A Generalized Lorentz Group-Based Adaptive Control for DC Drives Driving Mechanical Components

József K. Tar
Budapest Polytechnic
Institute of Mathematical and Computational Sciences
H-1081 Budapest, Népszínház utca 8.Hungary
E-mail: Tar.Jozsef@nik.bmf.hu

János F. Bitó
Budapest Polytechnic
Centre of Robotics and Automation
John von Neumann Faculty of Informatics
H-1081 Budapest, Népszínház utca 8.Hungary
E-mail:Jbito@zeus.banki.hu

Imre J. Rudas
Budapest Polytechnic
Institute of Mathematical and Computational Sciences
H-1081 Budapest, Népszínház utca 8.Hungary
E-mail: Rudas@bmf.hu

Karel Jezernik
University of Maribor, Faculty of Electrical Eng. and Computer Science, Institute of Robotics
Smetanova ul. 17, SI-2000, Maribor, Slovenia
E-mail: Karel.Jezernik@uni-mb.si

Abstract — an efficient approach recently was invented for the adaptive control of approximately and partially known mechanical systems. Like traditional soft computing it is based on "uniform structures" for modeling, but these structures are obtained from certain Lie groups as the Symplectic Group characteristic to inner symmetries of the mechanical systems. This approach considerably reduced the number of free parameters in the model in comparison e.g. with neural networks or fuzzy systems. It also replaces the process of parameter tuning with simple, lucid, and explicit algebraic operations of limited steps. Till now its efficiency was investigated for mechanical uncertainties and external dynamic interactions. The present paper concentrates on its application for controlling electric DC motors driving mechanical components: the inductance of the motor armature sets limit to the speed of the change in the motor torque in the case of a voltage generator-controlled motor. The complex non-linear interaction of the mechanical, the electrical and the software components prescribing the control rule for the mechanical parts are considered. To achieve adaptive control the Generalized Lorentz Group is applied as an especially convenient algebraic representation for both the mechanical and the current control even for SISO sub-systems. Simulation results are presented for the control of a CTC controlled pendulum.

I. INTRODUCTION

Insufficient and inaccurate knowledge regarding the dynamic properties of the robot arm, the dynamic interaction between the arm and its environment as well as the behavior of the electric drives makes the adaptive control of electromechanical systems an interesting task. The need for developing a universally useful controller makes it undesirable to include some particular model of the robot-work-piece interaction in the control software. Instead of this a more intelligent control being able to "learn" the main features specific to the technological operation under consideration would be much more expedient. This indefinite nature of the task makes it unlikely to find a closed form analytical problem-formulation for which an elegant proof of convergence or bounded error etc. could be found. It is more likely to achieve results via applying simple modeling and learning technique as in the case of the traditional Soft Computing (SC) approaches.

The present state of this approach corresponds to the creation of a new branch of Soft Computing (SC) for particular problem classes possibly wider than that of the control of mechanical systems [1]. Like "traditional" SC it evades the development of analytical system models and tries using simple uniform structures, but in contrast to the traditions, these structures are obtained from various Lie groups used in different fields of Physics as the Symplectic Group [2] or the Generalized Lorentz Group [1]. The main advantages are drastic reduction in size and increase in lucidity in comparison with the "conventional" architectures being the subject of ample investigations [e.g. 3-4]. Further advantage is that the generally "obscure" --either strictly causal, semi-stochastic or fully stochastic-- "learning" or parameter tuning seems to be replaceable by simple explicit algebraic procedures of limited steps in the case of the new structures. The basic idea originated from the field of mechanical systems' control, and later it was further developed via considering certain general features of this internal symmetry group in a much wider scale.

To follow this program in the present paper a 1 DOF mechanical systems driven by non-ideal electric DC motors are considered as possible extension of the investigations. First typical convergence possibilities are considered for SISO systems in the case of the adaptive law suggested. Following this the model of a CTC controlled pendulum driven by a DC motor is developed. Finally simulation investigations are presented for the case of the proposed adaptive control and conclusions are drawn.

II. THE IDEA OF THE ADAPTIVE CONTROL BASED ON ABSTRACT LIE GROUPS

From purely mathematical point of view the control problem can be formulated as follows: there is given some *imperfect model of the system* on the basis of which some *excitation* is calculated for a desired input $\mathbf{i}^d$ as $\mathbf{e}=\varphi(\mathbf{i}^d)$. The system has its *inverse dynamics* described by the *unknown function* $\mathbf{i}^r=\psi(\varphi(\mathbf{i}^d))=f(\mathbf{i}^d)$ and resulting in a realized $\mathbf{i}^r$ instead of the desired one $\mathbf{i}^d$. (In Classical Mechanics these values are the *desired* and the *realized joint accelerations*, while the external free forces and the joint velocities serve as the parameters of this temporarily valid and changing function.) It is evident, that normally one can obtain information via observation only on the "net" function $f()$, and that this function considerably varies in time. Furthermore, we do not have practical tools to "manipulate" the nature of this function directly: we can manipulate or *deform* its actual input $\mathbf{i}^{d*}$ in comparison with the *desired one* in general. The aim is to achieve and maintain the $\mathbf{i}^d=f(\mathbf{i}^{d*}):=\mathbf{g}(\mathbf{i}^d)$ state, that is when the original function altered by the deformation possesses the prescribed fixed point. [We can directly manipulate only the nature of the *model function* $\varphi()$.]

On the basis of the idea of the renormalization transformation applied in Chaos Theory as e.g. in [5-8] a "scaling iteration" as its modification was suggested for finding the proper deformation factor for a SISO

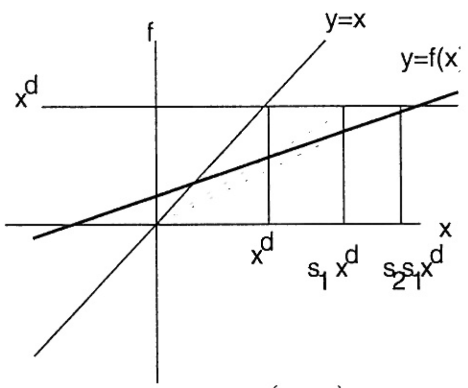

Fig. 1: Example A/1: $\{s_n|>1\}, s_n \to 1$, monotone, properly convergent case

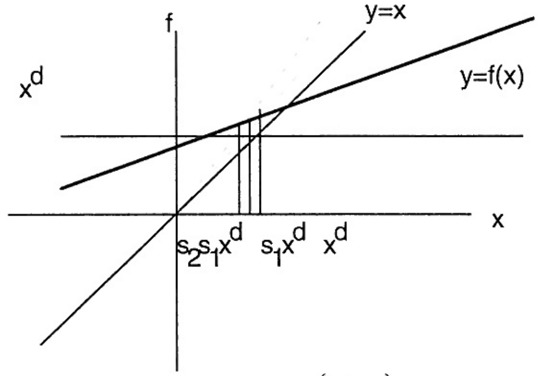

Fig. 2. "Example A/2": $\{s_n|<1\}, s_n \to 1$, monotone, properly convergent case

system in [1]. The original problem for which renormalization can be applied is given as follows: on the basis of some physical considerations there is given a function $g(x)$ with which a series $x_{n+1}=g(x_n)$ is generated. Other considerations indicate that the fixed point of this series $x=g(x)$ should be a given value. It may happen that the above series is divergent or converges to a different fixed-point. For "amending" or manipulating $g(x)$ a renormalization parameter s is so introduced that the modification is $\gamma_s(x):=s^{-1}g(sx)$. It can be shown that if $g(x)$ is contractive for a given region around x the series defined by the equation $x=s_{n+1}^{-1}g(s_n x)$ converges to a proper renormalization value. This problem is quite analogous with our control task with the exception that the answer of the composite system cannot be manipulated by s^{-1}. Therefore in [1] the following modification of the original algorithm was suggested:

$$s_{n+1} f\left(s_n s_{n-1} \dots s_1 x^d\right) = x^d \tag{1}$$

If the situation of $s_n \to 1, \dots s_n s_{n-1} \dots s_1 \to s$ occurs sx^d just corresponds to the desired deformation of the input according to the above program. This series has a lucid

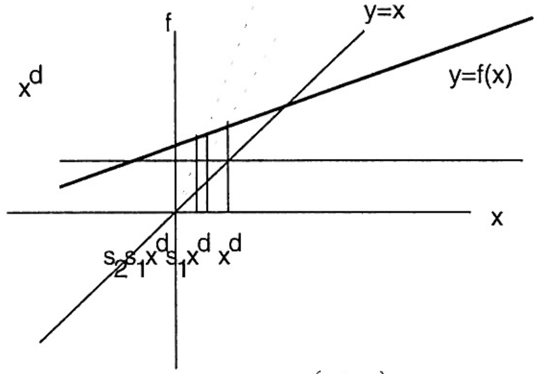

Fig.3. "Example B/1" $\{s_n|<1\}, s_n \to 1$, convergence to a false solution 0 instead of a negative value with a positive initial (that is desired) value

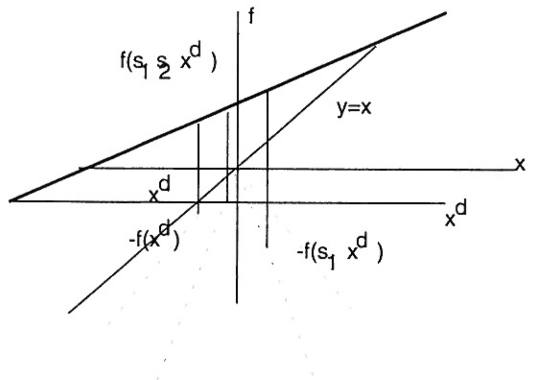

Fig. 4. Example B/2 $\{|s_n|<1\}, s_n \to 1$, convergence to a false solution 0 instead of a negative value with a negative initial (that is desired) value

geometric interpretation shown Fig.1 for a typical case "Example A/1": Convergent series that converges to the desired solution. For this $f(x)$ must be strictly monotone increasing, flat, and the solution must be in the positive quarter of the plain. The monotone and proper convergence of the scaling factors can be either increasing as in Fig. 1 or decreasing as in Fig. 2. of "Example A/2". In the next example the function is not "flat enough", so the "s"-factors can be greater and smaller than one. (However, though their series is not monotone, convergence to the proper value occurs). The above series can work for positive x^d till the solution of the equation $x = f(x) \geq 0$. If this solution is negative, we obtain a series $\{s_n s_{n-1} \dots s_1 x^d\} \to 0$, which is convergent but doesn't converge to the solution of the problem. This corresponds to Fig. 3 ("Example B/1"), that is to convergence to a false value, to zero. Another example of false "convergence to zero" can be considered when the solution is not located in the "proper quarter" though the initial (desired value) is negative (a counterpart of "Example B/1" Fig. 3 "Example B/2"). (To take into account the effect of the negative scaling numbers double mirroring for the vertical axis and alteration of the sign of the function were applied).

The above considerations are quite "illustrative" for SISO systems in which a scalar value can be either increased or decreased in order to approach a desired solution. For MIMO systems the situation can be more ambiguous due to the richer set of possibilities as a consequence of the greater degree of freedom of the system. For a continuous $\mathbf{f}: \Re^n \to \Re^n$ mapping the above scalar factors can be replaced by certain specially constructed linear transformations as $\mathbf{s}_{n+1}\mathbf{f}(\mathbf{s}_n \mathbf{s}_{n-1} \mathbf{s}_{n-2} \dots \mathbf{s}_1 \mathbf{x}^d) = \mathbf{x}^d$, $\{\mathbf{s}_{n+1}, \mathbf{s}_n, \mathbf{s}_{n-1}, \dots, \mathbf{s}_1\}$ in which appropriate convergence is required not only in the norm of the vectors but in their direction, too. Since in this case the task is only ambiguously defined, it is expedient to utilize the available freedom for constructing convenient and useful solutions: *a) let the matrices be non-singular; b) let their inverses easily computable; c) let their product be of the same nature as the original matrices are; d) let they form a continuous and a smooth set also containing the identity operator, so they can be used as slowly and continuously changing corrections to a given model.*

For this purpose matrices belonging to any Lie group defined by a basic quadratic expression as $\Lambda^T \mathbf{G} \Lambda = \mathbf{G}, \det \mathbf{G} \neq \mathbf{0}, \det \Lambda = 1$ with a constant $\mathbf{G}$ is appropriate since $\Lambda^{-1} = \mathbf{G}^{-1} \Lambda^T \mathbf{G}$. If for instance $\mathbf{G} = \mathbf{I}, \langle 1, \dots, 1, -c \rangle, \Im = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{I} & \mathbf{0} \end{bmatrix}$ (c is the velocity of light) correspond to the Orthogonal Group, the Generalized Lorentz Group, and the Symplectic Group describing the inner symmetry of the Euclidean Geometry, the Electrodynamics (when the size of $\mathbf{G}$ is 4×4), and that of the Classical Mechanics, respectively. (Positive definite Riemannian metrics can be associated only with the first case.) In the case of the Orthogonal Group it is easy to create a rotation which rotates a given vector to be parallel with an other one and leaves their orthogonal sub-space unchanged. This means that only the minimum of the necessary transformations is executed, so in this sense the given transformation is as close to the identity operator as possible. In similar way the "Minimum Operation Symplectic Group" and the "Generalized Lorentz Group" were invented and used in [2] to obtain such "minimum transformations". Preliminary steps also were done for investigating the convergence of the solution of this series in general and in details in [1]. In the next part these ideas are applied for a SISO control of both parts of a connected 1 DOF mechanical and 1 DOF electrical system.

III. SIMULATION INVESTIGATIONS FOR A GIVEN ELECTROMECHANICAL SYSTEM

Actually this system consists of a pendulum of mass "m", length "ℓ". From the Euler-Lagrange equations as the equation of motion of the mechanical system is described by

$$\ddot{q} = -\frac{g}{\ell}\sin(q) + \frac{1}{m\ell^2} Q \tag{2}$$

in which "q" corresponds to the pendulum's angular position, "g" is the gravitational acceleration and "Q" is the motor's torque exerted on the pendulum's shaft. As a "rough model" of that

$$Q = \hat{\alpha} \ddot{q}^D + \hat{\beta} \tag{3}$$

with which the relation between the "desired" and the "realized" accelerations (superscripts D and R, respectively) --at least in the case of perfect or `ideal` motor-- will be:

$$\ddot{q}^R = \frac{\hat{\alpha}}{m\ell^2}\ddot{q}^D + \frac{\hat{\beta}}{m\ell^2} - \frac{g}{\ell}\sin(q) \quad (4)$$

It is trivial that for $\hat{\alpha} > 0$ and $\hat{\beta}/m\ell^2 > g/\ell$ positive desired acceleration will result in positive required acceleration. However, a similar statement is not true for arbitrary negative acceleration. It is easy to see that if an "artificial mechanical model" is chosen as

$$\hat{\beta} = 12mg\ell\,\mathrm{sign}(\ddot{q}^D), \text{ and } \hat{\alpha} = \frac{1}{2}m\ell^2 \quad (5)$$

the desired and the "realized" accelerations will have the same sign. Via excluding near zero desired acceleration (*qDpp*) within a thin interval around 0 as

*if sign(qDpp)≠0 qDpp=qDpp+sign(qDpp)*1e-3;*
else
qDpp=1e-3;
end;

as far as the mechanical part of the control is concerned convergence can be expected according to the general considerations made for SISO systems (via application of (1) for the desired acceleration as input, and the realized acceleration as the output of the function this excludes zero scaling factor, too).

The motor's equation of motion expressed in the term of the $Q(t)$ torque exerted on the pendulum's shaft in the case of a voltage-generator-based command signal is given as

$$\dot{Q}(t) + \frac{R}{L}Q(t) + \frac{\mu^2 KK_b}{L}\dot{q}(t) = \frac{\mu K}{L}U(t)$$
$$\dot{Q}(t) + AQ(t) + B\dot{q}(t) = CU(t) \quad (6)$$

in which

- $U(t)$ is the motor voltage (provided by a voltage generator, used for control purposes);
- L denotes the armature inductance (constant, characteristic to the coil in the armature);
- R stands for resistance of the armature coil (constant);
- μ is the gear ratio (constant, exaggerates the torque at the robot's joint);
- K_b is the electromotive self-induction constant;
- K means the torque constant of the DC motor;

It also is supposed that the exact motor torque cannot be directly measured: for instance it can be estimated via measuring the motor current with a near unit factor β as $Q^{Real} = \beta Q^{Meas}$ If we distinguish between the "exact" motor parameters {A,B,C} and their approximately known values $\{\tilde{A}, \tilde{B}, \tilde{C}\}$ in the case of CTC control --that is when the "desired torque" is prescribed as $\dot{Q}^{Des} = \alpha(Q^{Des} - Q^{Meas})$ and the model parameters are used to calculate the $U(t)$ control signal from (6)-- this leads to the equation of motion as

$$\frac{d}{dt}\left(T^{Meas} - T^{Des}\right) = -\alpha\frac{C}{\beta\tilde{C}}\left(T^{Meas} - T^{Des}\right) + \left[\left(\frac{C}{\tilde{C}}\tilde{A} - A\right)T^{Meas} + \frac{1}{\beta}\left(\frac{C}{\tilde{C}}\tilde{B} - B\right)\dot{q}\right] \quad (7)$$

If the model values well approximate the actual A, B, and C values, and $\beta \cong 1$, and α is great enough the dominating term on the right hand side corresponds o an exponential decay of the difference between the torque "ordered from the motor" on the basis of the mechanical model, while the terms in the [] brackets are small errors or perturbations in this exponential behavior.

For improving this control, exactly in the same way as in the case of the mechanical system's control generalized Lorentz matrices associated with the vectors in the MIMO variant of (1) defined as (**f** is the vector, $\mathbf{e}^{(f)}$ is the unit vector parallel with **f**, c=1, the other $\mathbf{e}^{(i)}$s are pairwisely orthogonal unit vectors, each of them is orthogonal to $\mathbf{e}^{(f)}$, too) can be used with a diagonal matrix **g**=<1,...,1,-c^2>. For a SISO system the first component of **f** contains the physically interpreted value, its second component is a "dummy factor" equal to 1, and the appropriate scaling matrix can be obtained as $\mathbf{s}=[\mathbf{D}]\mathbf{g}^{-1}[\mathbf{R}]^T\mathbf{g}$ from the "Desired" and the "Realized" accelerations:

$$\left[\begin{array}{c|c|c|c|c} \mathbf{e}^{(f)}\sqrt{f^2/c^2+1} & \mathbf{e}^{(2)} & \ldots & \mathbf{e}^{(DOF)} & \dfrac{\mathbf{f}}{c} \\ \hline f/c^2 & \mathbf{0} & \ldots & \mathbf{0} & \sqrt{f^2/c^2+1} \end{array}\right] \quad (8)$$

equations (4,5,7) --and in the case of the application of the adaptivity in (1)-- a closed system of equations is obtained. It describes the complicated non-linear interaction and coupling between the mechanical, the electrical, and the control software components. As in the case of the mechanical system, for more efficient behavior for scaling the model of the electrical components can be "completed" with a jumping term yielding equation of motion for the torque as

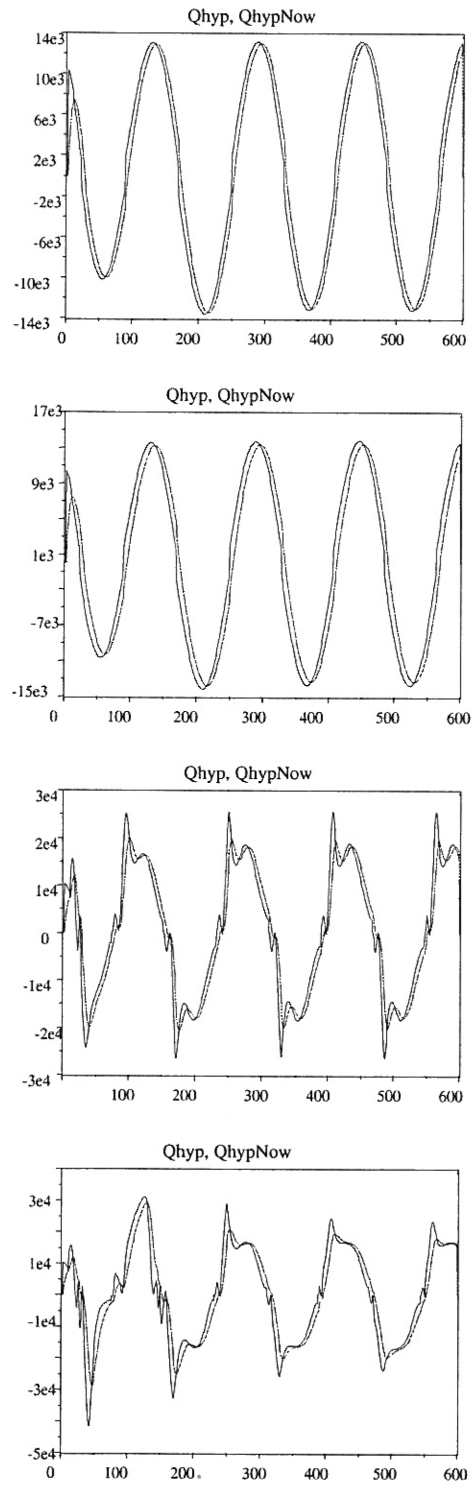

Fig. 5. Ordered and exerted torques without adaptation, with motor adaptation only, with mechanical adaptation only, and with full adaptation

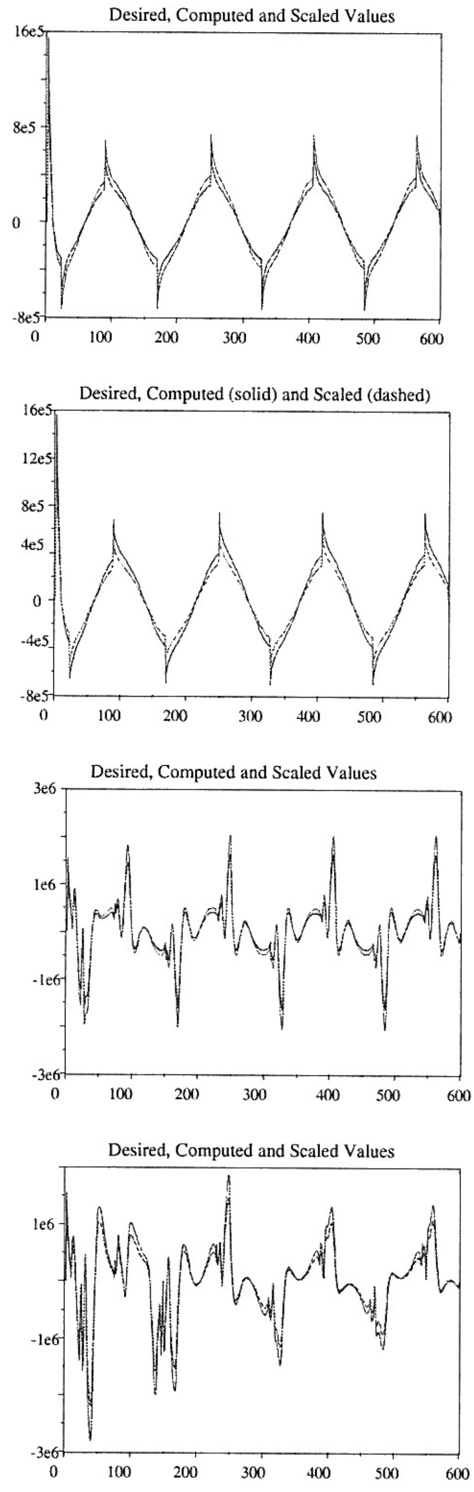

Fig. 6. The speed of the change in the motor torque (corresponds to Fig. 5)

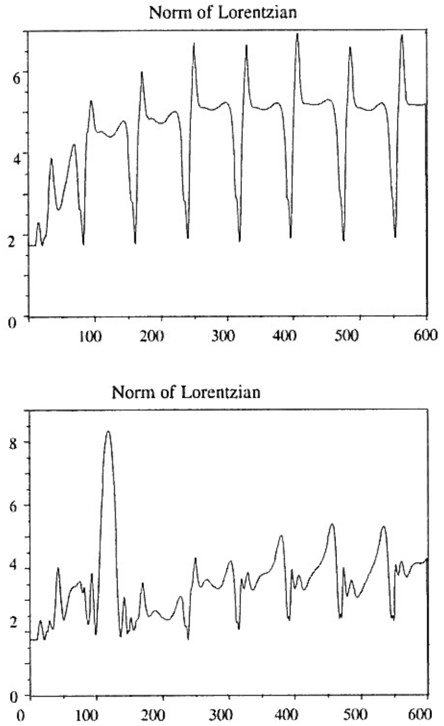

Fig. 7. The norm of the Lorentzian of the mechanical identification without and with motor identification

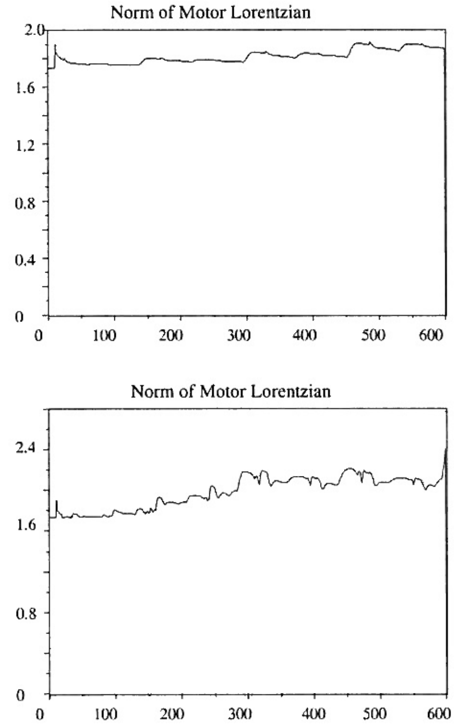

Fig. 8. The norm of the motor's Lorentzian without and with mechanical identification

$$\dot{Q}^{Meas} = \frac{C}{\beta\tilde{\tilde{C}}}\dot{Q}^{Des} + \left(\frac{C}{\tilde{\tilde{C}}}\tilde{A} - A\right)Q^{Meas} + \\ + \frac{1}{\beta}\left(\frac{C}{\tilde{\tilde{C}}}\tilde{B} - B\right)\dot{q} + \frac{C}{\beta\tilde{\tilde{C}}}\Gamma sign\left(\dot{Q}^{Des}\right) \qquad (9)$$

In the simulations A=1, B=1, and C=1 was used for the "actual motor", and 1.3, 1.4, and 0.8 values for their inaccurate approximation in the motor model, and β=0.8, and Γ=5000. It was required to have two times bigger exponent for the motor torque tracking than for the mechanical trajectory tracking to make the motor fast enough for acting in a computed torque control.

Typical simulation results are given in Fig. 5 describing the ordered and exerted torques at different levels of adaptation. As it was expected the essential difference occurs when the identification of the mechanical system is turned on or off. The motor adaptation only slightly influences the exerted torques.

In Fig. 6 the speed of change in the motor torque is described in the cases represented in Fig. 5. It is clear that the essential difference happens when the mechanical identification is switched on or off. The interaction of the adaptation algorithms also is apparent in that figure but it becomes more visible in Fig. 7 describing the norm of the Lorentzian of the mechanical identification with and without the motor identification.

It is clear that the motor identification can decrease the "burden" of the mechanical identification to some extent. In Fig. 8 the counterpart of these curves, the norm of the Lorentzian of the motor-identification is described without and with mechanical identification. Switching on the identification loop for the mechanical system imposes more burden on the motors adaptive current control.

Regarding the trajectory reproduction essential difference occurs only between switching on or off the identification of the mechanical part only.

IV. CONLUSIONS

In this paper a generalized Lorentz transformation based adaptive control was investigated for controlling an electromechanical device consisting of a DC motor driving the mechanical shaft of a physical pendulum. It was shown via simulation that the same adaptive control philosophy can simultaneously be applied to the mechanical and the electrical parts. For further investigations it seems to be expedient to consider similar MIMO electro-mechanical systems for further proofing.

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