H_2 Control for a Two Revolute Joints Robot Arm

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Abstract

In this paper we present the H_2 methodology in order to compute the controller for a robot arm with two revolute joints. We consider a general method of formulating control problem, which makes use of linear fractional transformation as introduced by Doyle. The formulation makes use of the general two-port configuration of the generalized plant with a generalized controller. The H_2 norm is the quadratic criterion used in optimal control as LQG. The overall control objective is to minimize the H_2 norm of the transfer matrix function from the weighted exogenous inputs to the weighted controlled outputs. The advantage of H_2 control technique, which uses the linearized model of the robot structure, is that it is completely automated and very flexible. The order of the linear model can be reduced so we obtain a low order controller for the real structure without the loose of performances. Finally, we prove that the closed loop control structure has very good inner robustness.

1. Introduction

Rigid robot systems are subjects of the research in a both robotic and control fields. The reported research leads to a variety of control methods for rigid robot systems. The present paper is addressed to robotic manipulator control. High speed and high precision trajectory tracking are frequently requirements for applications of robot arms.

Conventional controllers for robot arm structures are based on independent control schemes in which each joint is controlled separately by a simple servo loop. This classical control scheme is inadequate for precise trajectory tracking. The imposed performances for industrial applications require the consideration of the complete dynamics of the robot arm. Furthermore, in real-time applications, the ignoring parts of the robot dynamics, or errors in the parameters of the robot arm may cause the inefficiency of this classical control (such as PD controller). Even in well-structured industrial applications, manipulators are subject of the structured uncertainty, i.e. the parameter uncertainty due to unknown load, friction coefficients and so on. When the dynamic model of the system is not known a priori or is not available, a control law is designed based on an estimated model. (the basic idea behind adaptive control strategies) or on the specific robust control methods.

The organization of this paper is the following. In section 2 the nonlinear model of a vertical planar robot arm with two revolute joints is presented. Also we state the two-port representation of the generalized plant for H_2 design. Section 3 deals with H_2 control technique for the robot arm, which represent the generalization of the classical LQG control by introducing the so-called *weighting functions.* In section 4 we apply the reduction for the balanced robot model. The original and reduced models have matching DC gains (steady-state response) in order to fulfill all the tracking requirements. The section 5 is dedicated to the computer simulation and comparisons. Finally, the section 6 collects the conclusions.

2. H_2 Design Architecture

2.1. The Robot Arm Model

The robot arm is modeled as a set of n rigid bodies connected in series with one end fixed to the ground and the other end free. The bodies are connected via either revolute or prismatic joints and a torque actuator acts at each joint. The dynamic equation of an n-link robot arm is given by ([2]; [8]):

$$T = J(q)\ddot{q} + V(q,\dot{q})\dot{q} + G(q) + F(\dot{q}) \tag{1}$$

where

- T is an $(n \times 1)$ vector of joint torques;
- $J(q)$ is the $(n \times n)$ manipulator inertia matrix;
- $V(q,\dot{q})$ is an $(n \times n)$ matrix representing centrifugal and Coriolis effects;
- $G(q)$ is an $(n \times 1)$ vector representing gravity;

- $F(\dot{q})$ is an $(n \times 1)$ vector representing friction forces;
- $q, \dot{q}, \ddot{q}$ are the $(n \times 1)$ vectors of joint positions, speed and accelerations, respectively.

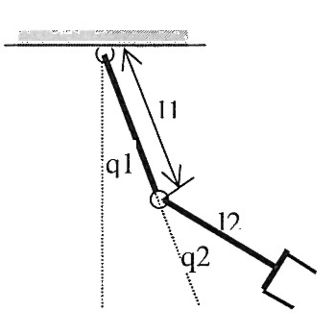

Fig. 1. Two link planar manipulator arm

The equations (1) form a set of coupled nonlinear ordinary differential equations, which are quite complex, even for simple robotic arms. Is considered the control of the simple vertical planar robot arm with two revolute joints shown in Fig. 1.

The elements of the dynamic equation (1) for this robot arm with electrical motor dynamics are found to be:

$$J(q)=\begin{bmatrix} l_1^2(m_1+m_2)+m_2l_2^2+2m_2l_1l_2c_2+J_1n_1^2 & m_2l_2^2+m_2l_1l_2c_2 \\ m_2l_2^2+m_2l_1l_2c_2 & m_2l_2^2+J_2n_2^2 \end{bmatrix} \quad (2)$$

$$V(q,\dot{q})=m_2l_1l_2s_2\begin{bmatrix} -2\dot{q}_2 & -\dot{q}_2 \\ \dot{q}_1 & 0 \end{bmatrix} \quad (3)$$

$$G(q)=\begin{bmatrix} (m_1+m_2)gl_1s_1+m_2gl_2s_{12} \\ m_2gl_2s_{12} \end{bmatrix} \quad (4)$$

$$F(\dot{q})=\begin{bmatrix} v_1\dot{q}_1+C_1 sign(\dot{q}_1) \\ v_2\dot{q}_2+C_2 sign(\dot{q}_2) \end{bmatrix} \quad (5)$$

with $c_i = cos\ (q_i)$; $s_i = sin\ (q_i)$;

$c_{12} = cos\ (q_1 + q_2)$; $s_{12} = sin\ (q_1 + q_2)$

J_i = moments of inertia for electrical motor i.
n_i = factor of reduction gear i.
v_i = viscous friction for joint i.
C_i = Coulomb friction for joint i.

The robot arm starts at position ($q_1 = 0$, $q_2 = 0$) and the control objective is to track the desired trajectories given by a step modification in angle references or by a sinusoidal reference trajectory. Adding the dynamic of the DC-motors, gears and variators, we find an eight order nonlinear equations system: the four-order robot model (the mechanical dynamics) presented above with the states

$$[q_1, \dot{q}_1, q_2, \dot{q}_2]^T = [x_1, x_2, x_3, x_4]^T \quad (6)$$

and a four-order actuator model (the electrical dynamics) described by:

$$\begin{cases} \dot{x}_5 = \dfrac{x_7 - R_1x_5 - n_1K_{e1}x_2}{L_1} \\ \dot{x}_6 = \dfrac{x_8 - R_2x_6 - n_2K_{e2}x_4}{L_2} \\ \dot{x}_7 = \dfrac{K_{v1}U_{c1} - x_7}{T_{v1}} \\ \dot{x}_8 = \dfrac{K_{v2}U_{c2} - x_8}{T_{v2}} \end{cases} \quad (7)$$

The state vector will have eight elements:

$$[q_1, \dot{q}_1, q_2, \dot{q}_2, I_1, I_2, U_{m1}, U_{m2}]^T = [x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8]^T$$

In (7) (with appropriate electrical parameters) U_{c1}, U_{c2} are the voltages supplied by the designed controller (inputs for the general plant model).

2.2. Two-port representation of robot arm model

Generally, if we consider w = {r, d, n} the exogenous input of the system (r for reference signals, d for disturbances and n for measurement noises), z = {z_1, z_2, z_3} the quality output of the system and u the control input (controller output) and (y = e) the controller input (system error), we have the representation from Fig. 2 (see [4])

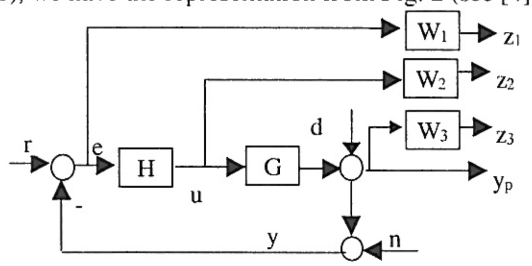

Fig. 2 The H_2 controller design architecture

W_1, W_2 and W_3 are appropriate weighting functions, used in the controller design process. We consider a general method of formulating control problem, which makes use of linear fractional transformation as introduced by Doyle. The formulation makes use of the general two-port configuration of the generalized plant with a generalized controller. The classical control loop from Fig. 2. is equivalent with the so-*called two-port representation* of the generalized plant from Fig. 3.

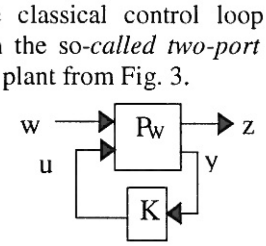

Fig. 3. Two-port representation of control loop

The generalized plant P_w has different order (McMillan number) and different number of inputs and outputs with respect to the original plant, according with weighting functions and designer strategies.

The overall control objective is to minimize the norm of the transfer matrix function from the weighted exogenous inputs w to the weighted controlled quality outputs z, so the optimization problem is:

$$\inf_{K \in RH_\infty} \left\| H_w^z \right\|_2 \tag{8}$$

where

$$H_w^z = F_L(P,K) = P_{11} + P_{12} K (I - K P_{21})^{-1} P_{22} \tag{9}$$

$P = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}$ is the original linear model of the plant partitioned according to the dimensions of w, u and z, y, respectively.

In our specific case, we propose the following two-port control architecture from Fig. 4.

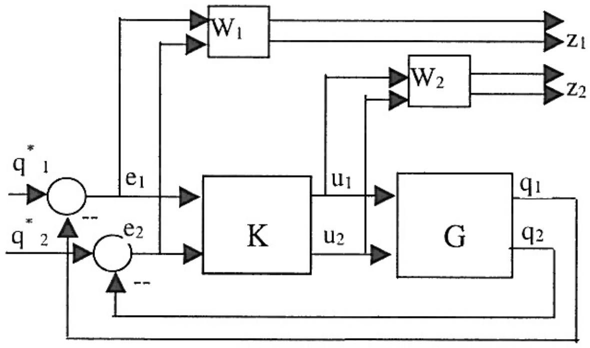

Fig. 4. H_2 design architecture

- the quality outputs are:

$$\begin{cases} z_1 = W_1 \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} \\ z_2 = W_2 \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \end{cases} \tag{10}$$

- the measurements are only the angles, and we shall prove that this is a reasonable choice.
- the controller inputs are the angular errors
- the exogenous plant inputs are the angle references
- the controller outputs are the command voltages

With reference to Fig. 3., we define

$$w = \begin{bmatrix} q_1^* \\ q_2^* \end{bmatrix};\ u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix};\ e = \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} q_1^* - q_1 \\ q_2^* - q_2 \end{bmatrix} \tag{11}$$

$$z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix};\ z_1 = W_1 e;\ z_2 = W_2 u;\ y = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$$

3. H_2 Controller Design

We consider a general method for formulating H_2 control problem, which makes use of linear fractional transformation as introduced by Doyle. The formulation makes use of the general two-port configuration of the generalized plant presented in Fig. 3. The overall control objective is to minimize the norm of the transfer matrix function from the weighted exogenous inputs to the weighted controlled outputs.

The H_2 optimal problem can be stated as

$$\inf_{K \in RH_\infty} \left\| H_w^z \right\|_2 \tag{12}$$

and is suitable for linear systems. We shall use in the next sections only the linearized model G of the nonlinear two-joint robot arm around the equilibrium point $q_1 = q_2 = 0$. For the moment we shall not consider dry or viscous friction.

Beginning with this linear model G, we shall derive the generalized plant P_W (called also the augmented plant) using the following general formulas:

$$P_W(s) = H_{\begin{bmatrix} w \\ u \end{bmatrix}}^{\begin{bmatrix} z \\ y \end{bmatrix}}(s) = \left[\begin{array}{c|c} W_1 & -W_1 G \\ 0 & W_2 \\ 0 & W_3 G \\ \hline I & -G \end{array}\right] \tag{13}$$

The design of the H_2 controller is based on this augmented plant matrix transfer $P_W(s)$. From (12) we can see that the augmented plant can be represented in state-space form:

$$P_W(s) \stackrel{s}{=} \left[\begin{array}{cc|c} A & B_1 & B_2 \\ C_1 & D_{11} & D_{12} \\ \hline C_2 & D_{21} & D_{22} \end{array}\right] \tag{14}$$

We can solve the ***H^2*-norm optimal control problem** by observing that it is equivalent to a conventional *Linear-Quadratic Gaussian optimal control problem*. The $\mathbf{H}^2$ optimal controller K(*s*) is thus feasible in the usual LQG manner as a full-state feedback K_c and a Kalman filter with residual gain matrix K_f with following relations:

a) Kalman Filter:

$$\dot{\hat{x}} = A\hat{x} + K_f (y - C_2 \hat{x} - D_{22} u)$$

$$K_f = \left(\Sigma C_2^T + B_1 D_{21}^T\right)\left(D_{21} D_{21}^T\right)^{-1}$$

$$\Sigma A^T + A\Sigma - \left(\Sigma C_2^T + B_1 D_{21}^T\right)\left(D_{21} D_{21}^T\right)^{-1}\left(\Sigma C_2^T + B_1 D_{21}^T\right)^T$$

b) Full-state feedback

$$u = K_c \hat{x}$$

$$K_c = \left(D_{12}^T D_{12}\right)^{-1}\left(B_2^T P + D_{12}^T C_1\right)$$

$$A^T P + PA - \left(B_2^T P + D_{12}^T C_1\right)^T \left(D_{12}^T D_{12}\right)^{-1}\left(B_2^T P + D_{12}^T C_1\right)$$

$$K(s) \stackrel{s}{=} \left[\begin{array}{c|c} A - K_f C_2 - B_2 K_c + K_f D_{22} K_c & K_f \\ \hline -Kc & 0 \end{array}\right] \tag{15}$$

4. Model Order Reduction

From (13) and (15) we can see that the controller order is the sum of the original plant order and the orders of each weighting functions. The controller and the augmented (generalized) plant have the same order. Since the linearized model is an eight order model, even by using scalar weighting functions we can expect an at least eight order optimal H_2 controller. From the real implementation point of view, this is unacceptable. One can either reduce the order of the plant model prior to controller design, or reduce the controller order in the final stage (after the controller design), or to use both methods.

By reducing the robot arm model order we are able to reduce the controller order. We have three possibilities for the model order reduction: balanced truncation, balanced residualization and optimal Hankel norm approximation. Residualization, unlike truncation and optimal Hankel norm approximation, preserves the steady-state gain of the system (which is very important in our case since we are interested in tracking capabilities), it is simple and computationally inexpensive. Since our controller has the same order as the generalized plant P, eight, we shall prove that we are able to dramatically reduce the controller order to two. In the next we shall use the so-called balanced realization. A balanced realization is an asymptotically stable minimal realization in which the controllability and observability Gramians are equal and diagonal. Any minimal realization of a stable transfer function can be balanced by a simple state similarity transformation. We define the *Hankel singular values* of G(s) making use of the minimal state space representation of G(s):

$$G(s)\overset{s}{=}\left(A_g, B_g, C_g, D_g\right) \tag{16}$$

Then

$$G_b(s)\overset{s}{=}(A, B, C, D) \tag{17}$$

is *balanced* if the solutions to the following Lyapunov equations are equal:

$$\begin{cases} AP+PA^T+BB^T=0 \\ A^TQ+QA+C^TC=0 \end{cases} \Rightarrow P=Q=diag(\sigma_1,\sigma_2,\ldots,\sigma_n)=\Sigma \tag{19}$$

P and Q are the controllability and observability Gramians, also defined by

$$P\overset{\Delta}{=}\int_0^\infty e^{At}BB^Te^{A^Tt}dt;\ Q\overset{\Delta}{=}\int_0^\infty e^{A^Tt}C^TCe^{At}dt \tag{20}$$

The σ_i 's are the ordered Hankel singular values of G(s), more generally defined as

$$\sigma_i\overset{\Delta}{=}\left[\lambda_i(PQ)\right]^{\frac{1}{2}},\ i=1,\ldots,n \tag{21}$$

In a balanced realization the value of each σ_i is associated with the state x_i of the balanced system. The size of σ_i is a relative measure of the contribution x_i makes to input-output behaviour of the system.
After balancing a system, each state is just as controllable as it is observable, and a measure of state's joint observability and controllability is given by its associated Hankel singular values.

Let (A, B, C, D) be a balanced minimal realization of a stable system G(s), and partition the state vector x, of dimension n, into $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ where x_2 is the vector of n-k states which we wish to remove. With appropriate partitioning of A, B and C, the state-space equations become

$$\begin{cases} \overset{\circ}{x}_1 = A_{11}x_1 + A_{12}x_2 + B_1u \\ \overset{\circ}{x}_2 = A_{21}x_1 + A_{22}x_2 + B_2u \\ y = C_1x_1 + C_2x_2 + Du \end{cases} \tag{22}$$

4.1. Truncation

A k-th order truncation of the realization $G\overset{s}{=}(A, B, C, D)$ is given by

$$G_a\overset{s}{=}(A_{11}, B_1, C_1, D) \tag{23}$$

The truncated model G_a is equal to G at infinite frequency because

$$G(\infty)=G_a(\infty)=D \tag{24}$$

but apart from this there is little that can be said about the relationship between G and G_a .

An advantage of model truncation is that the poles of truncated model are a subset of the poles of the original model and therefore retain any physical interpretation they might have.

4.2. Residualization

In truncation, we discard all the states and dynamics associated with x_2. Suppose that instead of this we simply set $\overset{\circ}{x}_2 = 0$, i.e. we *residualize* x_2, in state-space equations. One can solve for x_2 in terms of x_1 and u and back substitution of x_2 , then gives

$$\begin{cases} \overset{\circ}{x}_1 = \left(A_{11} - A_{12}A_{22}^{-1}A_{21}\right)\cdot x_1 + \left(B_1 - A_{12}A_{22}^{-1}B_2\right)\cdot u \\ y = \left(C_1 - C_2A_{22}^{-1}A_{21}\right)\cdot x_1 + \left(D - C_2A_{22}^{-1}B_2\right)\cdot u \end{cases} \quad (25)$$

Let assume A_{22} is invertible and define

$$\begin{cases} A_r \overset{\Delta}{=} A_{11} - A_{12}A_{22}^{-1}A_{21} \\ B_r \overset{\Delta}{=} B_1 - A_{12}A_{22}^{-1}B_2 \\ C_r \overset{\Delta}{=} C_1 - C_2A_{22}^{-1}A_{21} \\ D_r \overset{\Delta}{=} D - C_2A_{22}^{-1}B_2 \end{cases} \quad (26)$$

The reduced order model

$$G_a(s) \overset{s}{=} \left(A_r, B_r, C_r, D_r\right) \quad (27)$$

is called residualization of $G(s) \overset{s}{=} (A, B, C, D)$.

In our case, we find the following Gramian for the balanced system:

$$\Sigma = diag(2.3946, 0.1945, 0.0001, 0.0001, 0, 0, 0, 0) \quad (28)$$

It is clear that we keep only the first two balanced states, the remainder elements having no importance in the input-output system behaviour. It is necessary to detect the physical signification of these two principals states by computing the power of the state signal:

$$x_b = Tx \Rightarrow x_b^T x_b = x^T T^T T x \cong x^T \left[\begin{array}{ccc|c} 74 & .5 & 0 & \\ .5 & 0 & 0 & 0_{3x5} \\ 0 & 0 & 74 & \\ \hline & & 0_{5x3} & 0_{3x3} \end{array}\right] x$$

Retaining the first two elements of the balanced state,

$$x_b^T x_b \cong x_{b1}^2 + x_{b2}^2 \cong 74x_1^2 + 74x_3^2 \quad (29)$$

It is clear that only the states $[x_1 \quad x_3] = [q_1 \quad q_2]$, the measurement outputs, are important from the input-output point of view because these states appear magnified in the two main components of the balanced reduced order state vector. It follows that we can use for design the second order balanced reduced model of the robot arm.

5. Controller Design, Computer Simulation and Results

With reference to Fig. 4., we chose

$$W_1 = 10^6 \begin{bmatrix} \frac{1}{s} & 0 \\ 0 & \frac{1}{s} \end{bmatrix}; W_2 = 10^{-2} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (30)$$

and from (15) we obtain a four-order H_2 controller.

In this moment we are able to test the control system performances. The tracking capabilities are tested for step angle references and for low-frequency sinusoidal angle references. In order to prove the inner robustness properties, we apply a strong step modification for mass m_2, simulating the real manipulator function of robot arm. The simulation results are the following (neglecting the dry and viscous friction):

a) Step response. The system has the initial position $q_1^* = q_2^* = 30^o$ and we apply a step reference $q_1^* = q_2^* = 60^o$. (angle in radians).

From Fig. 5 we observe a very good behaviour of the control system. Both robot arms are driven in the desired position in less than half second, with no overshoot. The evolution of current intensity of DC-motors (actuators) is represented in Fig. 6.

The next simulated experiment is presented in Fig. 7. using different low frequencies for the two robot arms.

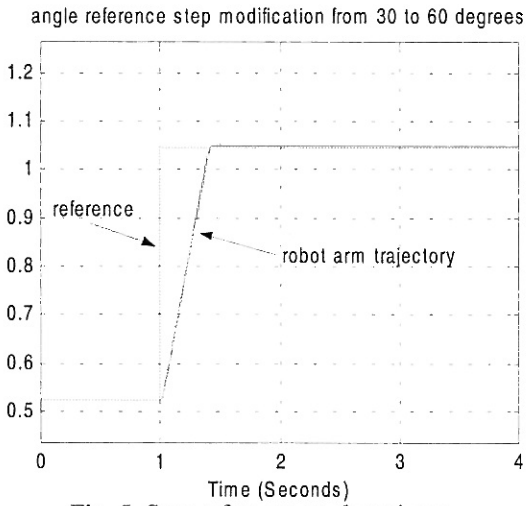

Fig. 5. Step reference angle trajectory

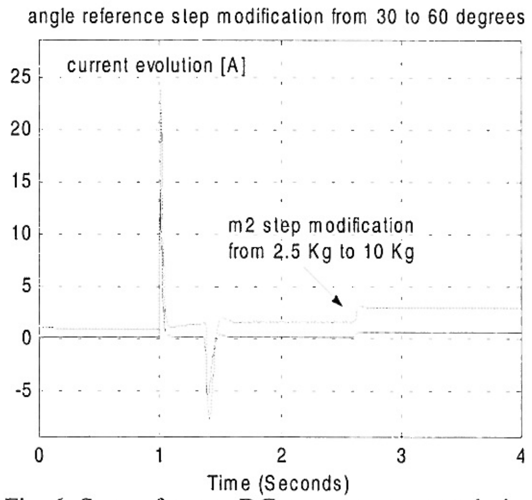

Fig. 6. Step reference DC-motor current evolution

b) Low-frequency sinusoidal references: We apply a sinusoidal reference $\begin{cases} q_1^* = 30^o \cdot \sin(t) \\ q_2^* = 30^o \cdot \sin(2t) \end{cases}$

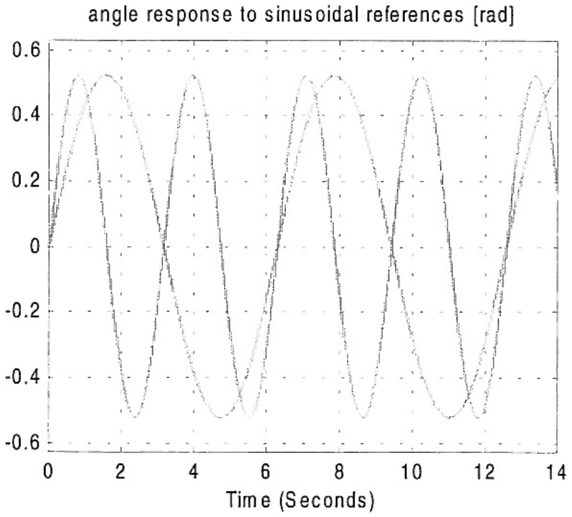

Fig. 7. Angle response to sinusoidal reference

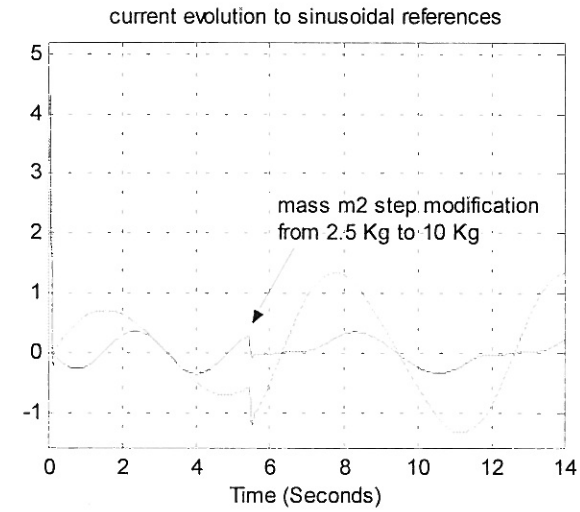

Fig. 8. Current evolution to sinusoidal reference

6. Conclusions

In this paper we presented H_2 methodology for designing a reduced order controller for a two-joint robot arm. Starting with the nonlinearly eight-order model, we proposed the following methodology for the controller design:

- Obtain the linear model from the set of nonlinearly differential equation around the equilibrium point $q_1 = q_2 = 0$.
- Obtain the balanced realization of the linear model in order to compute the reduced order model using balanced residualization. Starting with an eight order linear model we find a two-order reduced model.
- Compute the H_2 controller using appropriate weighting functions. In our case, we do not use W_3 and W_2 is chosen a scalar matrix only for methodological reasons.
- Test the control system behaviour for evaluating the tracking performances.

Following these steps, we found a four-order 2x2 H_2 controller with very good dynamical performances. The command voltages (controller outputs) are limited to 100 V. Simulating the input step response, we obtain a very good response, with no overshoot and less than half second transitory time. The low-order frequency response is also very good, with no important phase lag. The control system proved a good robustness performance, tested by step modification of the mass m_2, simulating the real function of the gripper. Other research direction may include the H_2 controller design in presence of dry friction.

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