

ICINCO 2007

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Keynote Lecture

Real Time Diagnostics, Prognostics, & Process Modeling Industrial Perspective

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Ford Motor Company

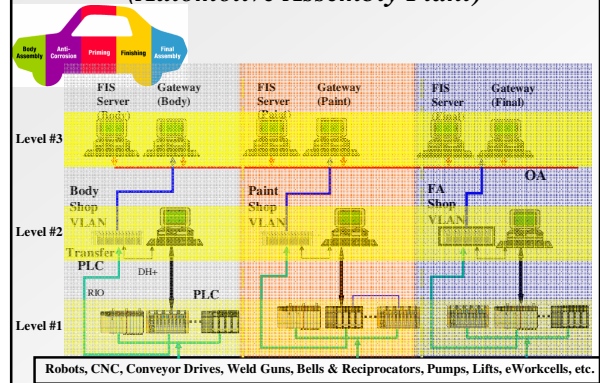


- Global automotive industry leader based in Dearborn, MI.
- Manufactures and distributes automobiles in 200 markets across six continents.
- 300,000 employees & 108 plants worldwide
- Ford Motor Company celebrated its 100th anniversary on June 16, 2003.

Outline

- **Introduction**
- **Practical Process Modeling**
 - Applied Indirect Adaptive Control
 - Rule-Base Guided Adaptive Control
 - Opportunities for Process Optimization
- **Diagnostics & Prognostics in Industrial Setting**
 - Research Motivation for Condition-Based / Predictive Maintenance (CBM / PdM)
 - Autonomous Diagnostics & Prognostics – the Novelty Detection Approach
 - Estimation of Machine Health
 - Remaining Useful Life Prediction
 - Applications
- **Evolving Systems**
- **Concluding Comments**

Generic Plant Information Infrastructure (Automotive Assembly Plant)



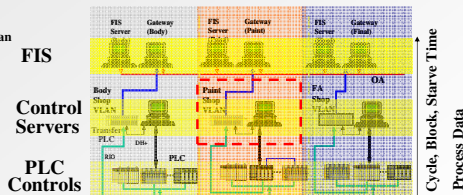
Generic Plant Informatics Applications

• ActivPlant

- Ford, Toyota, DCX, CocaCola, Bosch, Gillette

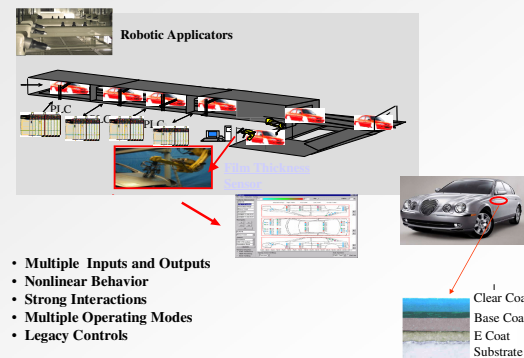
• Cimplicity

- GM, Nissan

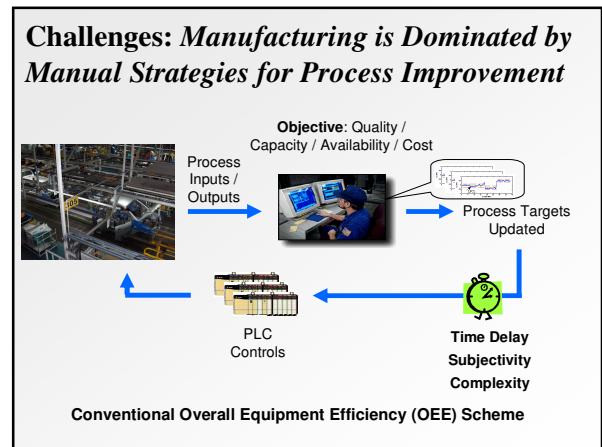
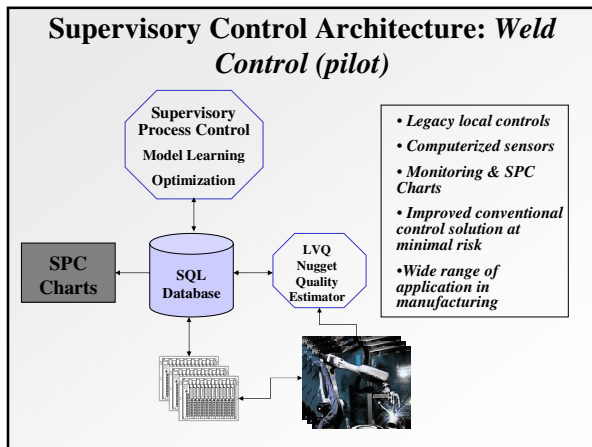
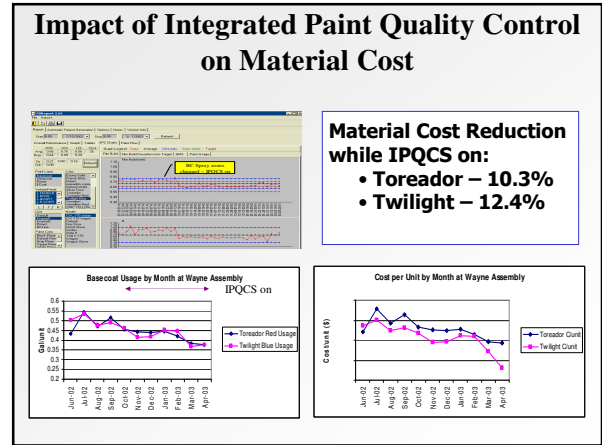
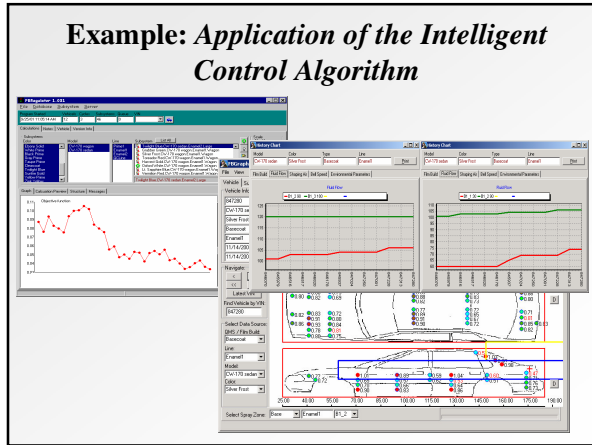
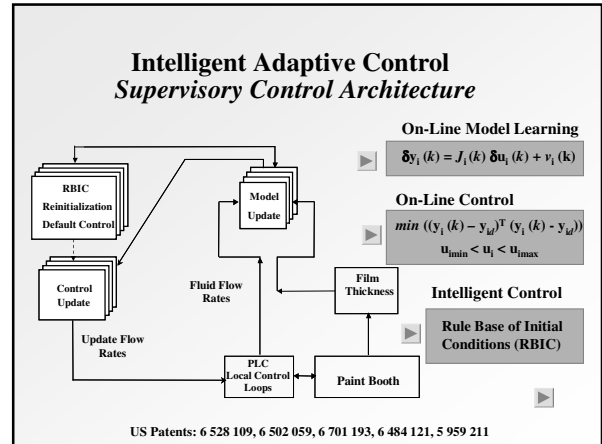
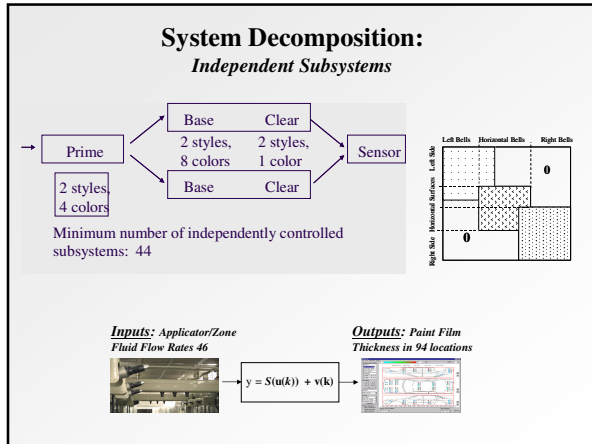


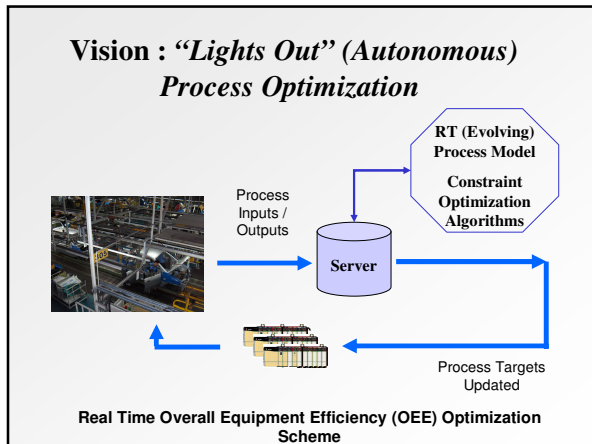
- Low, PLC level: local control algorithms (ladder logic, PID control)
- Intermediate level: information sharing and process performance enhancement
- Upper, FIS level: machine operating attributes for Overall Equipment Efficiency (OEE) assessment & limited set of process data
- **Opportunities:** machine health monitoring, process modeling, supervisory control, adaptive control, process optimization, autonomous agents, etc.

Applied Intelligent Control: Control of Automotive Paint Process



- Multiple Inputs and Outputs
- Nonlinear Behavior
- Strong Interactions
- Multiple Operating Modes
- Legacy Controls





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 - Case Study: *Control of Automotive Paint Process*
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Predictive Maintenance (PdM): Impact & Opportunities

- Reactive > 50-60%
- Preventive > 40-50%
- Predictive < 5%

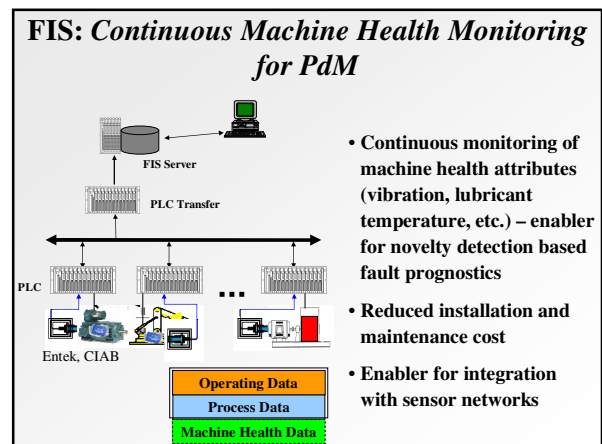
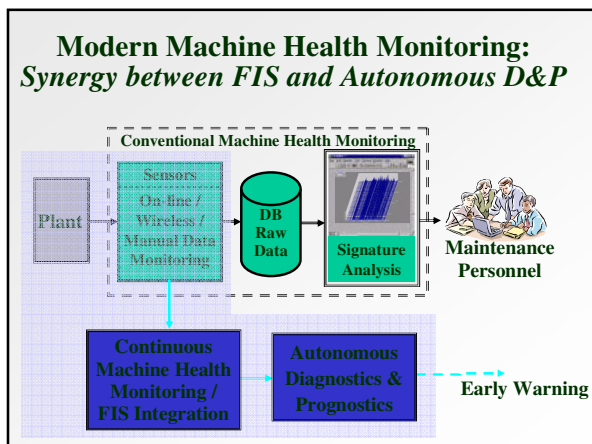
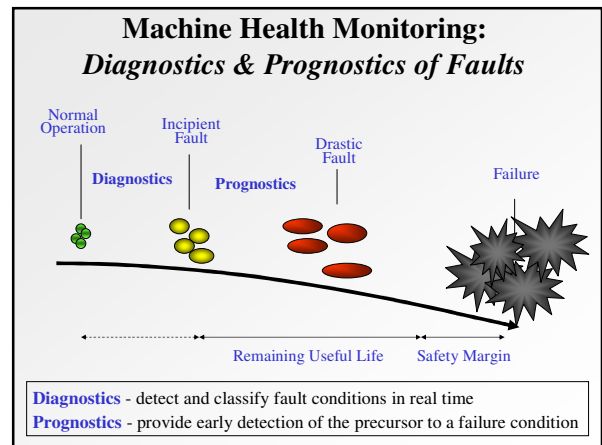
* 69% of all planned maintenance is unnecessary. ARC Advisory Group

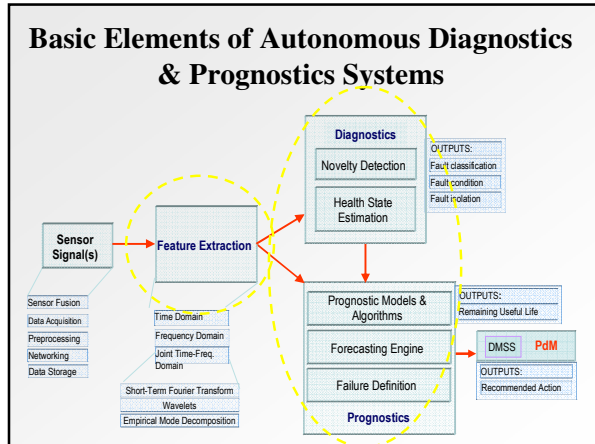
"More than \$1 trillion is spent each year to replace perfectly good equipment because no reliable and cost-effective method is available to predict the equipment's remaining life". (McLean & Wolf, Sensor Magazine Online, June, 2002)

Current Status

- Machine health monitoring is in its early stage
- Conventional diagnostics methods & systems are cost prohibitive
- Lack of reliable fault prognostics algorithms
- **Great research opportunity with significant potential impact**

PdM Systems – one of the main drivers for autonomous D & P



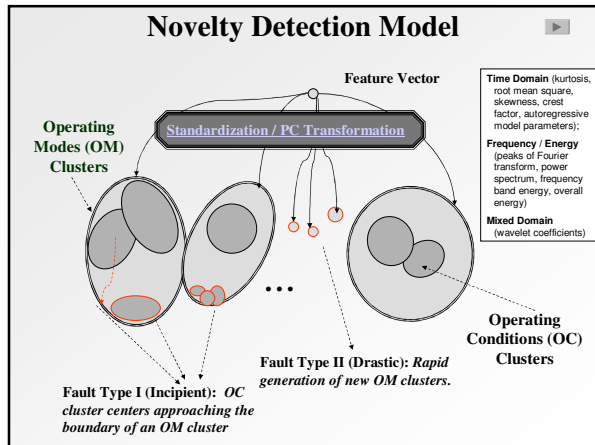


The Novelty Detection Challenge

- How to recognize (and compare numerically) the **difference** between the different machine operations?
- How identify the **difference** between a normal machine operation and an anomaly?

Hydraulic Excavator of Shim-Caterpillar Mitsubishi

Code	Label	OperationDescription
1	F1	Fuel Spray Nozzle deactivated
2	F2	TurboCharger Deterioration
3	F3	Valve Clearance Changed
4	F4	Air Filter Obstruction (High)
5	F5	Air Filter Obstruction (Low)
6	NH	Normal Operation (High Load)
7	NM	Normal Operation (Medium L.)
8	NL	Normal Operation (Light Load)



Learning OM Clusters to Approximate Equipment Operating Modes

If {Feature vector is within the closest OM cluster}

$$(\underline{y} - \underline{y}_i^*)' S_i^{-1} (\underline{y} - \underline{y}_i^*) < \chi_{p,\alpha}^2$$

Then {Update cluster parameters & transformation mapping}

$$\underline{y}_i^*(k) = \alpha \underline{y}_i^*(k-1) + (1 - \alpha) \underline{y}(k)$$

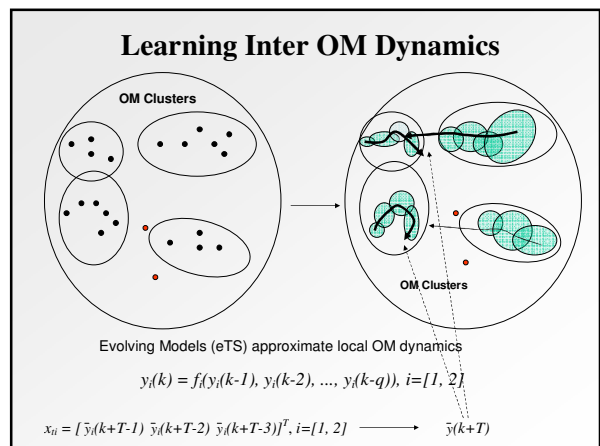
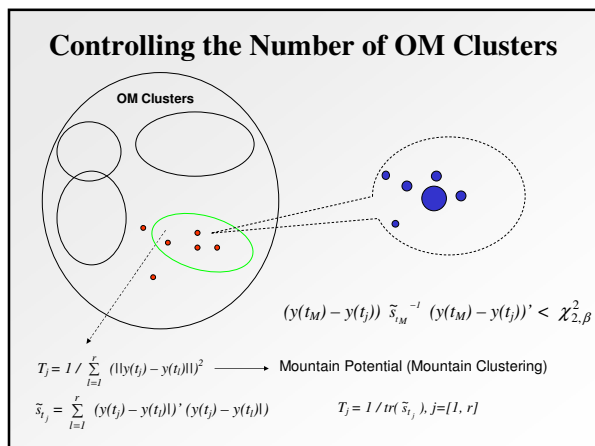
$$S_i^{-1}(k) := Q_i(k)$$

$$Q_i(k) = \alpha^{-1} (Q_i(k-1) - Q_i(k-1) y(k) (\alpha + y'(k) Q_i(k-1) y(k))^{-1} y'(k) Q_i(k-1))$$

$$T_i: \quad Q_i = T_i^{(u)} Q_i^{(s)} T_i^{(v)}$$

Else {Start a new OM cluster}

$$\underline{y}_{i+1}^* = \underline{y}(k)$$

$$S_{i+1} = Q_i(0)$$


Real Time Diagnostics

- Diagnostics is carried out based on three independent methods of analysis

– Diagnostics based on classification (PC space)

$$(y(k) - y_j^*) s_j^{-1} (y(k) - y_j^*)' < \chi_{2,\beta}^2 \quad r_c$$

– Diagnostics based on feature/signal enveloping (feature space)

$$|Y(k) - \bar{Y}_j| \leq c \bar{\sigma}_j \quad r_{SPC}$$

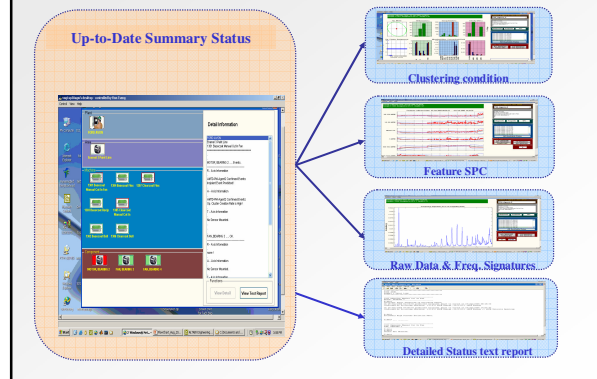
– Diagnostics based on velocity threshold

$$(y(k+1) - y(k))' (y(k+1) - y(k)) < v^* \quad r_v$$

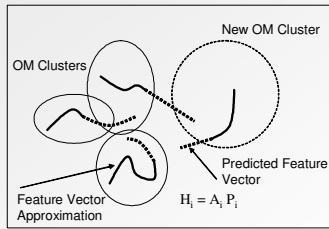
- Above three results are combined through a weighted voting rule to come up with a severity rating

$$S_R = w_c r_c + w_{spc} r_{spc} + w_v r_v$$

Summarized Machine Health Report



Prediction of Type 1 (Incipient) Equipment Faults



$$I = \arg \min_j (d_j(\tilde{y}(k+T) - y_j) s_j^{-1} (\tilde{y}(k+T) - y_j)'), j=[1, m].$$

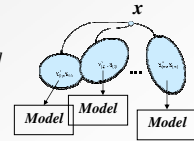
$$(\tilde{y}(k+T) - y_j^*) s_j^{-1} (\tilde{y}(k+T) - y_j^*)' < \chi_{2,\beta}^2$$

Failure Modes Type I: OM feature models approaching the boundary of an OM cluster with a low health factor

Real Time Prognostics: eTS Model Based Prediction

- Fuzzily blended sub-models:

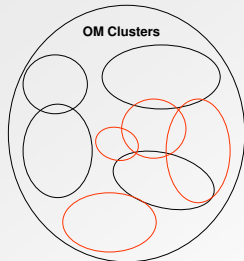
\mathcal{R}^i : IF $(x_j \text{ is } \mathfrak{R}_j^i)$ AND ... AND $(x_n \text{ is } \mathfrak{R}_n^i)$
 THEN $(y^i = x^T \pi^i) \quad i=[1, R]$



- Pseudo-linear universal approximators
- Approximate non-linear dynamics
- Multiple operating modes
- Parameter and structure learning
- NN-like computational efficiency (ANFIS)

$$y = \sum_i v_i(x) y^i(x)$$

Prediction of Type 2 (Drastic) Equipment Faults



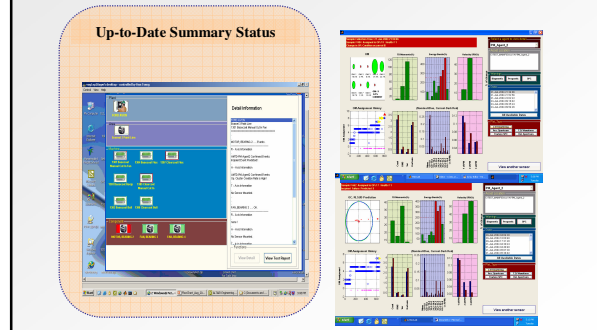
Dramatic increase of the number of created clusters with low health factors

$$(T^*_O(k) - T_O(k))^2 / V^*_O(k) < 9$$

$$T^*_O(k+1) = \alpha T^*_O(k) + (1 - \alpha) T_O(k)$$

$$V^*_O(k+1) = \alpha V^*_O(k) + (1 - \alpha) (T_O(k) - T^*_O(k))^2$$

Summarized Status of Monitored Equipment: A Step Towards Immune Systems



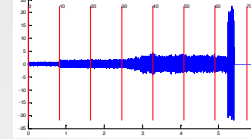
Validation: Accelerated Testing of Bearings under Severe Conditions



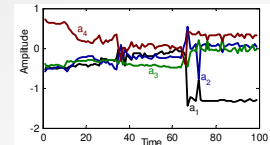
- 6309 DGBB bearing
SKF Condition Local Monitoring Unit (LMU).
- Bearing life
 - Normal Condition: Several years assuming 24 hours/day operation
 - Accelerated testing conditions (increasing load from 2.6 KN to 18 KN)
- Sensor: An accelerometer in the vertical axis
- Sampling: 213 points snapshot at 5 kHz at 15 min interval

Example: Data and Features

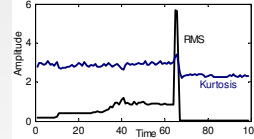
Data: Concatenated signal segments



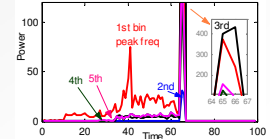
4 coefficients of AR(4) model



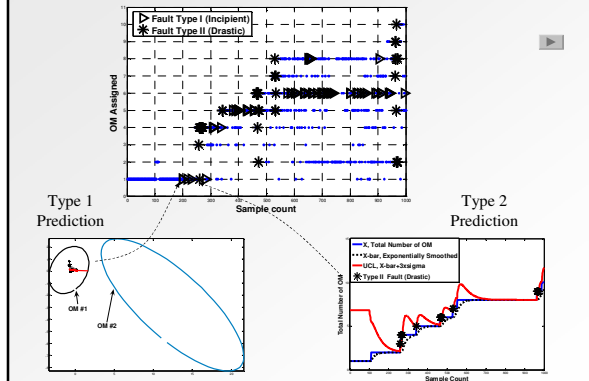
RMS and Kurtosis



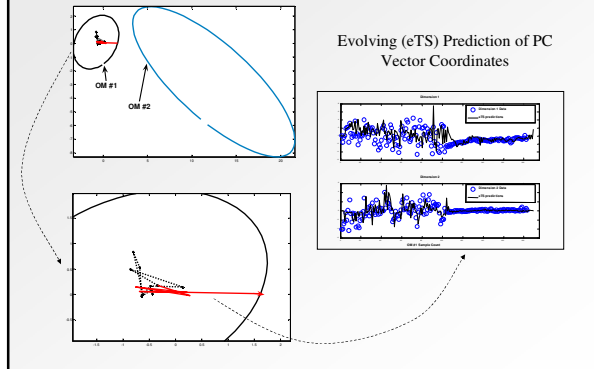
5 power features calculated from FFT



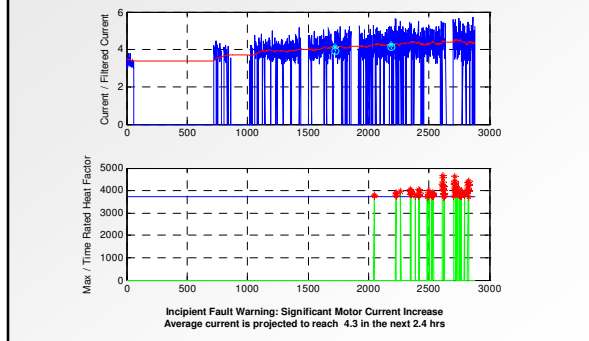
Example: Incipient & Drastic Fault Warnings



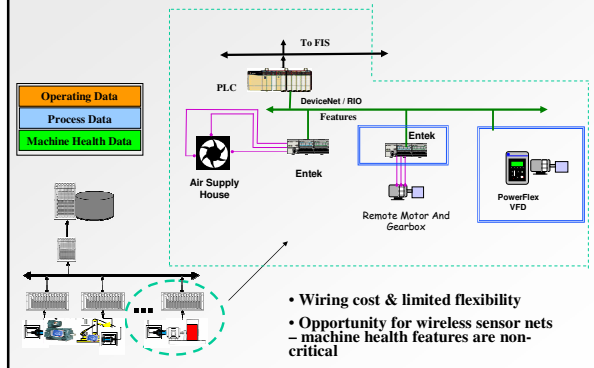
Example: Zoomed-In View of Incipient Fault Prediction

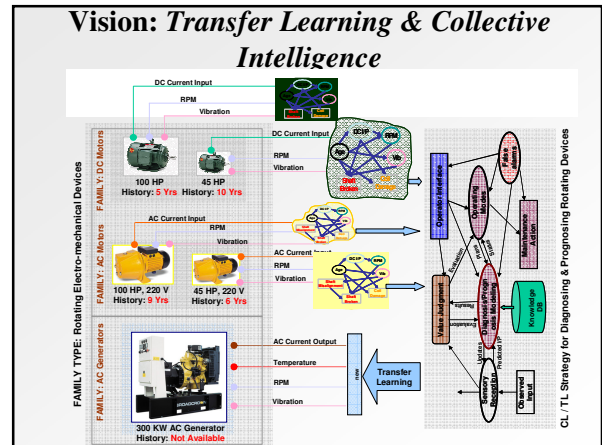
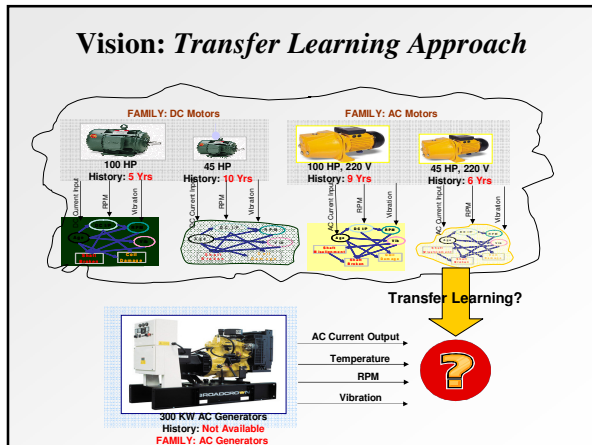
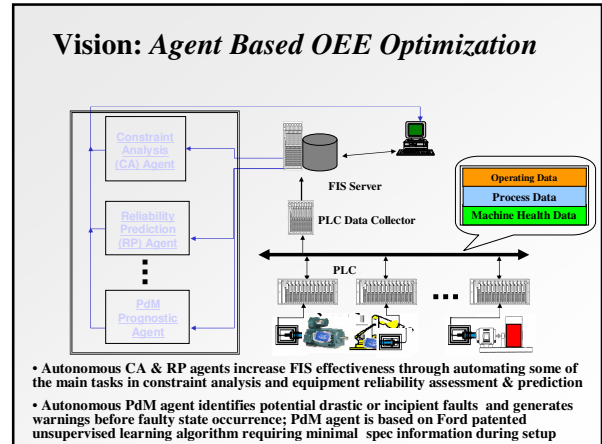
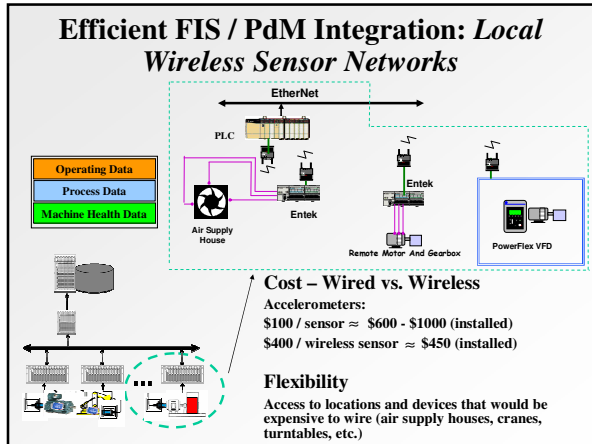


Example: Single Feature (Current Monitoring of VFD Controlled Electric Motors)

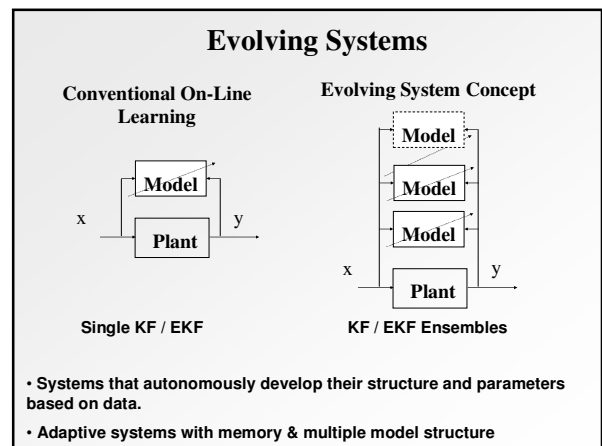


Efficient FIS / PdM Integration: Local Sensor Networks





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Fuzzy Systems That Can Evolve



Takagi-Sugeno (TS) Models

$$\mathcal{R}^i: \text{IF } (x_1 \text{ is } \mathfrak{K}_1^i) \text{ AND } \dots \text{ AND } (x_n \text{ is } \mathfrak{K}_n^i) \\ \text{THEN } y^i = f_i(x) \quad i = [1, R]$$

- Real time parameter and structure learning
- Fuzzily blended local sub-models
- Open multimodel structure
- Universal approximators
- Multiple operating modes
- NN-like computational efficiency

Applications:

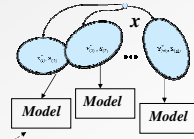
- Soft Sensors
- Prognostics
- Control

Alternative Interpretations of TS

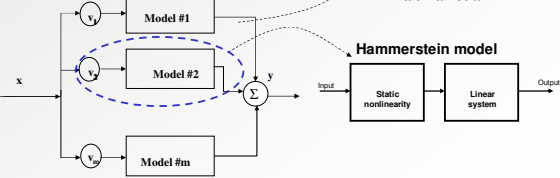
$$\mathcal{R}^i: \text{IF } (x_1 \text{ is } \mathfrak{K}_1^i) \text{ AND } \dots \text{ AND } (x_n \text{ is } \mathfrak{K}_n^i) \\ \text{THEN } (y^i = x_i^T \pi^i) \quad i = [1, R]$$

$$y = \sum_i v_i(x) y^i(x)$$

$$v_i = \frac{\tau^i}{\sum_{j=1}^N \tau^j}$$

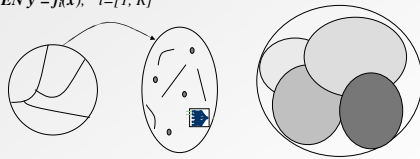


Raibman et al.



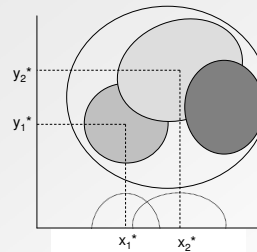
Flexible TS: Geometric Interpretation

$$\mathcal{R}^i: \text{IF } (x_1 \text{ is } \mathfrak{K}_1^i) \text{ AND } \dots \text{ AND } (x_n \text{ is } \mathfrak{K}_n^i) \\ \text{THEN } y^i = f_i(x); \quad i = [1, R]$$



- Fuzzy partitioning of the input space
- Rules are granules (bundles, clusters) in the input-output space
- Fuzzy model structure identification problem can be transformed to clustering (similarly to RBF NN)
- Granular structure implies system decomposition and simpler subsystem models

Evolving Modeling Paradigm



IF x is close to x_1^* THEN y is y_1^*

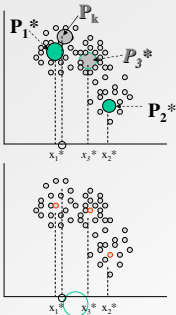
IF x is close to x_2^* THEN y = $a_0 + a_1 x$

- Structure Learning through Incremental Clustering the Input-Output Space (K-NN, SOM, K-NN-M, Evolving Mountain/Subtractive)

- Input-Output Cluster Covariance Defines the Consequent Subsystem Structure

- Parameter Learning (RLS, LMS, BP, EKF, Clustering)

eTS: On-Line Learning Model Structure Using Incremental Mountain Clustering



1. IF the potential of $z(k)$ is significantly different than all cluster centers potentials THEN
IF the new data input $x(k)$ is close to the existing antecedent centers x_i^* THEN $x(k)$ replaces this center x_i^* ELSE a new rule with center x_{M+1}^* is created
2. IF the potential of $z(k)$ is within the range of cluster centers potentials THEN the new data point $z(k)$ is assigned to the closest center and model is updated

eTS: Recursive Least Square (RLS, KF / EKF / EKF+) Model Learning

$$\psi = [\lambda_1 x^T, \lambda_2 x^T, \dots, \lambda_N x^T]^T \quad \theta_k = [\pi_{1(k-1)}^T, \pi_{2(k-1)}^T, \dots, \pi_{R(k-1)}^T, \pi_{(R+1)k}^T]^T$$

- Parameter Learning (No New Rule)

$$y_k = \psi_k^T \theta_{k-1} \quad \theta_k = \theta_{k-1} + C_k \psi_k (y_k - \psi_k^T \theta_{k-1}) \quad \theta_0 = 0$$

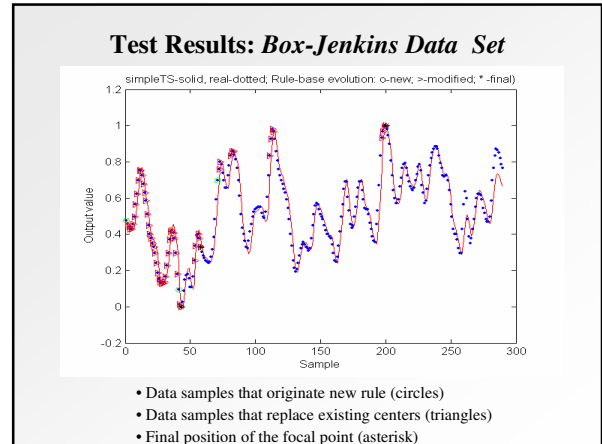
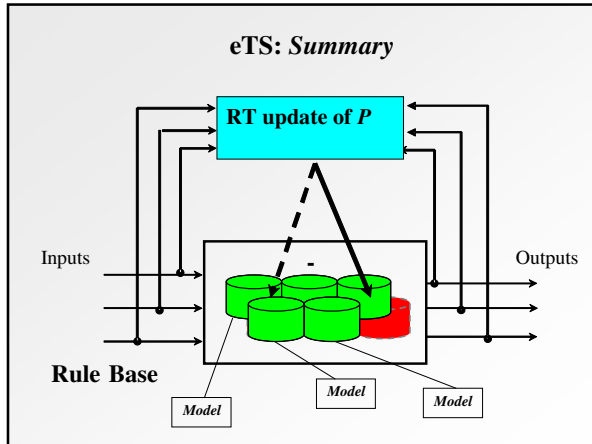
$$C_k = C_{k-1} \frac{C_{k-1} \psi_k \psi_k^T C_{k-1}}{1 + \psi_k^T C_{k-1} \psi_k}; k = [1, TD] \quad C_0 = \Omega I$$

- Structure Learning (New Rule Added):

$$C_k = \begin{bmatrix} \rho \zeta_{11} & \dots & \rho \zeta_{1R(n+1)} & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \rho \zeta_{R(n+1)1} & \dots & \rho \zeta_{R(n+1)R(n+1)} & 0 & \dots & 0 \\ 0 & 0 & 0 & \Omega & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & \dots & \Omega \end{bmatrix}$$

$$\pi_{R+1k} = \frac{1}{\sum_{i=1}^R \frac{1}{1 + (z_k - z_i)^2}} \pi_{Rk-1}$$

$$\rho = \frac{R^2 + 1}{R^2}$$



Real Time Diagnostics, Prognostics, & Process Modeling: *Industrial Perspective*

Drivers

- Advances in information technology, sensor development, and communications
- Network architectures, sensor nets, industrial web
- Exponentially increasing amount of information

Challenges

- Need for real time autonomous algorithms for diagnostics, prognostic, process control & optimization to increase
- Fast, and cost effective development process
- Robustness
- Performance self-assessment capability
- Low cost of ownership and maintenance

Reference Material

Adaptive Control Strategy
On-Line Model Learning

$$y(t) = S(\mathbf{u}(t)) : \quad \begin{aligned} \mathbf{J}(k+1) &= \mathbf{J}(k) + w(k) \\ \delta \mathbf{y}(k) &= \mathbf{J}(k) \delta \mathbf{u}(k) + v(k) \end{aligned}$$

LMS (Widrow-Hoff) Model Learning

$$\mathbf{J}(k) = \mathbf{J}(k-1) + \alpha (\delta \mathbf{y}(k) - \mathbf{J}(k-1) \delta \mathbf{u}(k)) \delta \mathbf{u}^T(k) / (\delta \mathbf{u}^T(k) \delta \mathbf{u}(k))$$

RLS (Kalman Filter) Model Learning

$$\begin{aligned} \mathbf{J}_f^T(k) &= \mathbf{J}_f^T(k-1) + \mathbf{L}_f(k-1) (\delta \mathbf{y}_f(k) - \mathbf{J}_f^T(k-1) \delta \mathbf{u}(k)) \\ \mathbf{L}_f(k-1) &= \mathbf{P}_f(k-1) \delta \mathbf{u}(k) (\mathbf{R}_f + \delta \mathbf{u}^T(k) \mathbf{P}_f(k-1) \delta \mathbf{u}(k))^{-1} \\ \mathbf{P}_f(k-1) &= \mathbf{L}_f(k-1) \delta \mathbf{u}^T(k) \mathbf{P}_f(k-1) + \mathbf{Q}_f \end{aligned}$$

Adaptive Control Strategy
On-Line Control (Constrained Optimization)

$$\min_{\mathbf{u}_{\min} < \mathbf{u} < \mathbf{u}_{\max}} ((\mathbf{y}(k) - \mathbf{y}_d)^T (\mathbf{y}(k) - \mathbf{y}_d))$$

Levenberg-Marquardt (LM) Control Update

$$\begin{aligned} \mathbf{u}(k+1) &= \mathbf{u}(k) + \mathbf{K}(k) (\mathbf{y}_d - \mathbf{y}(k)) \\ \mathbf{K}(k) &= \mathbf{J}^T(k) \mathbf{G} (\rho \mathbf{I} + \mathbf{J}(k) \mathbf{J}^T(k))^{-1} \\ \mathbf{u}(k+1) &= \text{sat}(\mathbf{u}(k) + \mathbf{J}^+(k) (\mathbf{y}_d - \mathbf{y}(k))) \end{aligned}$$

Constrained Optimization Control Update

$$\begin{aligned} \mathbf{u}(k) &= \text{argmin} (\|\mathbf{y}_d - \mathbf{y}(k)\|^2 + \alpha \|\mathbf{u}(k) - \mathbf{u}(k-1)\|^2) \\ \text{s.t.} & \\ \mathbf{y}(k) &= \mathbf{y}(k-1) + \mathbf{J}(\mathbf{u}(k) - \mathbf{u}(k-1)) \end{aligned}$$

Stability of IPQC Control Algorithm

$$\Delta y(k) = J(k) \Delta u(k)$$

$$J_j(k+1) = J_j(k) + w_j(k)$$

$$\Delta y_j(k) = \Delta u^T(k) J_j(k) + v_j(k)$$

IPQC Control Laws:

- Levenberg-Marquardt Control Update
- LQR Control Update
- Constraint Optimization Control Update

• LQR Control Update

$$u(k+1) = u(k) + K(k) (y_d - y(k))$$

$$K(k) = (\hat{J}^T(k) \bar{P} \hat{J}(k) + \bar{R})^{-1} \hat{J}^T(k) \bar{P}$$

$$y(k+1) = y(k) + \hat{J}(k) \Delta u(k+1)$$

$$J = \sum_{j=1}^m (y_d - y(k))^T \bar{Q} (y_d - y(k)) + \Delta u^T(k+1) \bar{R} \Delta u(k+1)$$

$$\bar{Q} = \bar{P} \hat{J} / (\hat{J}^T \bar{P} \hat{J} + \bar{R})^{-1} \hat{J}^T \bar{P}$$

• Closed Loop Dynamics: LQR Control Update

$$y(k+1) = y(k) + J(k+1) \Delta u(k+1)$$

$$= (I - J(k+1)(\hat{J}^T(k) \bar{P} \hat{J}(k) + \bar{R})^{-1} \hat{J}^T(k) \bar{P}) y(k) + (\hat{J}^T(k) \bar{P} \hat{J}(k) + \bar{R})^{-1} \hat{J}^T(k) \bar{P} y_d$$

$$A = (I - J(k+1)(\hat{J}^T(k) \bar{P} \hat{J}(k) + \bar{R})^{-1} \hat{J}^T(k) \bar{P})$$

Stability Condition

• Closed Loop Dynamics: LM Control Update

$$A = I - J(k+1) \hat{J}^T(k) G (\rho I + \hat{J}(k) \hat{J}^T(k))^{-1}$$

• Closed Loop Dynamics: LQR Control Update

$$A = (I - J(k+1)(\hat{J}^T(k) \bar{P} \hat{J}(k) + \bar{R})^{-1} \hat{J}^T(k) \bar{P})$$

Stability Condition

$$J(k+1) \approx \hat{J}(k) \Rightarrow A^T(k) I A(k) < I$$

Intelligent Control Strategy

The Rule-Base of Initial Conditions

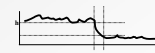
IF $x(k)$ is close to x_i^*

THEN $\tilde{J}(k)$ is J_i^* ; $\tilde{u} = u_i^*$

$$\tilde{J}(k) = \sum_{i=1}^m \tau_i J_i^* / \left(\sum_{i=1}^m \tau_i \right);$$

$$\tilde{u}(k) = \sum_{i=1}^m \tau_i u_i^* / \left(\sum_{i=1}^m \tau_i \right)$$

$$E(k) = \sum_{j=k+1}^{\infty} (y(j) - y_d)^T (y(j) - y_d)$$



Update the RBIC with the current operating point & model



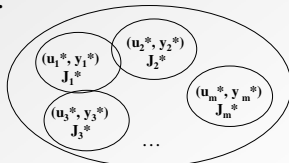
Infer a new initial condition from the RBIC

Synergy Between Conventional & Intelligent Control

The Rule-Base of Initial Conditions (RBIC)

IF $x(k)$ is close to x_i^* THEN $\tilde{J}(k)$ is J_i^* and \tilde{u} is u_i^*

...



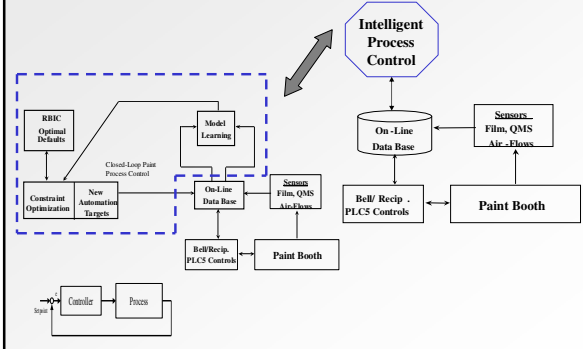
$x(k) = (u^T(k), y^T(k))$ current operating point

$x_i^* = (u_i^{*T}, y_i^{*T})^T$ prototypical operating point

$t_i = \exp(-(\mathbf{x}(k) - \mathbf{x}_i^*)^T (\mathbf{x}(k) - \mathbf{x}_i^*) / \sigma^2)$, $i=[1, m]$

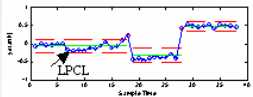
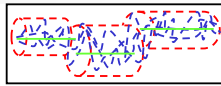
Implementation of Intelligent Process Control Systems

Supervisory Control Architecture: Integration with Legacy Controls



Novelty Detection: Inspired from Process Monitoring

- Statistical Process Control Charts
 - Statistical process control (SPC) involves using statistical techniques to measure and analyze the variation in processes
- Pattern Recognition Approach
 - Conventional classification methods require enough examples for all classes; problem – limited fault data
- Novelty detection is the process of learning the *normality* of a system by fitting a model to the set of normal examples





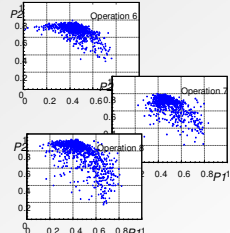
$$|y - y_i^*| < 3 \sigma_{y_i}$$

$$(\underline{y} - \underline{y}_i^*)^T S_{y_i}^{-1} (\underline{y} - \underline{y}_i^*) < (1/n) \chi^2_{p, \alpha}$$

Novelty Detection Paradigm: Main Problem #1

The Problem: How to recognize (and compare numerically) the **difference** between the different machine operations?
 Once this problem is solved, we would be able to discover the slow (often: invisible) tendency of **performance deterioration** of the machine/system.

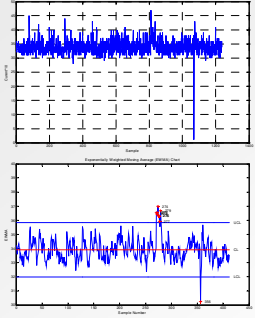




Code	Label	OperationDescription
1	F1	Fuel Spray Nozzle deactivated
2	F2	TurboCharger Deterioration
3	F3	Valve Clearance Changed
4	F4	Air Filter Obstruction (High)
5	F5	Air Filter Obstruction (Low)
6	NH	Normal Operation (High Load)
7	NM	Normal Operation (Medium L.)
8	NL	Normal Operation (Light Load)

Novelty Detection Paradigm: Main Problem #2

Example: VFD Control of Electric Motors



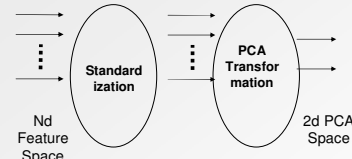
VFD Control:
 $\Phi = V / f = \text{const}$

Major Reasons for High Current:

- Load increase
- Voltage drop
- Friction increase (jam, brake, gearbox / bearing failures)
- Insulation degradation

VFD Time Rated Heat Factor Window:
 $HF = P^2 T = (1.5 I_n)^2 180$
HF Range:
 $I = [102\% - 250\% I_n]; T = [1000s - 40s]$

Preprocessing & Transformation of Feature Space



Recursive Standardization

$$Y(k) = (Y^R(k) - \bar{Y}) (\text{diag}(\bar{S}))^{-0.5}$$

$$\bar{Y}(k+1) = \alpha \bar{Y}(k) + (1 - \alpha) Y^R(k+1)$$

$$\bar{S}(k+1) = \alpha \bar{S}(k) + (1 - \alpha) (Y^R(k+1) - \bar{Y}(k+1))' (Y^R(k+1) - \bar{Y}(k+1))$$

Recursive PCA Transformation

$$S(k+1) = \alpha S(k) + (1 - \alpha) Y(k+1)' Y(k+1)$$

$$S = T * S_0 * T^T$$

$$y(k) = Y(k) * T^T$$

Constraint Analysis and Reliability Prediction Agents

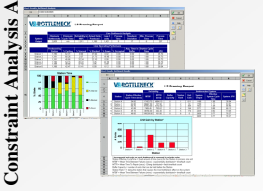
Cycle Time
Down Time

Block Time
Starve Time

FIS Database


Constraint Analysis Agent

- Update MTBF / MTBR Distributions
- Identify Bottleneck Constraints




Reliability Prediction Agent

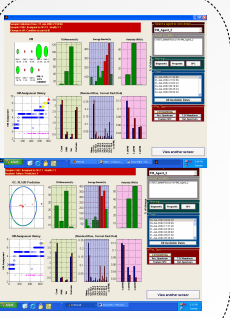
- Update Survival Function Models
- Predict Asset Fault Probabilities



Predicted Fault Warning is Combined with a Summarized Status of Energy Features

Up-to-Date Summary Status





EKF* Learning of Nonlinear Systems

Consider a static nonlinear system with unknown parameters:

$$d_n = h(x_n)$$

If the analytical form (not the parameters) of the nonlinear input/output mapping h is known then assuming slow changing parameters the learning problem can be viewed as estimation of the state of the dynamic nonlinear system:

$$\begin{aligned} x_{n+1} &= x_n \\ d_n &= h(x_n) \end{aligned}$$

The result of this assumption is the EKF learning rule:

$$\begin{aligned} K_n &= P_n H_n (R_n + H_n^T P_n H_n)^{-1} \\ \hat{x}_n &= \hat{x}_{n-1} + K_n [d_n - h(\hat{x}_{n-1})] \\ P_{n+1} &= P_n - K_n H_n^T P_n + Q_n \end{aligned}$$

where $H_n^T = \left. \frac{\partial h(x)}{\partial x} \right|_{x=\hat{x}_n}$

In a more realistic setting the nonlinear input/output mapping h is not known. In this case we can use another Kalman filter (conventional) to learn the Jacobian H of the unknown mapping h :

$$H_n^T = \left. \frac{\partial h(x)}{\partial x} \right|_{x=\hat{x}_n}$$

To learn H we assume a linear system with a state vector formed by the elements of H :

$$\begin{aligned} H(k+1) &= H(k) + w(k) \\ \delta d(k) &= H(k) \delta h(k) + v(k) \end{aligned}$$

Then the conventional (linear) Kalman filter:

$$\begin{aligned} H(k) &= H(k-1) + L(k-1) (\delta d(k) - H(k-1) \delta \hat{x}(k)) \\ L(k-1) &= S(k-1) \delta \hat{x}(k) (R + \delta \hat{x}(k) S(k-1) \delta \hat{x}(k))^{-1} \\ S(k-1) &= L(k-1) \delta \hat{x}^T(k) S(k-1) \end{aligned}$$

Provides the real time estimation of H that is substituted in the EKF learning rule.