RESILIENCE OF TRANSPORTATION NETWORKS

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Cascades in Infrastructure Networks

FLOODING IN THAILAND COULD CAUSE INDUSTRY-WIDE HARD DRIVE SHORTAGE

BY AMY FREELAND / 31 OCTOBER 2011 / 0 COMMENTS

Heavy monsoon rains that have left much of Thailand literally under water could impact the computer industry this holiday season and beyond. According to All Things D, the flooding already has affected the Thailand operations of two major hard drive manufacturers, Western Digital and Seagate Technology.

The New York Times

December 27, 2008

Flight Delays Radiate From Chicago and Atlanta

Major power outage hits New York, other large cities

August 14, 2003

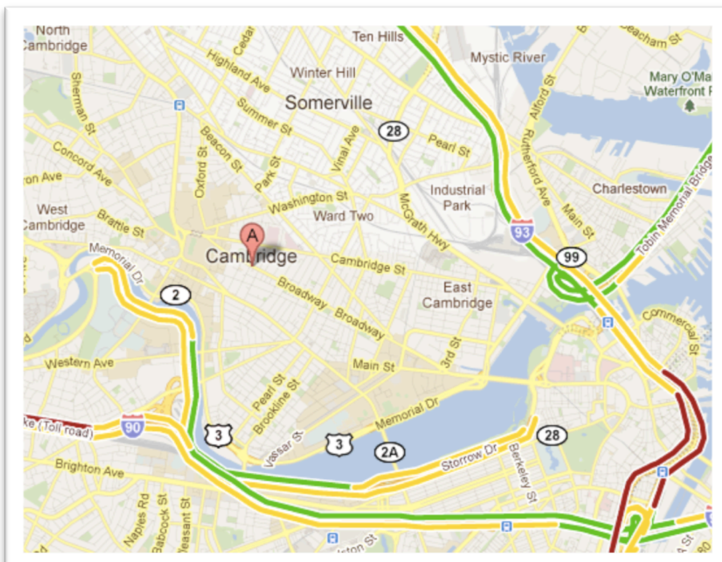
Vulnerability of Transportation Systems

"The Transportation Sector's components are susceptible to the consequences of natural disasters and can also make attractive terrorist targets. The sector's size, its physically dispersed and decentralized nature, the many public and private entities involved in its operations, the critical importance of cost considerations, and the inherent requirement of convenient accessibility to its services by all users - these aspects combine to make transportation vulnerable to security threats."

- Volpe National Transportation Systems Center Report '03

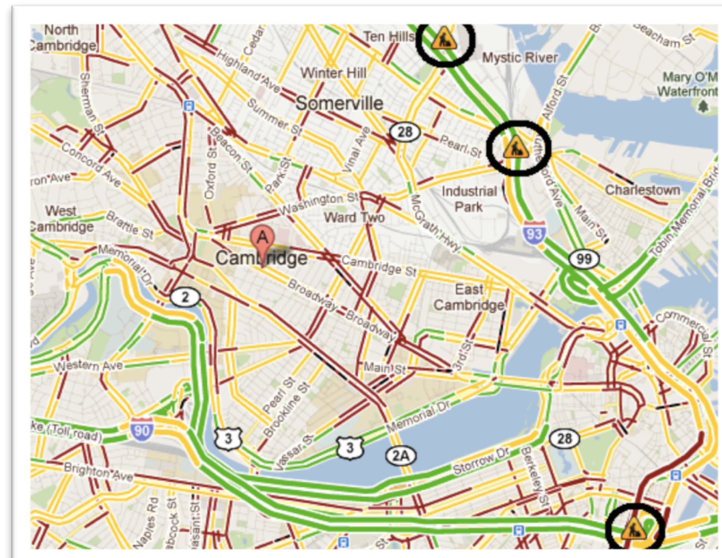
Disturbances in Urban Transportation Networks

- Accidents, road closures, inclement weather, etc.
- Load balancing related to adaptive road choice behavior of drivers
- Cascade effects can magnify the effect of disturbance



Typical Monday at 6:30 p.m.

disturbance



Monday November 7, 2011, 6:30 p.m.

(Courtesy: Google Maps)

Urban Transportation Network

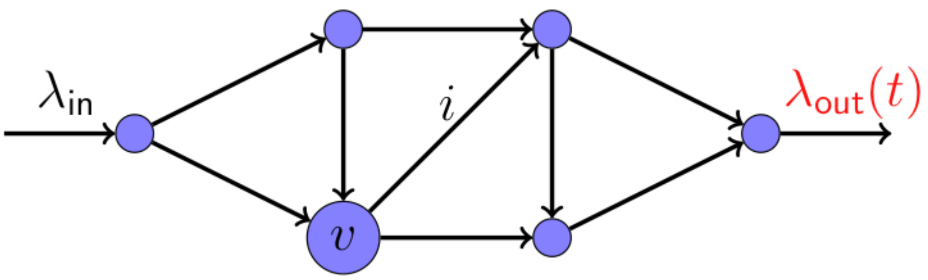
[A traffic jam in China]

Objective: Develop a dynamical model for transportation and derive metrics for their resilience

Outline

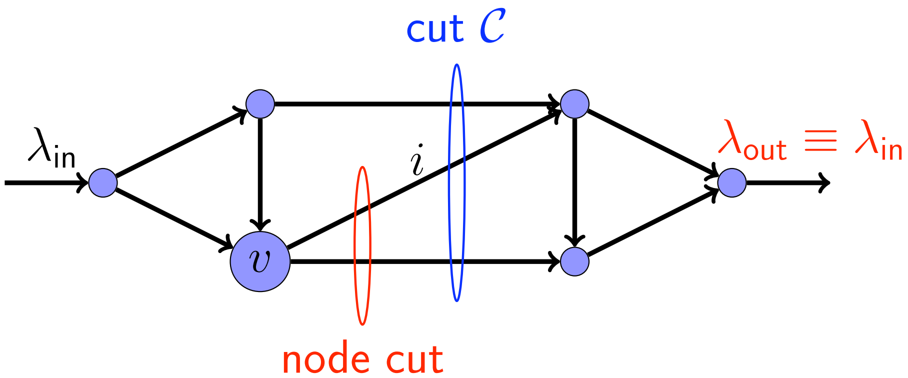
- Dynamical network flow formulation
- Stability of equilibria
- Margins of resilience
- Cascade effects
- Conclusions

Transportation as Network Flow



- Directed acyclic graph with single O/D pair
- Constant arrival rate λ_{in} at the origin
- Driver route choice decisions + traffic physics determine $\lambda_{\text{out}}(t)$

Static Network Flow



- Link flow capacity: $f_i^{\max}$
- $\lambda_{\text{out}} = \lambda_{\text{in}} \longleftrightarrow$ feasible f :

$$f_i \leq f_i^{\max} \quad \forall i$$

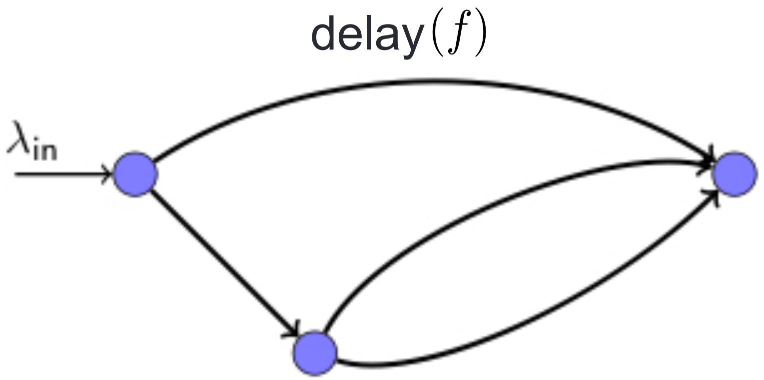
$$\sum_{\text{incoming}} f_i = \sum_{\text{outgoing}} f_j$$

- Max flow min cut theorem:

$$\lambda_{\text{in}} \leq \text{ min-cut capacity } \implies \text{ feasible } f$$

- Static perspective: link outflow always equals inflow

Wardrop Equilibrium

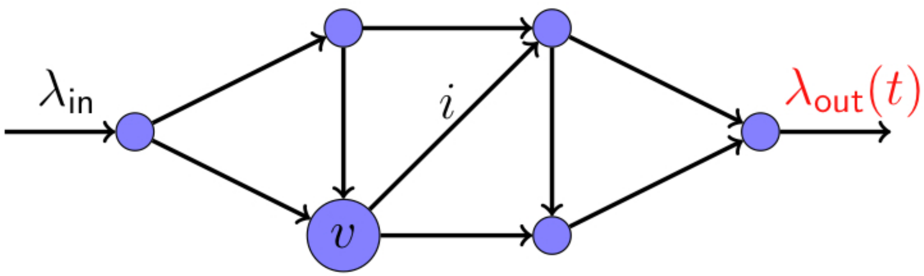


- π : distribution of driver population by route preference
- π induces static f^{π}

Wardrop equilibrium:

- delay (f) on any used path is no greater than the delay on any other path
- globally stable under best response dynamics if $\lambda_{\text{in}} <$ min-cut capacity
- π (and hence f^{π}) evolves as per global best response strategy by drivers

Transportation physics



ρ_i : density on link i

- Congestion dynamics

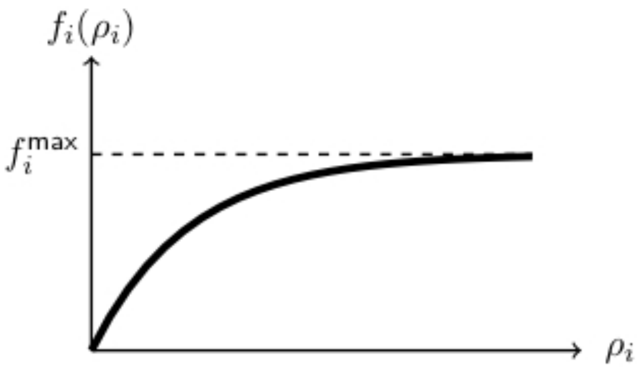
Rate of change of ρ_i = flow into link i – flow out of link i

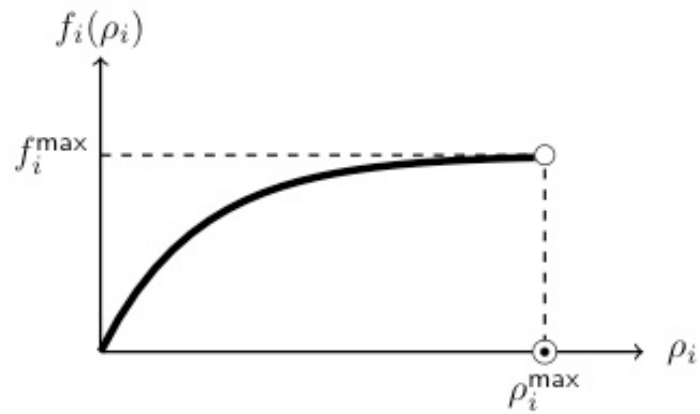
- Flow conservation

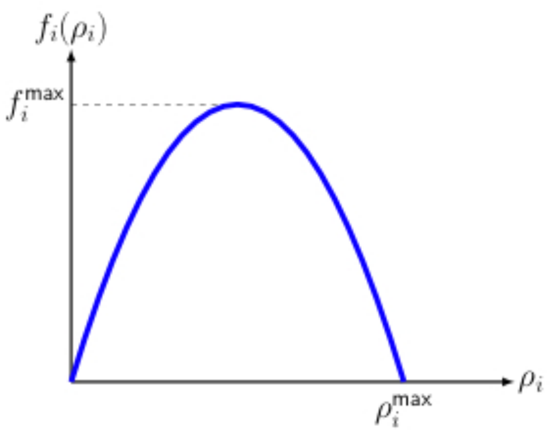
$$\sum_{i \text{ incoming to } v} f_i = \sum_{j \text{ outgoing from } v} f_j \qquad \forall v$$

Flow function

- Outflow on a link depends on the traffic density on that link: $f_j(\rho_j)$







ρ_i : density on link

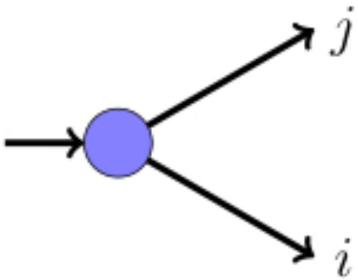
Outflow is not necessarily equal to inflow on a link

Multi-scale driver decision model

- Drivers take decision at every node
- Node-wise decisions influenced by:
  - global information available infrequently
  - real-time node-specific information

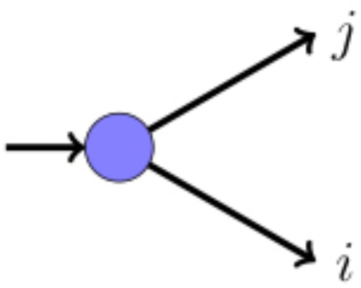
Local route choice decisions

- At node v, $G : \underbrace{\rho}_{\text{local info}} \times \underbrace{\pi}_{\text{(old) global info}} \rightarrow \text{prob. vector}$



- Locally responsive routing policy G^* :
  - Consistency: $G_i^*(\rho^\pi, \pi) \sim \pi$
    - if local observations match expectation, then follow suit
  - Sensitivity: $\partial G_i^* / \partial \rho_j \geq 0, \quad i \neq j$
    - locally prefer links with less congestion

Example: i-logit

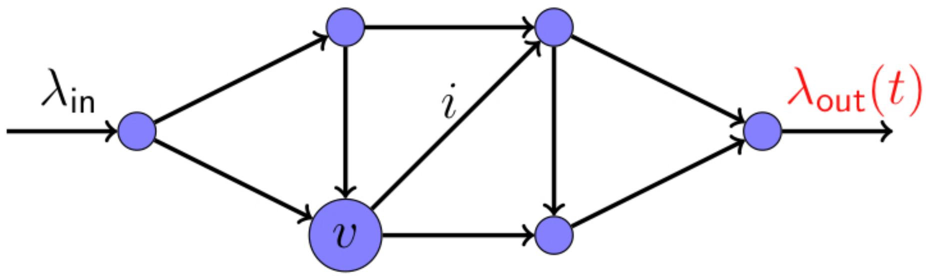


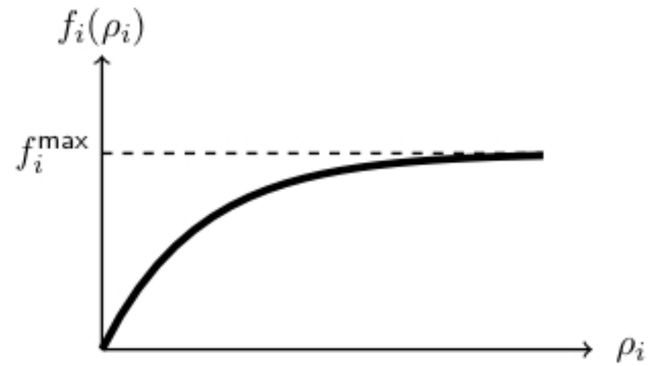
$$G_i^*(\rho) \propto f_i^\pi \exp\left(-\beta(\rho_i - \rho_i^\pi)\right), \quad \beta \geq 0$$

$$\text{i.e., utility}_i = \underbrace{\rho_i^\pi - \rho_i}_{\text{myopia}} + \underbrace{\frac{\log f_i^\pi}{\beta}}_{\text{inertia}} + \text{noise}(\beta)$$

- Myopia prevents passiveness; inertia prevents aggressiveness

Dynamical network flow





- Congestion dynamics (fast scale)

$$\dot{\rho}_i(t) = \text{inflow at } v \quad \cdot G_i(\rho, \pi) - f_i(\rho_i)$$

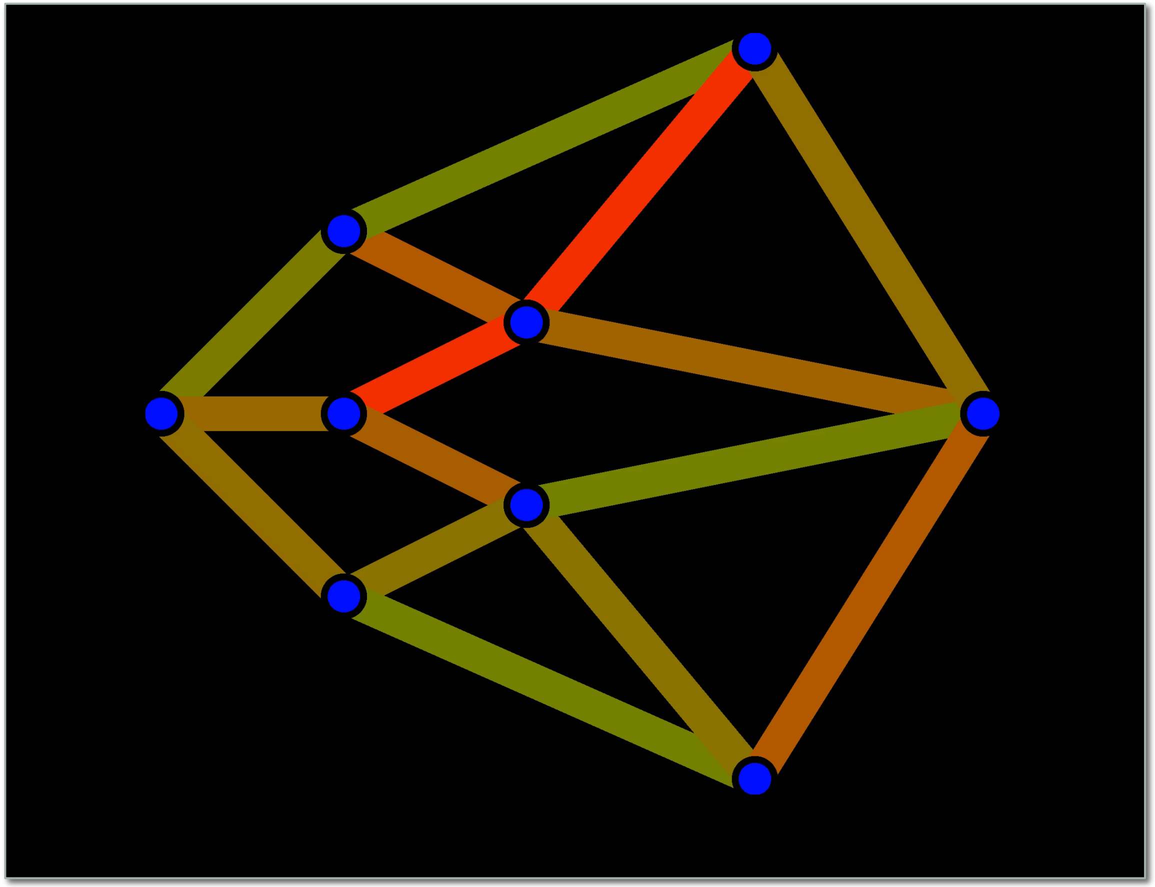
- Global decision dynamics (slow scale)

$$\dot{\pi} = \eta\,(\text{best response}(\rho) - \pi)$$

- Flow conservation

$$\sum_{i \text{ incoming to } v} f_i = \sum_{j \text{ outgoing from } v} f_j \qquad \forall v$$

Illustration of Network Flow Dynamics



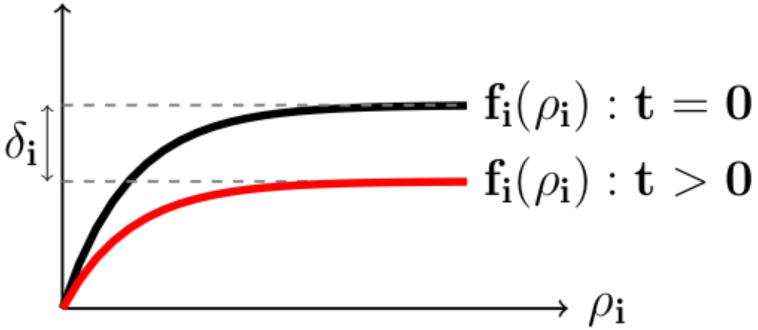
Stability of Wardrop equilibrium

Theorem: If

- $\lambda_{\text{in}} <$ min-cut capacity
- Drivers do not update their global decisions sufficiently fast w.r.t. traffic dynamics (small η)

- Then Wardrop equilibrium is globally stable.

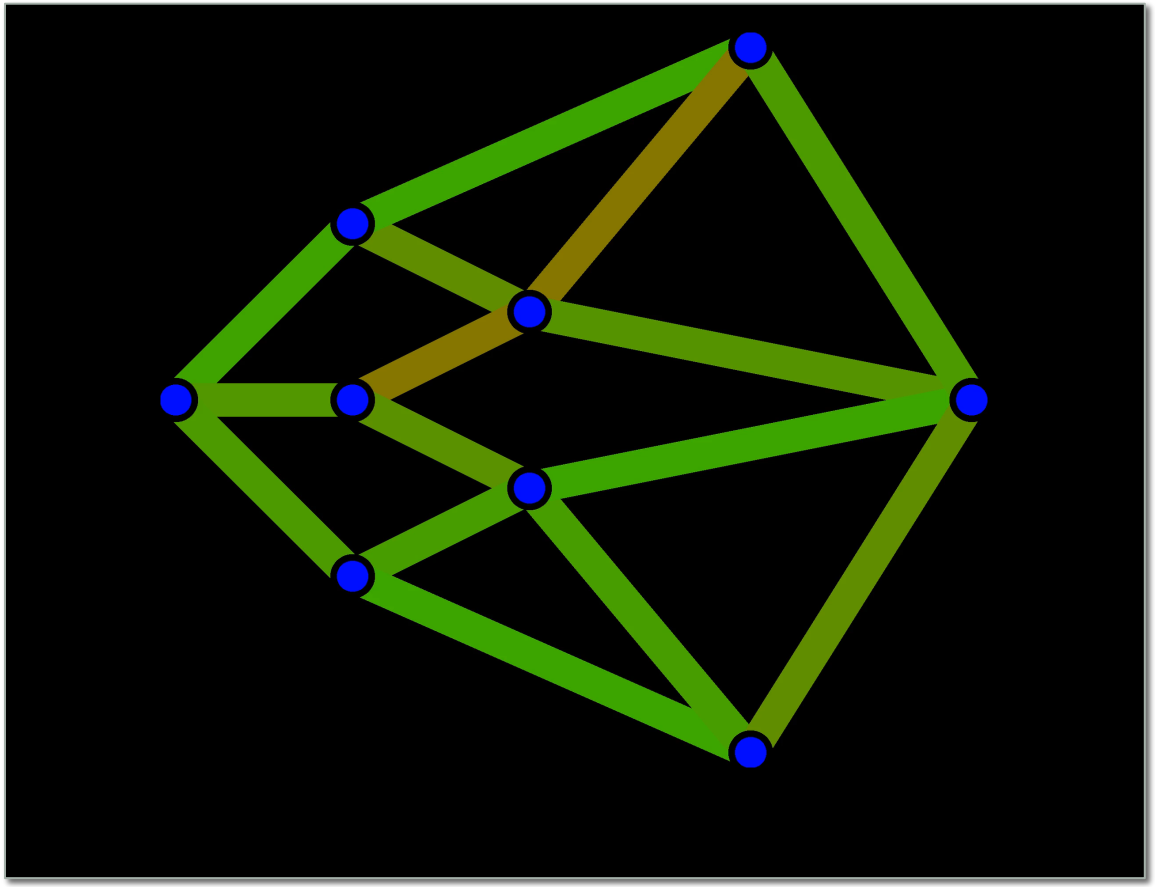
Perturbations: infinite density capacity



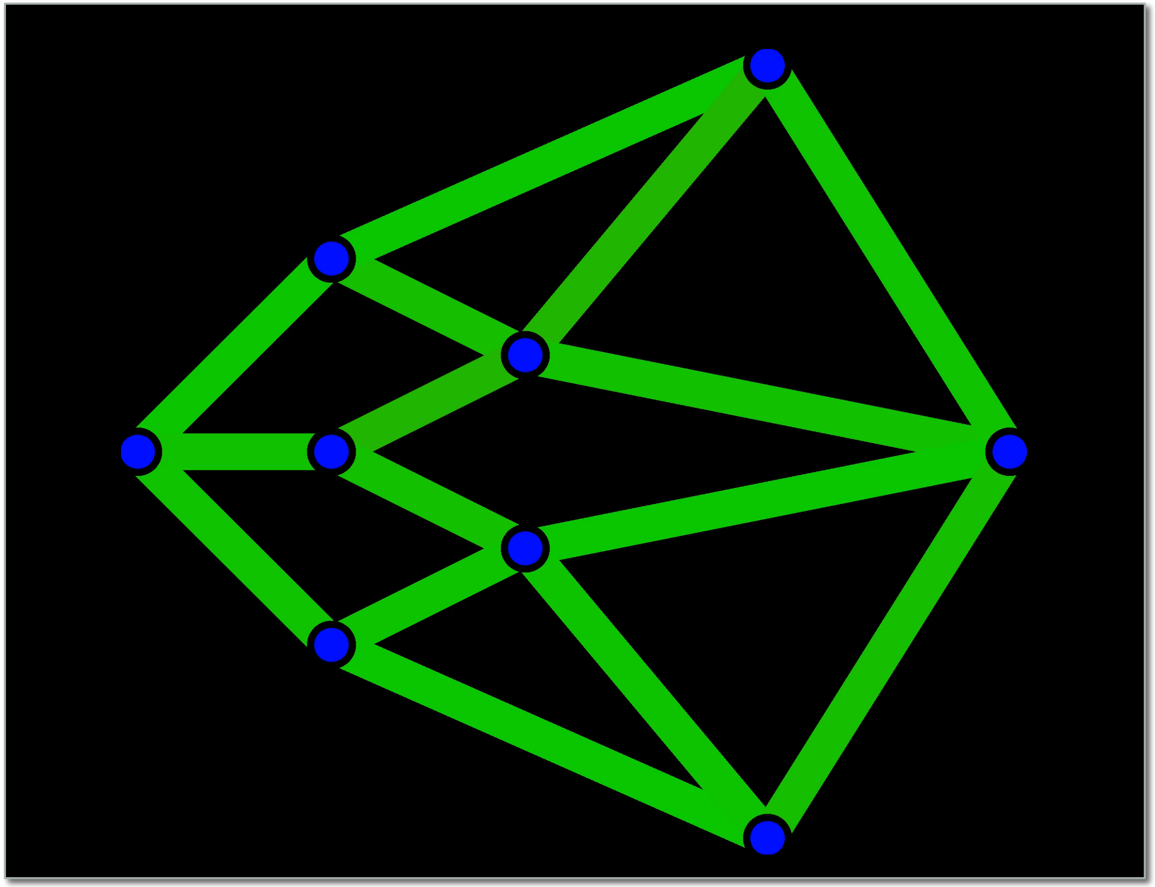
$$\delta_i = \|f_i - \tilde{f}_i\|_\infty$$

$$\delta = \sum_{i \in \mathcal{E}} \delta_i$$

Network Response to Small Perturbation



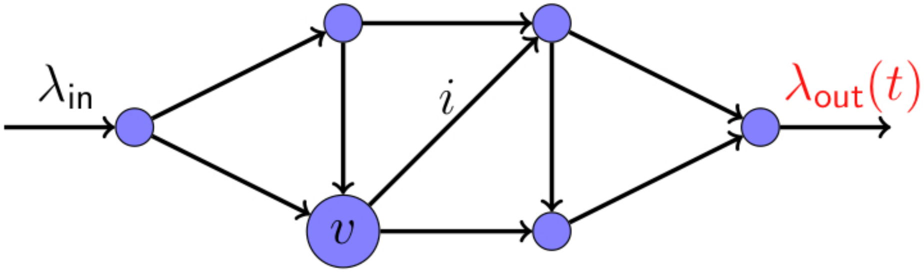
Network Response to Large Perturbation



Transferring Property

- The perturbed network is fully transferring w.r.t. eqm f^{eq} (not necessarily Wardrop) under G if :

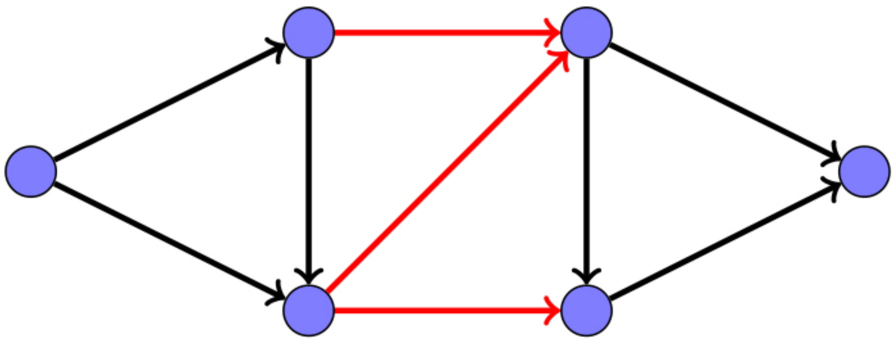
$$\liminf_{t\to\infty} \lambda_{\text{out}}(t) = \lambda_{\text{in}} \quad \text{with initial condition } f^{\text{eq}}$$



- Margin of resilience for a given G and f^{eq}

$$:= \inf_{\delta} \text{ perturbed network is not fully transferring w.r.t. } f^{\text{eq}} \text{ under } G$$

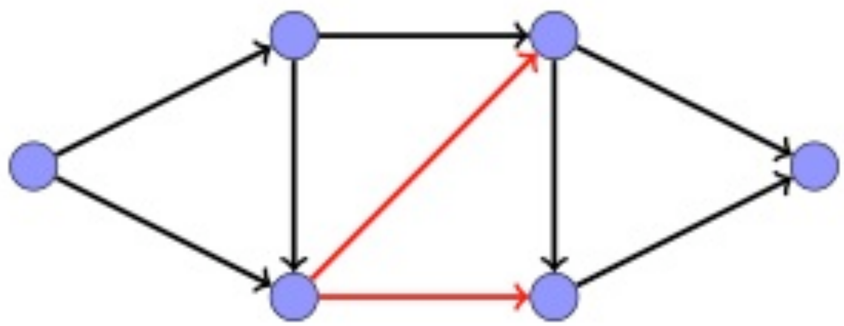
Upper Bound on Margin of Resilience



- $\forall G$, margin of resilience ≤ min cut residual capacity

$$:= \min_{\text{cut } \mathcal{C}} \sum_{i \in \mathcal{C}} (f_i^{\text{max}} - f_i^{\text{eq}})$$

A Tighter Upper Bound



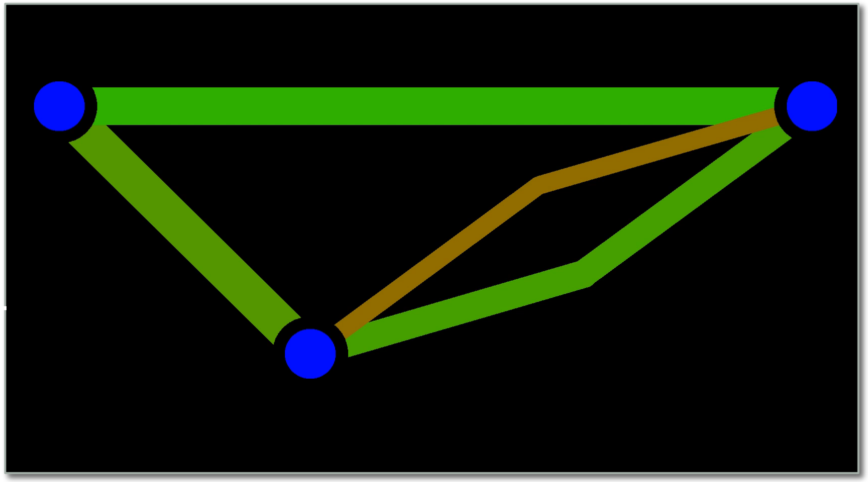
- $\forall G$, margin of resilience ≤ min node cut residual capacity

$$:= \min_{v} \sum_{i \text{ outgoing from } v} \left(f_i^{\mathsf{max}} - f_i^{\mathsf{eq}}\right)$$

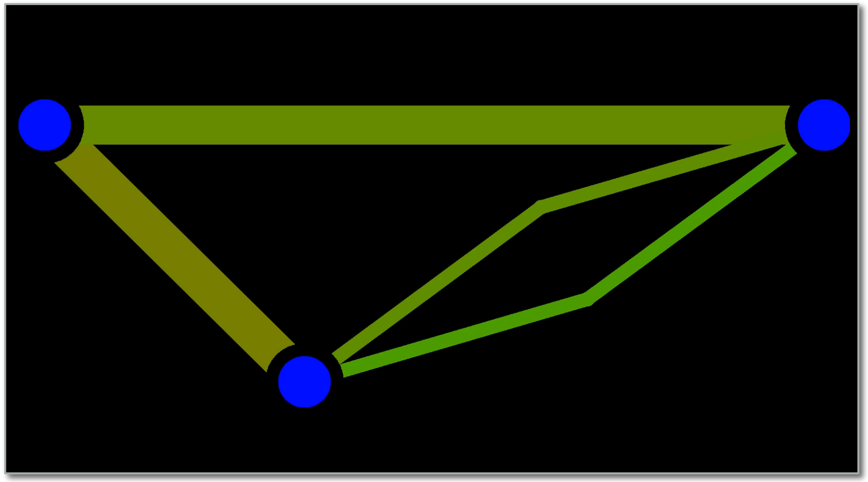
Sufficiency for Margin of Resilience

Possible loss of resilience due to:

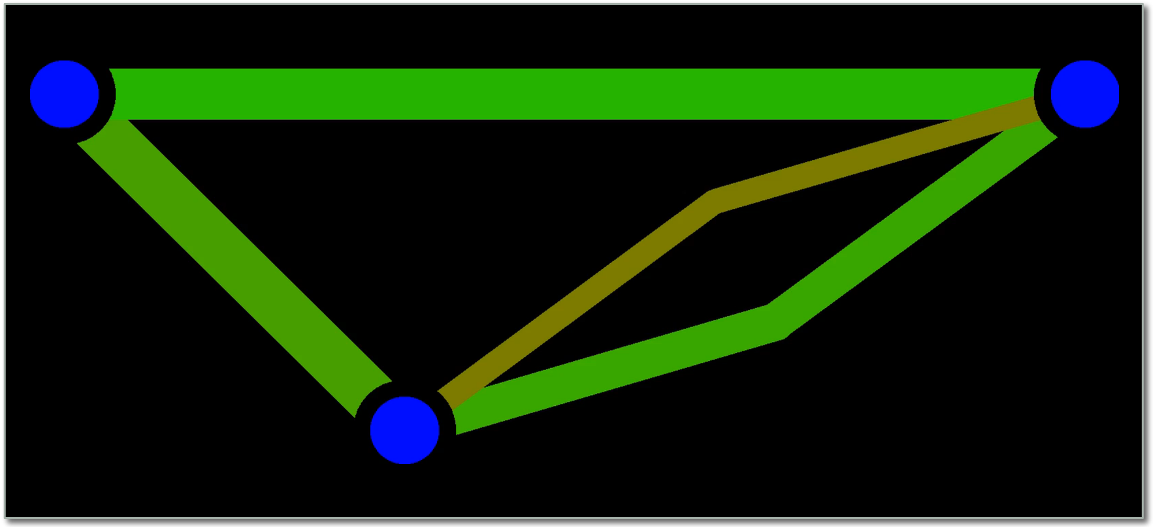
- Passive routing



- Aggressive routing

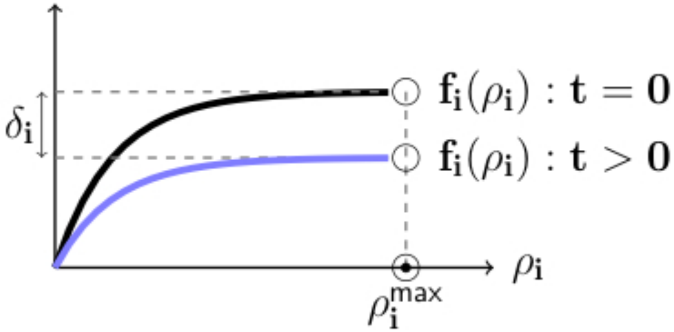


Optimality of Locally Responsive Routing



- G^* creates the perfect balance between passive and aggressive routing
- For G^*, margin of resilience = $\min_v$ residual capacity of node v

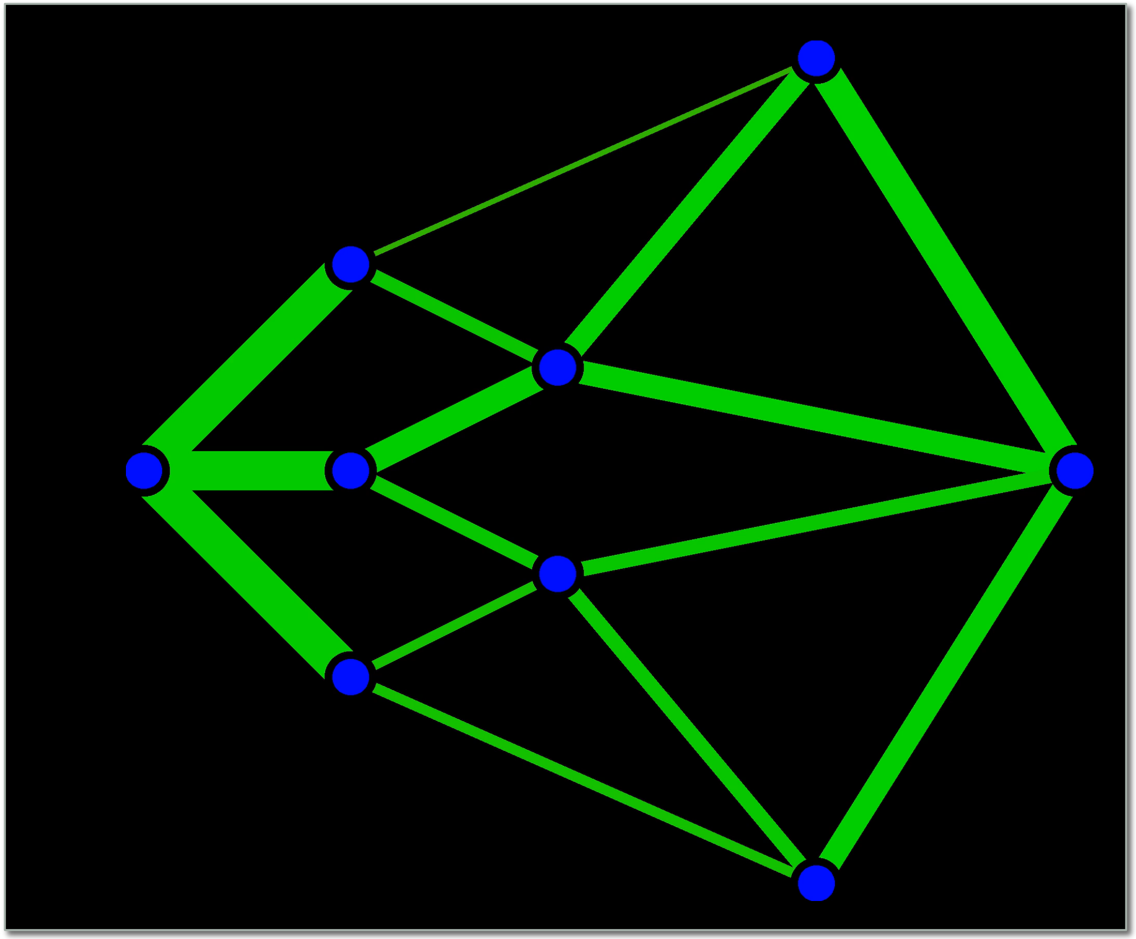
Perturbations: finite density capacity



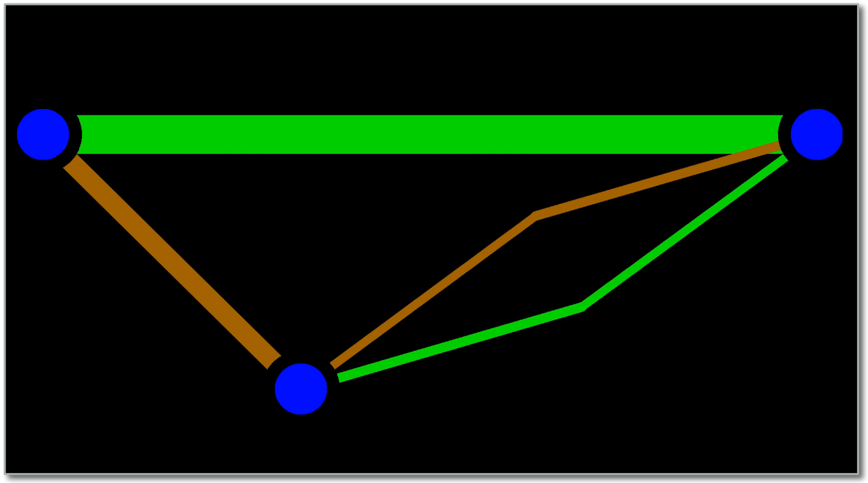
$$\dot{\rho}_i = \mathbf{1}_{\text{link } i \text{ open}} \cdot \text{inflow at node } v \cdot G_i - \mathbf{1}_{\text{downstream open}} \cdot f_i$$

Finite density capacity constraints cause upstream cascades

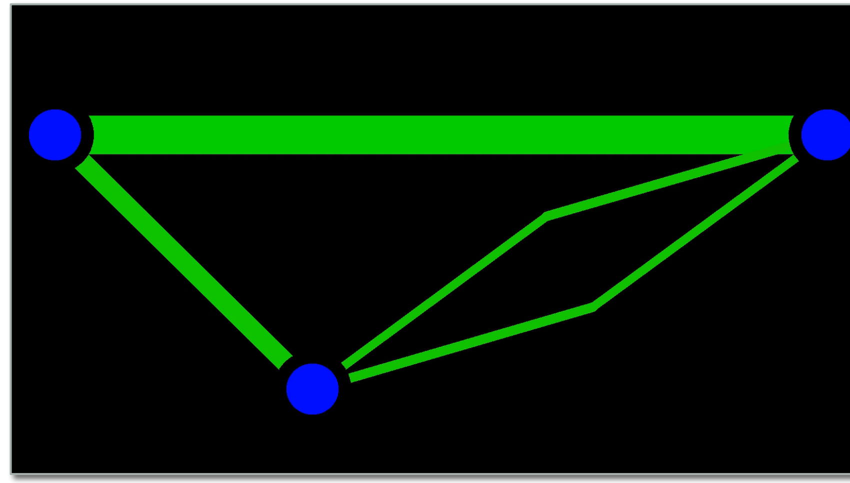
Upstream Cascades



Upstream Cascades can Increase Resilience



Unbounded density capacity



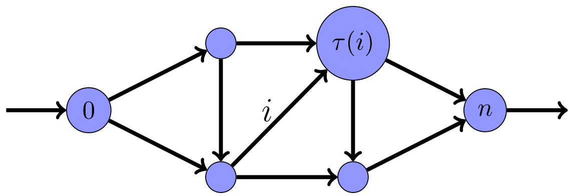
Upstream cascades due to bounded density capacity

Upstream cascades compensate for lack of downstream information

Upper Bound on Margin of Resilience

- Backward recursion algorithm:
  - d_v : min downstream perturbation needed to shut down node v
  - $c_i(x_i)$: min perturbation to remove capacity x_i from link i

$$c_i(x_i) = \min\{x_i, d_{\tau(i)}\}$$



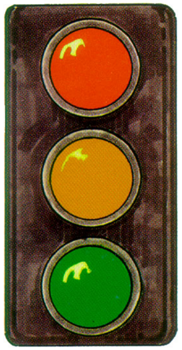
$d_n := +\infty$. For $v = n-1, \ldots, 1, 0$, iteratively let d_v be the solution to

$$\begin{aligned} \text{minimize} \quad & \sum_{i \in \mathcal{E}_v^+} c_i(x_i) \\ \text{subj. to} \quad & \sum_{i \in \mathcal{E}_v^+} x_i = \sum_{i \in \mathcal{E}_v^+} (f_i^{\mathsf{max}} - f_i^{\mathsf{eq}}), \\ & x_i \in [0, f_i^{\mathsf{max}}] \quad \forall i \in \mathcal{E}_v^+ \end{aligned}$$

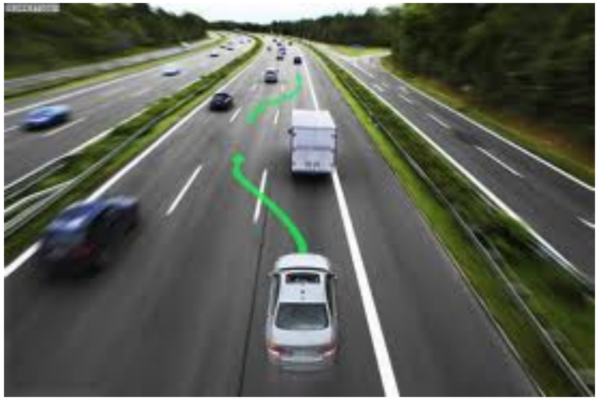
- Margin of resilience $\leq d_0$

Implications for Intelligent Transportation Systems

- Green light control
  - to influence routing G



- Congestion pricing
  - to influence equilibrium
- Automated driving
  - to influence the flow function



Conclusions

- Dynamical model for transportation networks
- Stability of equilibria under multiscale driver decisions
- Robust route choice behavior
- Characterization of margins of resilience
- Effect of cascades on the margins

Future Work

- Multiple origins and destinations
- Micro foundations: spatial queuing networks
- Control and mechanism design: green light control, dynamic tolls