Observers with unknown inputs for Estimation of the Road Profile

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Abstract— **This paper deals with a new method for estimation of the road profile using partial measurements of vehicle dynamics. This method is based on a robust observer designed with a nominal dynamic model of the vehicle. This model is a linear half-car model whose inputs have been considered as unknown states to be estimated. The estimation accuracy of our observer has been validated experimentaly using a trailer equiped with position sensors and accelerometers.**

Keywords— **Profile, Pavement, Observer, Estimator, Vehicle.**

I. Introduction

Road profile unevenness through road/vehicle dynamic interaction and vehicle vibration affects safety (Tyre contact forces), ride comfort, energy consumption and wear. The road profile unevenness is consequently a basic information for road maintenance management systems.

Measurement of road unevenness has been a subject of numerous research for more than 70 years. Methods developed can be classified in two types : response type and profiling method. Nowadays profiling method giving a road profile along a measuring line are generally preferred. These methods pertain to two basic techniques : rolling beam or inertial profiling method. The last method which was first proposed in 1964 [4]. is now worldwide used. Inertial profiling methods consist in analyzing the signal coming from displacement sensors and accelerometers. One problem with the inertial profiling method, as currently used, is the impossibility to build up a 3D profile from elementary measurements needed for road/vehicle interaction simulation package.

Finding a way to get a 3D profile from the dynamic response of an instrumented car driven on a chosen road section is the general purpose of a research engaged at Roads and Bridges Central Laboratory (in French : LCPC) in cooperation with the Robotics Laboratory of PARIS (in French : LRP).

The method proposed here, developed at the LRP, is the use of observers [5] [6]. Design of such observers requires a dynamic model. In a first step, we built up a model for a trailer comprising a chassis (sprung mass) and two wheels linked to the chassis through a RENAULT 21 rear suspension. This model has been experimentally validated in comparing estimated and measured dynamics response of the trailer.

The second paragraph of this paper deals with the vehicle description and modeling. The third section is devoted to the validation of the half-car model. Then the observer design is presented in the fourth section. The main results are presented in the fifth section to show the accuracy of the estimated road profile coming from our observer based method. Finally, the last section concludes on effectiveness of the method.

II. System description and modeling

We find in the literature many different kind of this model. We will mention one proposed by the Task Group [2], where the two wheels are modeled as a Mass-Spring system without coupling. This model is very interesting since the number of degrees of freedom is reduced.

Other types of models have been studied [3], which are more adapted while using a coupling between the left wheel and right vehicle part. The experimental bench considered is a two wheels trailer. This system is represented by figure(1). It consists of two wheels with masses m_1 and m_2 respectively, coupled to a chassis with mass m through a suspension. k_i and B_i represent the stiffness and the damper constant respectively.

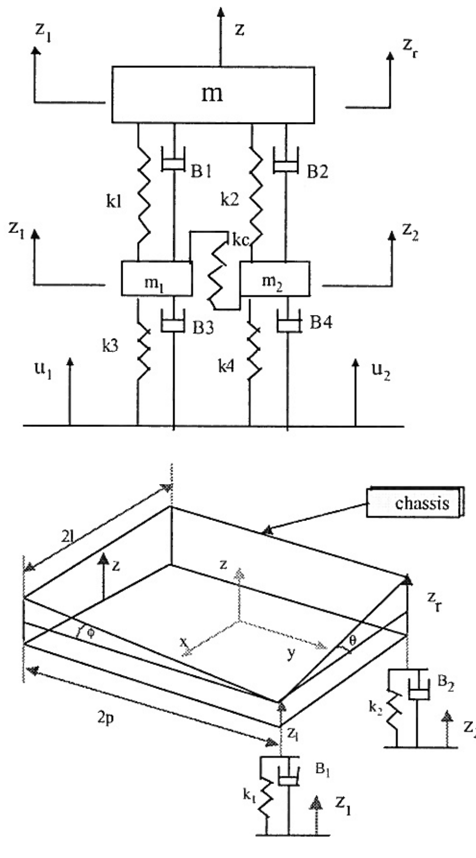

Fig. 1. half-car model

Let z be the displacement of the chassis and z_1, z_2 be the vertical displacement of the left and the right wheels respectively. Let z_l and z_r be the vertical displacement of the left and the right sides of the chassis respectively. The road profile u_1 and u_2 are the two inputs signals of the system. θ and ϕ represent the chassis orientations. The distance between the two wheels is $2l$ and the length of the trailer is $2p$. The two wheels are coupled through a stiffness k_c. I_x and I_y are the lateral and longitudinal moments of inertia respectively. We have then:

$$\begin{aligned} z_l &= z - l\sin\theta - p\sin\phi \\ z_r &= z + l\sin\theta - p\sin\phi \end{aligned} \qquad (1)$$

If the angular displacements are small, then we can assume $\sin\theta \simeq \theta$ and $\sin\phi \simeq \phi$. We note by $\dot{\theta}, \dot{\phi}, \ddot{\theta}$ and $\ddot{\phi}$ the angular velocities and accelerations of the chassis. When we consider the translation motion of the mass m_1, we obtain:

$$\begin{aligned} m_1\ddot{z}_1 = & -(k_1+k_3+k_c)z_1+k_c z_2+ \\ & k_1 z - k_1 l\theta - k_1 p\phi - \\ & (B_1+B_3)\dot{z}_1+B_1\dot{z} - B_1 l\dot{\theta} - \\ & B_1 p\dot{\phi}+k_3 u_1+B_3\dot{u}_1 \end{aligned} \qquad (2)$$

The translational motion of the mass m_2 obeys to the following equation:

$$\begin{aligned} m_2\ddot{z}_2 = & -(k_2+k_4+k_c)z_2+k_c z_1 \\ & +k_2 z - k_2 l\theta - k_2 p\phi \\ & -(B_2+B_4)\dot{z}_2 + B_2\dot{z} - B_2 l\dot{\theta} \\ & -B_2 p\dot{\phi}+k_4 u_2+B_4\dot{u}_2 \end{aligned} \qquad (3)$$

Let us now consider the vertical translation of the center of mass m:

$$\begin{aligned} m\ddot{z} = & k_1(z_1 - z_l) + k_2(z_2 - z_r)+ \\ & B_1(\dot{z}_1 - \dot{z}_l) + B_2(\dot{z}_2 - \dot{z}_r) \end{aligned}$$

Replacing z_l and z_r by their values given by equations 1, we obtain:

$$\begin{aligned} m\ddot{z} = & k_1 z_1+k_2 z_2-(k_1+k_2)z \\ & +(k_1-k_2)l\theta + (k_1+k_2)p\phi \\ & +B_1\dot{z}_1+B_2\dot{z}_2-(B_1+B_2)\dot{z} \\ & +(B_1-B_2)l\dot{\theta}+(B_1+B_2)p\dot{\phi} \end{aligned} \qquad (4)$$

The roll motion of the center of mass m can be written:

$$\begin{aligned} I_x\ddot{\theta}/l = & k_2(z_2 - z_r)p - k_1(z_1 - z_l)p+ \\ & B_2(\dot{z}_2 - \dot{z}_r)p - B_1(\dot{z}_1 - \dot{z}_l)p \end{aligned}$$

According to 1, we obtain,

$$\begin{aligned} I_x\ddot{\theta}/l = {} & -k_1 z_1 + k_2 z_2 + (k_1 - k_2) z - \\ & (k_1 + k_2) l\theta + (k_2 - k_1) p\phi - \\ & B_1 \dot{z}_1 + B_2 \dot{z}_2 + (B_1 - B_2)\dot{z} - \\ & (B_1 + B_2) l\dot{\theta} + (B_2 - B_1) p\dot{\phi} \end{aligned} \quad (5)$$

The pitch motion of the center of mass m gives:

$$\begin{aligned} I_y\ddot{\phi}/p = {} & -k_2(z_2 - z_r)p - k_1(z_1 - z_l)p \\ & -B_2(-\dot{z}_r + \dot{z}_2)p - B_1(\dot{z}_1 - \dot{z}_l)p \end{aligned}$$

$$\begin{aligned} I_y\ddot{\phi}/p = {} & -k_1 z_1 - k_2 z_2 + (k_1 + k_2) z \\ & +(k_2 - k_1) l\theta - (k_2 + k_1) p\phi \\ & -B_1 \dot{z}_1 - B_2 \dot{z}_2 + (B_2 + B_1)\dot{z} \\ & +(B_2 - B_1) l\dot{\theta} - (B_2 + B_1) p\dot{\phi} \end{aligned} \quad (6)$$

Finally, we can write the dynamic model in the following form:

$$M\,\ddot{q} + B\,\dot{q} + Kq = Cu + D\dot{u} \quad (7)$$

With $u = (u_1, u_2)^T$ and $q \in R^5$ the following coordinate vector

$$q = [z_1, z_2, z, \theta, \phi]^T$$

Where M is the inertia matrix, B the damping effects and K is the spring effects dues to the suspension.

III. Validation of the half-car model

For experimental validation of our model, we consider that a trailer provided of an Axel of RENAULT 21 and equipped of Dunlops the Mans air is used (see (2)). An excitation (vibrating) system of 100KN has been placed under a vertically guided table corresponds to the level of soil. Different signals are then generated to excite the system and the sensors record the trailer's dynamics. We compare then the measured signals with our simulations. As an example, we consider the excitation signal which represents the motion of the trailer with a speed of 20m/s rolling on a plate of 1 cm of thickness and 1m of large. The signals coming from the vertical displacement of the chassis and the two wheels are represented in the figure (3). We can see that, the measured sizes are near of the simulated sizes. The steady state error is due to sensor calibration and integration of velocities for obtention of positions.

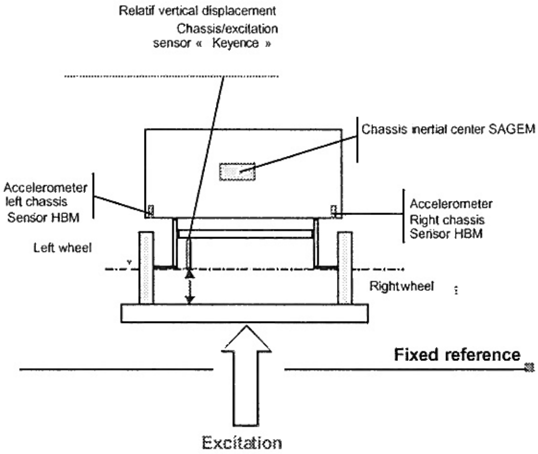

Fig. 2. The trailer with different sensors

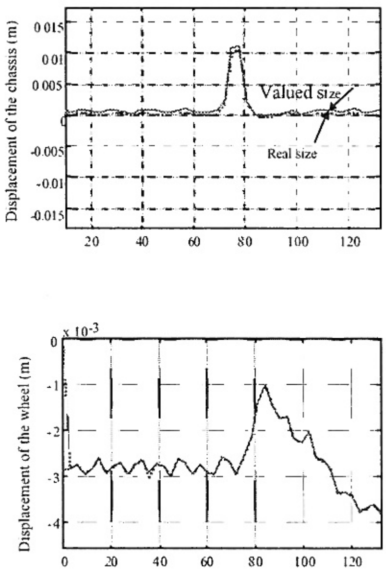

Fig. 3. Estimated and measured sizes

IV. Observation and estimation of the road profile

In order to estimate the profile of the pavement while using observers, we must have an efficient informations and measures which allow to estimate the unknown input states of the system. The different measurable outputs have to be appropriately chosen. Our idea is to parametrize the system model in order to compute the signals to be estimated. The system (7) can be rewritten as follows with y as output vector and $x_1 = [q, \dot{q}]^T$:

$$\begin{aligned} \dot{x}_1 &= Ax_1 + C_1 u + D_1 \dot{u} \\ y &= C' x \end{aligned} \quad (8)$$

Where:

$$A = [A_{11} \quad A_{12}]$$

The sub matrix A_{11} and A_{12} are:

$$A_{11} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ (-k_1-k_3-k_c)/m_1 & k_c/m_1 & k_1/m_1 & (-B_1-B_3)/m_1 & 0 \\ k_c/m_2 & (-k_2-k_4-k_c)/m_2 & k_2/m_2 & 0 & (-B_2-B_4)/m_2 \\ k_1/m & k_2/m & (k_1+k_2)/m & B_1/m & B_2/m \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -k_1l/I_x & k_2l/I_x & (k_1-k_2)l/I_x & -B_1l/I_x & -B_2l/I_x \\ -k_1p/I_y & k_2p/I_y & (k_1+k_2)p/I_y & -B_1p/I_y & -B_2p/I_y \end{bmatrix}$$

$$A_{12} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ B_1/m_1 & -k_1l/m_1 & -k_1p/m_1 & -B_1l/m_1 & -B_1p/m_1 \\ B_2/m_2 & k_2l/m_2 & -k_2p/m_2 & B_2l/m_2 & -B_2p/m_2 \\ (-B^1-B^2)/m & (k^1-k^2)l/m & (k^1+k^2)p/m & (B^2-B^1)l/m & (B^1+B^2)p/m \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ (B^1-B^2)l/I_x & (k^2-k^1)l^2/I_x & (k^2-k^1)pl/I_x & (-B^1-B^2)l^2/I_x & (-B^1+B^2)lp/I_x \\ (B_1+B_2)p/I_y & (-k_1+k_2)lp/I_y & (-k_1-k_2)p^2/I_y & (-B_1+B_2)lp/I_y & (-B_1-B_2)p^2/I_y \end{bmatrix}$$

$$C_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ k_3/m_1 & 0 \\ 0 & k_4/m_2 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad D_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ B_3/m_1 & 0 \\ 0 & B_4/m_2 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

The input dependant part of the system model can be written:

$$\dot{u} = A_2 x_1 + C_2 u + D_2 \begin{bmatrix} \ddot{z}_1 \\ \ddot{z}_2 \end{bmatrix} \qquad (9)$$

With:

$$A_2 = [A_{21} \quad A_{22}]$$

The sub matrix A_{21} and A_{22} are:

$$A_{21} = \begin{bmatrix} (k_1+k_c+k_3)/B_3 & -k_c/B_3 & k_1/B_3 & k_1l/B_3 & k_1p/B_3 \\ -k_c/B_4 & (k_2+k_4+k_c)/B_4 & -k_2/B_4 & -k_2l/B_4 & k_2p/B_4 \end{bmatrix}$$

$$A_{22} = \begin{bmatrix} (B_1+B_3)/B_3 & 0 & -B_1/B_3 & B_1l/B_3 & B_1p/B_3 \\ 0 & (B_2+B_4)/B_4 & -B_2/B_4 & -B_2l/B_4 & B_2p/B_4 \end{bmatrix}$$

$$C_2 = \begin{bmatrix} \square k_3/B_3 & 0 \\ 0 & \square K_4/B_4 \end{bmatrix}$$
$$D_2 = \begin{bmatrix} \square m_1/B_3 & 0 \\ 0 & \square m_2/B_4 \end{bmatrix}$$

In equation (9) the accelerations will be considered as inputs, since they can be measured.

A. Observer Design

Let us assume, in a first step, an estimation $(\hat{u}, \dot{\hat{u}})$ of u and $\dot{u}$ available. We can develop a state observer as follows:

$$\begin{aligned} \dot{\hat{x}}_1 &= A\hat{x}_1 + C_1\hat{u} + D_1\dot{\hat{u}} \\ &\quad + GC'(x_1 \square \hat{x}_1) \end{aligned} \qquad (10)$$

This observer needs estimation of the road profile and its derivatives. These will computed by use of the following estimator, where the available measurements of accelerations $\ddot{z}_1$ and $\ddot{z}_2$ are used as inputs in this step.

$$\dot{\hat{u}} = A_2\hat{x}_1 + C_2\hat{u} + D_2 \begin{bmatrix} \ddot{z}_1 \\ \ddot{z}_2 \end{bmatrix} \qquad (11)$$

The integration of this equation gives $\hat{u}$. The principle of this observer is given by figure (4). The different outputs that constitute the vector y are the vertical displacement of the right and left wheel, the vertical displacement and the orientations of the chassis.

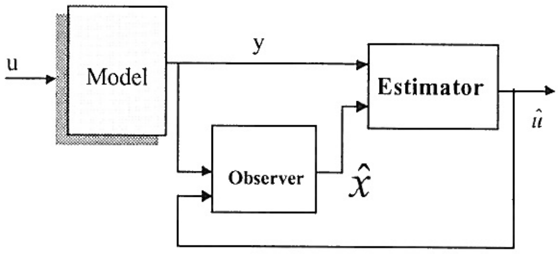

Fig. 4. Principle of the observer and estimator

B. Convergence analysis

The dynamic of the state estimation error can be written:

$$\dot{e}_1 = \dot{x}_1 \square \dot{\hat{x}}_1 = (A \square GC' + D_1.A_2)e_1 \qquad (12)$$

$$\dot{e}_2 = \dot{u} \square \dot{\hat{u}} = A_2 e_1 + C_2 e_2 \qquad (13)$$

Gathering the dynamic error equations, we obtain:

$$\dot{e} = \begin{pmatrix} \dot{e}_1 \\ \dot{e}_2 \end{pmatrix} = \begin{pmatrix} A_3 & 0 \\ A_2 & C_2 \end{pmatrix} \begin{pmatrix} e_1 \\ e_2 \end{pmatrix} \qquad (14)$$

Let us define :

$$A_3 = A + D_1.A_2 \square GC'$$
$$L = \begin{pmatrix} A_3 & 0 \\ A_2 & C_2 \end{pmatrix}$$

We have then:

$$\dot{e} = Le \tag{15}$$

This error e converges towards zero if the matrix L is stable. let us define the Lyapunov function:

$$V = e^T e \tag{16}$$

The time derivative of this function is given by:

$$\dot{V} = \dot{e}^T e + e^T \dot{e} \tag{17}$$

Using the relation (15), it becomes:

$$\dot{V} = e^T (L^T + L) e \tag{18}$$

We chose then the diagonal positive matrix

$$G = diagonal(g_1, g_2, g_3 ... g_{10})$$

in order to have a convergent error e, the eigenvalues of the matrix $(L + L^T)$ must be in the left-half open plane. We deduce then some conditions on gains g_i:

$$g_i > 0, i = 1, 2, 3, 8 \tag{19}$$

and:

$$\begin{aligned} g_4 &> (B_3 + B_4)/m_1 \\ g_5 &> (B_2 + B_4)/m_2 \\ g_6 &> (B_1 + B_2)/m - (B_1 + B_3)/m_1 \\ g_7 &> (k_2 B_4)/m^2 \\ g_9 &> (B_1 + B_2) l / I_x \\ g_{10} &> (B_1 + B_2) p / I_y \end{aligned} \tag{20}$$

Under these conditions, we have $\dot{V} < 0$ and the error e converges asymptotically towards zero.

V. Main Results

A. Simulation Results

We give in this paragraph some simulation results to test our estimator. The algorithm of Rung-Kutta developed here is of order 4. The signals used in our simulations are obtained from measures done on the trailer. These signals are digital on 2559 points delivered by a rotating memory. The parameters of the model used in our simulations are supposed constant and well known. The sampling frequency is 128 Hz. We consider for example that the profile of the pavement have the shape of the chirp sinus of figure(5). We try by our estimator to retrieve this input signals.

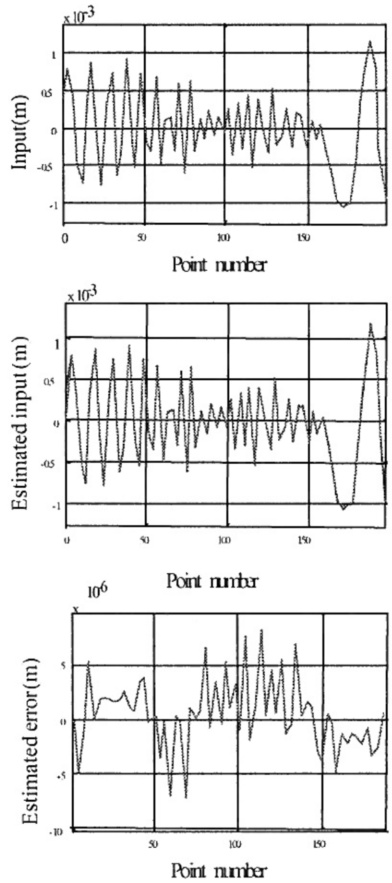

Fig. 5. Estimation results

We note that the estimated signal follows the variations of the real signal. The difference between these two signals that represents the estimation error is between -5.10^{-6} and $5.10^{-6} m$ (In percentage, it gives 0.5%).

B. Experimental Results

To illustrate the performance of the proposed approach, we consider a trailer shown in figure (6) which is excited by real road signals. The outputs of the system are the displacements of the wheel and the chassis which correspond to signals given by sensors. The position of the right wheel is fixed. Only the left part of the vehicle is excited. Our estimator uses then the measurement coming from the sensor 1 (for z_1) and the sensor 2 (for z_2) to estimate the input u. This treatment is shown on the figure (7):

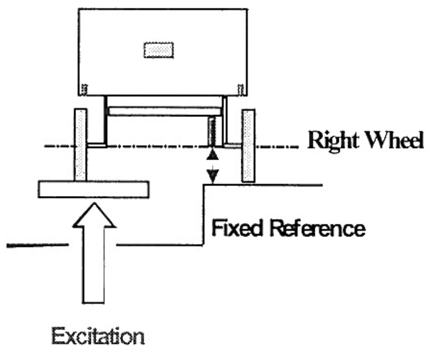

Fig. 6. The trailer: fixed right wheel

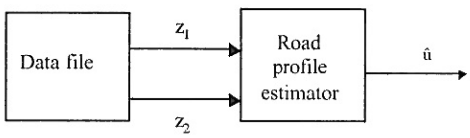

Fig. 7. Principle of estimation of the road profile

B.1 Left quarter-vehicle

The two signals plotted in figure (8) and obtained from trailer are applied to our estimator in order to have the profile of the pavement. The results are then represented by figure(9). We show the convergence of the road profile to actual value with only a maximum of 0.4 % error.

B.2 Right quarter-vehicle

We suppose now that our quarter-vehicle model is on a fixed reference. We are going to see, if our estimator is able to determine the unknown input of this system. The signal of figure (10) represents the displacement of the vehicle body. This signal is injected to our estimator.

We can see that the estimated signal shown in figure (11), varies around the real signal (zero). The estimation error is around 2mm.

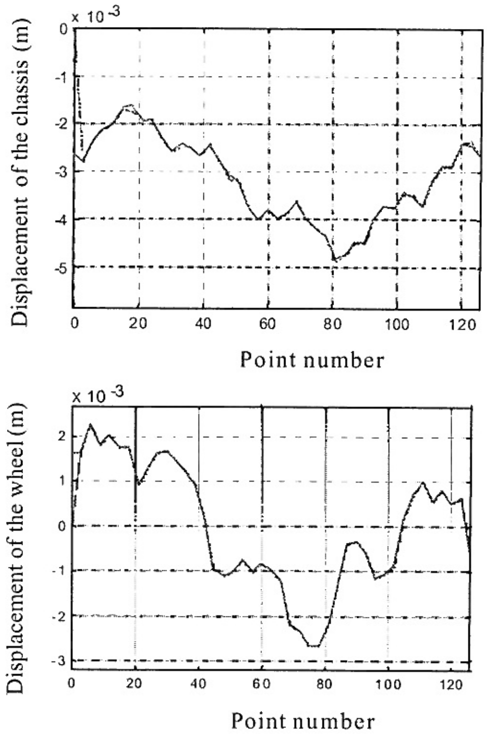

Fig. 8. left-car input data

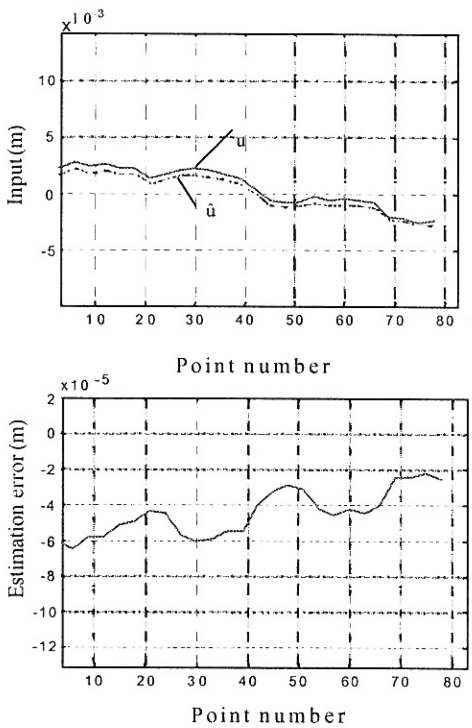

Fig. 9. Input estimation

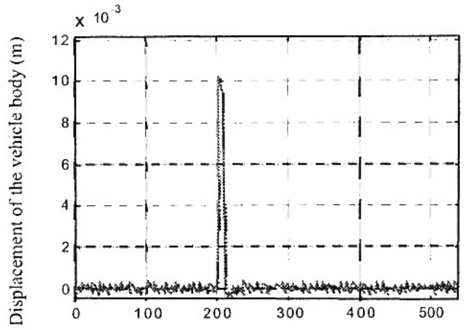

Fig. 10. Right-car input data

From these results, one can conclude to the effectiveness of our estimation method.

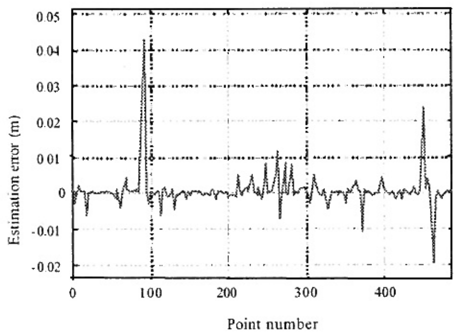

Fig. 11. Estimation error

VI. Conclusion

In this paper, a new approach on on-line estimation of the road profile has been developed. The proposed approach is based on an observer and requires only few sensors in comparison with the methods currently in use. Moreover, the vehicle dynamics are explicitly considered. The signals of the road profile represent the unknown inputs of the model to be estimated. The reconstruction (observation) of these inaccessible inputs is validated using a trailer. It is shown that the estimated profile converges to its reference with an error less than 0.6%. Future research will investigate the estimation of the road profile using a robust estimator based on the sliding mode observers. In this method, the four road profile signals will be considered as unknown inputs to be estimated. The requirement to measure all the parameters of the model is the most important limitation of the proposed approach. Thus, in our future work, we intend to identify on line these parameters. Finally, it's planned to validate the robust estimator approach on an instrumented vehicle.

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