is approximately equal to zero.

The expression (29) will go to:

$$\begin{aligned} \dot{\widetilde{w}} &= -\gamma.K.j_0.\Delta\,\dot{\theta}\,.(-e^{-\gamma.|\Delta\theta|}).sign^2(\Delta\theta) \\ &\quad -\gamma.K.j_0.\Delta\,\dot{\theta}\,.(sign^2(\Delta\theta) - sign^2(\Delta\theta)) \end{aligned} \quad (30)$$

with the add of the term: $-\gamma.K.j_0.\Delta\,\dot{\theta}\,.(sign^2(\Delta\theta) - sign^2(\Delta\theta))$ in order to include the différence $w - \hat{w}$ in the last equation. The error will be expressed as follows:

$$\dot{\widetilde{w}} = -\gamma.\Delta\,\dot{\theta}\,(w - \hat{w}).sign(\Delta\theta) \quad (31)$$

when $\Delta\theta$ and $\Delta\,\dot{\theta}$ take the same sign, then the last expression will be expressed as:

$$\dot{\widetilde{w}} = -\gamma.\left|\Delta\,\dot{\theta}\right|.\widetilde{w} \quad (32)$$

then the estimation error can be represented by a first order differential equation:

$$\dot{\widetilde{w}} + \gamma.\left|\Delta\,\dot{\theta}\right|.\widetilde{w} = 0 \quad (33)$$

Knowing that by compensation the friction force, the value of $\Delta\,\dot{\theta}$.approaches zero (i.e constant). Then $\widetilde{w}$ converge asymptotically to zero.

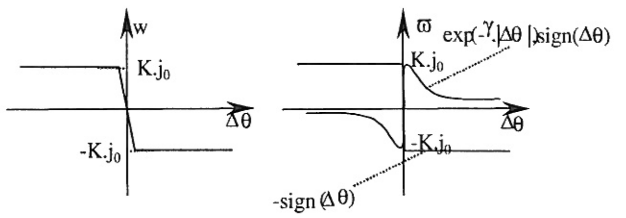

Fig. 6. (left):Approximation of w - (right):Torque estimation $\widehat{w}$

B.2.b Case of reverse motion. In this case, $\Delta\theta$ and $\Delta\,\dot{\theta}$ have different sign, that's mean parts 1 and 2 move in different ways Figure(4-right).

Then, the representation of the estimated torque is given by the following equation:

$$\widehat{w} = -K.j_0.(e^{-\gamma|\Delta\theta|} - 1).sign(\Delta\theta) \quad (34)$$

The estimated torque error is defined by:

$$\widetilde{w} = w - \widehat{w} \quad (35)$$

when $\Delta\theta \neq 0$, the variation of this error will be expressed as follows

$$\dot{\widetilde{w}} = -\,\dot{\widehat{w}} \quad (36)$$

then:

$$\dot{\widetilde{w}} = \gamma.K.j_0.\Delta\,\dot{\theta}\,.(-e^{-\gamma.|\Delta\theta|}).sign^2(\Delta\theta) + K.j_0.(e^{-\gamma.|\Delta\theta|}-1).\frac{2.\gamma.\Delta\,\dot{\theta}\,.e^{-\gamma.\Delta\theta}}{(1+e^{-\gamma.\Delta\theta})^2} \quad (37)$$

expression $\frac{2.\gamma.\Delta\dot{\theta}.e^{-\gamma.\Delta\theta}}{(1+e^{-\gamma.\Delta\theta})^2}$ goes to zero when $\Delta\theta \neq 0$, and goes to $\frac{1}{2}.\Delta\,\dot{\theta}$ when $\Delta\theta = 0$ with γ is a high and a positive value. We know also that when $\Delta\theta = 0$ the transmitted torque is approximately equal to zero.

The expression (37) will go to:

$$\dot{\widetilde{w}} = \gamma.K.j_0.\Delta\,\dot{\theta}\,.(-e^{-\gamma.|\Delta\theta|}).sign^2(\Delta\theta) - \gamma.K.j_0.\Delta\,\dot{\theta}\,.(sign^2(\Delta\theta) - sign^2(\Delta\theta)) \quad (38)$$

with the add of the term: $-\gamma.K.j_0.\Delta\,\dot{\theta}\,.(sign^2(\Delta\theta) - sign^2(\Delta\theta))$.

the error will be expressed as follows:

$$\dot{\widetilde{w}} = \gamma.\Delta\,\dot{\theta}\,(w - \hat{w}).sign(\Delta\theta) \quad (39)$$

when $\Delta\theta$ and $\Delta\,\dot{\theta}$ have different signs ($sign(\Delta\theta) = -sign(\Delta\,\dot{\theta})$), then the last expression will be:

$$\dot{\widetilde{w}} = -\gamma.\left|\Delta\,\dot{\theta}\right|.\widetilde{w}$$

then the estimation error can be represented by a first order differential equation:

$$\dot{\widetilde{w}} + \gamma.\left|\Delta\,\dot{\theta}\right|.\widetilde{w} = 0 \quad (40)$$

Then $\widetilde{w}$ converge asymptotically to zero.

IV. Position control

Our system in closed loop Figure (5) is described by the following equation:

$$U + w = I.\,\ddot{\theta} + F \quad (41)$$

where: F is the friction force, w represents the transmitted torque and U is the system control given by:

$$U = -K_P.e - K_D.\,\dot{e} + \widehat{F} - \widehat{w} + I.\,\ddot{\theta}_d \quad (42)$$

with: $\hat{F}$ is the estimated friction force, $\hat{w}$ represents the estimated transmitted torque, $e = \theta - \theta_d$ and K_P, K_D.represent the coefficients of a PD controller:

After replacing (42) into (41), equation (41) will be expressed as follows:

$$I.\,\ddot{e} + K_D.\,\dot{e} + K_p.e = -\widetilde{F} + \widetilde{w} \quad (43)$$

with: $\widetilde{F} = F - \widehat{F}$ is the estimated friction error and $\widetilde{w} = w - \widehat{w}$.is the estimated torque error

According to the estimated force from (15) and (20), we have:

$$\widetilde{F} = -\frac{\pi.b.a^2.\tilde{\mu}}{2} \quad (44)$$

Combination of equations (43) and (44) introduces a new representation:

$$\begin{cases} I.\,\ddot{e} + K_D.\,\dot{e} + K_P.e = \frac{\pi.b.a^2.\tilde{\mu}}{2} + \widetilde{w} \\ \dot{\tilde{j}} = -\frac{\alpha_2.|\theta_0|}{I}.\,\dot{\tilde{j}} + k.e \\ \dot{\widetilde{w}} = -\gamma.\left|\Delta\,\dot{\theta}\right|.\widetilde{w} \end{cases} \quad (45)$$

from (16), the dynamic of the estimation error of the coefficient μ is given by:

$$\dot{\tilde{\mu}} = -\frac{\lambda}{\left|\dot{\theta}_s\right|}.\tilde{\mu} - \rho.e \quad (46)$$

with: $\rho > 0$ and $\rho.e$ corresponds to the correction of the estimation error $\tilde{\mu}$.

Now, We define a representation state of our system after the choose of

$\epsilon^T = (\dot{e}, e, \dot{\tilde{j}}, \widetilde{w})$ as a vector state:

$$\begin{cases} \dot{\epsilon} = A.\epsilon + B.\tilde{\mu} \\ e = C.\epsilon \end{cases} \quad (47)$$

with: $A = \begin{pmatrix} -\frac{K_D}{I} & -\frac{K_P}{I} & 0 & \frac{1}{I} \\ 0 & 1 & 0 & 0 \\ 0 & k & -\frac{\alpha_2.|\theta_0|}{I} & 0 \\ 0 & 0 & 0 & -\gamma.\left|\Delta\,\dot{\theta}\right| \end{pmatrix}$,

$B^T = \begin{pmatrix} \frac{\pi.b.a^2}{2.I} \\ 0 \\ 0 \\ 0 \end{pmatrix}$.

K_P and K_D are chosen to obtain a negative reel part of the eigenvalues of matrix A.

We take: $V = \epsilon^T.P.\epsilon + \frac{\tilde{\mu}^2}{\rho} > 0$ as a Lyapunov function with an equilibrium state: $\epsilon = 0$ and $\tilde{\mu} = 0$, and P symmetric and positive definite matrix.

The variation of V will be calculated as follows:

$$\dot{V} = \dot{\epsilon}^T\,.P.\epsilon + \epsilon^T.P.\,\dot{\epsilon} + \frac{2}{\rho}.\,\dot{\tilde{\mu}}\,.\tilde{\mu} \quad (48)$$

$$\dot{V} = \epsilon^T.(A^T.P + P.A).\epsilon + 2.\epsilon^T.P.B.\tilde{\mu} + \frac{2}{\rho}.(-\frac{\lambda}{\left|\dot{\theta}_s\right|}.\tilde{\mu} - \rho.e).\tilde{\mu} \quad (49)$$

$$\dot{V} = -\epsilon^T.M.\epsilon + 2.e.\tilde{\mu} + \frac{2}{\rho}.(-\frac{\lambda}{\left|\dot{\theta}_s\right|}.\tilde{\mu} - \rho.e).\tilde{\mu} \quad (50)$$

$$\dot{V} = -\epsilon^T.M.\epsilon - \frac{2.\lambda}{\rho.\left|\dot{\theta}_s\right|}.\tilde{\mu}^2 < 0$$

with: $M = -(A^T.P + P.A)$ positive definite.

Then, the equilabrium state $\epsilon = 0$ and $\tilde{\mu} = 0$ is verified and the system is asymptotically stable in a closed loop.

A. Simulation results

The simulation tests have been done on a system representing an actuator driving by six electrical motors. The block diagram containing the different parts of the simulation shown in Figure(5). The system parameters taken into account for the simulations are: $\alpha_0 = 8N$, $\alpha_1 = 1.5N$, $\alpha_2 = 16N.M.s/rad$, $\frac{\lambda}{\theta_s} = 10s^{-1}$, $\gamma = 20s^{-1}$, $\sigma_0 = 10^4 N.m/rad$, $I = 10,77Kg/m^2$, and PD control law is characterized by: $K_P = 10$, $K_D = 5$.

Figure(7-left and right) represents the tracking of the output signal to the reference signal without and with compensation of the both backlash and friction respectively. We can notice the improvement of the tracking when applying our observers. Figure(11-left) describes the estimated dead zone magnitude. Figure(8-left and right) describes respectively the input and output signals of the reducer part without and with conpensation. We can notice the linearity of the input and output signals after compensation. Figures(9-left and right) shows respectively the output velocity signals before and after conpensation, where iit presents a little noises due to the numeric derivation of the output position signal. Figure(10-left and right) shows respectively the trasmitted torque to the load before then after compensation, where we notice the elimination of the dead zone and the linearization of the torque with the position. Figure(11-right) represents the reference and estimated friction force which describes the Tustin's model.

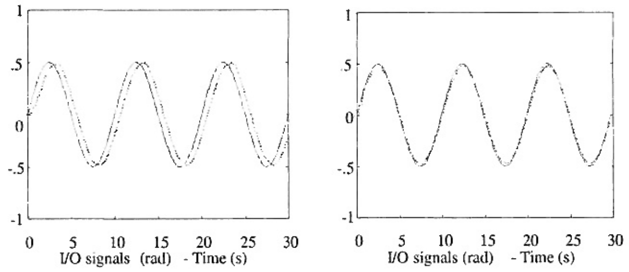

Fig. 7. Simulation results: I/O time signals: left: without compensation - right: I/O time signals with compensation

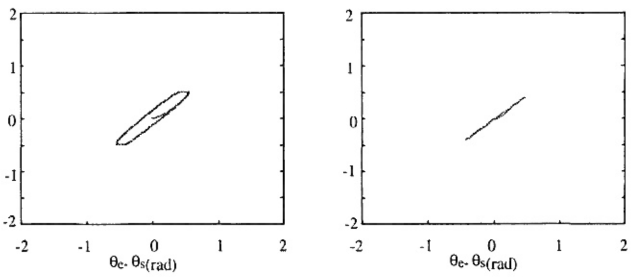

Fig. 8. Simulation results: Left: Input - Output positions signals before compensation - Right: Input - Output positions signals after compensation

Doted and continue lines describe respectively the output and input signals.

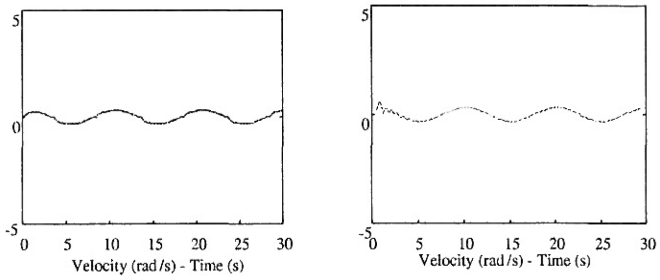

Fig. 9. Simulation results: Left: Velocity signal before compensation - Right: Velocity signal after compensation

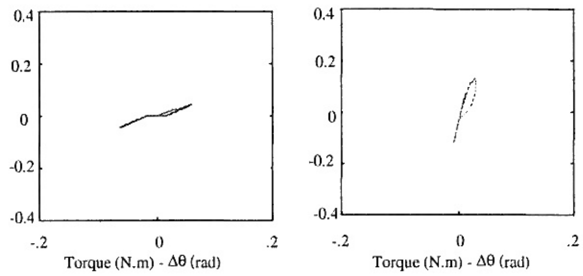

Fig. 10. Left: Simulation results: Output transmitted torque before compensation - Right: Output transmitted torque after compensation

V. Experimental results

The experimental tests have been applied on an actuator which is drived by six dc motors and is developed in the 'laboratoire de robotique de Paris'. This last presents several imperfections due to the presence of disruptive friction (static, stiction) and mechanical backlash. For a sinusoidal input with a frequency of 0.5Hz and a PD control with: $K_P = 1$ and $K_D = 0.1$, we get Figure (12-left) for the case without backlash and friction compensations and Figure (12-right) with the compensation case. The system presents good performances, besides only subsists a very weak static error. Figure (13) shows respectively the characteristic of the output velocity signal before and after compensation. The relation between the friction force and the velocity is described in Figures (14-left) whithout compensation and (14-right) with compensation, where an pseudo-hysteresis behaviour is present not due to an unknown phenomenon but is due to the transition when we change the sign of the position.

Doted and continue lines describe respectively the output and input signals.

VI. Conclusion

Two principals objectives have been treated in this paper. The first one is the estimation of the fricton force which is describing during a relative motion. This estimation is based on the estimation of the dynamic friction coefficient during the contact between the surfaces. The second is concerning the estimation of the dead zone magnitude describing the backlash effect in order to calculate

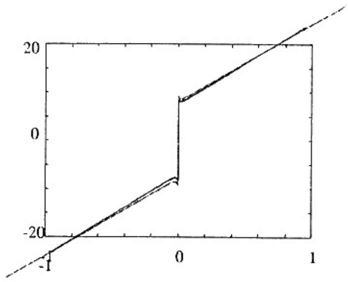

Fig. 11. Simulation results: left: Backlash magnitude estimation - right: reference and estimated friction force

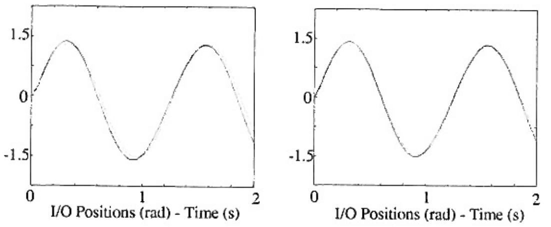

Fig. 12. Experimental results: Left: Input-Output signals without compensation - Right: Input-Output signals with compensation

the necessary trasmitted torque. The dead zone is defined by the variation of the magnitude during the motion inside the backlash interval. A compensation of the both friction and backlash is introduced in our control to eliminate the imperfections disturbances. An asymptotic stability of the system in a closed loop has been proofed theoretically firstly and secondly by simulation and experimental tests where the performences have been improved.

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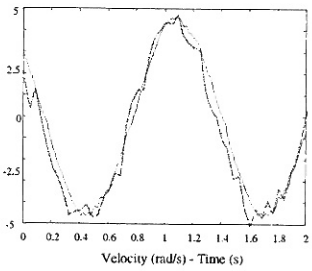

Fig. 13. Experimental results: output velocity signal before and after compensation

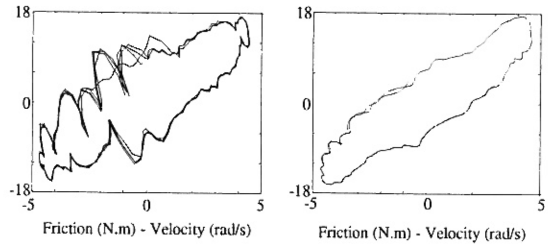

Fig. 14. Experimental results: Left: Friction force caracteristic without compensation - Right: Friction force caracteristic with compensation.

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