Trajectory Planning Based on Position Error Analysis and Fault Area Modelling

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ABSTRACT

A mobile-robot system with a very simplified 2D-rectangular workspace (2D-WS) and landmark-based positioning system is given. In this 2D-WS we establish a *position error function PEF*, which describes the error of the position measurement with respect to the landmarks over the surface of the working space. If an allowable positioning error maximum (ε_{max}) is given, we can calculate by use of *PEF* an appropriate set of trajectory, where the position measurement error will be lover than the given limit.

Moreover based on *PEF* we can easily select the trajectory between the start and goal positions for different optimum criteria as a compromise between speed, trajectory accuracy, safety, smoothest drive etc., for different traffic conditions. We demonstrate our results on computer-simulated case study.

1. INTRODUCTION

The models play very important role in learning from practice. Models of the controlled systems can be used to refine the commands on the basis of analyzing the errors. Better models lead to faster correction of command errors, requiring less practice to achieve a given level of performance. The benefits of accurate modelling are improved performances in all aspects of control.

Mobile robot systems have different kind of errors. One of the most significant errors is the uncertainty of the positioning. Positioning errors can be divided to more subgroups. Errors arisen from dead reckoning, from distance and angle measurement (i.e. errors from trilateration and triangulation) and from some other random effects are among others the most important problems. On the other hand additional to the sensing errors there are trajectory deviations caused by the control system, and by the uncertainty in the description of the environment. Obviously the error detection and error analysis have important role in mobile-robot system design.

The mobile robot should follow a well-defined path in the working space. Usually a position measurement system checks its actual position. The elapsed time between two checks is limited by the sensor system speed. Let us assume a maximal allowed position error (ε_{max}), which the mobile robot has to keep throughout its entire path, and a marker based positioning system.

Our goal is to develop an arrangement of the markers, which guarantees that the given maximal positioning error will not be exceeded. A path planning process will be performed based on the full knowledge of *PEF* .

Usually the position error is treated as a part of uncertainty in the environment's and robot's geometrical model. [1] Our approach is different. We discuss the *PEF* independently of the above-mentioned uncertainties.

We calculate the position errors with two different strategies.

In the first case both angle and distance measurement's error will be taken into account (combined triangulation-trilateration). The angle and distance errors will be handled as unified errors with different priority. We will prove that the higher number of discrete measurements (i.e. the use of more markers) will decrease the value of the *PEF*.

In the second case only distance errors will be analyzed. We will calculate the so-called segment area size, which corresponds to the calculated position error at a given position on the planned trajectory (it describes the possible locations of the robot). The segment area is formed with cutting of two (or more) sectors (see Fig.2.). Cutting of more sectors means distance measurement from more markers.

In both cases we have to take into consideration, that the use of more markers (i.e. more measurement) will significantly increase the time required for the position control. Therefore an optimal number of measurements should be defined for the practical use, which guarantees a position error below the given maximal error (ε_{max}) limit.

1.1. THEORETICAL BACKGROUND

Two different aspects of mobile robot navigation's error should be analyzed simultaneously. The first is the amount of uncertainty and the second is the presence of the errors in position measurement. B. R. Donald from the Cornell University has defined the following three kinds of uncertainty: uncertainty arising from sensing errors, control errors, and the uncertainty in the geometry of the environment [1]. He made the first systematic approach on the problem of error detection and recovery based on geometric and physical reasoning. The last of them he called: model uncertainty and in [2] he showed, that the model uncertainty can be represented by position uncertainty in a generalized configuration space. Therefore his motion strategies include sensor-based gross motion, compliant motions, and simple pushing motions. Further with presence of model error some plans may not even exist. For this reason he investigated the EDR (Error Detection and Recovery) strategies.

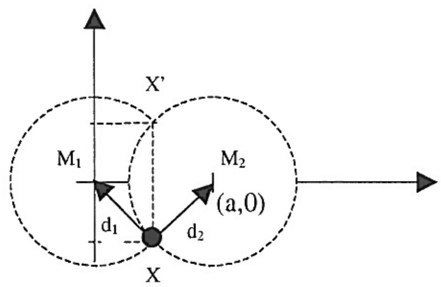
The initial assumption of our approach is similar to the above-mentioned one, but we chose other method for the analysis. Position error will be evaluated in three different cases:

1. Position error estimation based solely on distance measurement. (Chapter 2.)
2. Position error estimation based on angle and distance measurement. (Chapter 3.)
3. Position error estimation based on segment area modelling. (Chapter 4.)

2. POSITION ERROR CALCULATION BASED ON DISTANCE MEASUREMENT

This strategy is useful if distance is the only measured quantity. Let assume that we measure the distance from the marker 'M_1' with some relative error '$\pm\rho_{ml}$'. This measurement gives some segment area with flightiness '$2\rho_{ml}$' and the second measurement from the marker 'M_2' gives the second segment with flightiness '$2\rho_{m2}$'. If the distance to a landmark is the only available information, a single measurement constrains the robot's position to the arc of a circle (see Fig.1).

Fig. 1

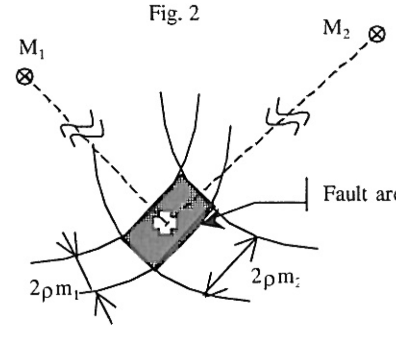


The robot, located at X, senses two landmarks (M_1, M_2) and measures the distances 'a' and 'b' to them respectively. The robot is in the intersection point (X) of two circles with radius 'a' and 'b'. Their centers are the points M_1 and M_2. Let assume for simplicity M_1 to be in the origin of the coordinate system.. M_2 is the point with coordinates (m, 0). Obviously the robot position X with position errors (ε) is:

$$X_{\pm\varepsilon} = \frac{m^2 + (a \pm \rho_{ml})^2 - (b \pm \rho_{m2})^2}{2m};$$
$$Y_{\pm\varepsilon} = \pm\sqrt{(a \pm \rho_{ml})^2 - x^2}; \quad (1)$$

The set of the possible positions of 'X' we call as *fault area* (what is the cut of the segments). The result describes among others the well-known method of the usage of active beacons at known map-locations and distance measurement (for example based on the time of flight method). The marker arrangement for the smallest possible fault area was investigated. The result is trivial; the smallest segment area can be achieved if a=b (see Fig.2).

Fig. 2



3. CALCULATION OF THE POSITION ERROR BASED ON ANGLE AND DISTANCE MEASUREMENT

The schematic model of the positioning procedure is on Fig. 3. We measure the distances (a and b) from the markers (M_1 and M_2). The position of the markers is known on the map, i.e. the distance (m) between them is known also.

Measurement is performed for example with a scanning laser distance measurement technology [4]. We measure the distance to the given 'M_I' marker first, and then the laser beam is deflected (with angle α) to the marker 'M_2', to which the distance will be measured also. This procedure is very similar to the *SAS (Side-Angle-Side)* triangulation strategy.

The mobile robot (R) is on its actual position. Further $m_{(real)}$ can be calculated from the given marker positions, and $m_{(measured)}$ from the measured 'a and b' distances and 'α' angle. From '$m_{(real)}$' and '$m_{(meas)}$' we can derive the *k – correction factor* what we can utilize for the trajectory modification later.

$$m_{(meas.)} = \sqrt{a^2 + b^2 - 2ab\cos\alpha}; \quad (2)$$

$$k = \frac{m_{(real)}}{m_{(meas)}}; \quad (3)$$

Absolute and relative errors of the system:

$$\Delta_{(asb)} = |m_{(real)} - m_{(meas)}|$$
$$\Delta_{(rel)} = R_e = \left|\frac{m_{(real)} - m_{(meas)}}{m_{(real)}}\right| \quad (4)$$

Furthermore generally known:

$$m_{(real)} = m_{(meas)}.(1 \pm R_e), \quad (5)$$

Where, R_e is a given system error (i.e. the relative error to the given distance). In our laboratory for the distance measurement with the laser technology we had 15[m] / 1[mm] = distance / error ratio [4].

Let be 'ξ' the angle error for the 'α^0', and respectively known '$m_{(meas)}$' from measured *'a'* and *'b'* distances. From (2) and (5) we get the final relation for $m_{(real)}$ distance:

$$m_{(real)} = \left(a_{(meas)}(1 \pm \mathrm{Re})\right)^2 + \left(b_{(meas)}(1 \pm \mathrm{Re})\right)^2 - \quad (6)$$
$$- 2[a_{(meas)}.b_{(meas)}(1 \pm \mathrm{Re})^2].\cos(\alpha_{(meas)})(1 \pm \xi));$$

With reducing equation (6), we examine the values of 'a, b, m' and 'α' for 'R_e' minimum. The result shows, that the minimal 'R_e' can be achieved by '$a=b$', and '$\alpha=90^0$'.

The equations can be formulated in the word co-ordinates too (all this is needed for the modelling in MATLAB environment). In this case 'R_e' have two components: 'Re^a_{XR}, Re^b_{XR}'. (Where Re^a_{XR} is the 'x' component of the relative error (R_e) belonging to the 'a' distance on the mobile robot (R) position). The final equation can be derived from the followings:

$$m^2 = (X_{m2} - X_{m1})^2 + (Y_{m2} - Y_{m1})^2; \quad (7)$$

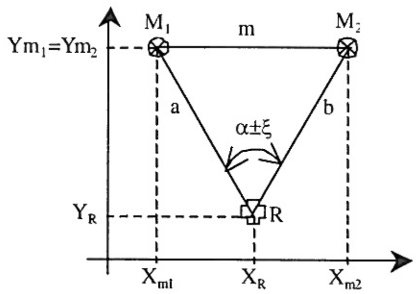
Simultaneously is valid the equation (2), where for distances 'a' and 'b' in word co-ordinates can be written:

$$a^2 = (X_R - X_{m1})^2 + (Y_{m1} - Y_R)^2;$$
$$b^2 = (X_{m2} - X_R)^2 + (Y_{m2} - Y_R)^2; \quad (8).$$

Where:

$$X_R \rightarrow X_R \pm \mathrm{Re}^a_{XR};$$
$$X_R \rightarrow X_R \pm \mathrm{Re}^b_{XR};$$
$$Y_R \rightarrow Y_R \pm \mathrm{Re}^a_{YR};$$
$$Y_R \rightarrow Y_R \pm \mathrm{Re}^b_{YR}; \quad (9).$$

Fig. 3.



With replacing (8) and (9) into the (2), from the comparison the equations (2) and (7), we get the final conditions for the minimal relative error divided for the 'X' and 'Y' co-ordinates.

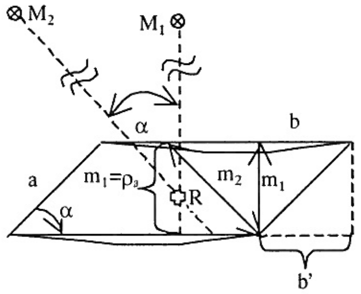
4. POSITION ERROR CALCULATION BASED ON THE SEGMENT AREA MODELLING

This is very similar method to the above mention (chapter 2.). The only difference is, in chapter 2. for distance measurement TOF method has been used, and now combination of *'light striping'* and *'laser eye'* methods will be investigated what we are using in our laboratory. Laser eye is a combined range and video sensor consisting of a camera and a laser range finder arranged so that the optical axis of the camera and the laser beam are parallel or coincident. Light striping: a line (laser beam) projected into the scene allows recovery of the depth of the point at which the light beam strikes (marker), by triangulation.

The calculation was prepared only for the cut of two segments belonging to the smallest error area. It can be proved, that it is enough to prepare the measurement from two markers in case of known trajectory. Measurements from additional markers (if they have different distance from the mobile robot position), will not significantly increase the accuracy of position determination, only the time consumption will be increased

Further we have to endeavor, that the cut of segments (see Fig.2) has more or less linear form, so the cut area could be form like a parallelogram. This is possible under 1% relative error of the distance, what can be easily kept with our *'light striping - laser eye'* technology measurements too [3]. On Fig. 4, where the relative error is more than 1%, the nearly linear fault area modell can be seen.

Fig. 4.



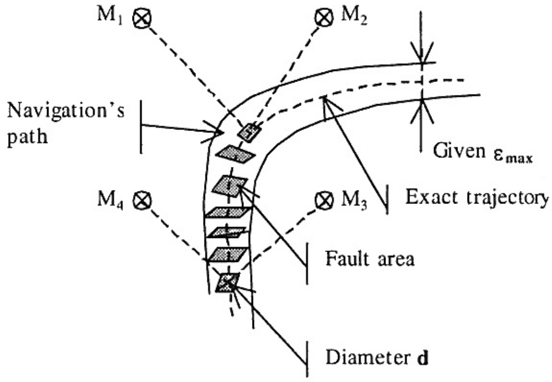
In our general calculation let, $M_1R \neq M_2R$ and 'α^0' is the angle between' M_1R' and 'M_2R'. The fault area (A) is performed with the following forms.

$$a = \frac{m_1}{\sin\alpha};$$
$$b = \frac{m_2}{\sin\alpha}; \qquad (10)$$
$$A = b.m_1 = \frac{m_1.m_2}{\sin\alpha};$$

Looking for the conditions of the minimal errors can be performed with the first derivation of (10) based on 'a', 'b' and 'α' angle. We get the result – *minimal fault area-*, when $a=b$ and the angle $\alpha=90^0$.

Further, if a maximal position error is given (ε_{max}), it is possible to calculate only with 'd' diameter of the parallelogram, because with given ε_{max} we can plot the *navigation's path,* along the trajectory (see Fig.5).

Fig.5.



The diameter (d) can be calculated based on the followings (See Fig.4.):

$$b'^2 = a^2 - m_1^2;$$
$$b'^2 = a.\cos\alpha; \qquad (11).$$

$$d^2 = (b+b')^2 + m_1^2;$$
$$d = \sqrt{[b + (a.\cos\alpha)]^2 + m_1^2}; \qquad (12).$$

With performing the first derivation of formula (12) based on 'a', 'b', and angle 'α' we get the conditions for the minimal diameter (d). From Fig.4, and after derivation can be seen, that the fault area is minimum, when the triangle M_1, R, M_2 is almost right-angled. (M_1, M_2 are the markers and R is the mobile robot.)

5. POSITION ERROR EVALUATION

The accuracy is calculated along the entire path. The minimal frequency of the position measurements is given by the system. Based on this, and in knowledge of the above-mentioned conditions of *(a, b, α)* would be divided the path, and we can determine the poses of the most accurate position measuring. The frequency of the position measuring will change (increase), if another agent(s) is on the scene.

Further, the relationship between the accuracy of the input measurements and the accuracy of the final estimate of the desired pose variables is formed by the notion of the *Geometric Dilution Of Precision (GDOP)* [5]. This metric expresses variation in the output estimate 'X' (geometric variables constituting the pose) with variations in the input parameters 'S' (sensor data).

$$GDOP = \frac{\Delta X}{\Delta S}; \qquad (13);$$

Where, we can take the limit for ΔS. In case, if ($\Delta S \rightarrow 0$), GDOP is equal with Jacobian (J) of the measurement equation.

In our case the pose estimation and the associated uncertainty based on triangulation using imprecise bearings, to the two specially displaced markers, where the ambiguous region is almost a parallelogram, see Fig. 2.

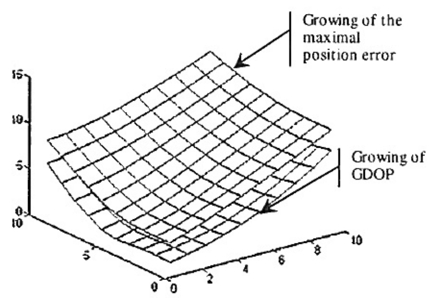
The GDOP for two markers located at the coordinates $M_1=(0,0)$, $M_2=(5,0)$ is given by:

$$GDOP = J = \begin{bmatrix} \dfrac{a}{m} & \dfrac{-b}{m} \\ \dfrac{a(m - x_R)}{m.y_R} & \dfrac{x_R.b}{m.y_R} \end{bmatrix} \quad (14);$$

Where, x_R, y_R are the robot coordinates, 'm' is the distance between markers, (m=5), and the magnitude of 'J' grows as the robot moves away from the 'x' axis. On Fig. 6, we can see the growing of *GDOP* and the calculated position error. Further we can compare, that the position error calculated from the segments area and the calculated *GDOP* are very similar to each other. This is verify, that our calculation with segment area model is correct and, it is possible to calculate the markers arrangement in the interest of the biggest accuracy of position measurements.

This is what we expected, because if a maximal position error is given (ε_{max}), can be determined the robot position, in which the robot must change the conditions of the next measurement.

Fig. 6.



6. TRAJECTORY PLANNING

Trajectory planning can be performed in knowledge of the calculated position error function. Each possible trajectory between the dock pairs has its own given maximal position error (ε_{max}), and calculated position errors (τ) in each points of the trajectory. Calculated position error is proportional to the above mentioned segment area. Real accuracy of the trajectory is given with calculated (τ), but they are predetermined distance measurements to this position error, from the given markers. For the relation between 'τ' and 'ε_{max}' is valid:

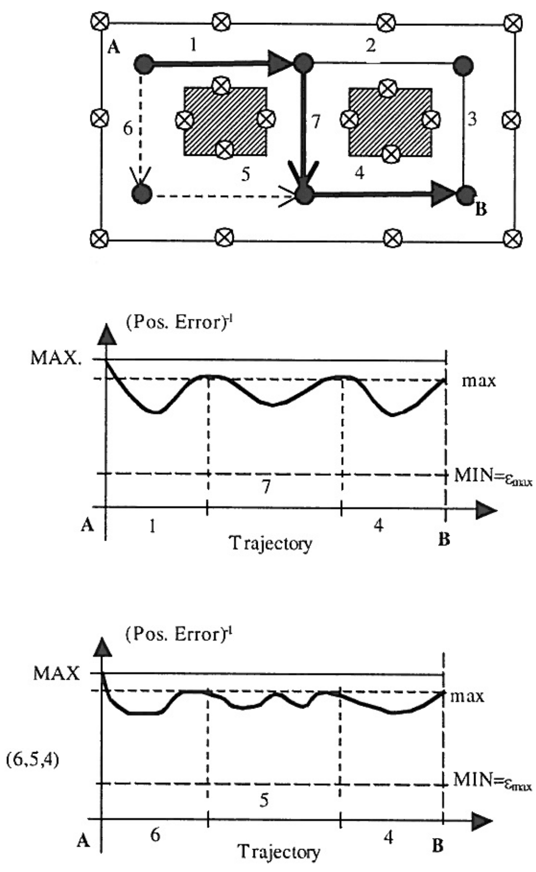
$$\tau < \varepsilon_{max} \quad (15).$$

The calculation can be performed with the above-mentioned strategies, see chapters 2., 3., 4.

In our laboratory we simulated the position error calculation in MATLAB environment.

Lot of trajectories exist between the given dock pairs which has different accuracy, traffic density and length (see Fig. 7.). Based on position error function (PEF), and given allowed maximal position error (ε_{max}), we can select the final trajectory between the start and goal positions for the different criteria of accuracy. The selection can be done on the following principle

Fig. 7.



Legend:
⊗ - markers;
1…7 – segments;
MAX – maximal accuracy (theoretically – exact trajectory)
max – maximal accuracy (achieved with measuring)
MIN – given minimal accuracy what is equal with max. pos.error ε_{max}
A / B – starting / goal dock positions

Conditions for trajectory selecting:

- The map is given in the memory of mobile robot, with known obstacles and markers positions.

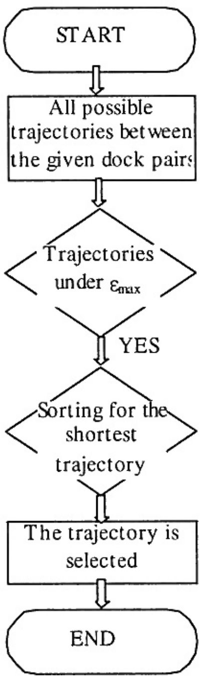
- The set of all navigation paths is known with a given allowed max. pos. errors (ε_{max}).

Trajectory selection process, see on Fig. 8.

7. CONCLUSION

In practical use some workspace with obstacles and the required accuracy of the robot is usually given. In this paper we described a procedure of markers' arrangement around the trajectories to keep the accuracy between the given intervals. Further we proved, that the best accuracy could be achieved, if the markers are equidistant from the mobile robot and from each other. Moreover we highlighted that the accuracy is susceptible with the angle (α) between the markers and the mobile robot. We verified, that the best accuracy can be achieved in case of $\alpha=90^0$. Finally we can define to a given accuracy, in given environment and trajectories, the needed number of markers and their displacement.

Fig. 8



REFERENCES

[1] Bruce Randall Donald: "Planning Multi-Step Error Detection and Recovery Strategies". Computer Science, Cornell University, '87.

[2] B. R. Donald: "A Geometric Approach to Error Detection and Recovery for Robot Motion Planning With Uncertainty". Computer Science, Cornell University '88.

[3] I. Nagy, L. Vajta, Cs. Gürtler: "Vision based navigation for multi-agent mobile robot system in the transport domain". Proc. IEEE. Int. Conf. INES '97, p. 171-174, Budapest, '97.

[4] M. Vogel, F. Vajda, L. Vajta: "LABrador-a Hierarchical Sensor System for Mobile Robot Navigation". Proc. IEEE Int. Conf. INES'99, Slovakia.

[5] G. Dudek, M. Jenkin: "Computational Principles of Mobile Robotics", Cambridge University Press, pp. 181.

[6] Cs. Gürtler, I. Nagy, L. Vajta,: "Trajectory Planning for Mobile Robots Based on Dynamical Models". Proc. IEEE. Int. Conf. INES 97, p. 171-174, Budapest, 1997.