Estimation and Propagation of Geometrical Parameters During Force-Controlled Execution of Polyhedral Contact Formation Sequences

Tine Lefebvre, Herman Bruyninckx and Joris De Schutter
Dept of Mechanical Engineering, Katholieke Universiteit Leuven
Celestijnenlaan 300B, B-3001 Leuven, Belgium
Tine.Lefebvre@mech.kuleuven.ac.be

Abstract

This paper improves previous work on the Kalman Filter based estimation of geometrical parameters (positions, orientations and dimensions) of polyhedral objects in contact during a force-controlled compliant motion task. Following improvements are achieved: (i) the derivation of the estimation equations is considerably simplified; (ii) the set of position closure constraints is minimized; (iii) the geometrical parameter estimator simultaneously performs a force decomposition which can be used in a force setpoint controller; (iv) the automatic generation of the estimation equations is easier; (v) the propagation of the estimates through sequences of contact formations becomes straightforward; and (vi) a faster measurement processing is obtained.

1 Introduction

Still today, industrial robots need very structured environments (i.e. exact positioning of tools and work pieces, of parts to assemble,...) in order to perform compliant motion tasks, i.e. tasks which involve contact between the manipulated object and the object(s) in the robot environment. Previous work by the authors [1] resulted in proof of concept solutions to perform *autonomous compliant motion* in less structured environments, i.e. environments in which some geometrical parameters are inaccurately known. Examples are the inaccurately known position, orientation and dimensions of the objects in contact.

Figure 1 shows an example of a force-controlled task that can be solved with the above-mentioned approach: the robot has grasped the *manipulated object* (a cube in this example), and has to put it in the *environment* (a corner with three walls in this example), but the relative position and orientation of both objects is not well known at the start of the task. The solution to this problem consists of two separate stages:

1. a *nominal* sequence of contact formations will lead the manipulated object from its initial free-space position to the desired final contact formation. A *task program* that specifies this nominal sequence has to be made *off-line*. (See, e.g., the work by Jing and co-workers, [2] for solutions to this problem.) For the example above, this sequence could consist of the following contact formations: no contact; one *vertex-face* contact; one *edge-face* contact; one *face-face* contact; one *face-face* contact plus one *edge-face* contact (i.e., at this moment, the cube has slid over the bottom of the corner until it contacts one of the walls); two *face-face* contacts; and finally the three *face-face* contacts of the cube in the corner.

2. during the *on-line task execution*, the robot controller uses the nominal model as input to a (hybrid or other) force control algorithm. The force and velocity setpoints are based on the estimates of the inaccurately known geometrical parameters. These estimates come from an estimator (e.g. a Kalman Filter [3]) that takes the force and velocity measurements as inputs and has a measurement equation (linking the measurements to the geometrical parameters) at its disposal. To every contact formation corresponds a different measurement equation. Transitions between contact formations can be detected by the measurements which become inconsistent with the current measurement equation.

As stated before, [1] describes the proof of concept solutions the authors have given to the above-mentioned on-line part of the problem in the case of contacts between objects with generally curved surfaces. This paper presents significant improvements over the material presented in [1] for contact formations between *polyhedral* objects. A large field of application is assembly, as the parts of the objects in contact during assembly are often polyhedral. The improvements are made in the following aspects:

1. The need for the modeling of the contact formation by a "Virtual Contact Manipulator" (VCM, [1]), describing the relative degrees of freedom between the manipulated object and the environment, is avoided. Next to the *reduced modeling effort* (Section 2.1), this

also results in a *minimal set of position closure constraints* (expressing that the objects are in contact), (Section 2.2).

2. The reciprocity based velocity and force measurement equations of [1] are replaced by a reciprocity based velocity equation and a consistency based force equation (Section 2.1). This *simplifies the measurement equations* and results in a simultaneous *force decomposition* by the Kalman Filter. This decomposition can be used as feedback to a force setpoint controller which independently controls the contact forces and moments in each of the individual contacts.

3. The contact formations between polyhedral objects can be modeled by a set of "elementary" *vertex-face* and *edge-edge* contacts (Section 4). The Kalman Filter estimator is made robust against modeling the contact formation with a non-minimal number of these elementary contacts. This leads to an *easy automatic generation of the equations*.

4. Consistently working with all geometrical parameters at all time, leads to an easy *propagation of estimates* from one contact formation to the next one (Section 3).

5. Finally, a *faster processing* of the measurements is obtained by estimating only (the instantaneously observable) part of the geometrical parameters by the Kalman Filter and updating the estimates for the other parameters outside the filter algorithm (Section 2.3).

The experiments (Section 5) are performed on a Kuka-IR 361 industrial robot arm, with a 6D Schunk force/torque sensor on its wrist. The presented experimental results are obtained by off-line processing of the measurements collected during a real force-controlled experiment. The compliant motion is realized by the hybrid force/position controller of [4].

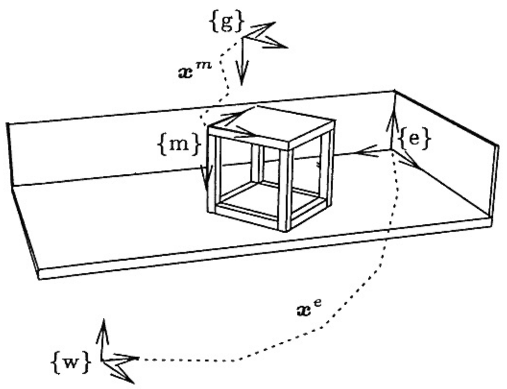

Figure 1: *Face-face* contact.

2 Estimation

Consider following reference frames (Figure 1): {w} attached to the world, {g} attached to the gripper, {m} attached to the manipulated object and {e} attached to the environment. The *sources of uncertainty* pervading the geometric parameters x are the inaccurately known position and orientation of {m} with respect to {g} and of {e} respect to {w}. These are called respectively *grasping uncertainties* x^m and *environment uncertainties* x^e. Also the *dimensions* of the manipulated object and the environment might be inaccurately known. All these uncertain geometrical parameters are gathered in the *state vector* x. The state is *static*, i.e. the geometrical parameters do not change during the task execution.

2.1 Measurement equations

The measurements z consist of (i) *twists* t of the manipulated object (translational velocities v and rotational velocities ω), derived from the robot joint positions and velocities, and of (ii) contact *wrenches* w (forces f and moments m), measured with a wrist force sensor:

$$z = \begin{bmatrix} t \\ w \end{bmatrix}; \qquad t = \begin{bmatrix} v \\ \omega \end{bmatrix}; \qquad w = \begin{bmatrix} f \\ m \end{bmatrix}. \qquad (1)$$

In the vector space of all possible contact wrenches (*wrench space*) a base G can be chosen. Every wrench w then corresponds to a coordinate vector ϕ: $w = G\phi$. Similarly, in the vector space of all possible twists (*twist space*) a base J can be chosen after which every twist t corresponds to coordinate vector χ: $t = J\chi$.

A *measurement equation* links the measurements z with the state x, and hence allows to estimate the state from the measurements. Previous work [1] presented a measurement equation based on the *reciprocity* of manipulated object twist and the contact wrench, i.e. no power is dissipated in the compliant motion. In other words, the measurement equation expressed that the measured twists t are reciprocal to the modeled wrench base G, and that the measured wrenches w are reciprocal to the modeled twist base J. [1] used the concept of a Virtual Contact Manipulator (VCM) to derive these wrench and twist bases G and J for contact formations between arbitrary objects. This VCM has the *relative* position and orientation between the manipulated object and the environment as joint values. For contact formations between *polyhedral* objects, it is easy to write G and J directly in function of the geometrical parameters x, without explicitly modeling the relative degrees of freedom between them. This reduces the modeling effort considerably. The reciprocity based measurement equations are then:

$$h_1(x, z) = \begin{pmatrix} (G(x))^T \; t \\ (J(x))^T \; w \end{pmatrix} = 0. \qquad (2)$$

$J(x)$, $G(x)$, t and w must be expressed with respect to the same frame. The wrench base $G(x)$ corresponding to the contact formation is the *union* of the wrench bases of the individual occurring contacts. On the other hand, the twist base $J(x)$ is the *intersection* of the twist bases of the individual contacts. Calculating this intersection (analytically) can be a tedious task, but the calculation of the union is cheap. Replacing the wrench based reciprocity equation by a *consistency* equation [5] avoids the calculation of J:

$$h_2(x, z) = \begin{pmatrix} (G(x))^T \, t \\ G(x) \, \phi - w \end{pmatrix} = 0. \qquad (3)$$

The reduced modeling effort leads to some extra parameter vector ϕ that needs to be estimated. ϕ contains the coordinates of the wrench w in the base $G(x)$. The estimates for ϕ can be used as feedback in a force setpoint controller.

2.2 Position closure constraints

The twist and wrench based measurement equations are first-order closure equations, expressing how the manipulated object can move to *keep* contact (compliant motion). However, these equations do *not* tell that the contact between the manipulated object and the environment is established in the first place. E.g. for a *vertex-face* contact, the twists and wrenches give information about the orientation of the face and the location of the force screw axis (i.e. information about the contacting vertex); however they do not impose that the vertex lies in the face (i.e. give information about the position of the face). The latter information is given by a zeroth-order *position closure constraint* $c(x) = 0$. This constraint can be processed by the Kalman Filter as a "measurement equation", [1]. As mentioned before (Section 2.1), the new measurement equations for contacts between polyhedral objects avoid the need for the VCM joint variables. This also reduces the number of position closure constraints to the minimum: the previous position closure constraints also included equations that express the dependence of the VCM joint variables on the geometrical parameters x.

2.3 Partial observation

During the task execution, the twist and wrench measurements give information about the state. A each time instant, the Kalman Filter produces (i) estimates of the state variables and (ii) a covariance matrix, indicating the covariances of, and the correlations between, the estimates.

In general, the measurement equation depends on only part of the state vector. E.g., for a cube in corner assembly the first contact formation is usually a vertex of the cube contacting a face of the corner object, during this contact formation the position of the corner is not observable. Further on, the state variables that occur in the measurement equation are called the *observable state variables*; the others are called the *unobservable state variables*[1]. Kalman Filters that keep track of the whole state vector can deal with this partial observation, but the estimator will be computationally more expensive than an estimator that only considers the observable state variables. The following Lemmas [6] state (i) that a (static) Kalman Filter can be run on only the observable state variables; and (ii) how the estimates and covariances of the unobservable state variables can be updated in this case.

Lemma 1 *The update for the estimate of the observable part of the state vector x_o and its covariance matrix P_{oo} are* independent *of the estimate of the unobservable part of the state vector x_u, its covariance matrix P_{uu} and the correlation between the estimates of the observable and unobservable part P_{uo}. Hence a Kalman Filter can be run on the reduced state x_o.*

Following symbols are adopted: $x_{o,k+n|k}$ and $x_{u,k+n|k}$ are the estimates of the observable and unobservable part of the state at time $k+n$, given the measurements up to time k; $P_{oo,k+n|k}$ and $P_{uu,k+n|k}$ are the covariance matrices on these estimates; and $P_{uo,k+n|k}$ is the correlation matrix between these estimates.

Lemma 2 *Suppose that from a time $k+1$ till a time $k+n$ only the parameters x_o are observable and a Kalman Filter calculates $x_{o,k+n|k+n}$ and $P_{oo,k+n|k+n}$. The updates for $x_{u,k+n|k+n}$, $P_{uu,k+n|k+n}$ and $P_{uo,k+n|k+n}$ are:*

$$x_{u,k+n|k+n} = x_{u,k|k} + K(x_{o,k+n|k+n} - x_{o,k|k}); \qquad (4)$$

$$P_{uo,k+n|k+n} = K P_{oo,k+n|k+n}; \qquad (5)$$

$$P_{uu,k+n|k+n} = P_{uu,k|k} - K(P_{oo,k|k} - P_{oo,k+n|k+n})K^T; \qquad (6)$$

where $K = P_{uo,k|k} P^{-1}_{oo,k|k}$.

3 Propagation

During each contact formation, different geometrical parameters are observable. The information gathered during one contact formation needs to be propagated to the following contact formations. However during these new contact

[1] In principle, the decomposition between the observable and unobservable *linear combinations* of the state variables can be made. These can be found by a singular value decomposition of the matrix H that describes the (linear) relation between the measurements and state: $z = Hx + \rho$. However, the measurement equations for the current problem are nonlinear in the state and the matrix H represents the derivative of the measurement equations with respect to the state vector, evaluated in the state estimate. Due to changing estimates, H changes each time step and other linear combinations of the state variables (occurring in the measurement equation) are observed by the linear filter.

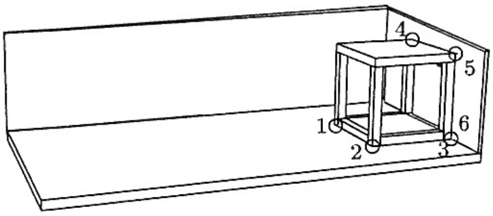

Figure 2: A double *face-face* contact can be modeled by the five *vertex-face* contacts 1 to 5. The system can deal with redundant constraints (e.g., choosing a 6th *vertex-face* contact).

formations, some of the previously observable geometrical parameters are unobservable and previously unobservable geometrical parameters have become observable. *Propagating* the information through sequences of contact formations is done by *considering all inaccurately known geometrical parameters (even the unobservable ones) in all contact formations.* Previous work always justified the use of a minimum number of variables as a means to obtain a faster measurement processing by the Kalman Filter. However, with the theory of Section 2.3, considering unobservable variables does not give any overhead for the Kalman Filter. When all geometrical parameters are estimated in all contact formations, the propagation of information is trivial: the state estimate and its covariance matrix at the end of a contact formation serve as initial values for the estimator of the next contact formation.

4 Unified description of polyhedral contacts

A contact formation between two polyhedral objects can be described as a number of *vertex-face* and/or *edge-edge* contacts in parallel. For polyhedral objects, the contact equations and wrench bases of these "elementary contacts" can easily be written in function of the state $\boldsymbol{x}$ [6]. A possible wrench base for the contact formation is:

$$G = \begin{bmatrix} G_1 & \dots & G_k \end{bmatrix}, \tag{7}$$

where G_i is the wrench base for the ith *vertex-face* or *edge-edge* contact. The position closure constraints are:

$$\begin{cases} c_1(\boldsymbol{x}) &= 0, \\ & \vdots \\ c_k(\boldsymbol{x}) &= 0, \end{cases} \tag{8}$$

where $\boldsymbol{c}_i(\boldsymbol{x}) = \mathbf{0}$ is the position closure constraint for the ith contact.

Figure 2 gives a possible choice for the location of the five *vertex-face* contacts (1 to 5) in which a double *face-face* contact can be decomposed. The "horizontal" *face-face* contact is represented by three *vertex-face* contacts (1 to 3). The "vertical" *face-face* contact is represented by only two *vertex-face* contacts (4 and 5), because one of the directions in which a moment can be exerted is already modeled by the *vertex-face* contacts 1 to 3.

Using these elementary contact formations is easier for the (automatic) task programmers, also because they should not worry about the consistency of the contact formation models they use in their off-line programming. That is, the programmers are allowed to represent two *face-face* contacts (Figure 2) with elementary *vertex-face* contacts 1 to 6, without bothering about whether they use the minimal number of constraints or not. The *extra position closure constraints* are linearly dependent on the others and do not introduce any new information, hence redundant position closure constraints can be added without problem. The Kalman Filter running on measurement Equation (3) can also be made robust against a *non minimal wrench base*: the extra base vectors in the wrench base result in

1. extra twist equations. The twists are also consistent with these. A statistically good inference by the Kalman Filter can still be assured. [2]

2. extra ϕ_i coordinates in the wrench equation. The linear combinations of ϕ_i corresponding to the null wrench space are physically meaningless, but unobservable. The estimation of $\boldsymbol{x}$ is not influenced by this.

Here, a trade-off between extensive off-line modeling and computational load during the task execution pops up: a simple implementation of a task programmer can easily return a redundant contact formation description; this however asks more computations of the Kalman Filter.

5 Experiment

A *face-face* contact formation between a cube (manipulated object) and a corner object (environment) is established (Figure 1). The uncertain geometrical parameters are six grasping uncertainties (three translations x^m, y^m, z^m and three orientations $\theta_x^m, \theta_y^m, \theta_z^m$) and six environment uncertainties (three translations x^e, y^e, z^e and three orientations $\theta_x^e, \theta_y^e, \theta_z^e$). During the *face-face* compliant motion, only two angular grasping uncertainties (θ_x^m and θ_y^m) that determine the orientation of the contacting faces are observable[3].

[2]Because the innovation covariance matrix is singular in this case, a pseudo-inverse of this matrix has to be used, [6].

[3]The measurement equation can also be written in function of θ_x^e and θ_y^e, which also determine the orientation of the contacting faces. The position closure constraints express the correlation between the angles θ_x^m, θ_y^m, θ_x^e and θ_y^e; this makes the different expressions of the measurement equations equivalent.

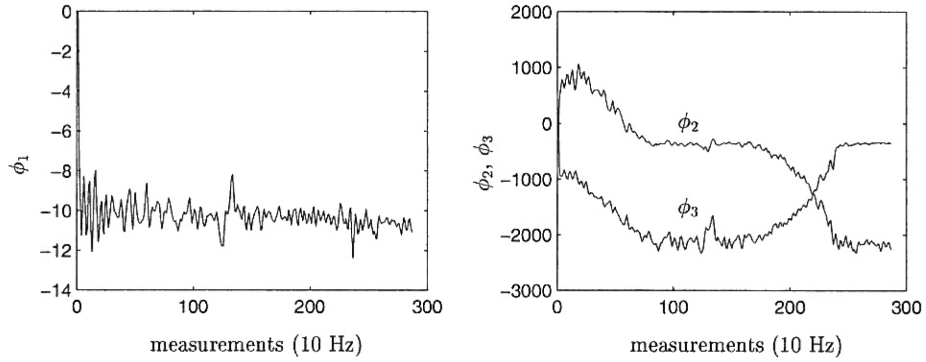

Figure 3: Estimates for ϕ during the *face-face* contact formation.

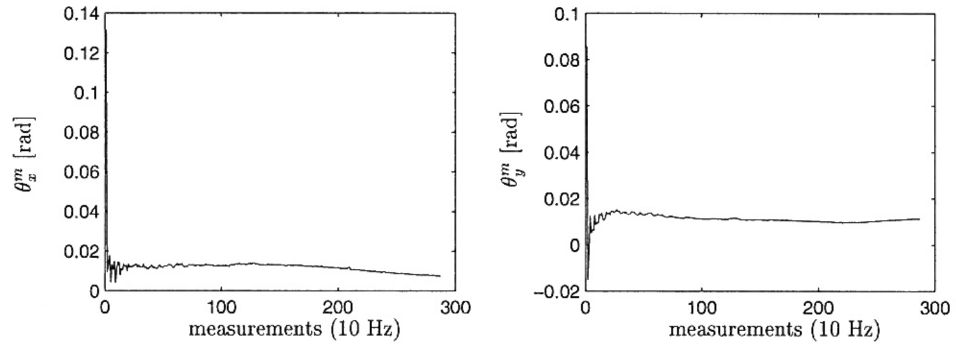

Figure 4: Estimates for the *observable state variables* θ_x^m and θ_y^m during a *face-face* contact formation.

The estimates of the geometrical parameters at the beginning of the compliant motion are correlated due to (i) correlation between the initial estimates, (ii) observations during the previous contact formations and (iii) the application of the position closure constraints. In this particular example, the estimates of x^m, y^m, z^m, z^e, θ_x^e and θ_y^e are correlated to the estimates of the observable variables θ_x^m and θ_y^m.

An Iterated Extended Kalman Filter [3] calculates estimates for the three wrench coordinates ϕ and *all* grasping and environment uncertainties. Figure 3 gives the estimates of the wrench coordinate vector ϕ. ϕ_1 corresponds to the wrench base vector expressing the contact force perpendicular to the face, ϕ_2 and ϕ_3 to the wrench base vectors expressing the moments around two axes in the face[4]. Figures 4 and 5 show the estimates of the observable variables and of the estimates correlated to these. The estimates of the unobservable state variables that are not correlated to the observable ones (i.e. x^e, y^e, θ_z^m and θ_z^e) do not change and are not plotted here.

The unobservable geometrical parameters are unimportant to perform the compliant motion, hence the calculation of the evolution of their estimates (Figure 5) is unnecessary during the contact formation. A Kalman Filter calculating the reduced state estimate $\boldsymbol{x} = \begin{bmatrix}\phi_1 & \phi_2 & \phi_3 & \theta_x^m & \theta_y^m\end{bmatrix}^T$ gives the same results for these estimates as the Kalman Filter calculating the full state estimate (Figures 3 and 4) . When however a new contact formation is reached, the estimates of the unobservable geometrical parameters can become valuable. Therefore, at that time, their estimates are updated with Equations (4)–(6). The results are marked by 'o' in Figure 5; they are identical to the ones calculated by the Kalman Filter that considers all the geometrical parameters at all time.

6 Conclusions

The presented paper improves previous work on the on-line estimation of geometrical parameters during force-controlled compliant motion execution of contact formations between polyhedral objects. The improvements are:

[4]Remark that the wrench base (one force direction and two moment directions in a chosen point on the *face-face* contact) is *not* obtained by the unified approach from Section 4. In the latter case, the *face-face* contact formation is decomposed into three *vertex-face* contacts. The corresponding wrench base vectors represent the three force directions, located in the points where the *vertex-face* contacts are chosen. The wrench coordinate vector ϕ then consists of the three coordinates corresponding to the contact forces in these three points.

The obtained estimates for the geometrical parameters $\boldsymbol{x}$ are independent of the chosen wrench base. The estimates for ϕ need to be interpreted in the chosen base. All possible wrench base vectors and corresponding ϕ are linear combinations of each other.

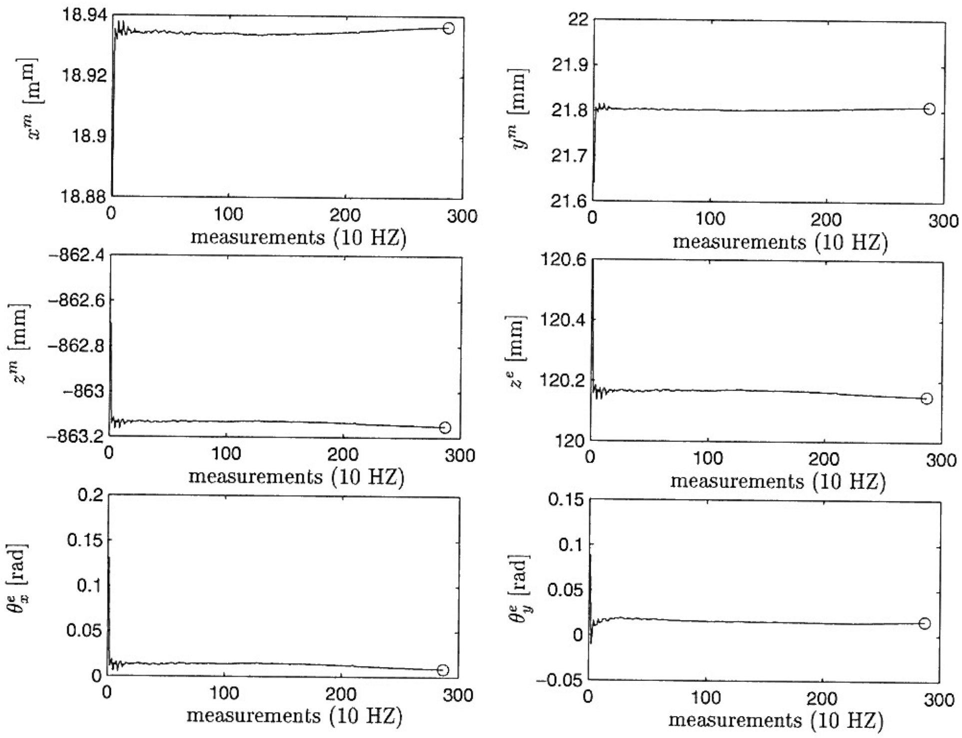

Figure 5: Estimates for the *unobservable state variables* of which the estimates are *correlated to the estimates of the observable state variables*. The final estimates (marked by a 'o') can be calculated with Equations (4)–(6).

1. avoiding the need of the Virtual Contact Manipulator, which simplifies substantially the derivation of the measurement equations and reduces the number of position closure constraints to the minimum;
2. avoiding the twist base $\boldsymbol{J}$ in the measurement equations, which simplifies even more the derivation of the measurement equations. Another advantage is that the estimator performs a wrench decomposition which can be used in a force setpoint controller;
3. modeling the contact formations between polyhedral objects in a unified way as a collection of *vertex-face* and *edge-edge* contacts which allows an easy automatic generation of the equations;
4. consistently using all inaccurately known geometrical parameters at all time, which makes the propagation of information about these parameters through sequences of contact formations straightforward;
5. processing the measurements faster by only considering the observable state variables in the Kalman Filter.

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