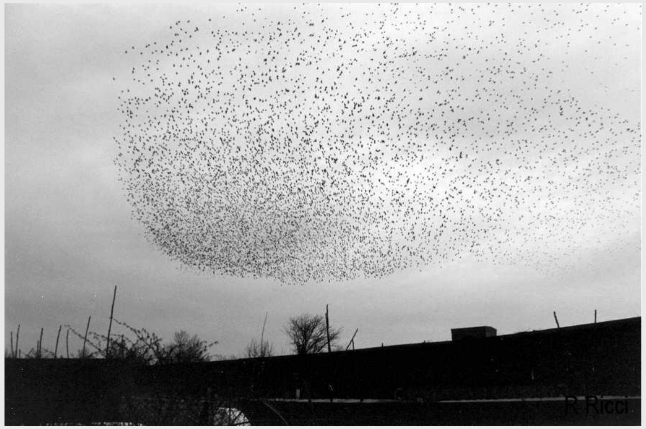
Synchronization of Multi-Agent Systems

Mark W. Spong
Center for Autonomous Engineering Systems and Robotics
Department of Electrical and Computer Engineering and
The Coordinated Science Laboratory
University of Illinois at Urbana-Champaign
1308 W. Main St.
Urbana, IL 61801

Joint work with Nikhil Chopra, Erick Rodriguez, Rogelio Lozano. This research is partially supported by the U.S. Office of Naval Research under grant N000140510186, and by the UIUC/CNRS collaborative agreement.

Introduction

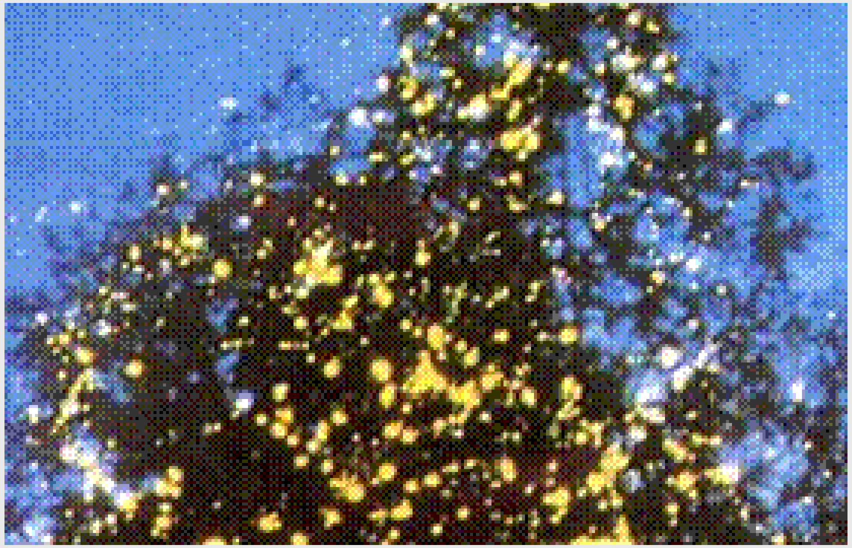
- The problem of synchronization of multiple agents arises in numerous applications, both in natural and in man-made systems.
- Examples from nature include:



Flocking of Birds

Schooling of Fish

More Examples from Nature

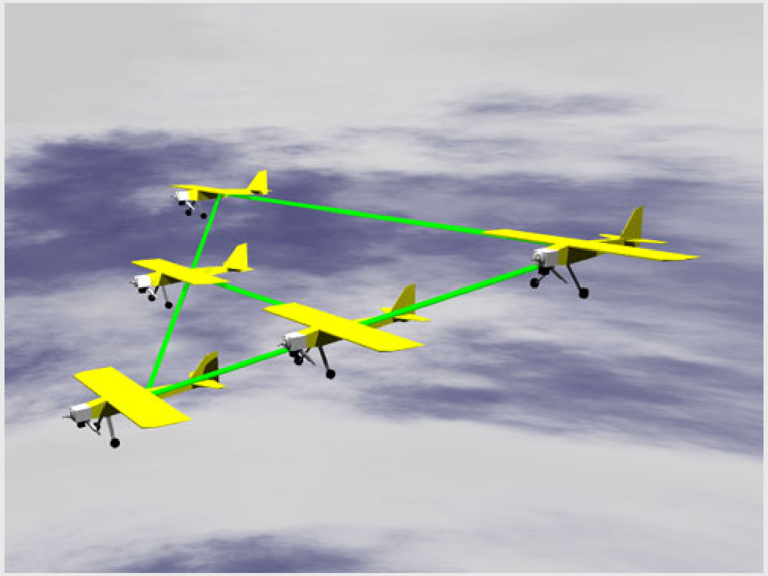


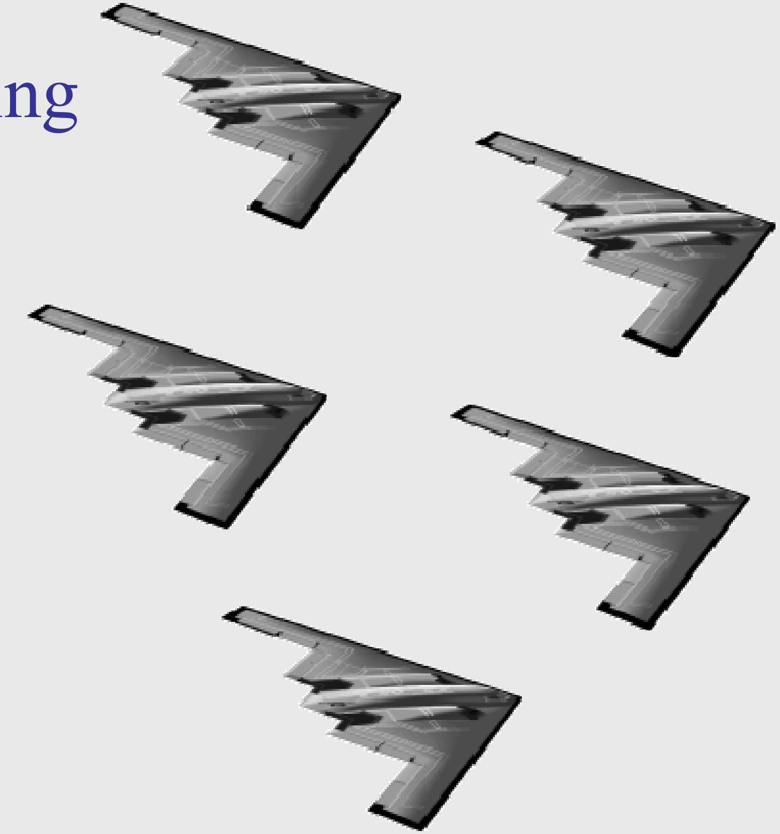
Synchronously Flashing Fireflies

A Swarm of Locusts

Examples from Engineering

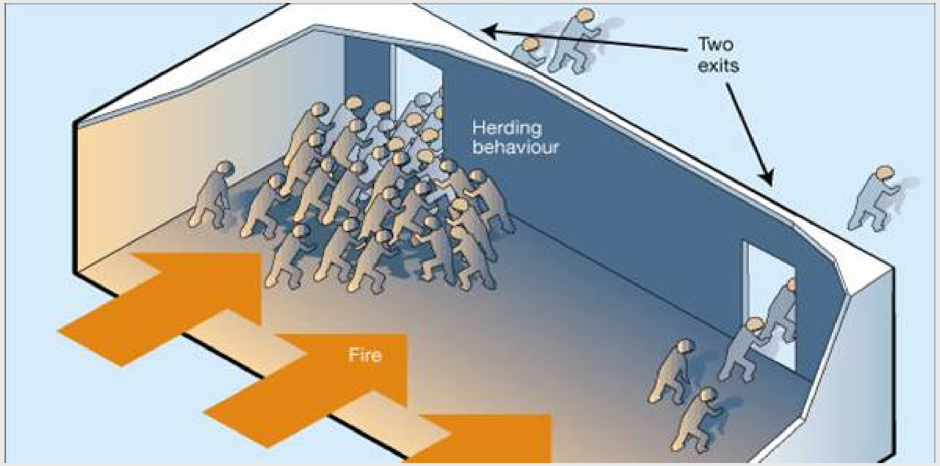
Autonomous Formation Flying and UAV Networks





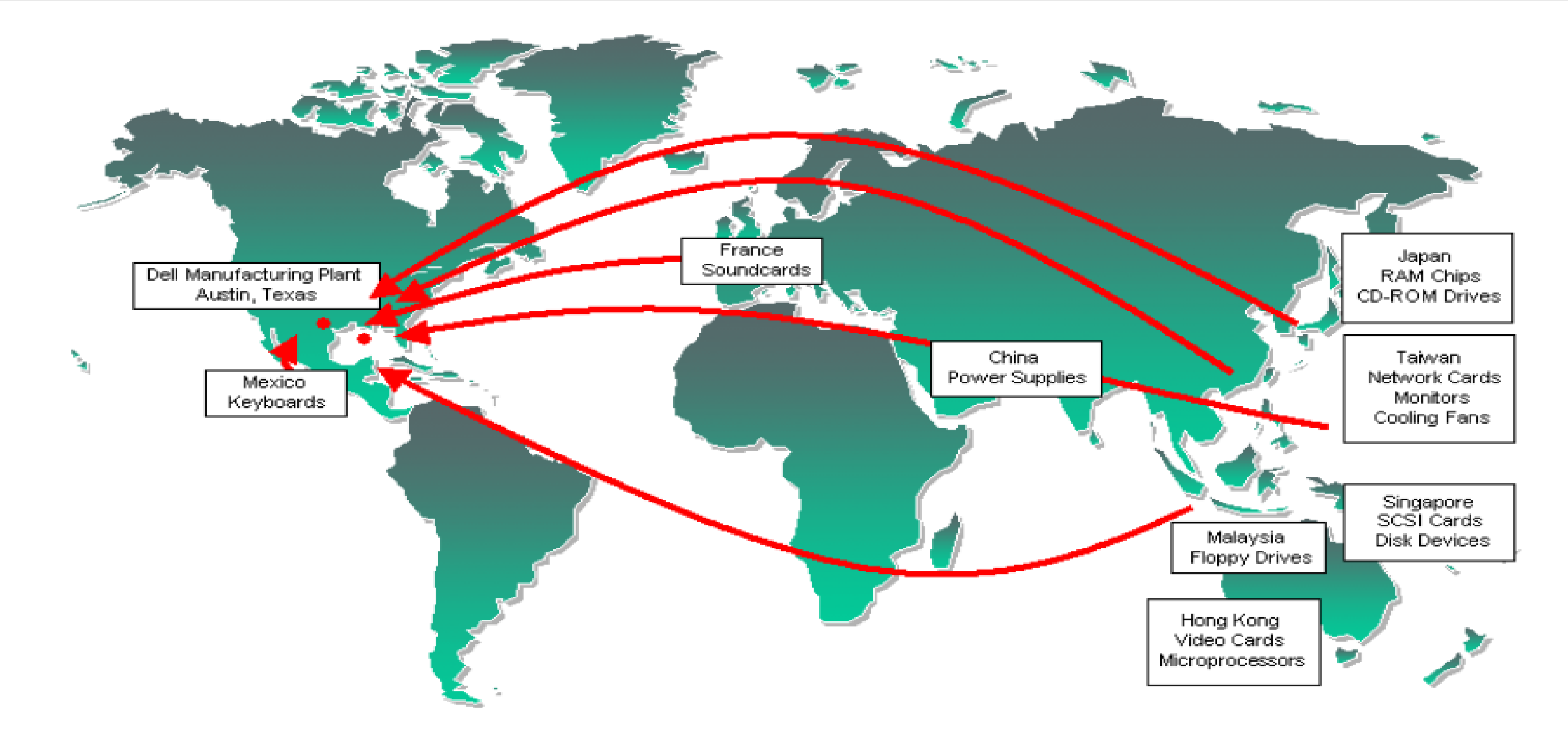
Examples from Social Dynamics and Robotics

Mobile Robot Networks



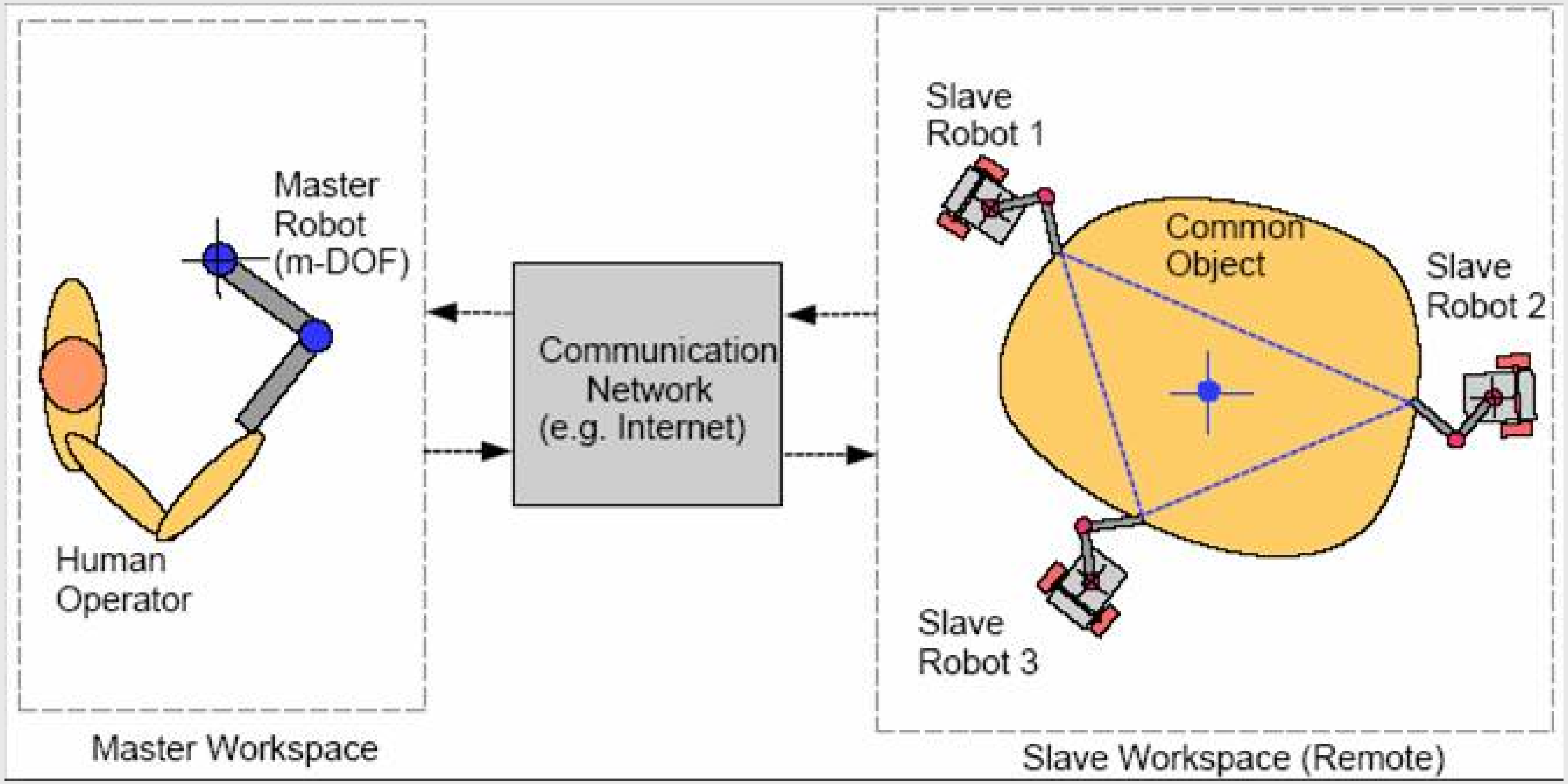
Crowd Dynamics and Building Egress

Examples from Economic Networks



Global Supply Chain-Dell Computer

Bilateral Teleoperation

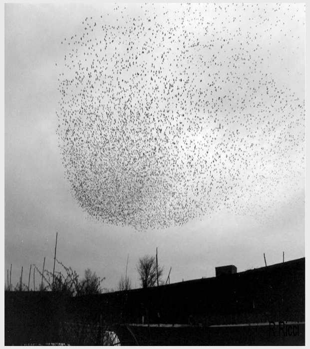


Multi-Robot Remote Manipulation

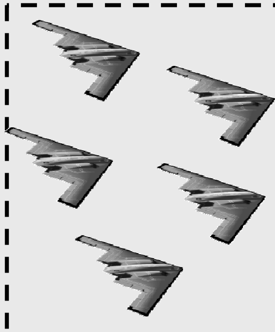
Other Examples

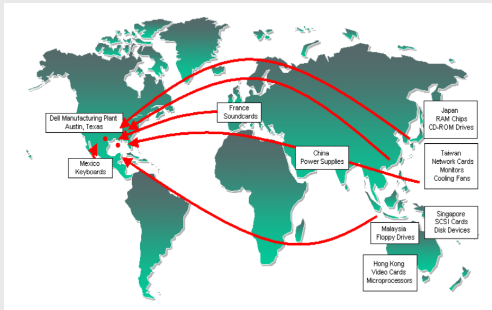
- Other Examples:
  - circadian rhythm
  - contraction of coronary pacemaker cells
  - firing of memory neurons in the brain
  - Superconducting Josephson junction arrays
  - Design of oscillator circuits
  - Sensor networks

The Big Picture

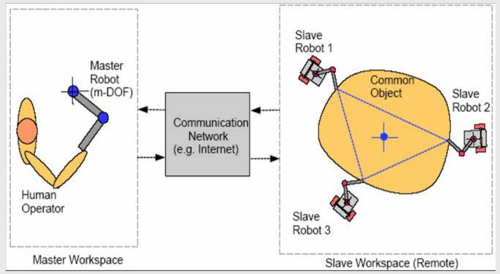


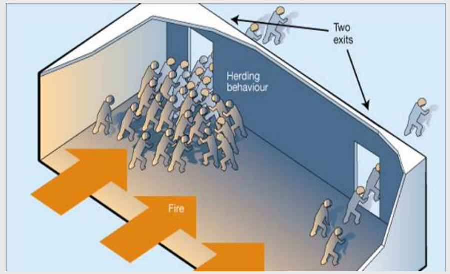
Inspiration from Nature





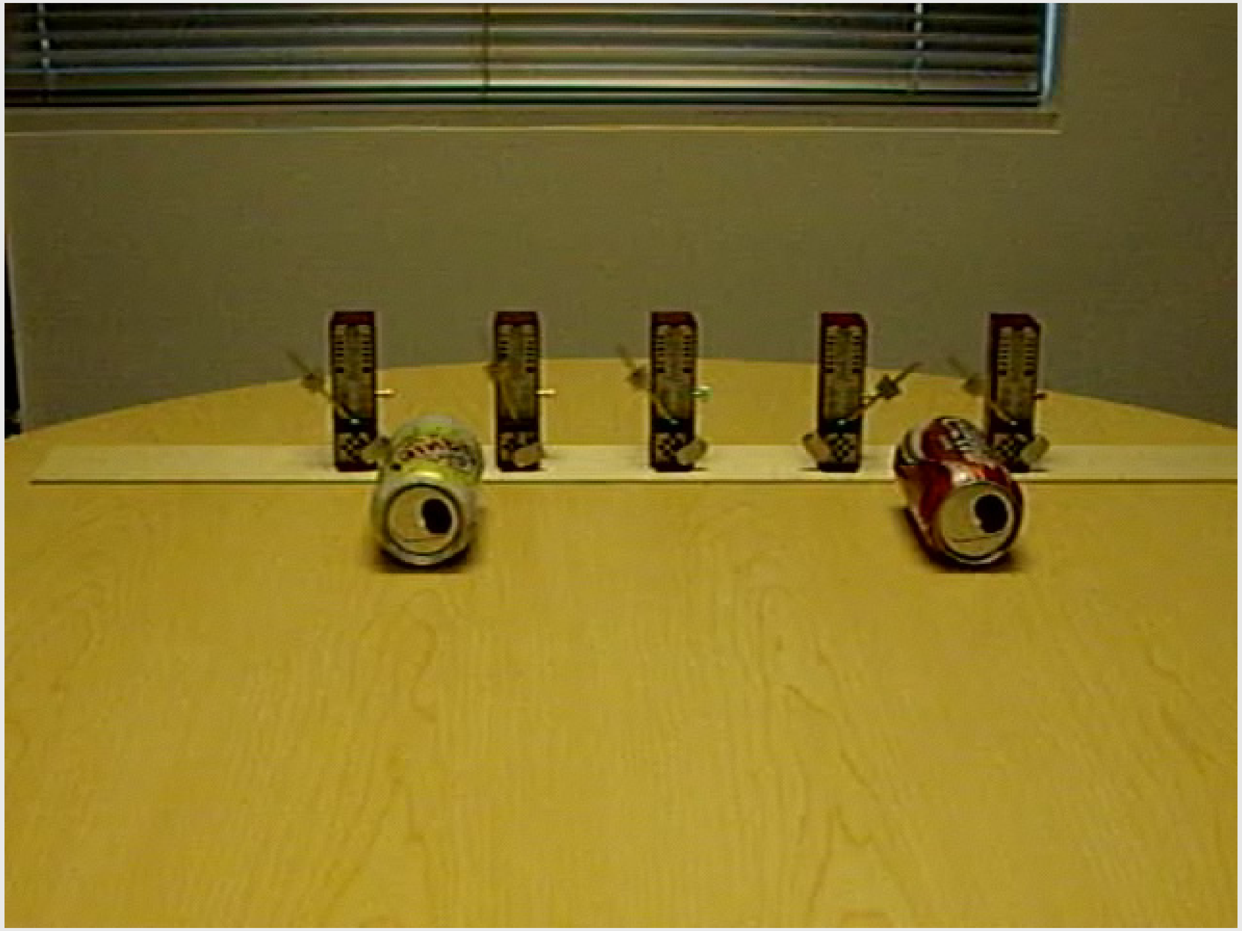
Coordinate Multi-agents





With Humans in the Control Loop

Example:
Synchronization of Metronomes



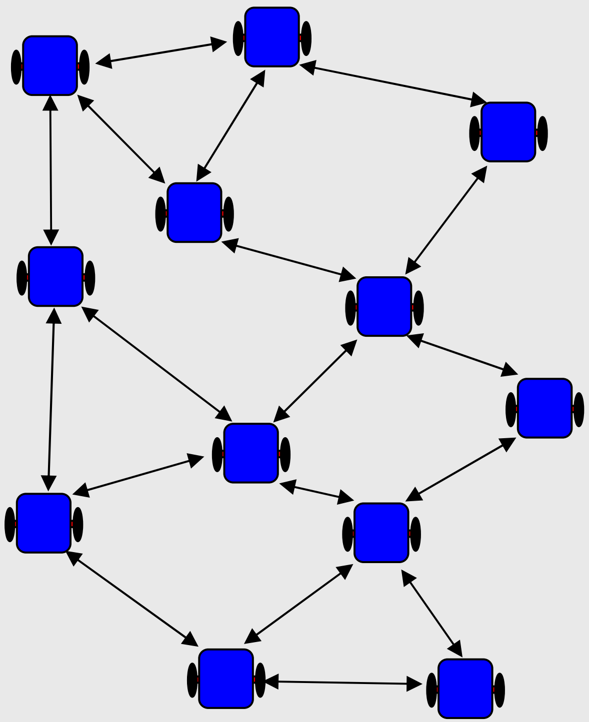
Some Background

The control of multi-agent systems is a very active area of research. Many methods are being applied, such as,

1. Incremental stability, contraction analysis, passivity (Angeli, Arcak, Nijmeijer, Pogromsky, Polushin, Slotine, Sepulchre, …)
2. Vehicle formations, graph theoretic methods, (Aeyels, Blondell, Francis, Jadbabaie, Justh, Krishnaprasad, Moreau, Morse, Murray, Olfati-Saber, Pappas, Rogge, Tsitsiklis, Tanner, Vicsek, …)
3. Coupled oscillators, mechanical systems, (Jadbabaie, Kuramoto, Leonard, Strogatz, Winfree, …)
4. Sensor networks, AI and stochastic methods, (Blondell, Bullo, Lavalle, Cortes, …)

Our work is more related to the approaches of 1. and 3. above.

Fundamental Questions



To analyze such systems several questions must be addressed:

- What are the dynamics of the individual agents?
- How do the agents exchange information?
- How do we couple the available outputs to achieve synchronization?

Outline of Presentation

- We will first present some general results on output synchronization of networked passive systems.
- We consider fixed and switching network interconnections and time delay in communication.
- We then specialize to the case of Lagrangian systems and show the stronger property of state synchronization.

The Theme

Passivity-Based Control

↓

Coordination of Heterogeneous Dynamical Networks

↓

- **Bilateral Teleoperation**
- **Multi-Agent Coordination**
- **Oscillator Synchronization**

Passivity

Consider a control affine nonlinear system of the form

$$\Sigma \begin{cases} \dot{x} &= f(x) + g(x)u \\ y &= h(x) \end{cases}$$

where $x \in R^n$, $u \in R^m$, and $y \in R^m$. The functions $f(.) \in R^n$, $g(.) \in R^{n \times m}$, and $h(.) \in R^m$ are assumed to be sufficiently smooth and we note that the dimensions of the input and output are the same. We assume that $f(0) = 0$ and $h(0) = 0$.

Definition 2.1 *The nonlinear system Σ is said to be passive if there exists a C^1 storage function $V(x) \geq 0$, $V(0) = 0$ and a function $S(x) \geq 0$ such that for all $t \geq 0$:*

$$\dot{V}(x(t)) \quad = \quad y^T(t)u(t) - S(x(t))$$

The system Σ is strictly passive if $S(x) > 0$ and lossless if $S(x) = 0$.

The Simplest Example

1. The passivity framework unifies and extends a number of results in multi-agent control.
2. In fact, most of the purely graph theoretic approaches model the agents as first-order integrators, which are the simplest passive systems, i.e.

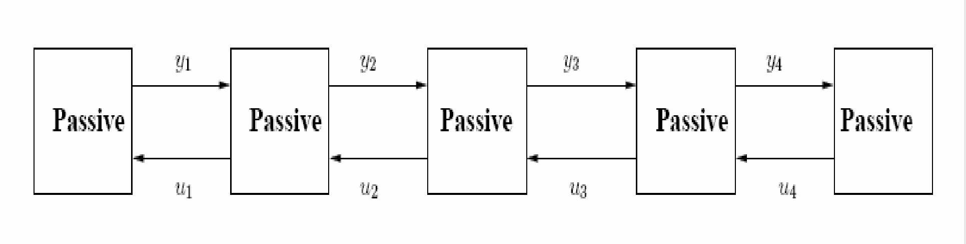
$$\begin{aligned} \dot{x} &= u \\ y &= x \end{aligned}$$

This is a lossless system with storage function $V(x) = \frac{1}{2}x^2$

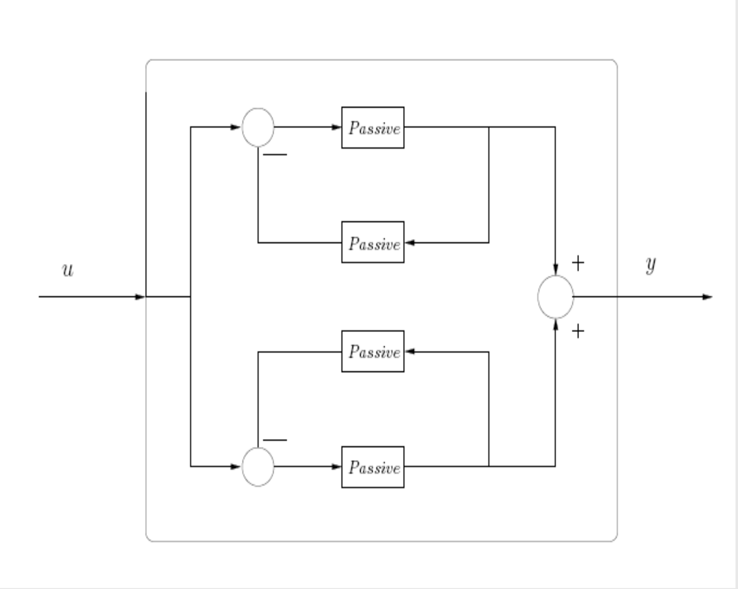
since $\dot{V} = uy$

Useful Properties of Passive Systems

Interconnection of Passive Two-Ports Networks is Passive



Feedback and Parallel Interconnection of Passive Systems is Passive



Useful Properties of Passive Systems

Another important feature of passive systems is that they can be stabilized with linear output feedback

$$u(t) = -ky(t)$$

since, in that case,

$$\dot{V}(x(t)) \leq -k||y(t)||^2 \leq 0$$

Under some additional conditions, for example, detectability, the equilibrium solution is asymptotically stable.

Nonlinear KYP Lemma

The following result, due to Moylan (IEEE TAC, 1974), is important for our general results:

Theorem: The following statements are equivalent.

1. The system Σ is passive
2. There exists a C^1 scalar storage function $V : R^n \rightarrow R$ such that $V(x) \geq 0$, $V(0) = 0$, and

$$\begin{aligned} L_f V(x) &= -S(x) \\ L_g V(x) &= h^T(x) \end{aligned}$$

where $L_f V(x)$ and $L_g V(x)$ are the Lie derivatives of V with respect to f and g, respectively.

Synchronization

Consider N agents described by passive dynamics

$$\begin{aligned} \dot{x}_i &= f_i(x_i) + g_i(x_i)u_i \\ y_i &= h_i(x_i) \end{aligned}$$

for $i = 1, \ldots, N$.

Definition:

1) The agents are said to state synchronize (or synchronize) if

$$\lim_{t \to \infty} |x_i(t) - x_j(t)| = 0 \quad \forall i, j = 1, \ldots, N$$

2) The agents are said to output synchronize if

$$\lim_{t \to \infty} |y_i(t) - y_j(t)| = 0 \quad \forall i, j = 1, \ldots, N$$

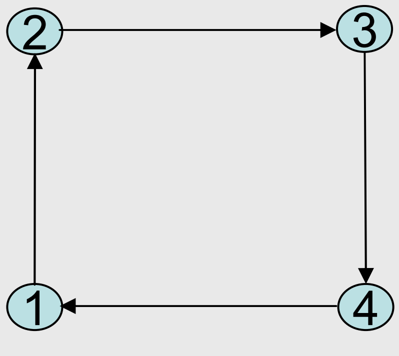
Background on Graph Theory

Information exchange between agents can be represented as a graph.

A graph $\mathcal{G}$ is a finite set $\mathcal{V}(\mathcal{G}) = \{v_i, \ldots, v_N\}$, whose elements are called nodes or vertices, together with set $\mathcal{E}(\mathcal{G}) \subset \mathcal{V} \times \mathcal{V}$, whose elements are called edges. An edge is therefore an ordered pair of distinct vertices.

If, for all $(v_i, v_j) \in \mathcal{E}(\mathcal{G})$, the edge $(v_j, v_i) \in \mathcal{E}(\mathcal{G})$ then the graph is said to be undirected. Otherwise, it is called a directed graph.

$$\mathcal{G} = \{\{1, 2, 3, 4\}, (1, 2), (2, 3), (3, 4), (4, 1)\}$$



Graph Connectivity

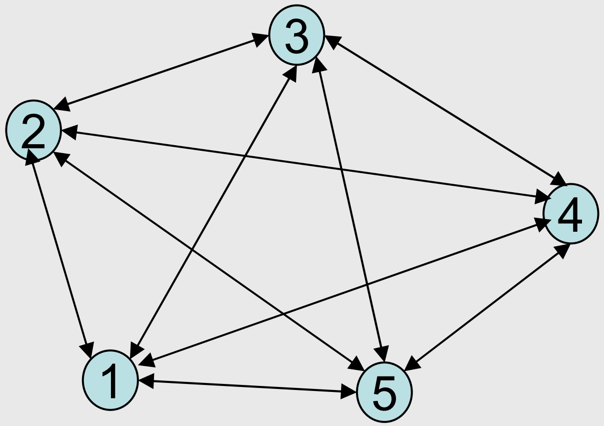
The in-degree of a vertex $v \in \mathcal{G}$ is the number of edges that have this vertex as a head. Similarly, the out-degree of a vertex $v \in \mathcal{G}$ is the number of edges that have this vertex as the tail.

If the in-degree equals the out-degree for all vertices $v \in \mathcal{V}(\mathcal{G})$, then the graph is said to be balanced.

A path of length r in a directed graph is a sequence $v_0, \ldots, v_r$ of $r + 1$ distinct vertices such that for every $i \in \{1, \ldots, r\}$, (v_i, v_{i+1}) is an edge.

A directed graph is connected if any two vertices can be joined by a path.

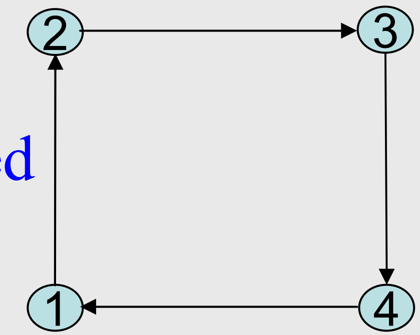
Examples of Communication Graphs

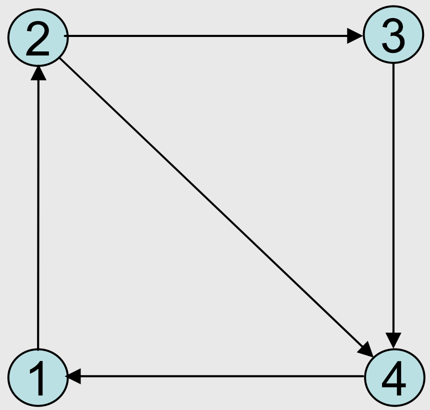


All-to-All Coupling

(Balanced -Undirected)

Balanced-Directed





Directed – Not Balanced

The Simplest Result

To begin, let's consider the simplest scenario. Assume that

1. The interconnection topology of the N passive agents is fixed and given as a balanced, connected graph.
2. The interconnection (control) is given as

$$u_i = \sum_{j \in \mathcal{N}_i} K(y_j - y_i), \quad i = 1, \ldots, N$$

where $\mathcal{N}_i$ is the set of neighbors of agent *i,* i.e., the agents that are communicating with agent *i*.

Theorem: All solutions of the closed loop system are bounded and the agents output synchronize.

Proof: Consider a storage function

$$V = 2(V_1 + \cdots + V_N)$$

where V_i is the storage function for agent i. Using the nonlinear KYP lemma and the above control yields

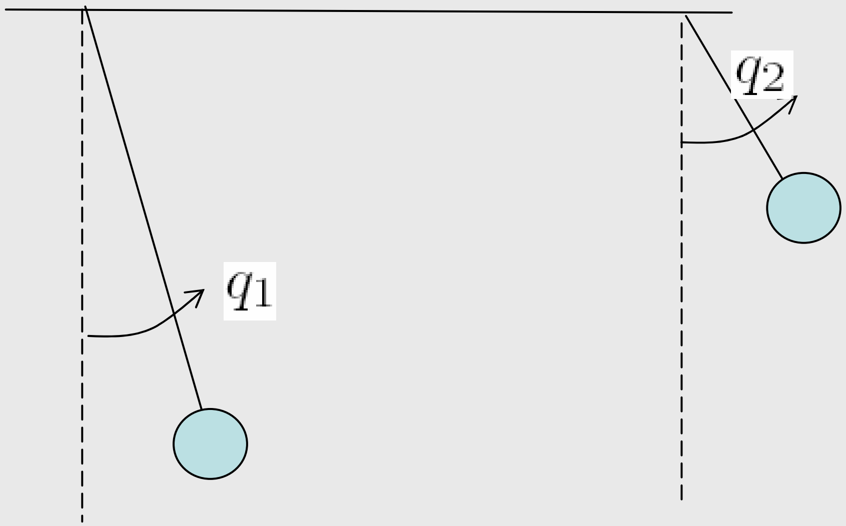
$$\begin{aligned} \dot{V} &= 2\sum_{i=1}^{N}(L_{f_i}V_i + L_{g_i}V_i u_i) \\ &= -2\sum_{i=1}^{N} S_i(x_i) + 2\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i} y_i^T K(y_j - y_i) \end{aligned}$$

As the information exchange graph is balanced, it can be shown, after some computation, that

$$\dot{V} = -2\sum_{i=1}^{N} S_i(x_i) - K\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i}(y_i - y_j)^T(y_i - y_j) \leq 0$$

and, hence, all signals are bounded. Lasalle's Invariance Principle and connectivity of the network can then be invoked to claim output synchronization.

Example: Coupled Pendula



Consider two coupled pendula with dynamics

$$\begin{aligned} \ddot{q}_1 + \frac{g}{L_1}\sin(q_1) &= u_1 \\ \ddot{q}_2 + \frac{g}{L_2}\sin(q_2) &= u_2 \end{aligned}$$

and suppose $u_1 = -u_2 = K(\dot{q}_2 - \dot{q}_1)$

Simulations

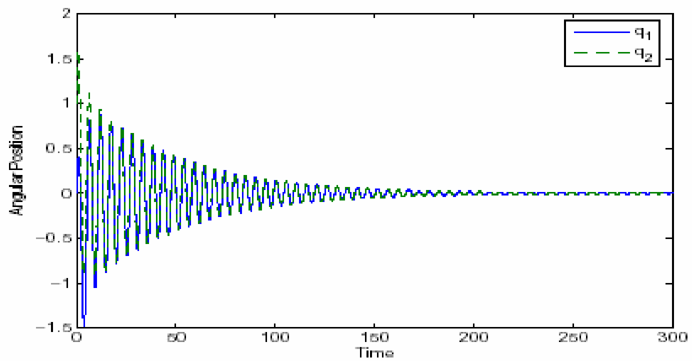


Fig. 3. The oscillations die out when the pendula have different lengths.

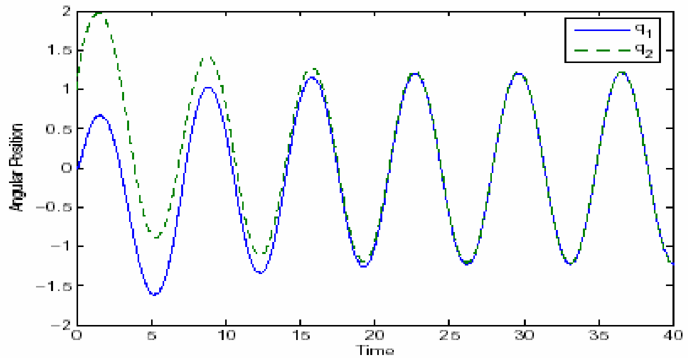


Fig. 4. The pendula move synchronously when they have identical lengths

Networks with Time Delay

Suppose there is a delay, T, in communication between any two nodes*. In this case output synchronization means

$$\lim_{t\to\infty} |y_j(t-T) - y_i(t)| = 0 \quad \forall i,j$$

We therefore choose the control inputs as

$$u_i(t) = \sum_{j\in\mathcal{N}_i} K(y_j(t-T) - y_i)$$

*The variable delay case is can also be treated within the passivity framework

Theorem: All solutions of the closed loop system are bounded and the agents output synchronize.

Proof: Consider a Lyapunov-Krasovskii functional for N-agent system as

$$V = K \sum_{i=1}^{N} \sum_{j \in \mathcal{N}_i} \int_{t-T}^{t} y_j^T(\tau) y_j(\tau) d\tau + 2(V_1 + \ldots + V_N)$$

As above, using the nonlinear KYP lemma and assuming that the graph is balanced, one can show that the derivative of V along trajectories of the system satisfies

$$\dot{V} \leq -K \sum_{i=1}^{N} \sum_{j \in \mathcal{N}_i} (y_j(t-T) - y_i)^T (y_j(t-T) - y_i) \leq 0$$

Using an extension of LaSalle's Theorem applicable to delay systems (Hale and Verduyn, 1993) we can show

$$\lim_{t\to\infty} |y_j(t-T) - y_i| = 0 \quad \forall j \in \mathcal{N}_i \quad i = 1, \ldots, N$$

Connectivity of the interconnection graph then implies output synchronization.

In many cases, such as nearest neighbor communications, the communication graph is not constant. Therefore, we next discuss the case of switching topology of the graph interconnection.

The Case of Switching Communications

In the case of switching graph topology we have

$$u_i(t) \;=\; \sum_{j \in \mathcal{N}_i(t)} K(y_j - y_i),\ i = 1, \ldots, N$$

where the set $\mathcal{N}_i$ of neighbors of agent *i* is time varying.

It turns out the output synchronization is still achieved provided the graph is balanced pointwise in time and satisfies a type of minimum dwell-time connectivity assumption. The key is to consider

$$V = 2(V_1 + \cdots + V_N)$$

as a common storage function for the switched (hybrid) system. (The case of time-delay in communication is more difficult.)

Application to Lagrangian Systems

We now consider a network of *N* Lagrangian systems

$$M_i(q_i)\ddot{q}_i + C_i(q_i,\dot{q}_i)\dot{q}_i + g_i(q_i) = \tau_i \qquad i = 1,\ldots,N$$

Lagrangian systems have a well-known passivity property from generalized velocity $\dot{q}$ to generalized torque τ , i.e.,

$$\dot{E} = \dot{q}^T \tau$$

where *E* is the total energy (kinetic plus potential). However, we will consider a different passivity relation in what follows using the notion of ***Feedback Passivation***.

Feedback Passivation

In the case of an N-degree-of-freedom Lagrangian system

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau$$

We define a preliminary feedback control

$$\tau = -M(q)\lambda\dot{q} - C(q,\dot{q})\lambda q + g(q) + u$$

where λ is a positive diagonal matrix, and u is an additional control input.

Feedback Passivation

Setting $r = \dot{q} + \lambda q$ we obtain the system

$$M(q)\dot{r} + C(q,\dot{q})r = u$$

It is follows, using the skew-symmetry of $\dot{M} - 2C$ that the above system is passive (lossless) with input u, output $y = h(x) = r$, and storage function

$$V(x) = r^T M(q) r$$

i.e.,

$$\dot{V}(x) = r^T u$$

Feedback Passivation

In state space we can write the system as

$$\begin{aligned}\dot{q} &= -\lambda q + r \\ \dot{r} &= -M^{-1}(q)C(q,\dot{q})r + M^{-1}(q)u\end{aligned}$$

which is of the form

$$\dot{x} = f(x) + g(x)u$$

with state vector $x = (q, r)$ and vector fields f and g given by

$$f(x) = \begin{pmatrix} -\lambda q + r \\ -M(q)^{-1}C(q,\dot{q})r \end{pmatrix} \; ; \; g(x) = \begin{pmatrix} 0 \\ M(q)^{-1} \end{pmatrix}$$

Synchronization of Lagrangian Systems

The problem of synchronization of Multi-Agent Lagrangian systems thus falls within our general framework of synchronization of passive systems. Defining the additional control inputs as

$$u_i(t) = \sum_{j \in \mathcal{N}_i} K(r_j(t-T) - r_i)$$

yields output synchronization if the interconnection graph is balanced and weakly connected.

State Synchronization

In fact, since

$$\begin{aligned} r_j(t) - r_i(t) &= (\dot{q}_j(t) + \lambda q_j(t)) - (\dot{q}_i(t) + \lambda q_i(t)) \\ &= \dot{e}_{ij}(t) + \lambda e_{ij}(t) \end{aligned}$$

where

$$e_{ij}(t) = q_j(t) - q_i(t)$$

it can be shown that output synchronization implies state synchronization.

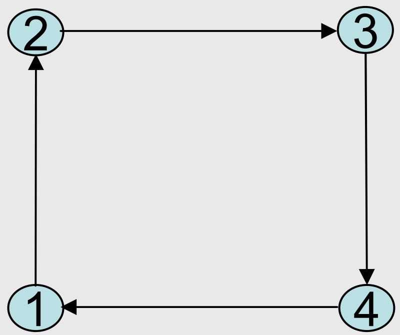
$$\begin{aligned} \lim_{t\to\infty} (q_j(t-T) - q_i(t)) &= 0 \\ \lim_{t\to\infty} (\dot{q}_j(t-T) - \dot{q}_i(t)) &= 0 \end{aligned}$$

Example

Consider a system of four point masses with second-order dynamics

$$m_i \ddot{q}_i = \tau_i \; ; \; i = 1, \ldots, 4$$

connected in a ring topology



Following our above derivation, applying the preliminary feedback

$$\tau_i = -m_i \lambda \dot{q}_i + u_i \ ; \ i = 1, \ldots, 4$$

the system becomes

$$m_i \dot{r}_i = u_i \ ; \ i = 1, \ldots, 4$$

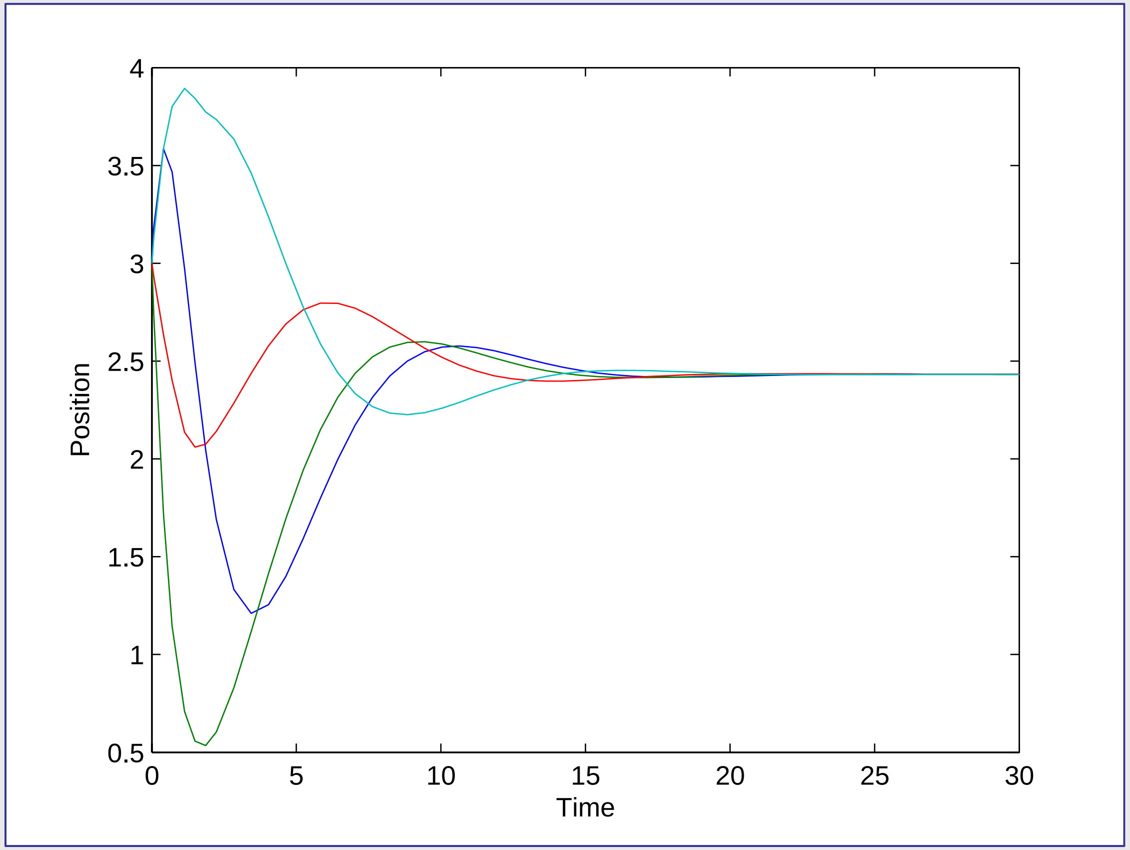
where $r_i = \dot{q}_i + \lambda q_i$. Let the communication among the agents be given by the above ring topology and suppose there are delays, T_{ij}, among the nodes.

Coupling the passive outputs leads to the closed loop system

$$\begin{aligned} m_1\dot{r}_1 &= K(r_2(t - T_{21}) - r_1) \\ m_2\dot{r}_2 &= K(r_3(t - T_{32}) - r_2) \\ m_3\dot{r}_3 &= K(r_4(t - T_{43}) - r_3) \\ m_4\dot{r}_4 &= K(r_1(t - T_{14}) - r_4) \end{aligned}$$

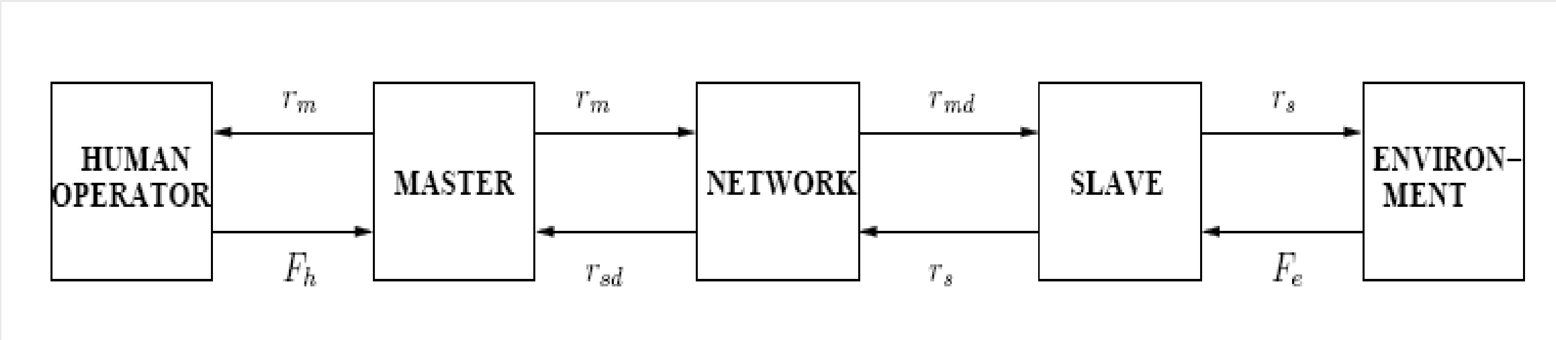
and the agents synchronize as shown below

Simulation Results



Application to Bilateral Teleoperation

A bilateral teleoperator consists of master and slave robots interconnected over a communication network. Viewing the teleoperation problem as one of synchronization leads to some interesting new results.



The standard approach to bilateral teleoperation relies on the passivity between velocity and force of Lagrangian systems. The system is then based on velocity commands from the Master to Slave. A drawback is that position is not directly controlled leading to the well-know problem of position drift.

Experimental Test

Velocity-Controlled Bilateral Teleoperator showing that position tracking is lost after contact with the environment.

Feedback Passivation/Synchronization approach

The master and slave dynamics are given as

$$\begin{aligned} M_m(q_m)\ddot{q}_m + C_m(q_m, \dot{q}_m)\dot{q}_m + g_m(q_m) &= \tau_m + F_h \\ M_s(q_s)\ddot{q}_s + C_s(q_s, \dot{q}_s)\dot{q}_s + g_s(q_s) &= \tau_s - F_e \end{aligned}$$

where F_h and F_e are the operator and environment forces, respectively. The preliminary feedback passivation is achieved with the control

$$\begin{aligned} \tau_m &= -M_m(q_m)\lambda\dot{q}_m - C_m(q_m, \dot{q}_m)\lambda q_m + g_m(q_m) + u_m \\ \tau_s &= -M_s(q_s)\lambda\dot{q}_s - C_s(q_s, \dot{q}_s)\lambda q_s + g_s(q_s) + u_s \end{aligned}$$

resulting in

$$\begin{aligned} M_m\dot{r}_m + C_m r_m &= u_m + F_h \\ M_s\dot{r}_s + C_s r_s &= u_s - F_e \end{aligned}$$

Feedback Passivation/Synchronization approach

where
$$\begin{aligned} r_m &= \dot{q}_m + \lambda q_m \\ r_s &= \dot{q}_s + \lambda q_s \end{aligned}$$
are the passive outputs.

Therefore, the coupling control inputs are chosen as

$$\begin{aligned} u_s &= K(r_m(t-T) - r_s) \\ u_m &= K(r_s(t-T) - r_m) \end{aligned}$$

Using the storage function

$$V = r_m^T M_m r_m + r_s^T M_s r_s + K \int_{t-T}^{t} (r_m^T r_m + r_s^T r_s) ds$$

Bilateral Teleoperation

we can show, under some assumptions on the human and environment forces,

- delay independent synchronization of the master and slave in free space, and
- tracking of steady-state contact force.

These results do not rely on the use of scattering/wave variables and, hence, do not suffer from position drift or wave reflections.

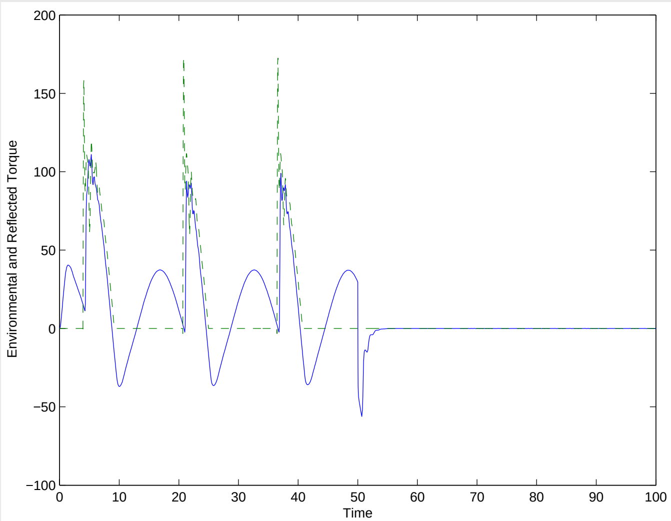
Experimental Test

Feedback-Passivity-Based Control of the Bilateral Teleoperator showing that position tracking is recovered After contact with the environment.

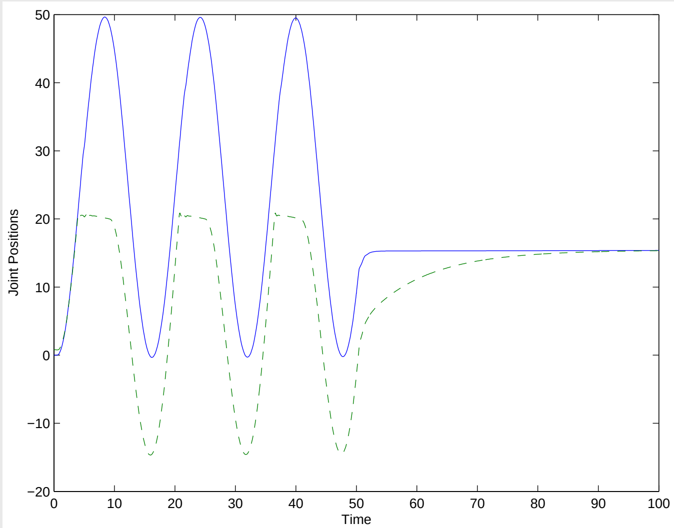
Simulations

Simulations of position and force tracking

Force



Position



Conclusions and Extensions

1. We can treat systems with nonlinear coupling

$$u_i = \sum_{j \in \mathcal{N}_i} K\phi(y_j - y_i) \quad i = 1, \ldots, N$$

   for a class of memoryless, passive (sector) nonlinearities.

2. We also obtain new results in the passivity context for so-called Kuramoto Oscillators

$$\dot{\theta}_i \;\; = \;\; \omega_i + \frac{K}{N} \sum_{j=1}^{N} \sin(\theta_j - \theta_i)$$

THANK YOU!

QUESTIONS?