Handling of an Object Exceeding Load Capacity of Manipulator Using Virtually Unactuated Joints

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Abstract

The load capacity of a manipulator depends on a load capacity of a joint or load capacities of some joints of the manipulator. Even if a manipulator could not handle an object because of its load capacity, some of its joints still have enough capabilities for the handling of the object. In this paper, we first propose a method to handle an object using available joints of the manipulator by introducing virtually unactuated joints. The virtually unactuated joints are controlled so that no load will be applied to the joints. The introduction of the virtually unactuated joints make the available degrees of freedom of the manipulator less than that of the original manipulator. We then propose a method to manipulate an object with dual manipulators.

1 Introduction

Various kinds of robots have been developed so far. In general, most of them have been used in an environment isolated from humans. However, with the coming of aging societies, robots are expected to execute various tasks in human environments, which could not be isolated from humans. The robots in human environments are expected to be driven by small actuators having lower outputs than the conventional ones for safety reasons. In this paper, we consider a problem to handle a heavy object by robots whose load capacities are lower than the ones required to handle it.

We assume that the load capacity of a manipulator depends on a capacity of a joint or capacities of some joints. In general, this assumption is satisfied, because the load capacity of a manipulator usually depends on the load capacities of several joints located close to the end the manipulator. Actuators with high power are heavy in general and could not be used to drive joints located close to the end of the manipulator. The joints located close to the end of the mechanism are to be driven by actuators having low power.

Several researches[1]~[3] for robots handling a heavy object have been done by introducing robots with special mechanisms. In this paper, we consider a problem to handle a heavy object by a serial link manipulator, which consists of revolute joints and rigid links. In the rest of this paper, the word "manipulator" means a serial link manipulator without any comment.

Under the assumption, we first propose a method to handle an object by a manipulator, whose load capacity is not enough to handle it, by introducing virtually unactuated joints. By the proposed method, the manipulator is controlled as if no load will be applied to the virtually unactuated joints by handling the object. The manipulator is to behave as if some of its joints are unactuated.

To manipulate the object freely, we need additional degrees of freedom. We propose a method to manipulate the object by dual manipulators in coordination. Some of their actuators could not support the load applied by handling the object and thus controlled virtually unactuated. We will derive a necessay and sufficient condition for the handling of the object by dual manipulators having virtually unactuated joints.

2 Handling of a Heavy Object by a Manipulator

Consider a problem to handle a heavy object by a manipulator, whose load capacity is not enough to handle it. As mentioned in Section 1, the load capacity of a manipulator often depends on the capacities of some of its joints, which are located close to the end of the manipulator. We could assume that the joints located close to the base of the manipulator have enough capacities to handle the object.

If the manipulator could handle the object by utilizing available joints that have still enough output to handle the object, we could extend the limitation of the manipulator. Small robots in human environment, thus, could handle a heavy object like a human without using several joints. In this section, we propose a method to handle an object by a manipulator without using joints whose load capacity are not enough high to handle the object.

2.1 Manipulation with One Joint Unactuated Virtually

When a manipulator handles an object, a force and a moment will be applied to the endpoint of the manipulator by grasping the object and additional torque will be applied to each joint. Let us first consider the case where only one joint could not support the load applied to the joint by handling the object.

As explained in Section 1, we consider to control the manipulator so that any additional torque will not be applied to the joint, which is referred to a virtually unactuated joint. We will derive the joint angle of the virtually unactuated joint, to which no torque will be applied by the force and the moment applied to the end of the manipulator by handling the object.

Fig.1 shows a n degrees of freedom manipulator. We attach the Cartesian coordinate systems Σ_i to the i-th link of the manipulator using the Denavit-Hartenberg notation[6]. When a force ${}^if(\in R^3)$ and a moment ${}^in(\in R^3)$ are exerted on the endpoint of the manipulator, a torque τ_i, which is applied to the i-th joint, is expressed as follows;

$$\tau_i = {}^ib_i \cdot ({}^ir_i \times {}^if + {}^in) \tag{1}$$

where ${}^ib_j(\in R^3)$ is a unit vector along the rotation axis of j-th joint, with reference to i-th coordinate frame. The j-th joint angle is defined around ${}^ib_j$ in the right-hand sense.

Let ${}^ir_j(\in R^3)$ be a vector with reference to i-th frame from j-th joint to an endpoint of a manipulator as shown in Fig.1. Assume that i-th joint is driven by an actuator without having enough maximum output torque to handle the object. Then, we have the following relation to zero the output torque applied to

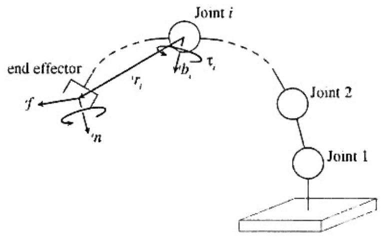

Figure 1: Serial link manipultor with a virtually unactuated joint

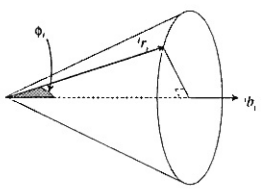

Figure 2: Angle ϕ_i between ${}^ir_i$ and ${}^ib_i$

the joint from Eq.(1) with $\tau_i = 0$;

$$\begin{aligned} -{}^ib_i \cdot {}^in &= {}^ib_i \cdot ({}^ir_i \times {}^if) \\ &= {}^ir_i \cdot ({}^if \times {}^ib_i) \end{aligned} \tag{2}$$

The manipulator can handle the object without having a torque applied to the i-th joint, as long as ${}^ir_i$ satisfies Eq.(2).

Let ${}^ir_{id}$ be ${}^ir_i$ satisfying Eq.(2) and let us derive ${}^ir_{id}$. Let the angle between ${}^ib_i$ and ${}^ir_i$ be ϕ_i. i-th joint revolves around ${}^ib_i$ (Fig.2) with ϕ_i constant. Taking the inner product of ${}^ir_i$ and ${}^ib_i$, we have the following relation;

$${}^ir_i \cdot {}^ib_i = |{}^ir_i||{}^ib_i|\cos\phi_i \tag{3}$$

We can calculate ϕ_i from Eq.(3), as follows;

$$\phi_i = \cos^{-1}\left(\frac{{}^ir_i \cdot {}^ib_i}{|{}^ir_i||{}^ib_i|}\right) \tag{4}$$

Let α_i (Fig.3) be the angle between ${}^ir_{id}$ and ${}^if \times {}^ib_i$, then, we can rewrite the right-hand side of Eq.(2) as follows;

$${}^ir_{id} \cdot ({}^if \times {}^ib_i) = |{}^ir_{id}||{}^if \times {}^ib_i|\cos\alpha_i \tag{5}$$

From Eq.(2) and (5), we have the following relation;

$$-{}^ib_i \cdot {}^in = |{}^ir_{id}||{}^if \times {}^ib_i|\cos\alpha_i \tag{6}$$

We can calculate α_i from Eq.(6), as follows;

$$\alpha_i = \cos^{-1}\left(\frac{-{}^ib_i \cdot {}^in}{|{}^ir_{id}||{}^if \times {}^ib_i|}\right) \tag{7}$$

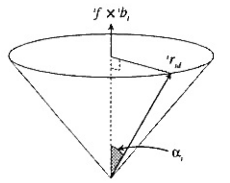

Figure 3: Angle α_i between ${}^i r_{id}$ and ${}^i f \times {}^i b_i$

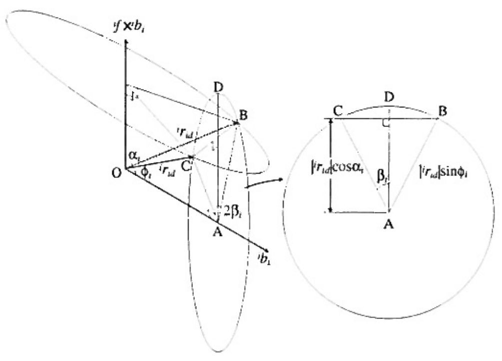

Figure 4: Derivation of ${}^i r_{id}$

${}^i r_{id}$ is a vector which intersects ${}^i b_i$ with the angle ϕ_i as shown Fig.2 and intersects ${}^i f \times {}^i b_i$ with angle α_i as shown in Fig.3. Fig.4 shows the relation among ${}^i r_{id}$, ${}^i b_i$, and ${}^i f \times {}^i b_i$. From Fig.4, two solutions exist for ${}^i r_{id}$, that is, $\overrightarrow{\mathrm{OB}}$ and $\overrightarrow{\mathrm{OC}}$ are solutions for ${}^i r_{id}$. $\overrightarrow{\mathrm{OB}}$ and $\overrightarrow{\mathrm{OC}}$, namely, ${}^i r_{id}$ are corresponding to $\overrightarrow{\mathrm{OD}}$ which rotates $\mp\beta_i$ around ${}^i b_i$, respectively. Thus, ${}^i r_{id}$ is given by

$$\begin{aligned} {}^i r_{id} &= R({}^i b_i, \pm\beta_i)\overrightarrow{\mathrm{OD}} \\ &= R({}^i b_i, \pm\beta_i)(\overrightarrow{\mathrm{OA}} + \overrightarrow{\mathrm{AD}}) \\ &= \overrightarrow{\mathrm{OA}} + R({}^i b_i, \pm\beta_i)\overrightarrow{\mathrm{AD}} \end{aligned} \quad (8)$$

where $R(a, \theta)$ is the 3×3 rotation matrix which rotates a vector around the arbitrary unit vector a by an angle θ in a right-hand sense. From Fig.4, an angle β_i is given by

$$\beta_i = \cos^{-1}\left(\frac{\sin\phi_i}{\cos\alpha_i}\right) \quad (9)$$

In Eq.(8), $\overrightarrow{\mathrm{OA}}$, whose length is $|{}^i r_{id}|\cos\phi_i$, is parallel to ${}^i b_i$. Similarly, $\overrightarrow{\mathrm{AD}}$, whose length is $|{}^i r_{id}|\sin\phi_i$, is parallel to ${}^i f \times {}^i b_i$. ${}^i r_{id}$ is rewritten as

$$\begin{aligned} {}^i r_{id} &= \frac{|{}^i r_{id}|\cos\phi_i}{|{}^i b_i|}{}^i b_i \\ &+ R({}^i b_i, \pm\beta_i)\left\{\frac{|{}^i r_{id}|\sin\phi_i}{|{}^i f \times {}^i b_i|}({}^i f \times {}^i b_i)\right\} \end{aligned} \quad (10)$$

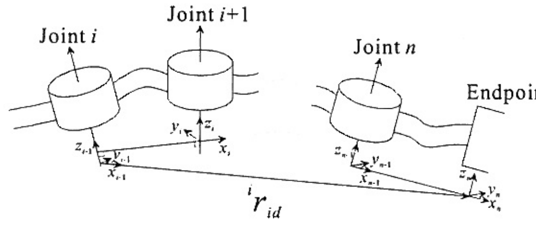

Figure 5: Derivation of θ_{id}

Thus, we obtained ${}^i r_{id}$ for zeroing the torque applied to the joint by manipulating the object.

Let θ_{id} be the joint angle of the virtually unactuated joint, i-th joint. Let θ_j be the joint angle of the j-th joint. Using joint angles, $\theta_{i+1}, \cdots, \theta_n$, and ${}^i r_{id}$, we could calculate θ_{id} from the Denavit-Hartenberg notation(Fig.5).

Since two solutions for ${}^i r_{id}$ exist as mentioned above, we have two solutions for θ_{id}. We select one solution considering the stability of the manipulation of the object.

$F = [f_x, f_y, f_z, n_x, n_y, n_z]^T$, a force and a moment applied to the endpoint of the manipulator, cause a torque, $\tau = [\tau_1, \cdots, \tau_n]^T$, around each joint axis. F and τ has the relation, $\tau = J^T F$. Assuming that i-th joint is the virtually unactuated joint, using the derivation explained above, we have $\theta_i = \theta_{id}$. Since the torque applied to the virtually unactuated joint by the force and moment applied to the end of the manipulator is zero, we have the following relation;

$$\begin{aligned} \tau_i &= {J^T}_i F \\ &= 0 \end{aligned} \quad (11)$$

where ${J^T}_i$ is the i-th row vector of J^T.

Let $\Delta\tau_i$ be a torque exerted on the joint i, when the i-th joint angle becomes $\theta_{id} + \Delta\theta_i$. From Eq.(11), we have the followling ralation;

$$\Delta\tau_i = {J^T}_i(\theta_1, \cdots, \theta_{id} + \Delta\theta_i, \cdots, \theta_n)F \quad (12)$$

If the joint motion caused by $\Delta\tau_i$ decreases $\Delta\theta_i$, then the virtually unactuated joint is stable. On the other hand, if the joint motion caused by $\Delta\tau_i$ increases $\Delta\theta_i$, then the joint is unstable. That is,

$\Delta\tau_i\Delta\theta_i < 0$: virtually unactuated joint is stable.

$\Delta\tau_i\Delta\theta_i > 0$: virtually unactuated joint is unstable.

We select a stable joint angle for the solution for θ_{id}.

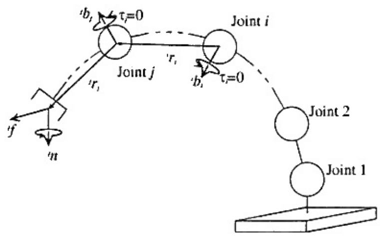

Figure 6: Serial link manipulator with several virtually unactuated joints

2.2 Manipulator with Multiple Virtually Unactuated Joints

In Section 2.1, we derived the joint angle of the virtually unactuated joint. In this section, we consider the case of multiple virtually unactuated joints when handling an object. We derive the joint angles of the virtually unactuated joints by extending the results obtained in Section 2.1.

We assume that multiple joints could not support the load applied to the joints by handling the object,

Let us consider the case of two virtually unactuated joints as shown in Fig.6.

We attach the Cartesian coordinate system to each link of the manipulator using the Denavidt-Hartenberg notation. Let i-th and j-th joints be the virtually unactuated joints. That is, i-th and j-th joints could not support the load exserted on the joints by the object. We assume that the load capacities of the joints other than i-th and j-th joints are high enough to handle the object and that these joints angles are controlled arbitrarily. Let ${}^hb_k(\in R^3)$, ${}^ir_i(\in R^3)$, ${}^jr_j(\in R^3)$, ${}^jf(\in R^3)$, and ${}^jn(\in R^3)$ be a unit vector along rotation axis of k-th joint with reference to h-th coordinate frame, a vector with reference to i-th frame from i-th joint to j-th joint, a vector with reference to j-th frame from j-th joint to an endpoint of a manipulator, a force vector acted on an endpoint with reference to j-th frame, and a moment vector acted on an endpoint with reference to j-th frame, respectively.

Assuming that $j > i$, the angle of the j-th joint is calculated by using the method proposed in Section 2.1. Let us consider how to find the angle of the i-th joint. For the force and motment exerted on the end of the manipulator, ${}^jf$ and ${}^jn$, a moment ${}^jm_j(\in R^3)$ exerted on the j-th joint is given by

$$ {}^jm_j = {}^jr_j \times {}^jf + {}^jn \tag{13} $$

From Fig.7, since the j-th joint is the virtually unactuated joint, the moment transmitted to the i-th joint is expressed by,

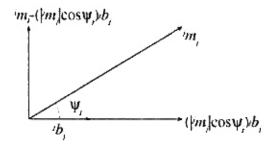

Figure 7: Moment ${}^jm_j$ acted on virtually unactuated joint

$$ {}^in = {}^iR_j\{{}^jm_j - (|{}^jm_j|\cos\psi_j){}^jb_j\} \tag{14} $$

with reference to the frame i, where ${}^iR_j(\in R^{3\times3})$ is a rotation matrix representing the orientation of frame j with reference to frame i and ψ_j is the angle between ${}^jm_j$ and ${}^jb_j$.

Since ${}^if(\in R^3)$, a force vector exerted on j-th joint with reference to i-th joint, is obtained from ${}^if = {}^iR_j{}^jf$. By applying the method proposed in Section 2.1, we can calculate the joint angle of the i-th joint. Thus, repeating the process, we can find the joint angles of the virtually unactuated joints using the method proposed in Section 2.1.

3 Manipulation of an Object by Dual Manipulators with Virtually Unactuated Joints

In Section 2, we derived angles of virtually unactuated joints. When the manipulator has virtually unactuated joints, the available degrees of freedom of the manipulator for the handling of the object reduces, because the joint angles of the virtually unactuated joints are uniquely determined by the force and the moment applied to the manipulator endpoint. The object motion, when the virtually unactuated joints are introduced, is limited within the available degrees of freedom of motion. To overcome this problem, we consider a problem to handle the object by dual manipulators. In this section, we consider the necessary and sufficient condition for the motion control of the object even if dual manipulators have virtually unactuated joints.

Assume that actuated joints do not move when dual manipulators handle an object. Then actuated joints are regarded as parts of links and we can consider closed loop chain consisting of only virtually unactuated joints. Let us consider a serial link manipulator obtained by cutting any position of this closed loop link mechanism(Fig.8).

We refer to this manipulator as the virtually unactuated manipulator. The necessity and sufficiency for constraining the object motion by deriving the Jacobian of the virtually unactuated manipulator and investigating its rank condition.

[Theorem] Let m and n_p be the dimension of the work space and the number of virtually unactuated joints, respectively. Then, necessary and sufficient condition for constraining the motion of the object completely, that is, for manipulation of all degrees of freedom of the object, is $\text{rank}J_p = n_p$, where $J_p(\in R^{m \times n_p})$ is the Jacobian of the virtually unactuated manipulator.

Proof[Sufficiency] Let $\theta_p(\in R^{n_p})$ and $x(\in R^m)$ be the joint angle vector of the virtually unactuated manipulator and the generalized coordinates describing the position and orientation of the endpoint of the manipulator, respectively. Then we have

$$\dot{x} = J_p \dot{\theta}_p \quad (15)$$

Since the position and the orientation of the virtually unactuated manipulator are constant when actuated joints do not move, we have

$$0 = J_p \dot{\theta}_p \quad (16)$$

In Eq.16, constraining the motion of the object completely is equivalent to $\dot{\theta}_p = 0$. The solution satisfying $0 = J_p\dot{\theta}_p$ is $\dot{\theta}_p = 0$ if and only if J_p is of full column rank, namely, $\text{rank}J_p = n_p$.

Proof[Necessity] Let the rank of J_p of the virtually unactuated manipulator be n_p. This means that the solution satisfying $0 = J_p\dot{\theta}_p$ is $\dot{\theta}_p = 0$. The solution of $0 = J_p\dot{\theta}_p$ is $\dot{\theta}_p = 0$ if and only if virtually unactuated joints do not move. Hence, the motion of an object is constrained completely.

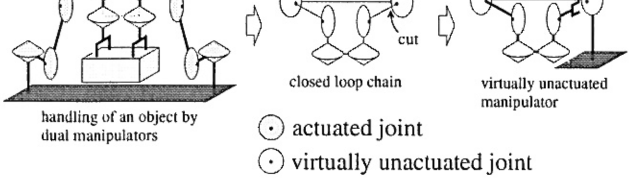

Figure 8: Serial link manipulator obtained by cutting virtually unactuated joint link mechanism

(Example) Let us consider the case where the dimension of the work space is six, namely, $m = 6$. Assume that the virtually unactuated manipulator configuration is not in a singular point. In case of $n_p = 7$, the dual manipulators cannot constrain the motion of the object because J_p is 6×7 matrix and $\text{rank}J_p = 6 < n_p$. Similarly, in case of $n_p = 6$, they can constrain the motion of the object because J_p is 6×6 matrix and $\text{rank}J_p = 6 = n_p$. Moreover, in case of $n_p = 5$, they can constrain the motion of an object, too, because J_p is 6×5 matrix and $\text{rank}J_p = 5 = n_p$.

4 Motion Control of an Object by Dual Manipulators with Virtually Unactuated Joints

In this section, we consider a problem to handle an object by the dual manipulators with virtually unactuated joints, provided that this dual manipulators satisfy the necessary and sufficient condition of the theorem in Section 3.

4.1 Control of Virtually Unactuated Joints

In this section, we consider the motion of virtually unactuated joints when the dual manipulators handle an object in coordination. As mentioned above, the joint angles of the virtually unactuated joints are controlled so as to have no load exerted on them. If the joints have back drivability like the direct drive arm, we do not have to actuate virtually unactuated joints. On the other hand, if the virtually unactuated joints do not have back drivability, the joint angles of the virtually unactuated joints are controlled to keep the desired joint angles obtained in Section 2.

4.2 Control of Actuated Joints

In order to handle an object by dual manipulators with virtually unactuated joints, we have to obtain the relationship between the motion of the object and of actuated joints. Several researches have been done for the cooperative control of dual manipulators with a passive joint or passive joints.

Liu et al. have presented kinematic and dynamic models of a multiple cooperative underactuated manipulator system handling an object[4]. Yeo et al. have proposed two-arm system consisting of a saw, four degrees of freedom SCARA robot and five degrees of freedom PT200 robot with a passive joint for sawing task[5]. In these researches, the relationships between the motion of the endpoints of the manipulators and of actuated joints have been derived. However, these researches have not described unified expressions of the internal and external forces exerted on the endpoints of the manipulators. In this section, we derive the unified expression of the internal and external forces exerted on the endpoints of the manipulators.

Let us consider handling of an object by dual manipulators with virtually unactuated joints. For convenience, left and right side manipulators are referred to as L-,R-manipulator, respectively. Let joints torques of L-,R-manipulators be $\tau_L(\in R^{n_L})$ and $\tau_R(\in R^{n_R})$, respectively. n_L and n_R are the nomber of joints of L-,R-manipulators, respectively. Let m be the dimension of the work space. Both manipulators have enough degrees of freedom for the work space, namely $m \leq n_L$ and $m \leq n_R$. Let the force and moment vectors of endpoints of L-,R-manipulators be $F_L(\in R^m)$ and $F_R(\in R^m)$, respectively. Let $J_L(\in R^{m\times n_L})$ and $J_R(\in R^{m\times n_R})$ be Jacobian matrices of L-,R-manipulators, respectively. Let $F_{ext}(\in R^m)$ and $F_{int}(\in R^m)$ be the external and internal force and moment vectors acted on the object, respectively. Let $F_{Le}(\in R^m)$ and $F_{Re}(\in R^m)$ be the force and moment vectors corresponding to F_{ext} in F_L and F_R, respectively. Let $F_{Li}(\in R^m)$ and $F_{Ri}(\in R^m)$ be the force and moment vectors corresponding to F_{int} in F_L and F_R, respectively. Then we have the following relations concerned with F_{ext} and F_{int}.

$$F_{ext} = F_{Le} + F_{Re} \tag{17}$$

$$F_{int} = F_{Li} = -F_{Ri} \tag{18}$$

From the static relation, we have

$$\begin{bmatrix} \tau_L \\ \tau_R \end{bmatrix} = \begin{bmatrix} J_L^T F_{Le} \\ J_R^T F_{Re} \end{bmatrix} + \begin{bmatrix} J_L^T F_{Li} \\ J_R^T F_{Ri} \end{bmatrix} \tag{19}$$

From Eq.(17) and (18), Eq.(19) is rewritten as

$$\begin{aligned} \begin{bmatrix} \tau_L \\ \tau_R \end{bmatrix} &= \begin{bmatrix} J_L^T G F_{ext} \\ J_R^T (E-G) F_{ext} \end{bmatrix} + \begin{bmatrix} -J_L^T F_{int} \\ J_R^T F_{int} \end{bmatrix} \\ &= \begin{bmatrix} J_L^T G & -J_L^T \\ J_R^T (E-G) & J_R^T \end{bmatrix} \begin{bmatrix} F_{ext} \\ F_{int} \end{bmatrix} \end{aligned} \tag{20}$$

where E is an identity matrix. G is a load sharing matrix and the external force is shared with the ratio $G : E - G$. In Eq.(20), since elements of τ_L and τ_R corresponding to virtually unactuated joints are zero, we can calculate F_{int} for any given F_{ext}. Using this relation, we can design a control system for dual manipulators.

5 Conclusion

In this paper, we considered a problem to handle a heavy object by manipulators. In general, the load capacity of a manipulator depends on the maximum output or outputs of a joint or joints. Other joints are still available for the handling of the object even if the load exceeds the manipulator's capacity. We proposed a method to handle an object by utilizing the available joints and introducing the virtually unactuated joints. The number of available joints is not enough to handle the object completely, because several joints, which could not support the load, are controlled virtually unctuated. We extended the method to the handling problem of an object by dual manipulators. Basic relations necessarily for designing a control system were designed. The use of multiple manipulators makes it possible to handle the object completely even with several virtually unactuated joints.

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