Formation Control and Vision Based Localization of a System of Mobile Robots

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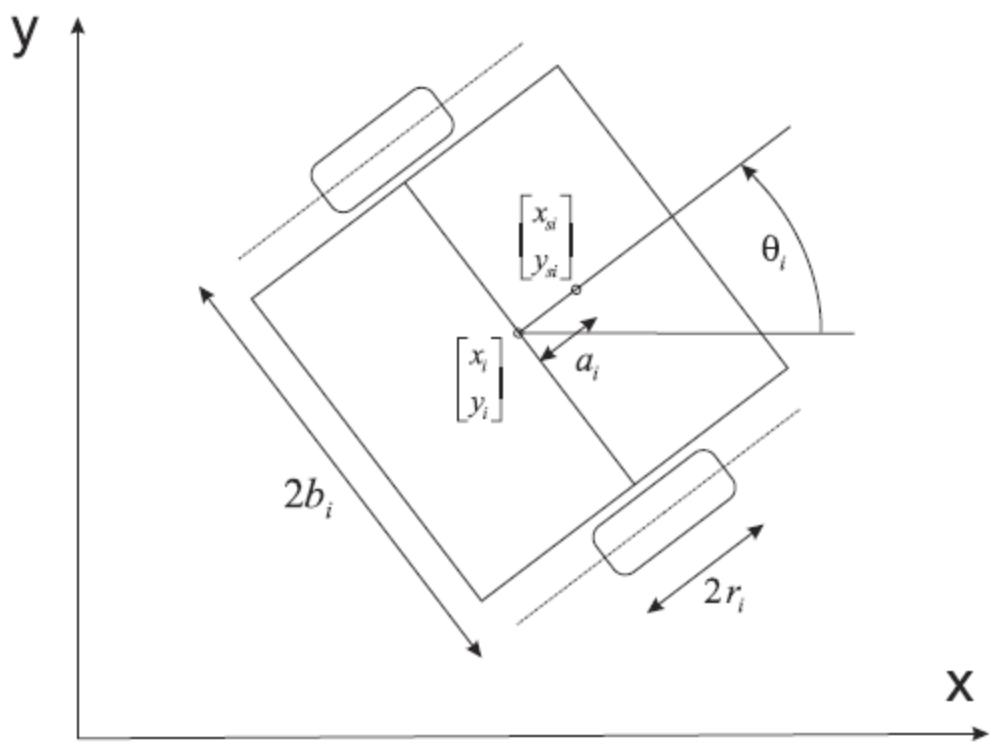
Outline

- Model of the system
- Control algorithms
  - tracking controllers
  - collision avoidance
- Robots and localization system (hardware)
- Simulations and experiments

Model of the system



i=1,...,M, M - number of differentially driven mobile robots

Kinematics of the system:

$$\begin{bmatrix} \dot{x}_i \\ \dot{y}_i \\ \dot{\theta}_i \end{bmatrix} = \begin{bmatrix} \cos\theta_i & 0 \\ \sin\theta_i & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_{vi} \\ u_{\omega i} \end{bmatrix}$$

Dynamics of the system:

$$\boldsymbol{M}_i\dot{\boldsymbol{\omega}}_{wi} + \mathrm{C}_i(\dot{\boldsymbol{q}}_i)\boldsymbol{\omega}_{wi} + \boldsymbol{D}_i\boldsymbol{\omega}_{wi} + \boldsymbol{G}_i(\boldsymbol{q}_i) = \boldsymbol{\tau}_i$$

Do algorithm

Characteristics of the method:
- takes into account the dynamics of robots,
- assumes that the task space free of other obstacles,
- robots follow desired trajectories,
- the shape of the robot is approximated with the circle.

Reference position of the *i*-th robot is sum of common position for the formation desired trajectory and the offset for the *i*-th robot:

$$\boldsymbol{q}_{di} = \begin{bmatrix} x_{di}(s) \\ y_{di}(s) \end{bmatrix} = \boldsymbol{q}_d + \begin{bmatrix} l_{xi} \\ l_{yi} \end{bmatrix}$$

Reference orientation:

$$\theta_{di} = \arctan\left(\frac{y'_{di}}{x'_{di}}\right)$$

Auxiliary variables are introduced:

$$\theta_{ie} = \theta_i - \alpha_{\theta i} \qquad v_{ie} = v_i - \alpha_{vi}$$

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Do algorithm

Virtual controls: $\alpha_{\theta i} = \theta_{di} + \arctan\left(\frac{\zeta_1}{\zeta_2}\right)$

$$\alpha_{vi} = \cos(\alpha_{\theta i})\left(-k_0 u_{di}^2 \psi(\Omega_{xi}) + \cos(\theta_{di}) u_{di}\right)$$

$$+ \sin(\alpha_{\theta i})\left(-k_0 u_{di}^2 \psi(\Omega_{yi}) + \sin(\theta_{di}) u_{di}\right)$$

where $\zeta_1 = -k_0 u_{di}\left(-\psi(\Omega_{xi})\sin(\theta_{di}) + \psi(\Omega_{yi})\cos(\theta_{di})\right)$

$$\zeta_2 = -k_0 u_{di}\left(\psi(\Omega_{xi})\cos(\theta_{di}) + \psi(\Omega_{yi})\sin(\theta_{di})\right) + 1$$

$$u_{di} = \dot{s}\sqrt{(x'_{di})^2 + (y'_{di})^2},$$

The composition of the position errors and collision avoidance term is:

$$\boldsymbol{\Omega}_i = \begin{bmatrix} \Omega_{xi} \\ \Omega_{yi} \end{bmatrix} = \boldsymbol{q}_i - \boldsymbol{q}_{di} + \sum_{j=1, j\neq i}^{N} \beta'_{ij} \boldsymbol{q}_{ij}$$

Where β'_{ij} is the partial derivative of the artificial potential.

Do algorithm

Control law for the i-th robot is as follows:

$$\boldsymbol{\tau}_i = \left(\bar{\boldsymbol{B}}_i\right)^{-1}\left(-\boldsymbol{L}_i\boldsymbol{\varpi}_{ie} - \boldsymbol{\Phi}_i\boldsymbol{\Theta}_i - \left[\begin{array}{cc}\chi_i & \theta_{ie}\end{array}\right]^T\right)$$

where $\chi_i = \boldsymbol{\Omega}_i^T - \theta_{ie}\frac{\partial\alpha_{\theta i}}{\partial\boldsymbol{q}_i} - \sum_{j=1, j\neq i}^{N}\left(\frac{\partial\alpha_{\theta i}}{\partial\boldsymbol{q}_{ij}}\theta_{ie} - \frac{\partial\alpha_{\theta i}}{\partial\boldsymbol{q}_{ji}}\theta_{je}\right)\bar{\boldsymbol{\Delta}}_{2i}$

$\boldsymbol{L}_i$ - matrix of coefficients,

$\boldsymbol{\Phi}_i\boldsymbol{\Theta}_i$ - adaptive component of the control.

$$\bar{\boldsymbol{\Delta}}_{2i} = \left[\begin{array}{c}\cos\theta_i \\ \sin\theta_i\end{array}\right] \quad \boldsymbol{\Delta}_{1i} = \left[\begin{array}{c}(\cos(\theta_{ie})-1)\cos(\alpha_{\theta i}) - \sin(\theta_{ie})\sin(\alpha_{\theta i}) \\ \sin(\theta_{ie})\cos(\alpha_{\theta i}) + (\cos(\theta_{ie})-1)\sin(\alpha_{\theta i})\end{array}\right]\alpha_{vi}$$

$$\alpha_{\omega_i} = -k\theta_{ie} - \frac{\boldsymbol{\Omega}_i^T\boldsymbol{\Delta}_{1i}}{\theta_{ie}} + \frac{\partial\alpha_{\theta i}}{\partial\boldsymbol{q}_{di}}\dot{q}_{od} + \frac{\partial\alpha_{\theta i}}{\partial\theta_{di}}\dot{\theta}_{od} + \frac{\partial\alpha_{\theta i}}{\partial u_{di}}\dot{u}_{od} + \frac{\partial\alpha_{\theta i}}{\partial\boldsymbol{q}_i}(\boldsymbol{u}_i + \boldsymbol{\Delta}_{1i})$$

$$+ \sum_{j=1, j\neq i}^{N}\frac{\partial\alpha_{\theta i}}{\partial\boldsymbol{q}_{ij}}\left(\boldsymbol{u}_i + \boldsymbol{\Delta}_{1i} - (\boldsymbol{u}_j + \boldsymbol{\Delta}_{1j})\right)$$

$$\boldsymbol{u}_i = -k_0 u_{di}^2\left[\begin{array}{c}\psi(\Omega_{ix}) \\ \psi(\Omega_{iy})\end{array}\right] + \dot{\boldsymbol{q}}_{di}$$

VFO (Vector Field Orientation)

Position error:

$$\mathbf{e}_i = \begin{bmatrix} e_{xi} \\ e_{yi} \end{bmatrix} = \mathbf{q}_{di} - \mathbf{q}_i$$

Auxiliary orientation error:

$$e_{ai} = \theta_{ai} - \theta_i.$$

Reference orientation:

$$\theta_{di} = atan2c(\dot{y}_{di}, \dot{x}_{di})$$

Auxiliary orientation variable:

$$\theta_{ai} = atan2c(h_{yi}, h_{xi}).$$

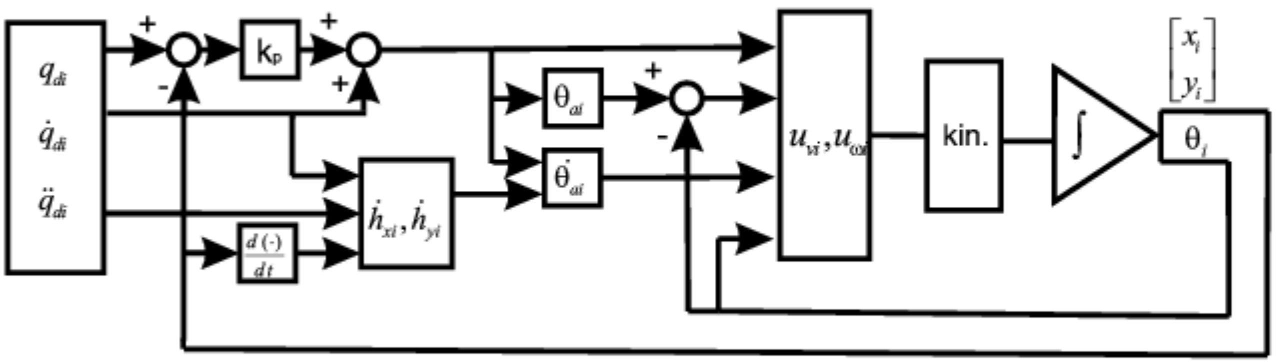
Convergence vector:

$$\mathbf{h}_i = \begin{bmatrix} h_{xi} \\ h_{yi} \\ h_{\theta i} \end{bmatrix} = \begin{bmatrix} k_p e_{xi} + \dot{x}_{di} \\ k_p e_{yi} + \dot{y}_{di} \\ k_\theta e_{ai} + \dot{\theta}_{ai} \end{bmatrix}$$

Control law:

$$u_{vi} = h_{xi} \cos\theta_i + h_{yi} \sin\theta_i$$
$$u_{\omega i} = h_{\theta i}$$

Virtual field orientation (VFO)



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$$u_{vi} = h_{xi} \cos\theta_i + h_{yi} \sin\theta_i$$
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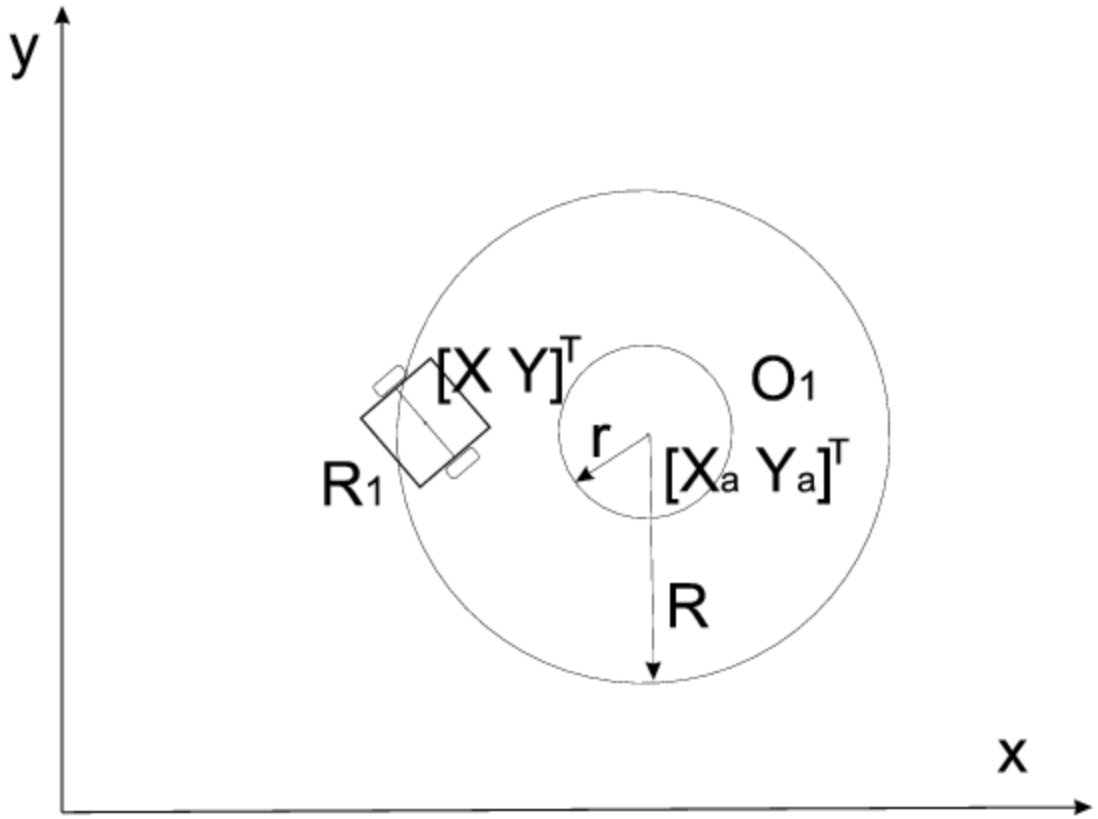
Collision avoidance



(X,Y) – coordinates of the robot R_1,

(X_a, Y_a) – coordinates of the O_1 obstacles center,

r – radius of the collision area,

R – radius of the artificial potential field.

Collision avoidance

The following sets of coordinates are introduced

Collision area:

$$\Delta = \left\{ \begin{bmatrix} x & y \end{bmatrix} : (x,y) \in \mathbb{R}^2, \left\| \begin{bmatrix} x & y \end{bmatrix}^T - \begin{bmatrix} x_a & y_a \end{bmatrix}^T \right\| \leqslant r \right\}$$

Interaction area:

$$\Gamma = \left\{ \begin{bmatrix} x & y \end{bmatrix} : (x,y) \notin \Delta,\ r < \left\| \begin{bmatrix} x & y \end{bmatrix}^T - \begin{bmatrix} x_a & y_a \end{bmatrix}^T \right\| < R \right\}$$

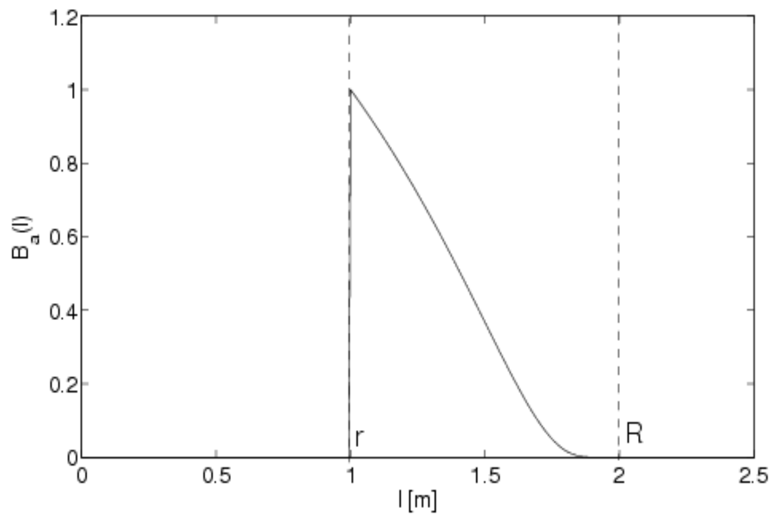
Set that include both above sets:

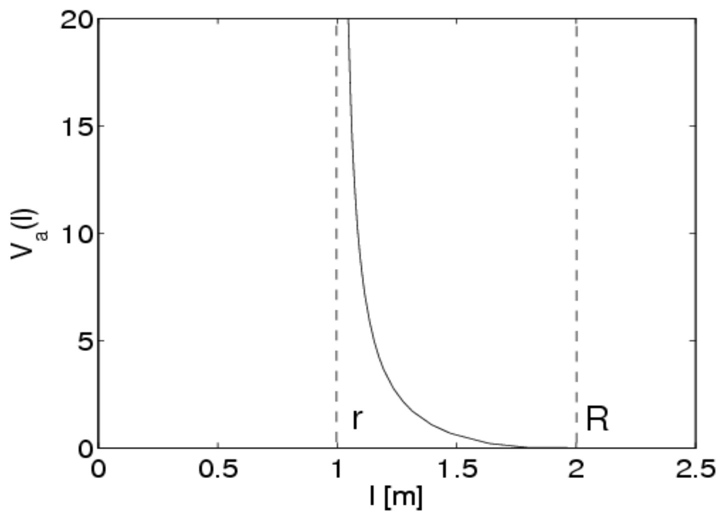
$$D = \Delta \cup \Gamma$$

Collision avoidance – artificial potential field (APF)

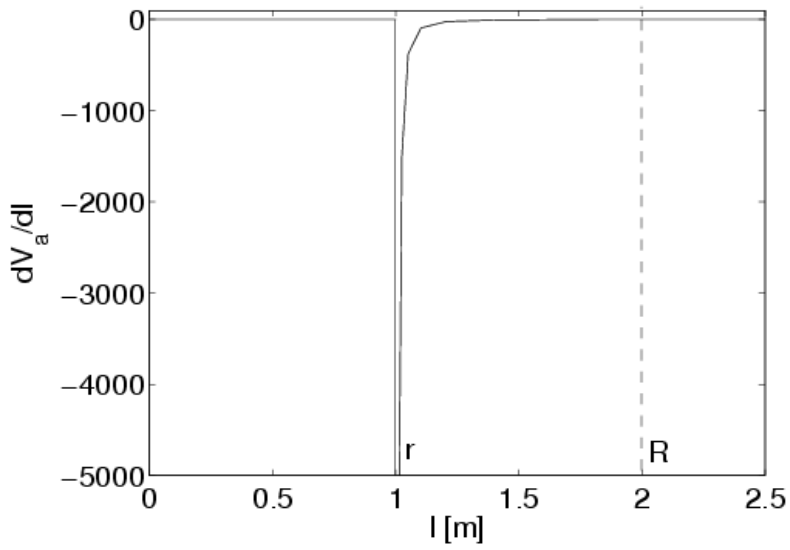
$$B_a(l) = \begin{cases} 0 & dla \quad l < r \\ e^{\frac{l-r}{l-R}} & dla \quad r \le l < R \\ 0 & dla \quad l \ge R \end{cases}$$

$$V_a(l) = \frac{B_a(l)}{1 - B_a(l)}$$





Collision avoidance – AFP



$$\frac{\partial V_a(l)}{\partial l} = \frac{(r-R)}{(l-R)^2\left(e^{\frac{l-r}{l-R}}-1\right)^2} e^{\frac{l-r}{l-R}}$$

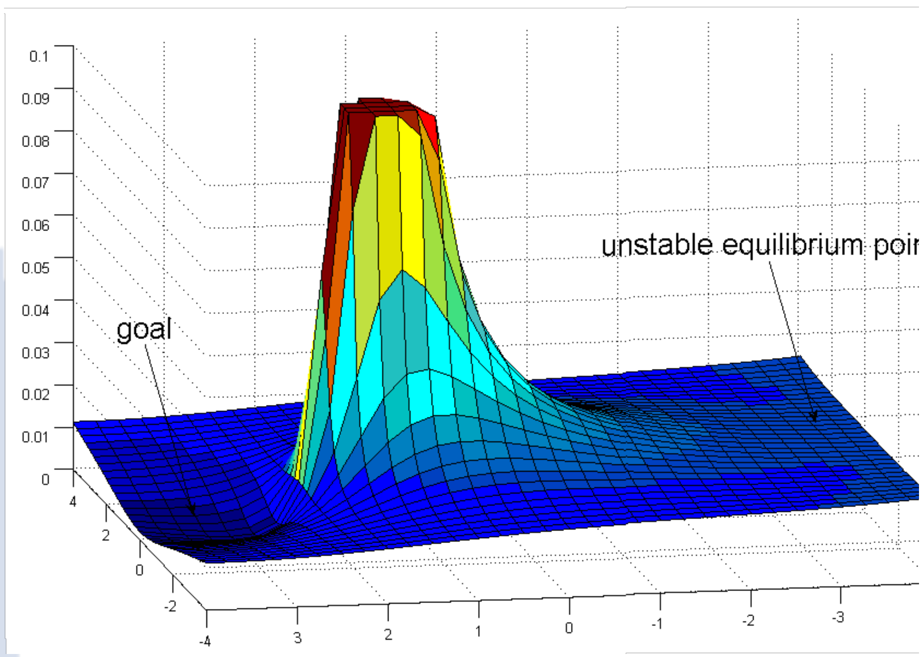
$$\frac{\partial V_a^2(l)}{\partial l^2} = \frac{(R-r)\left(r + re^{\frac{l-r}{l-R}} - 2l + 2le^{\frac{l-r}{l-R}} + R - 3Re^{\frac{l-r}{l-R}}\right)}{(l-R)^4\left(e^{\frac{l-r}{l-R}}-1\right)^3} e^{\frac{l-r}{l-R}}$$

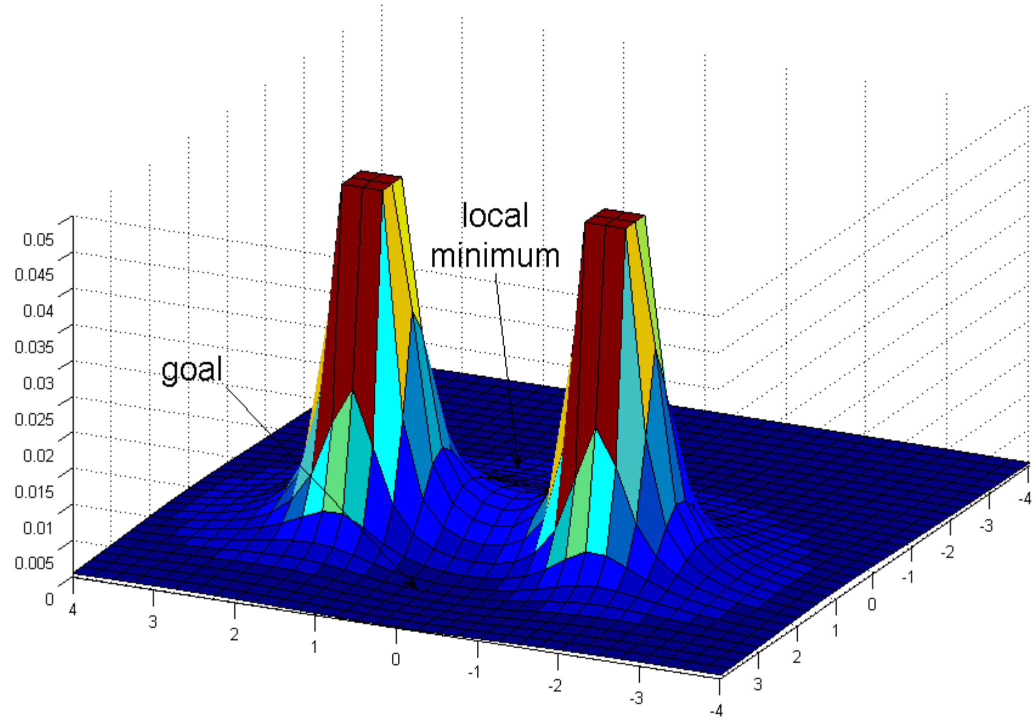
$$\frac{\partial V_a^3(l)}{\partial l^3} = \frac{(R-r)}{(l-R)^6\left(e^{\frac{l-r}{l-R}}-1\right)^4} e^{\frac{l-r}{l-R}}(-24lRe^{\frac{l-r}{l-R}} + 8Rre^{\frac{l-r}{l-R}} - 6lre^{2\,\frac{l-r}{l-R}}$$

$$+18lRe^{2\,\frac{l-r}{l-R}} + 8Rre^{2\,\frac{l-r}{l-R}} - 6l^2e^{2\,\frac{l-r}{l-R}} - r^2e^{2\,\frac{l-r}{l-R}} - 13R^2e^{2\,\frac{l-r}{l-R}} + 6rl$$

$$-r^2 - R^2 - 6l^2 + 6lR + 8R^2e^{\frac{l-r}{l-R}} - 4r^2e^{\frac{l-r}{l-R}} + 12e^{\frac{l-r}{l-R}}l^2 - 4Rr)$$

Saddle points and local minima





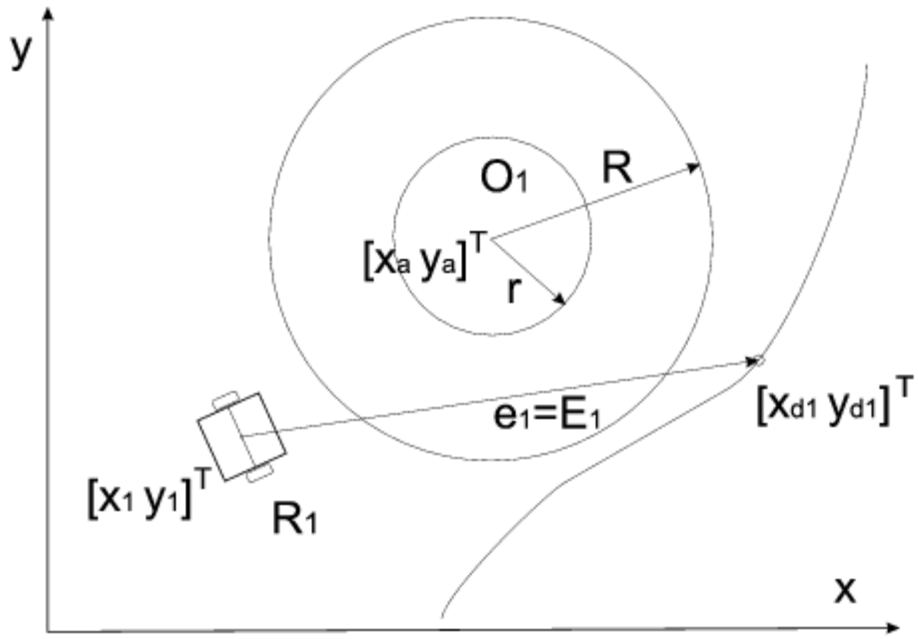
Saddle point – unstable equilibrium point

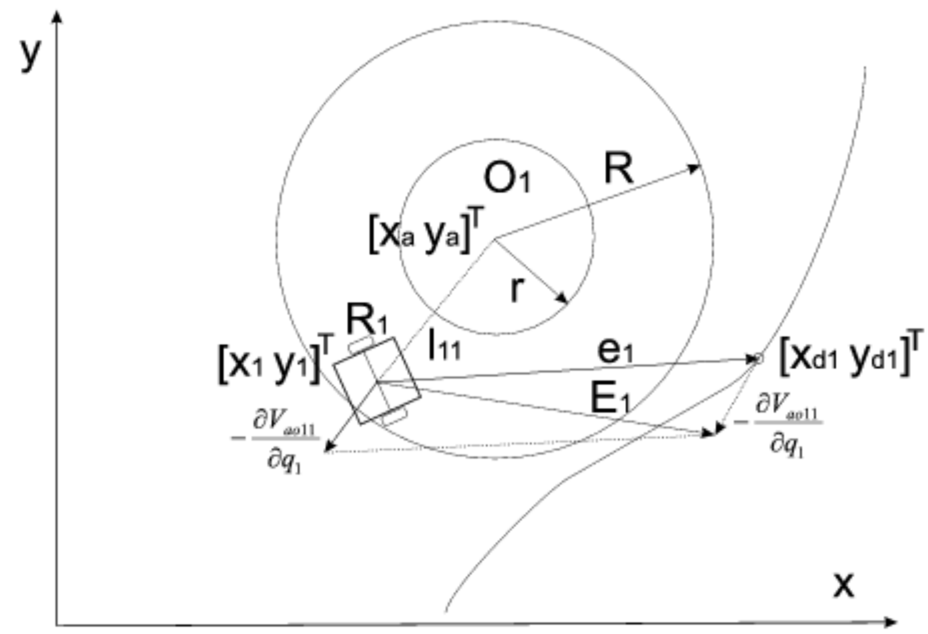
Local minimum – in case of overlapping APFs

Collision avoidance - modified VFO method

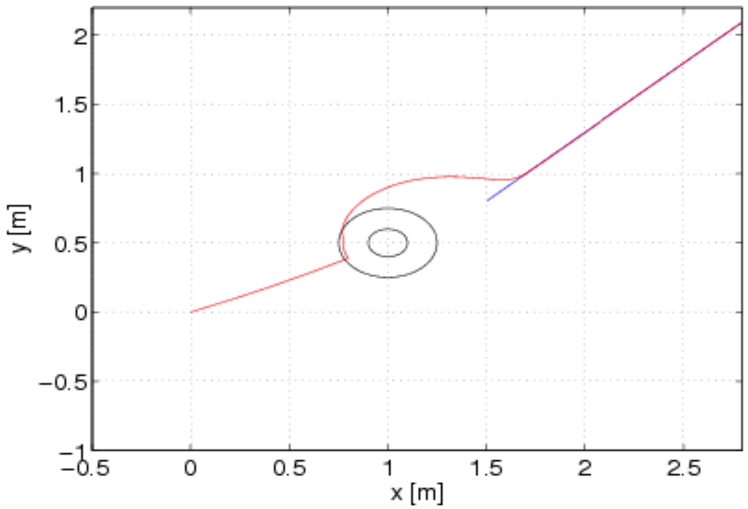
$$\mathbf{h}_i = \begin{bmatrix} h_{xi} \\ h_{yi} \\ h_{\theta i} \end{bmatrix} = \begin{bmatrix} k_p E_{xi} + \dot{x}_{di} \\ k_p E_{yi} + \dot{y}_{di} \\ k_\theta e_{ai} + \dot{\theta}_{ai} \end{bmatrix}$$

$$\mathbf{E}_i = \begin{bmatrix} E_{xi} \\ E_{yi} \end{bmatrix} = \mathbf{e}_i - \sum_{j=1, j\neq i}^{N} \left[\frac{\partial V_{arij}(l_{ij})}{\partial \mathbf{q}_i}\right]^T - \sum_{k=1}^{M} \left[\frac{\partial V_{aoik}(l_{ik})}{\partial \mathbf{q}_i}\right]^T$$

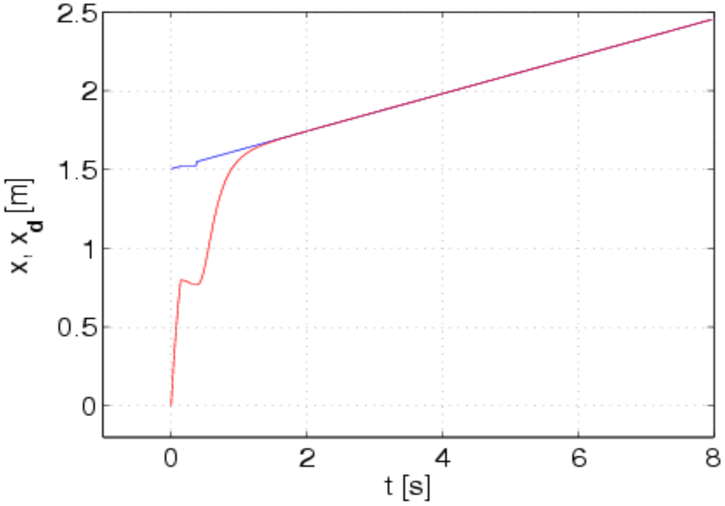




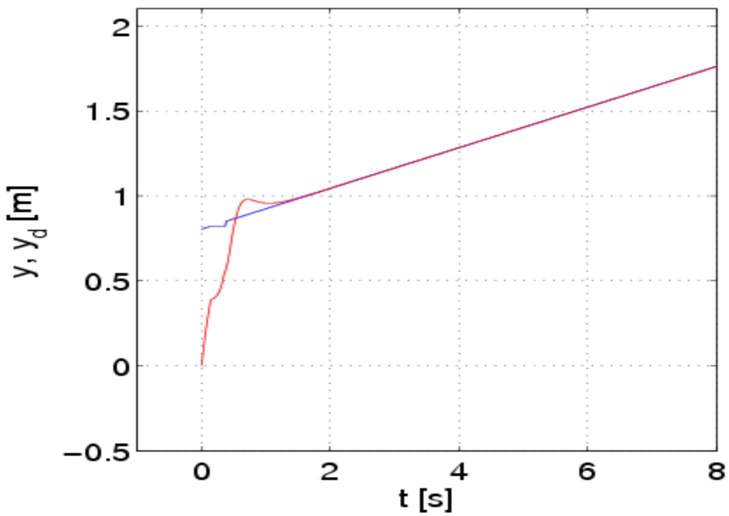
Simulation results



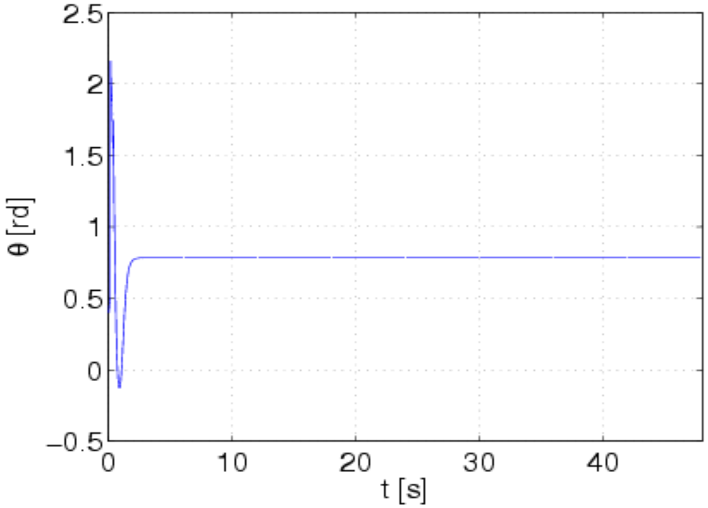
Position on (X,Y) plane



Time graph of X's variable

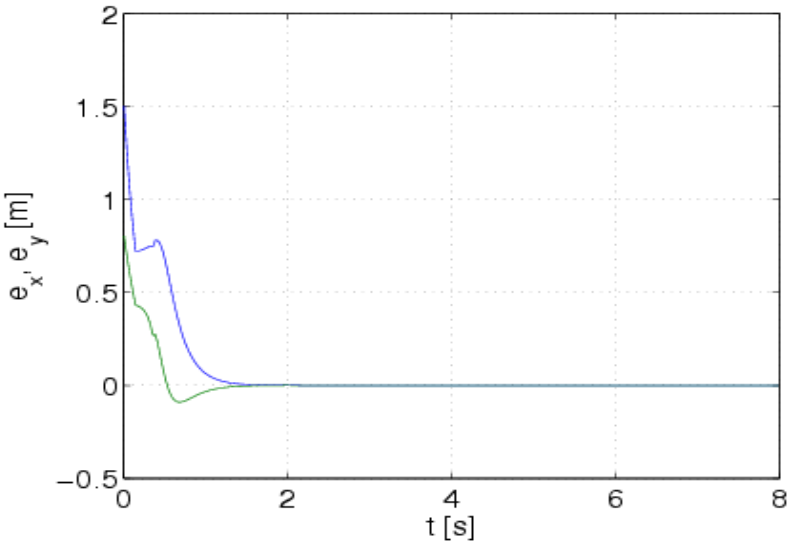


Time graph of Y's variable

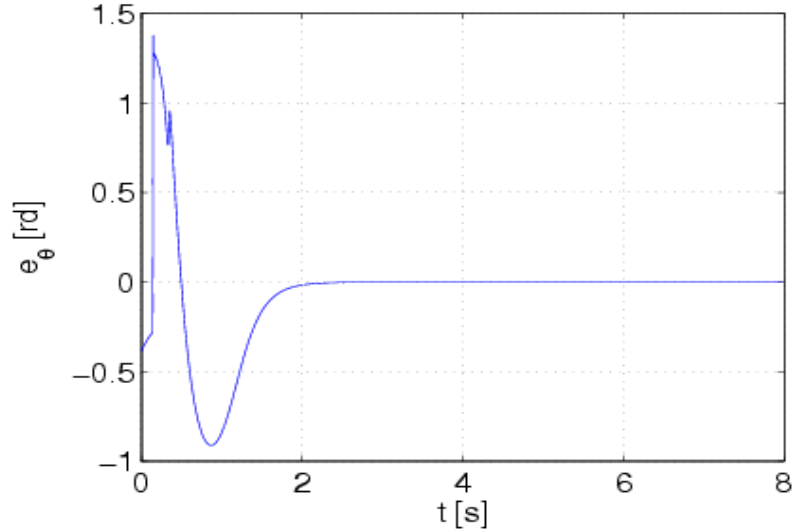


Time graph of orientation variable

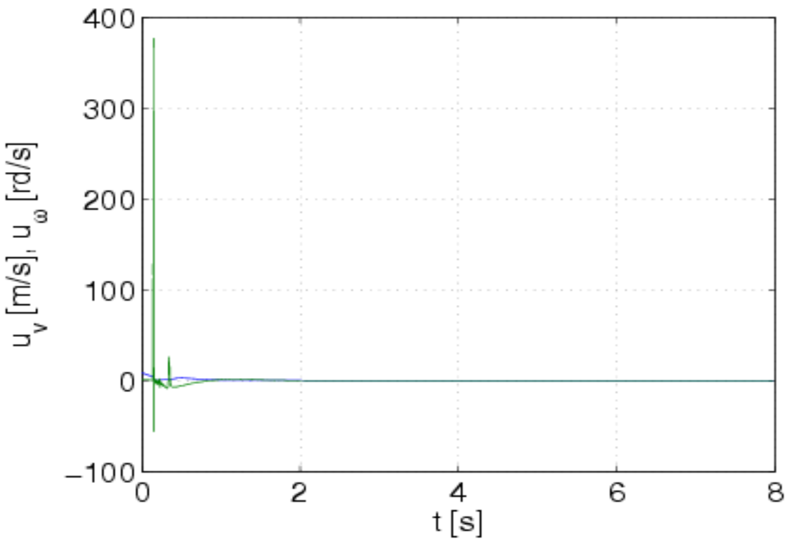
Simulation results (cont.)



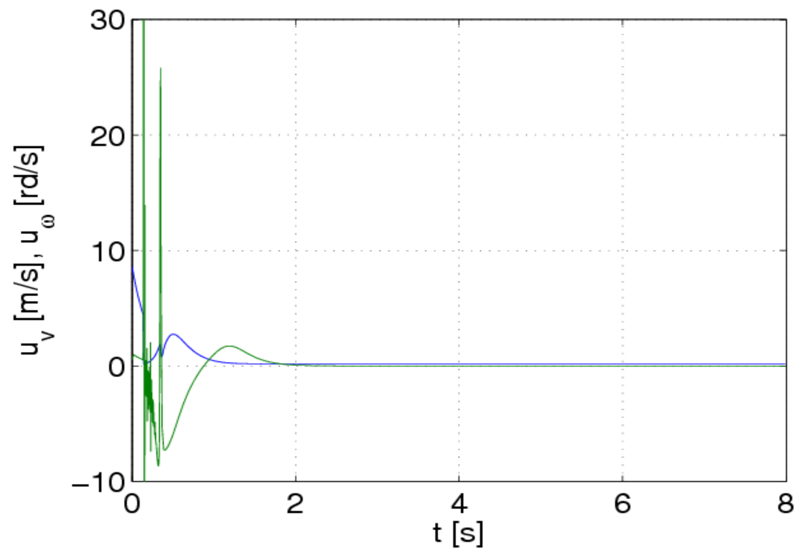
Time graph of position errors



Time graph of orientation errors

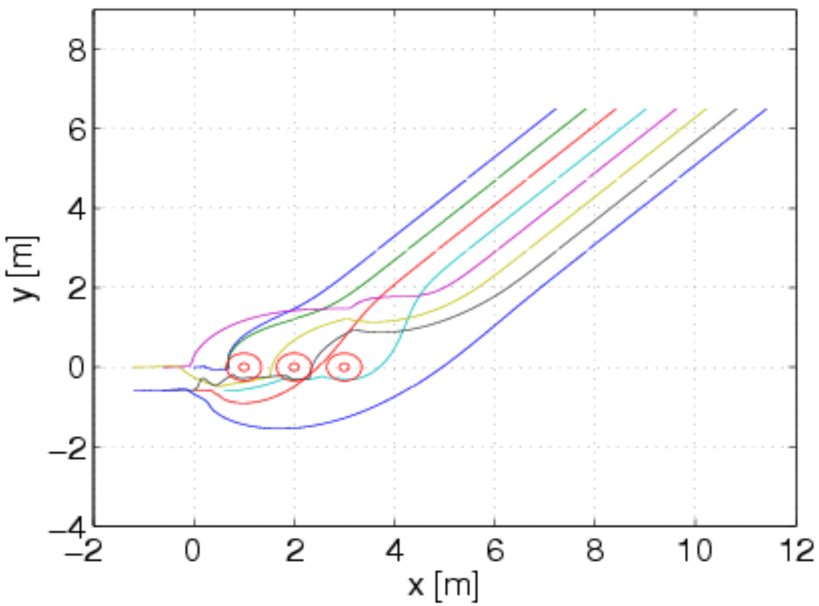


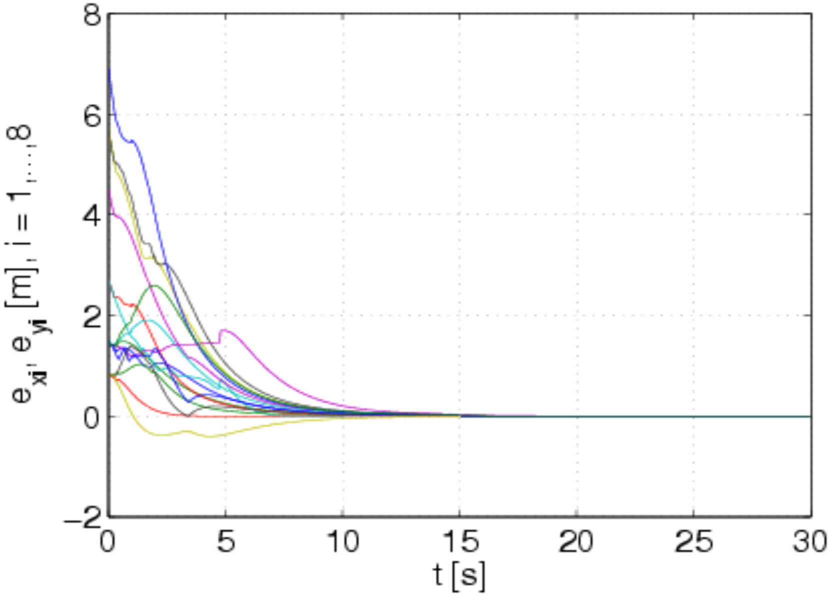
Time graph of velocities

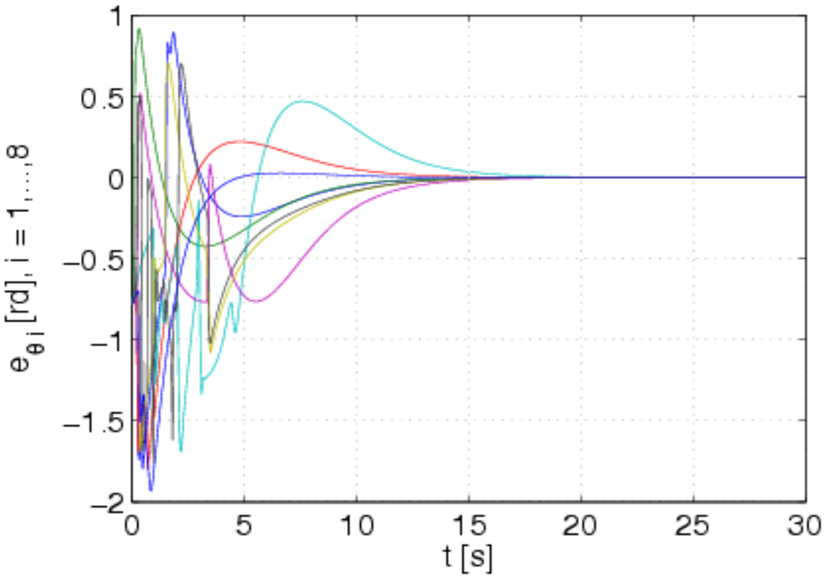


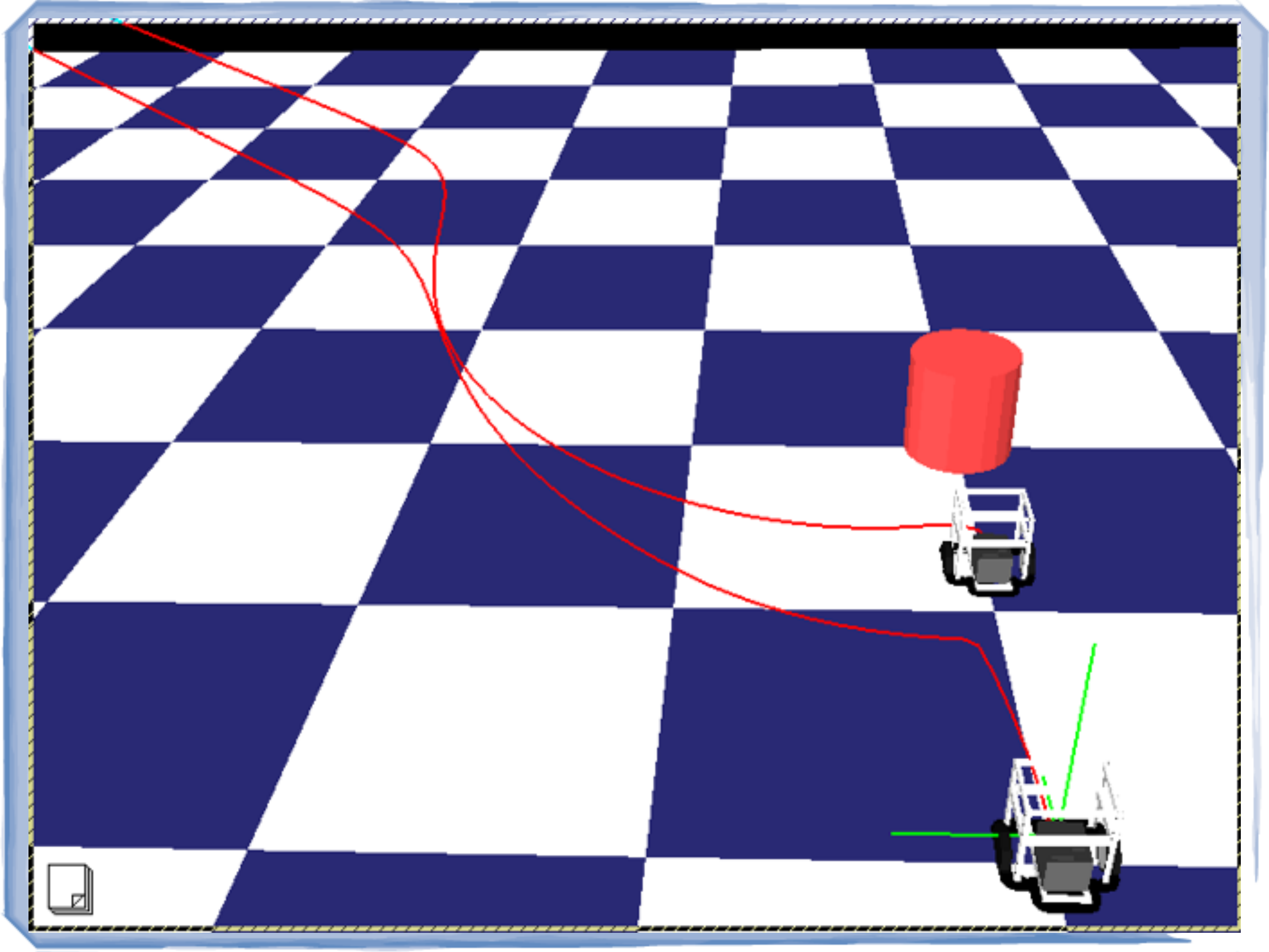
Time graph of velocities - scaled

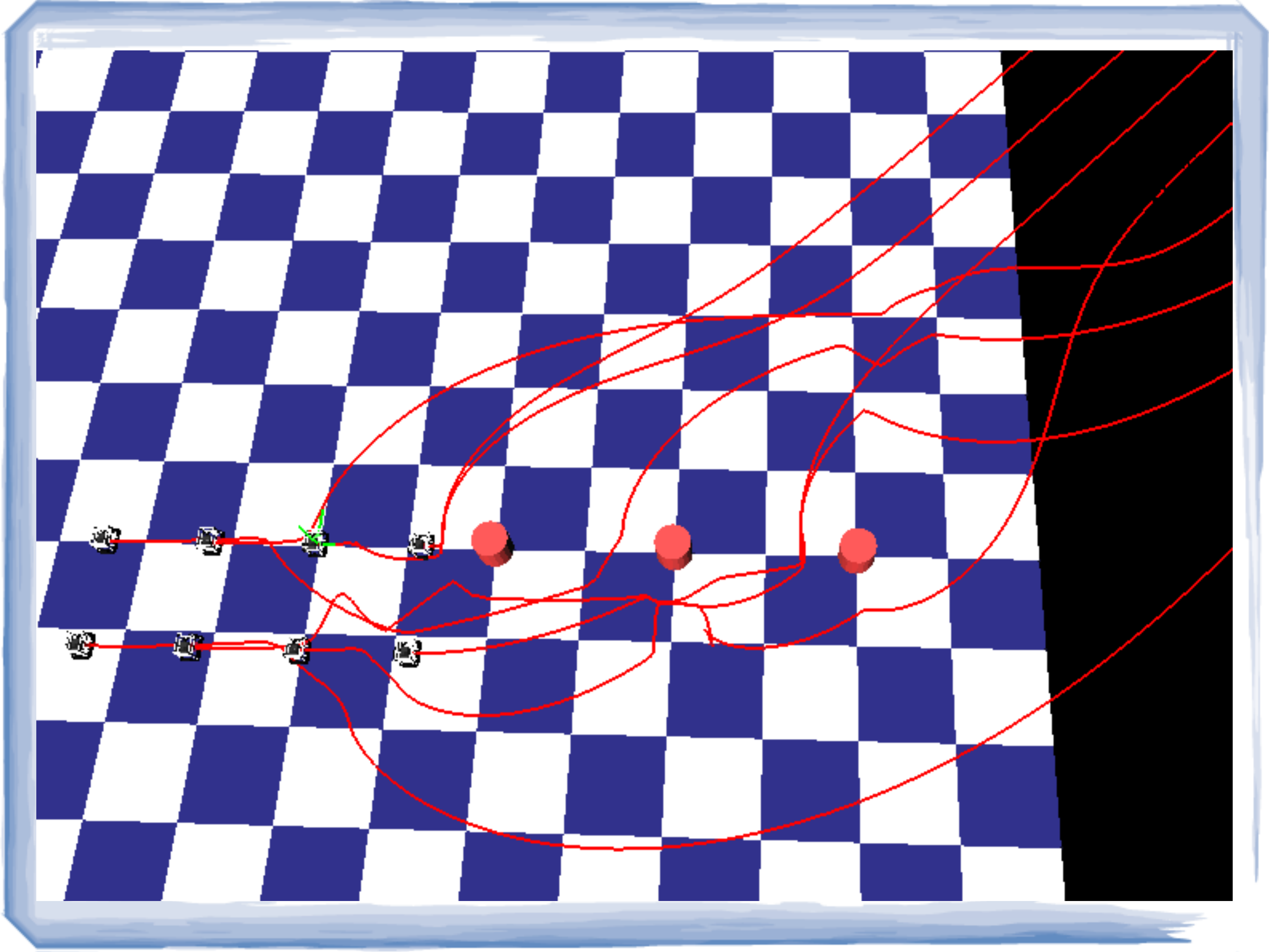
Simulation results (cont.)

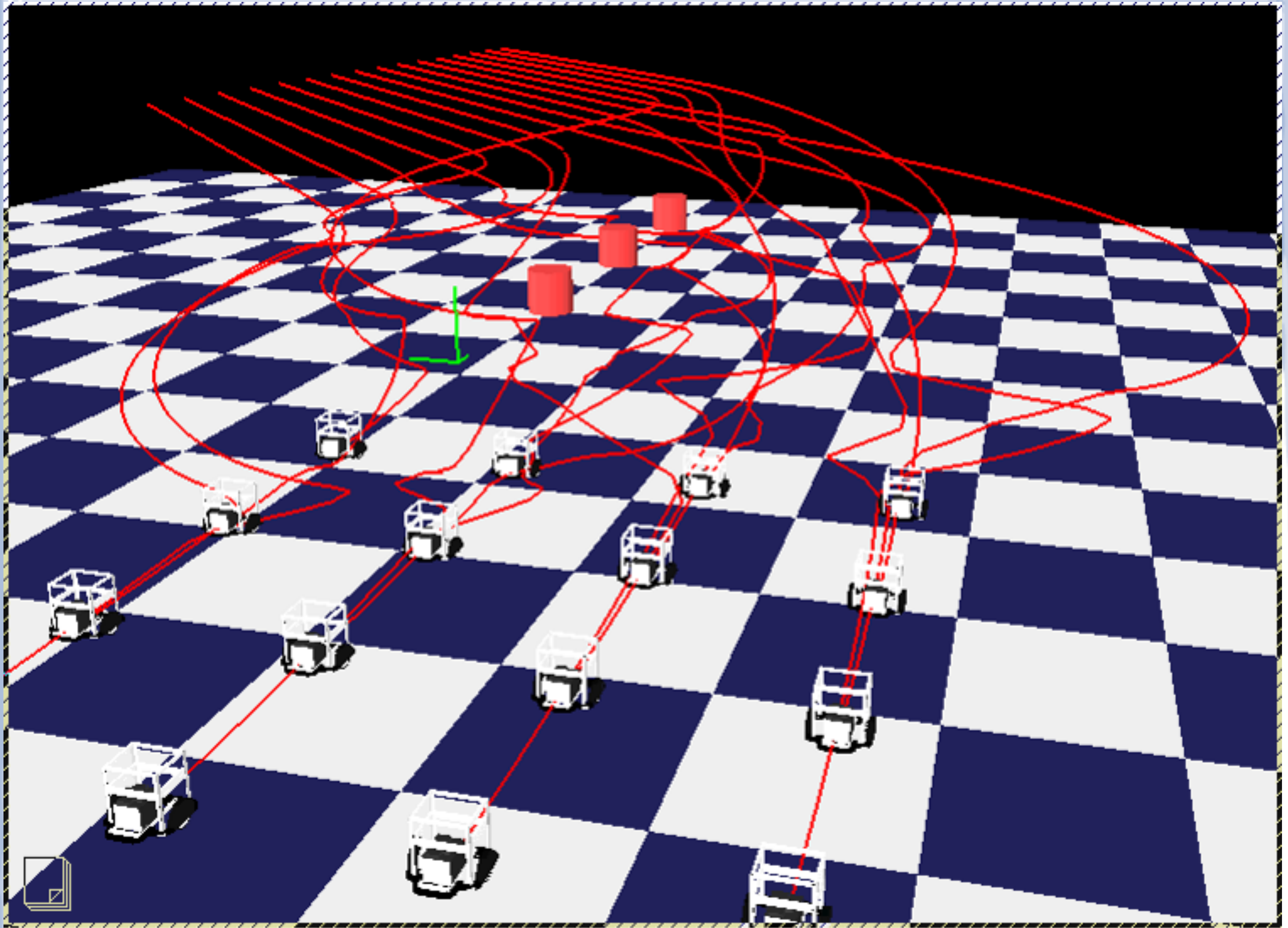






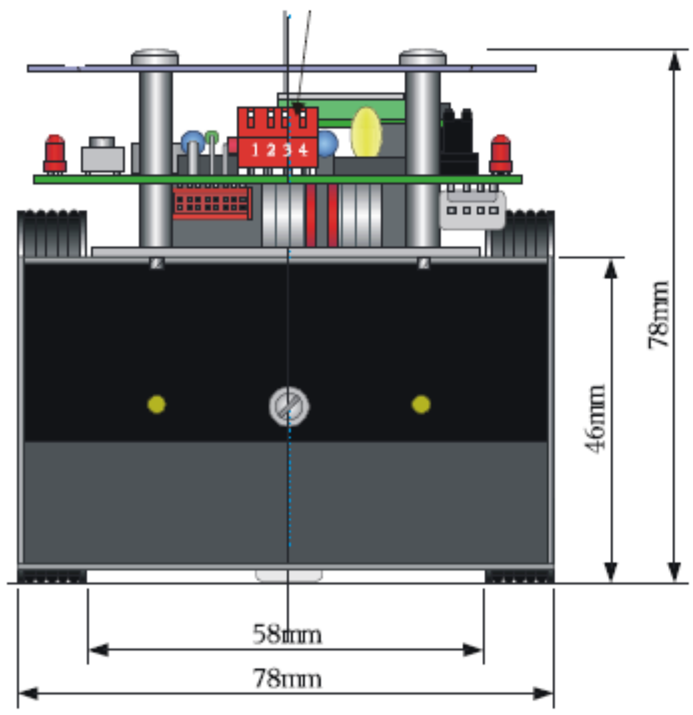


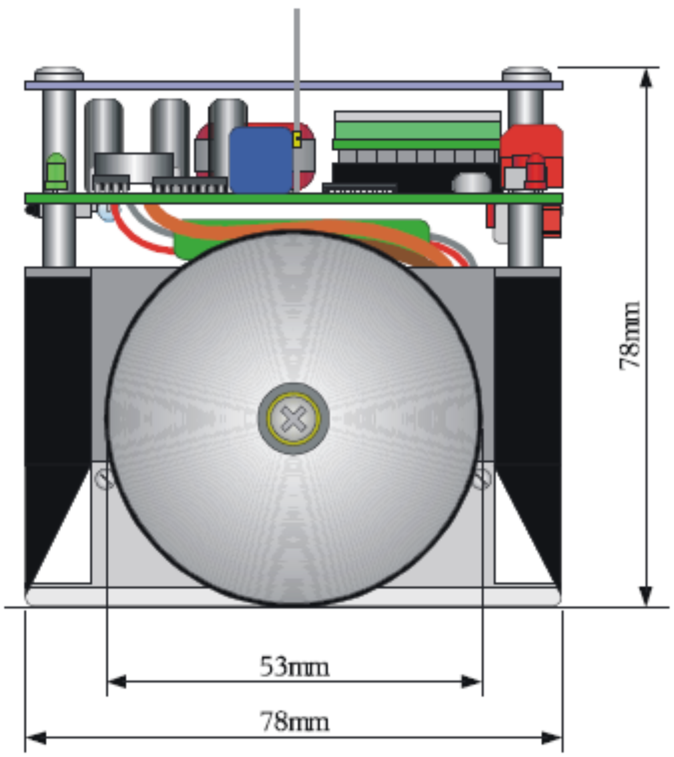




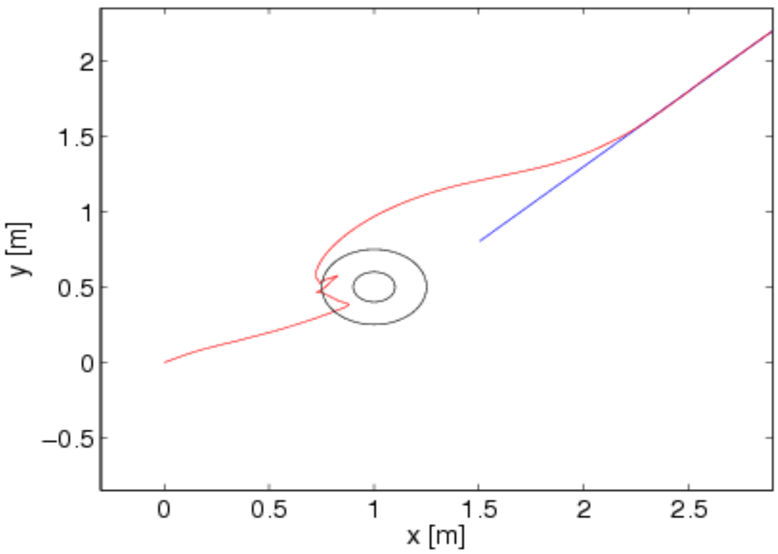
Experimental tests using Minitracker MTV3 robots

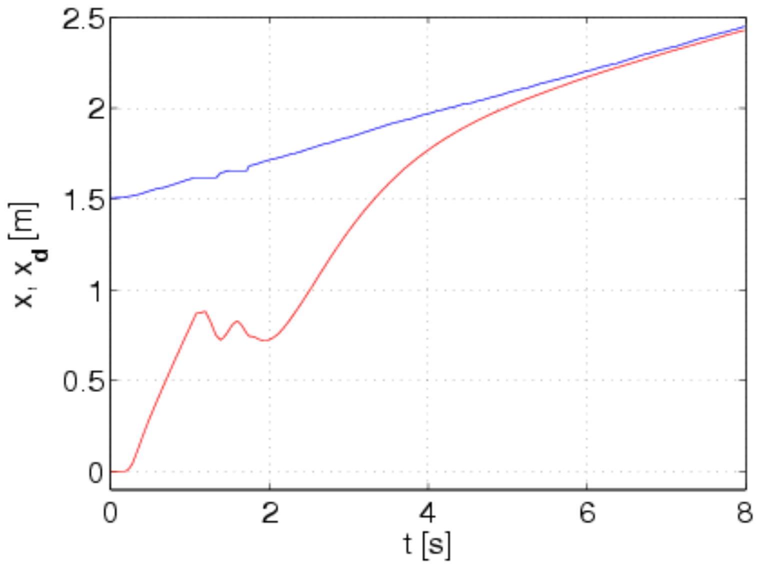
Mechanical dimensions

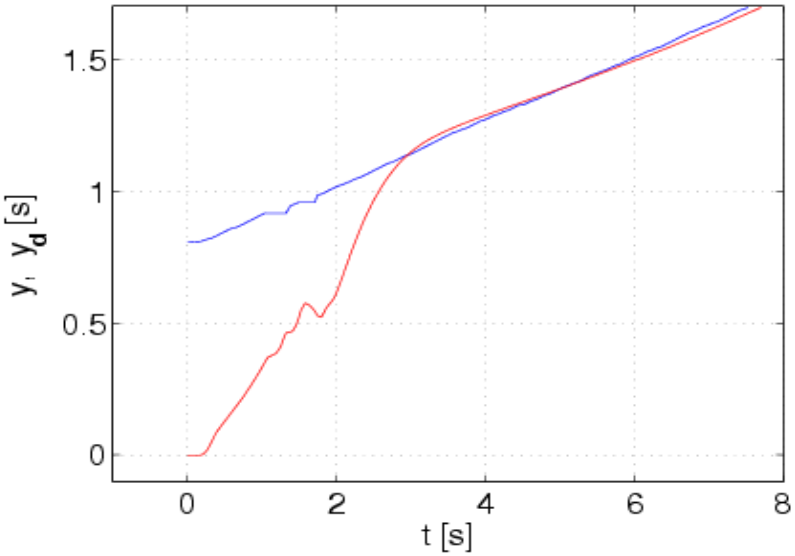


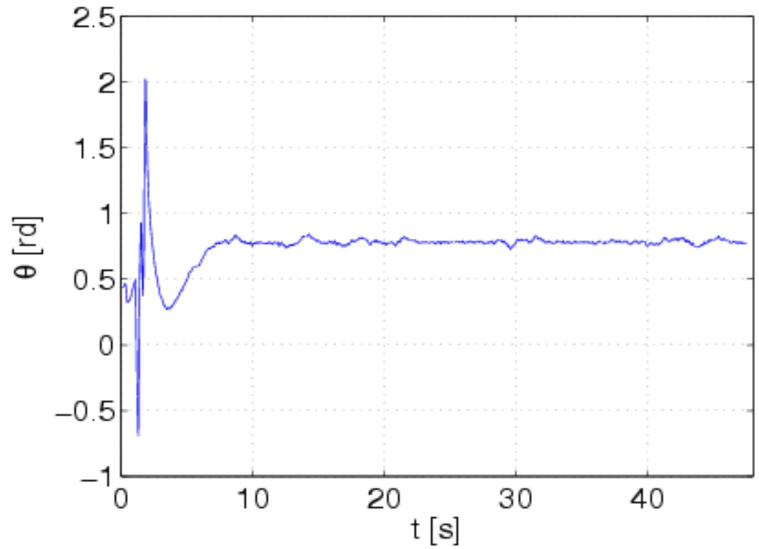


Experimental results

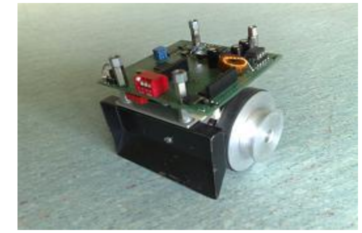


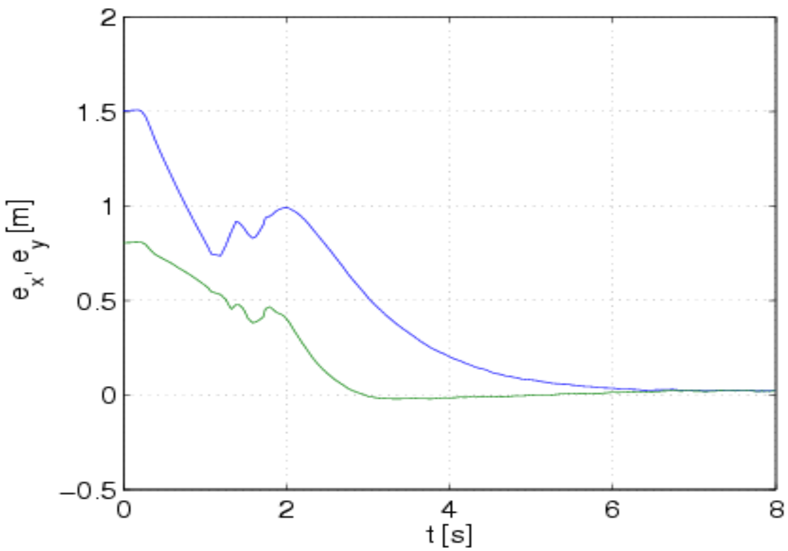


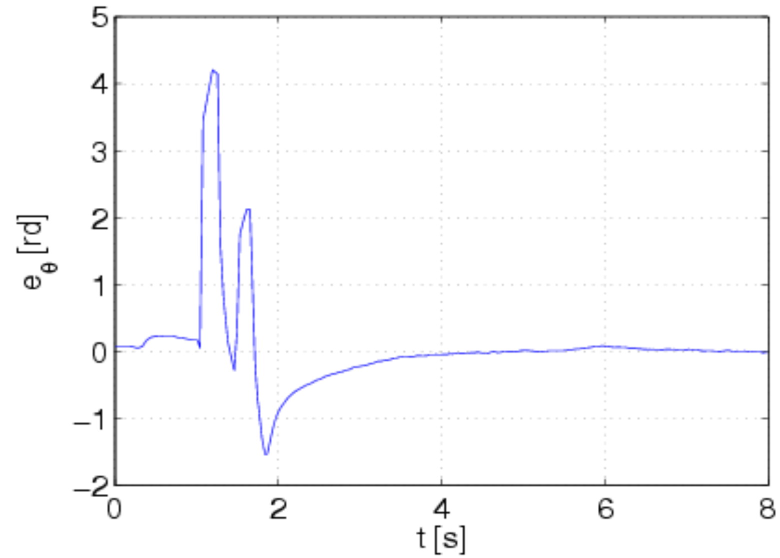


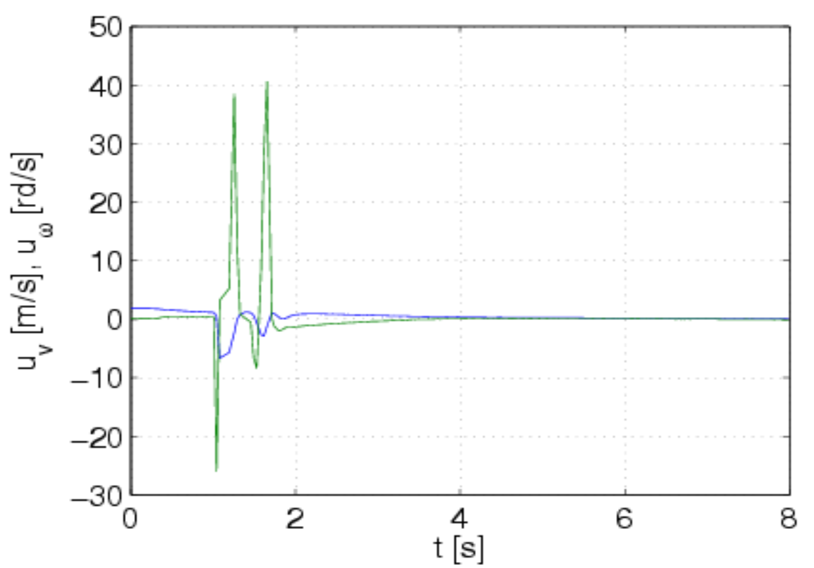


Experimental results (cont.)

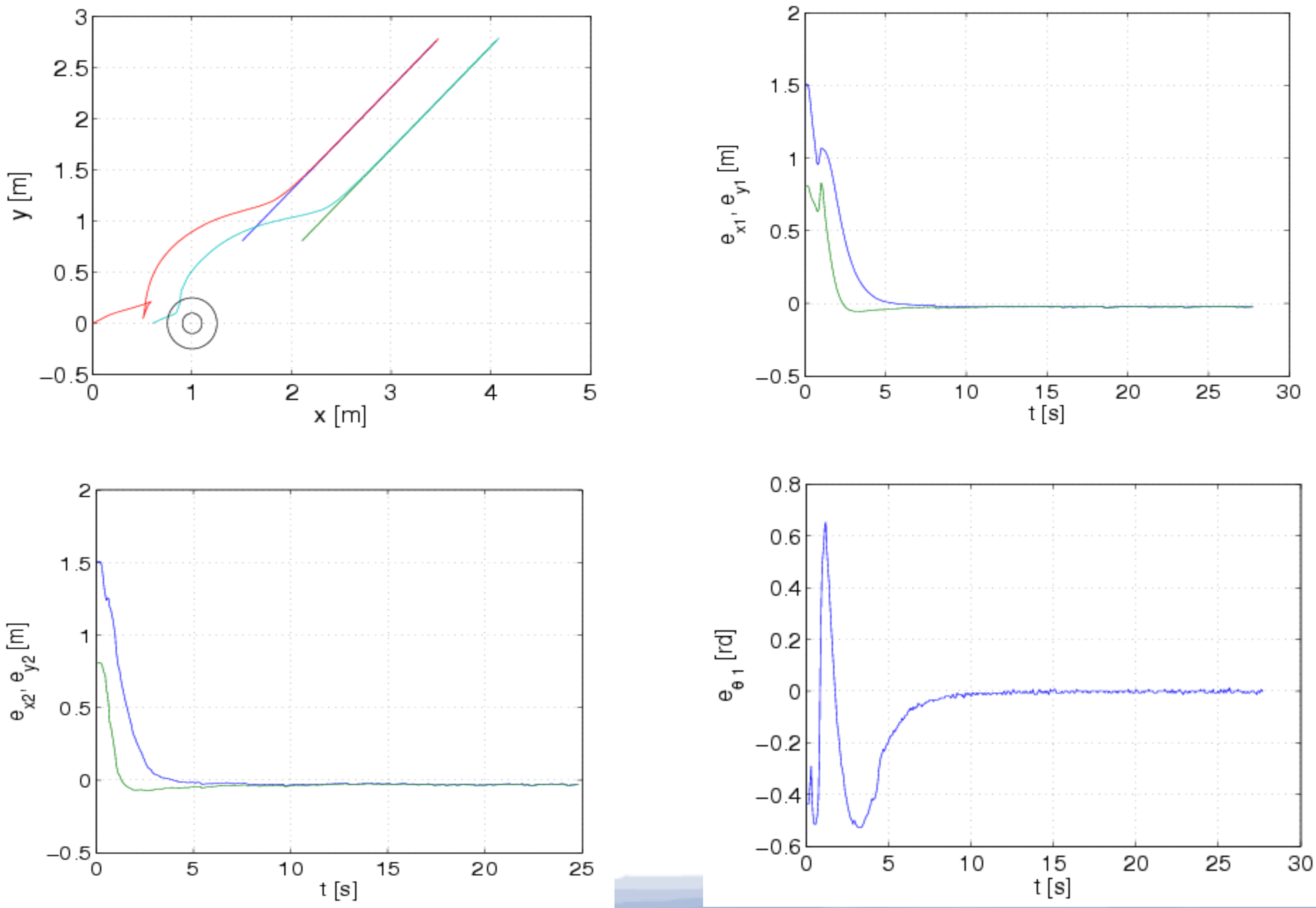




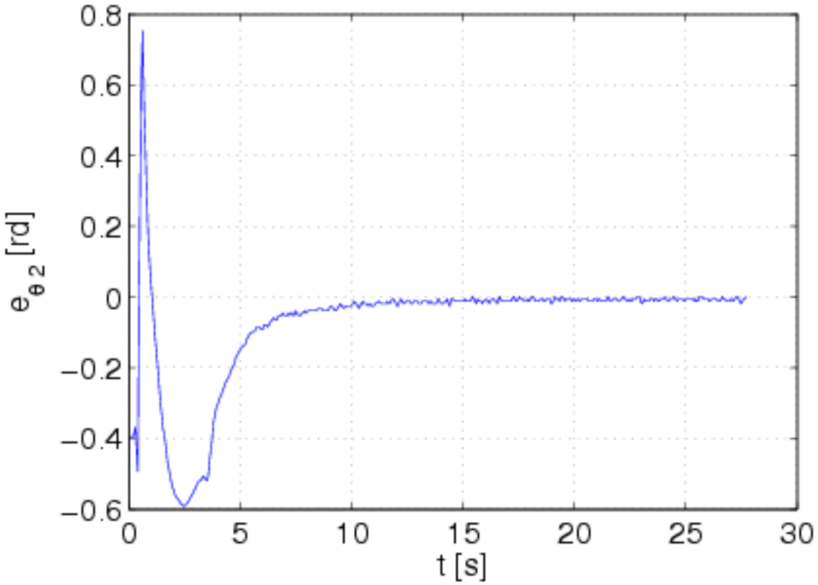




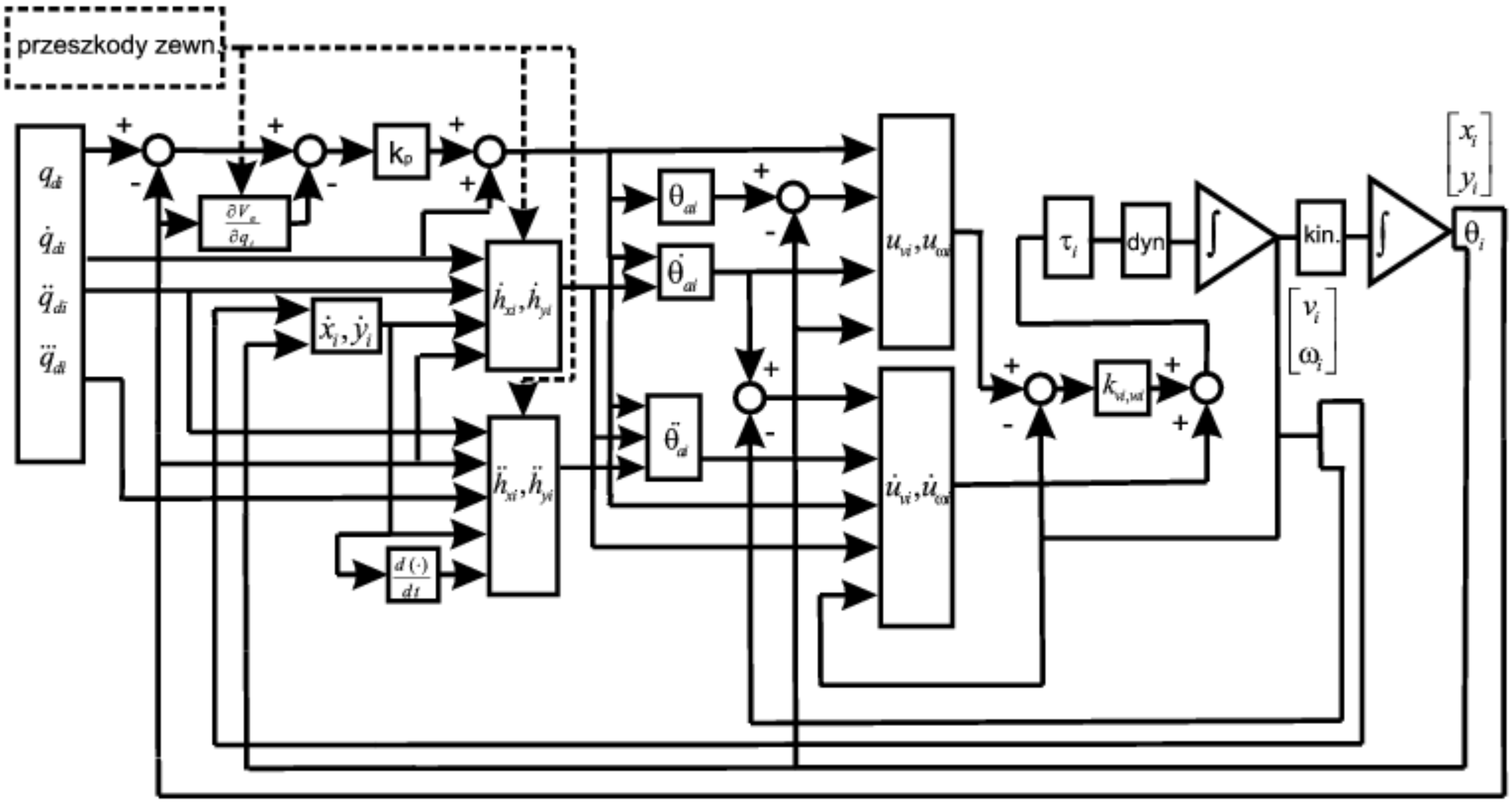
Experimental results (cont.)



Experimental results (cont.)



VFO with dynamics



$$\dot{u}_{vi} = \dot{h}_{xi}\cos(\theta_i) + \dot{h}_{yi}\sin(\theta_i) + \omega_i(-h_{xi}\sin(\theta_i) + h_{yi}\cos(\theta_i))$$

$$\dot{u}_{\omega i} = k_\theta \dot{e}_{ai}(t) + \ddot{\theta}_{ai}(t)$$

Linearizing control: $\boldsymbol{\tau}_i = \bar{\boldsymbol{B}}_i^{-1}(\bar{\boldsymbol{M}}_i \boldsymbol{\nu}_i + \bar{\boldsymbol{C}}_i \boldsymbol{\varpi}_i + \bar{\boldsymbol{D}}_i \boldsymbol{\varpi}_i)$

Linear control for the new input: $\boldsymbol{\nu}_i = \dot{\boldsymbol{u}}_i + \boldsymbol{k}_{\varpi}(\boldsymbol{u}_i - \boldsymbol{\varpi}_i)$ u_i – input from the VFO algorithm

Collision avoidance - assumptions

- Planned trajectories avoid collisions,
- When robot gets into APF area reference trajectory is frozen,
- Auxiliary orientation variable is disturbed when auxiliary orientation error is equal $\pm\frac{\pi}{2}$,
- Reference trajectory is disturbed when robot drives into saddle point.

VFO with collision avoidance - stability

Lypunov function: $V_l = V_{tl} + V_{al}$

Trajectory tracking term: $V_{tl} = \frac{1}{2}\mathbf{e}^T\mathbf{e} = \frac{1}{2}\left(e_x^2 + e_y^2\right)$

Collision avoidance term: $V_{al} = V_a(l)$

Lypunov function fulfills condition:

$$V_l(t) \leq (V_{l1}(0) - 2\frac{\varepsilon}{\lambda}\sqrt{V_{l1}(0)} + \frac{\varepsilon^2}{\lambda^2})e^{-\lambda t} + 2(\frac{\varepsilon}{\lambda}\sqrt{V_{l1}(0)} - \frac{\varepsilon^2}{\lambda^2})e^{-\frac{1}{2}\lambda t} + \frac{\varepsilon^2}{\lambda^2}$$

Where ε, λ are positive constants. In the steady state $V_{l1}(t) \leq \frac{\varepsilon^2}{\lambda^2}$ - Lyapunov function is bounded to sphere.

Orientation converges to $\lim_{t\to\infty} \theta_i = \theta_{di} + \varepsilon_\theta$

VFO with collision avoidance - stability

As the model of the dynamics is feedback linearized it is easy to show that

$$\lim_{t\to\infty} \varpi_i = u_i$$

- linear and angular velocities converge exponentially to velocity controls.

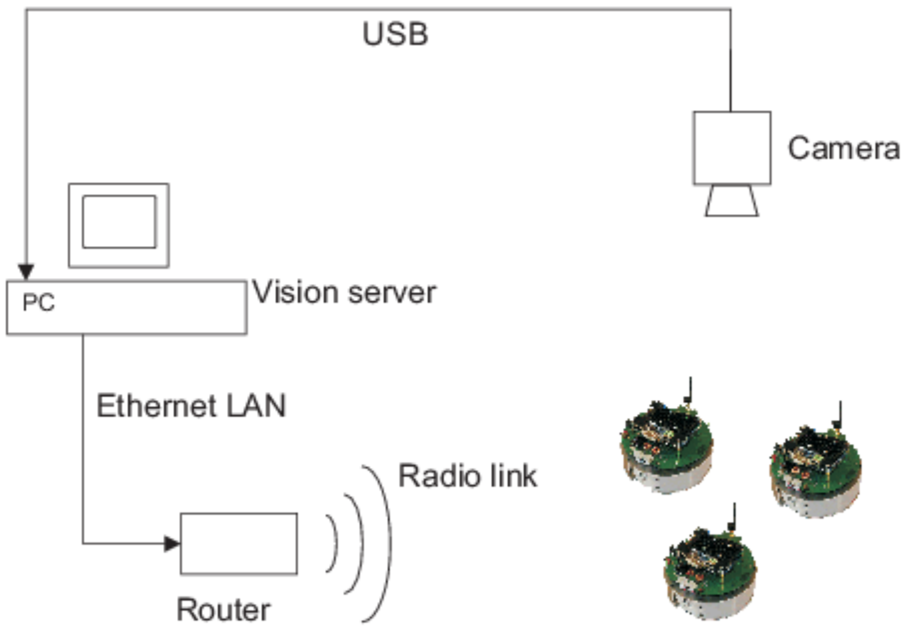
VFO - collision avoidance

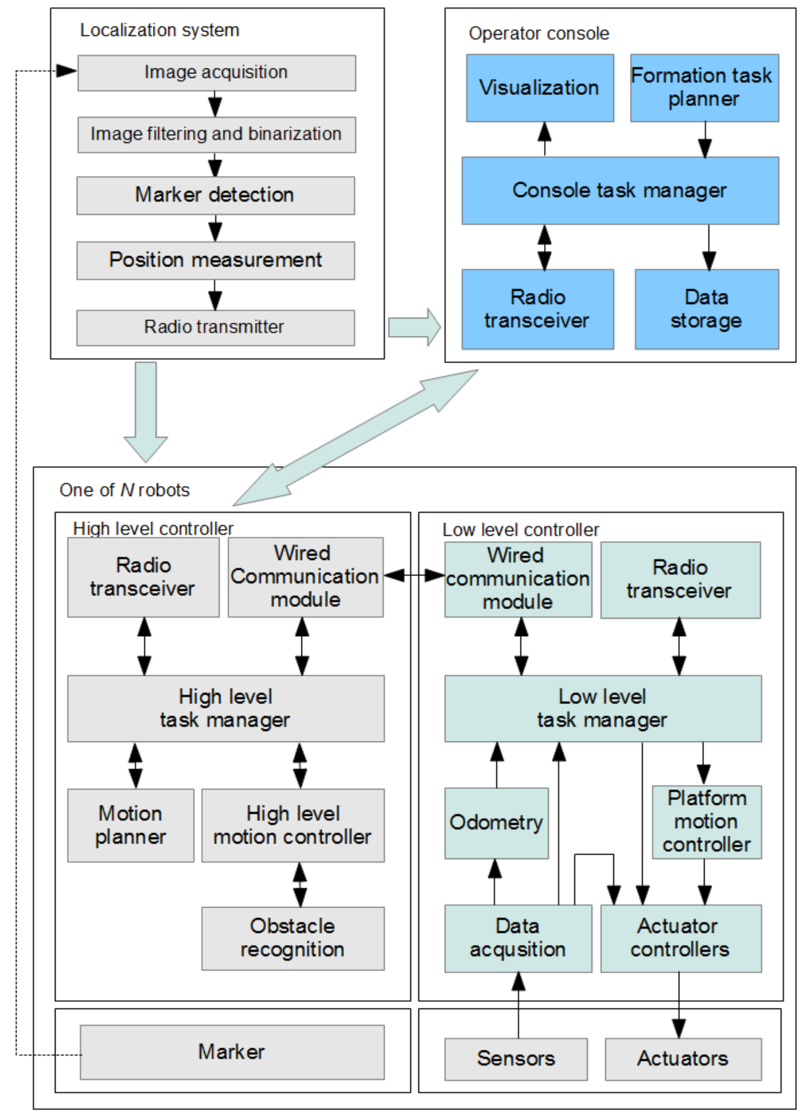
As shown in [1] the collision avoidance is guranteed if $\dot{V} \leqslant 0$ and $\lim_{\|\mathbf{q}-\mathbf{q}_a\| \to r^+} V_a = +\infty$

where q is position of the robot, q_a is position of the obstacle, *r* is the radius of the obstacle. Previously presented artificial potential function satisfy second condition.

[1] D. M. Stipanovic, P. F. Hokayem, M. W. Spong, D. D. Siljak, "Cooperative Avoidance Control for Multiagent Systems", Journal of Dynamic Systems, Measurement and Control, Vol. 127, pp. 699-707, septembr 2007.

Architecture of the system

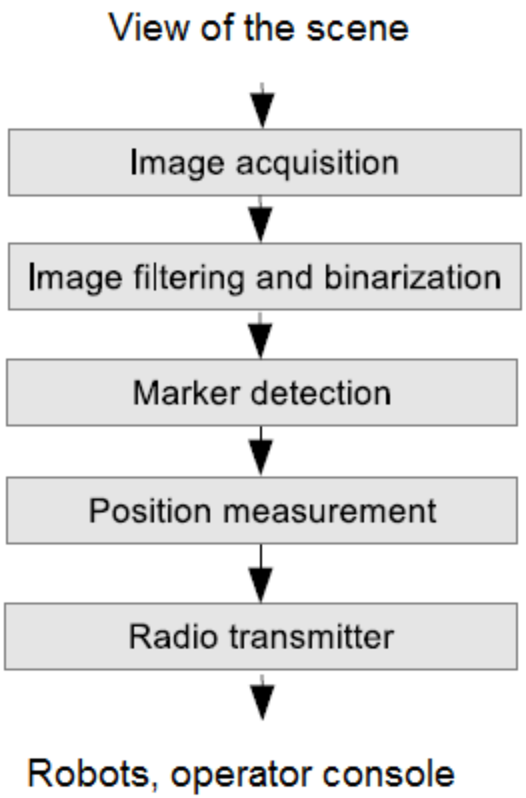


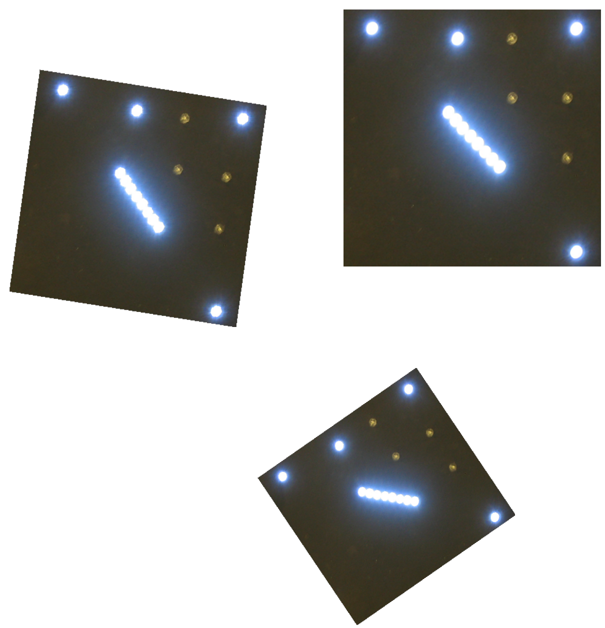


Optional components of the system:

- external localization system,
- operator console.

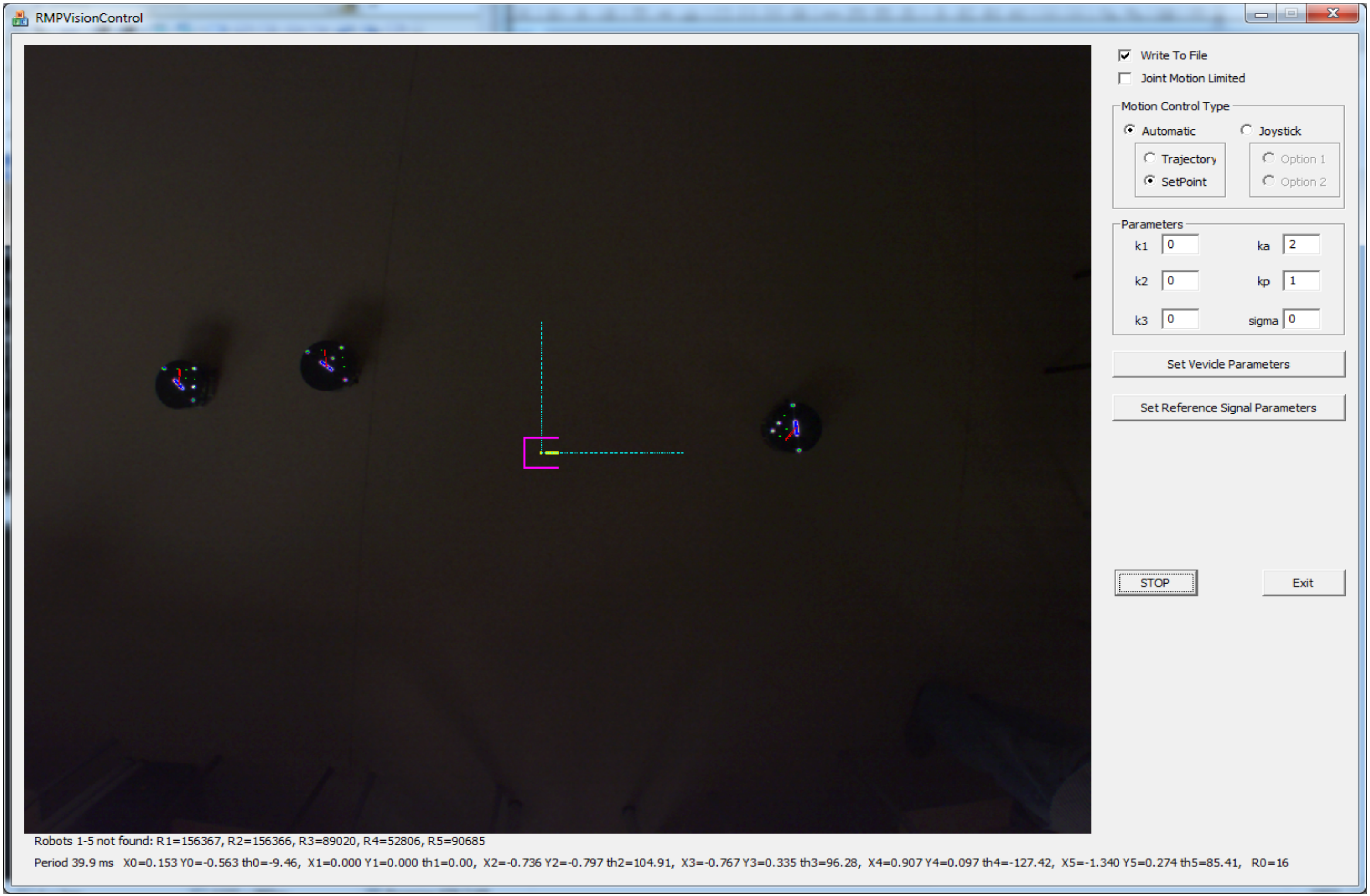
Localization system



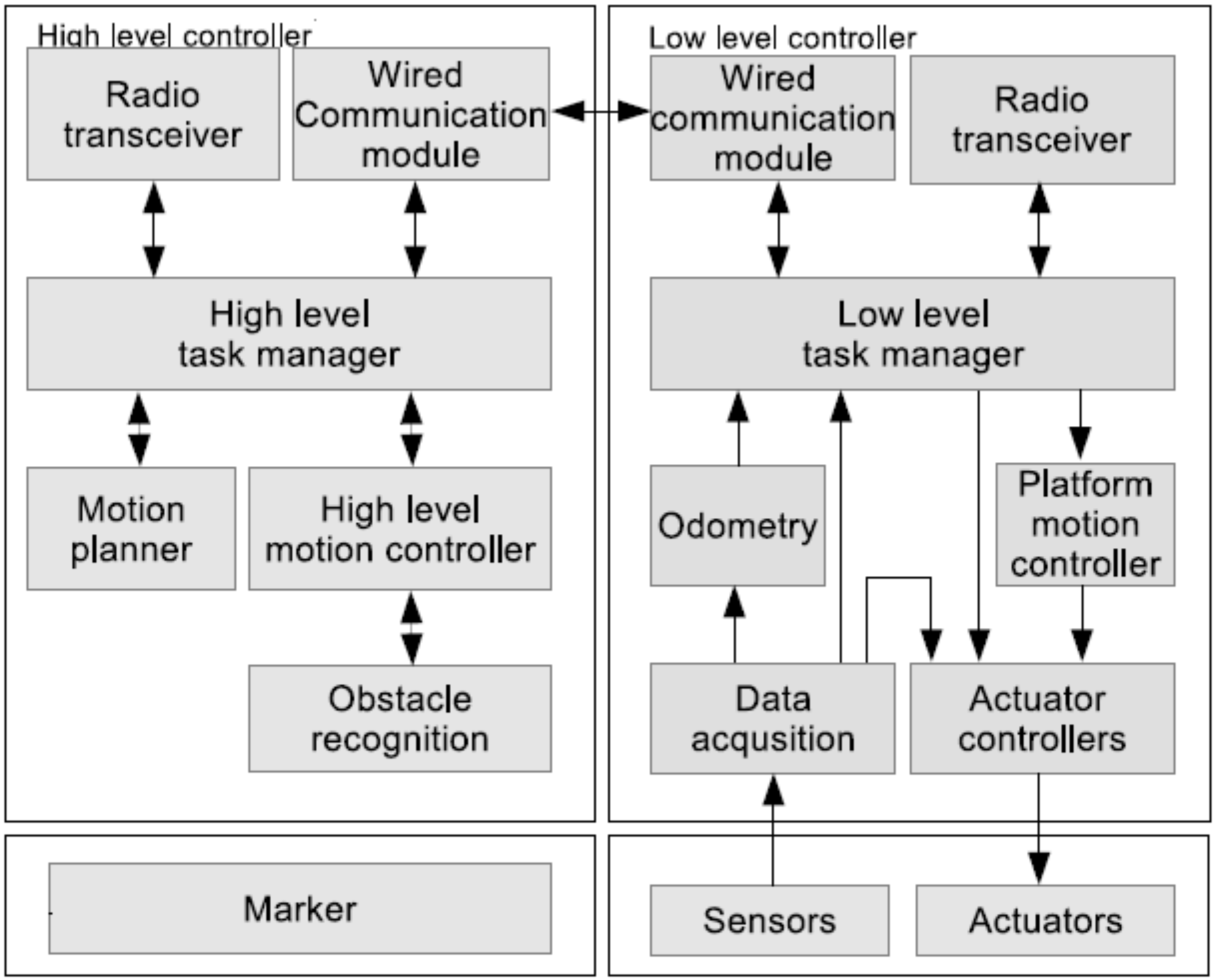


- uEye UI-1240SE-C USB camera,
- PENTAX C418DX lens,
- 1280×1024 pix resolution,
- frq 25Hz,
- LED markers,
- real-time operation,
- Scene 4x3.2m.

Localization system

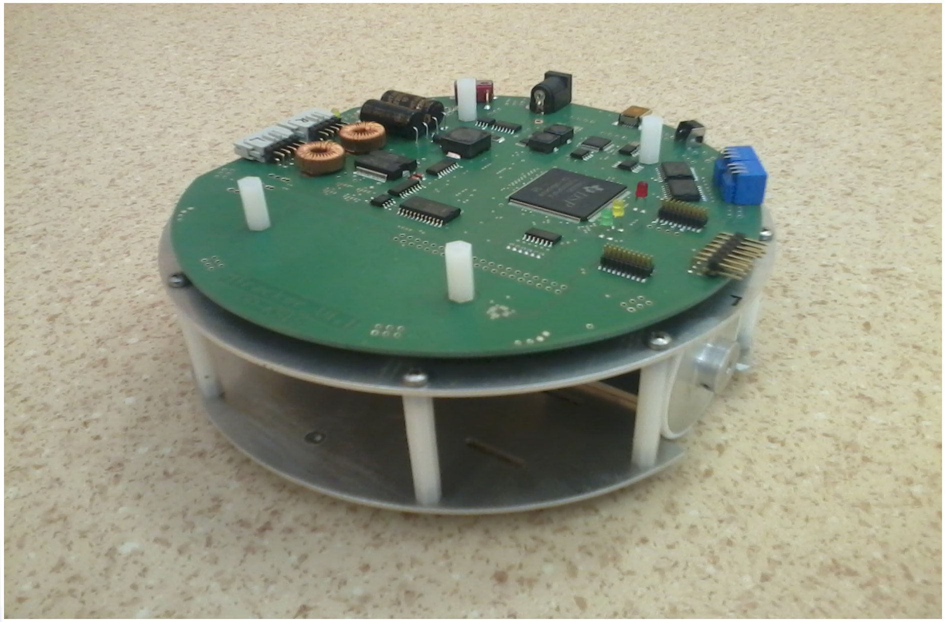


Robots



High-level controller and low-level controllers connected with RS232

Robots

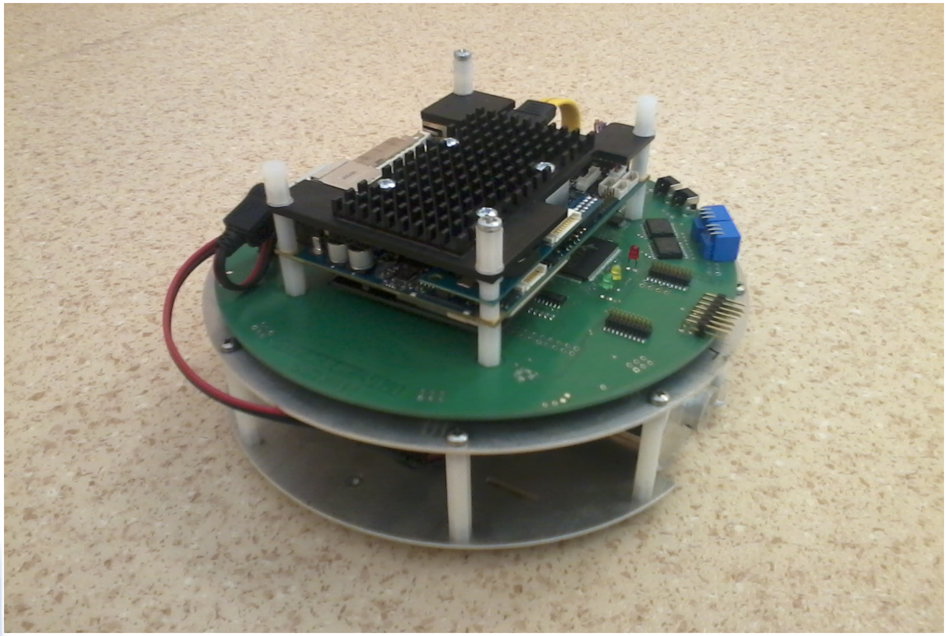


- Parameters
- Diameter: 170mm
- height (with low-level controller): 65mm
- max. speed 1m/s
- power supply: accu 8-15V, 2000-4400mAh

Low-level controller

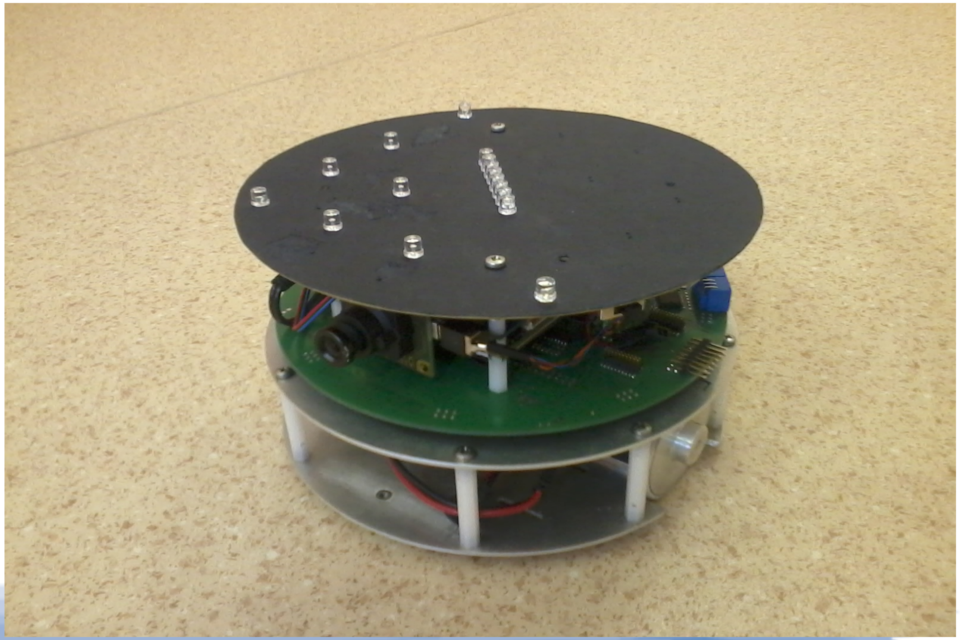
- TMS320F28335 150MHz
- RAM 256KB
- FLASH 128KB
- cc2500 radio transceiver 256kbps
- Sensors: IR proximity (200mm range), gyroscope, two-axis accelerometer, compass

Robots

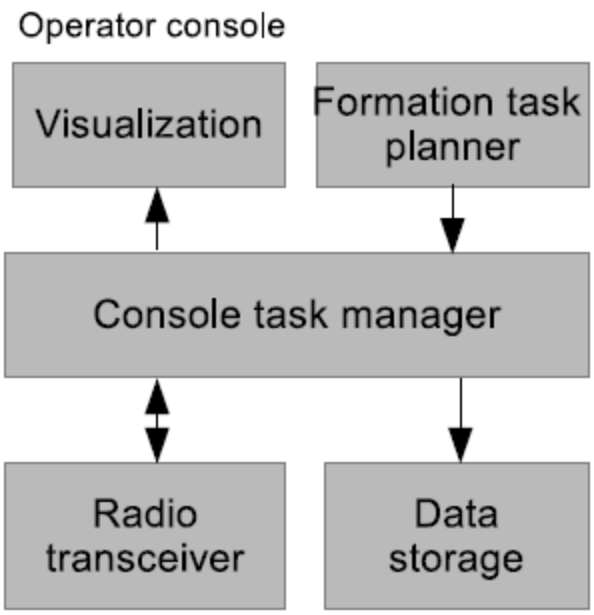


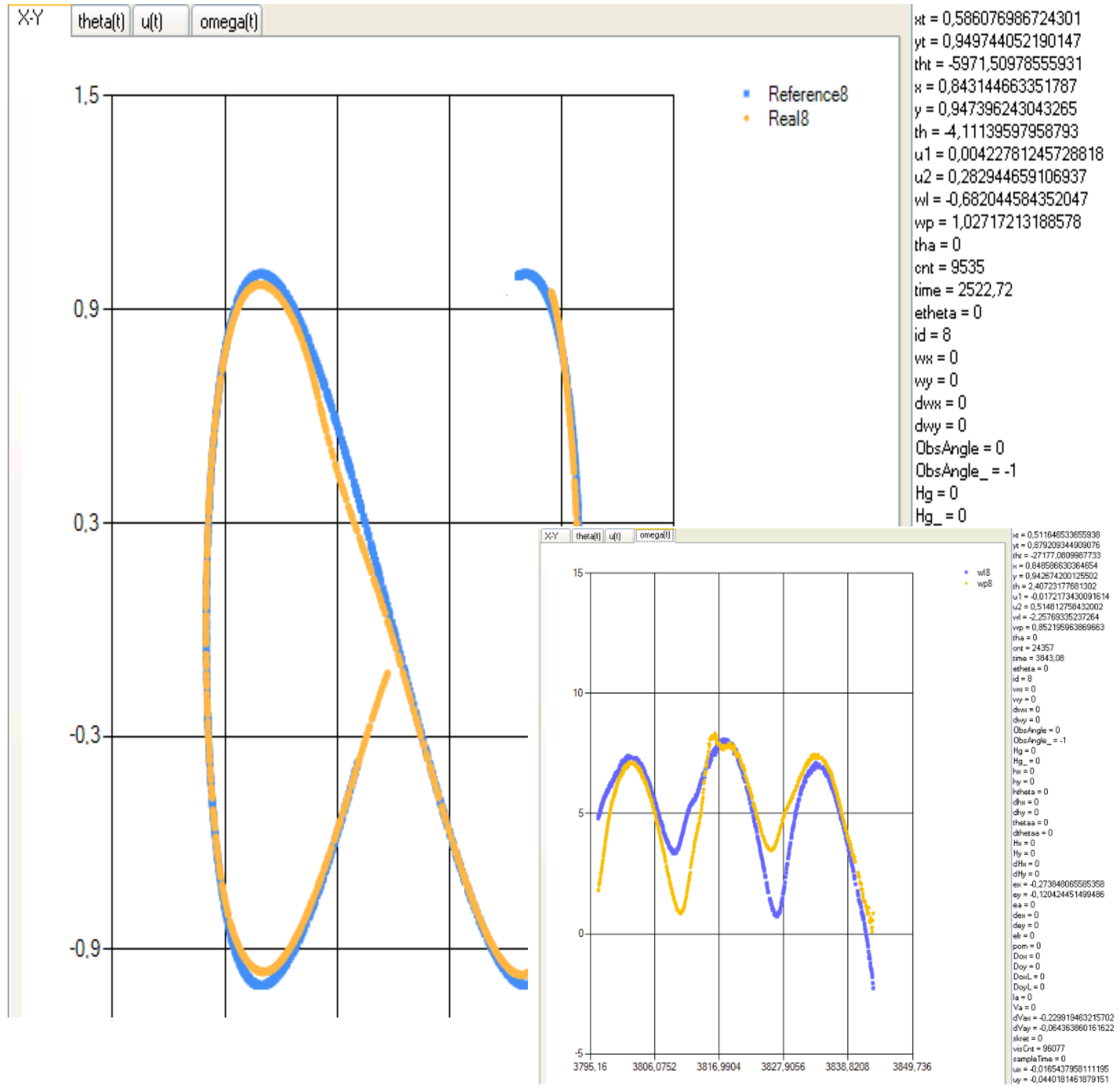
High-level controller
- Intel Atom
- 1,6GHz
- SSD 16GB disk
- Windows, Linux or QNX

Interfaces
- 2x USB interface
- RS232 port
- LAN



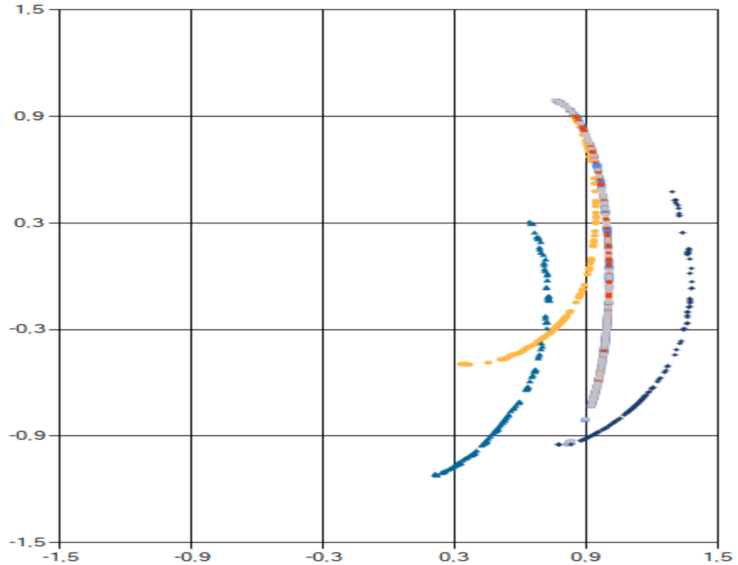
Operator console

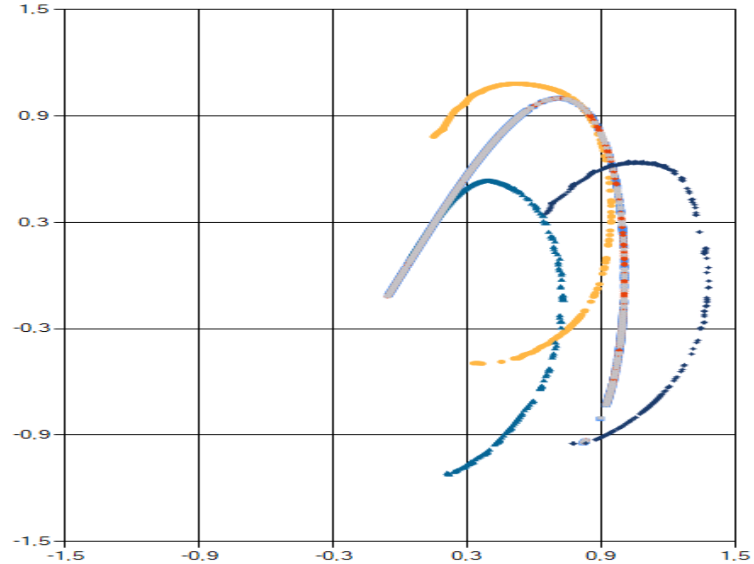


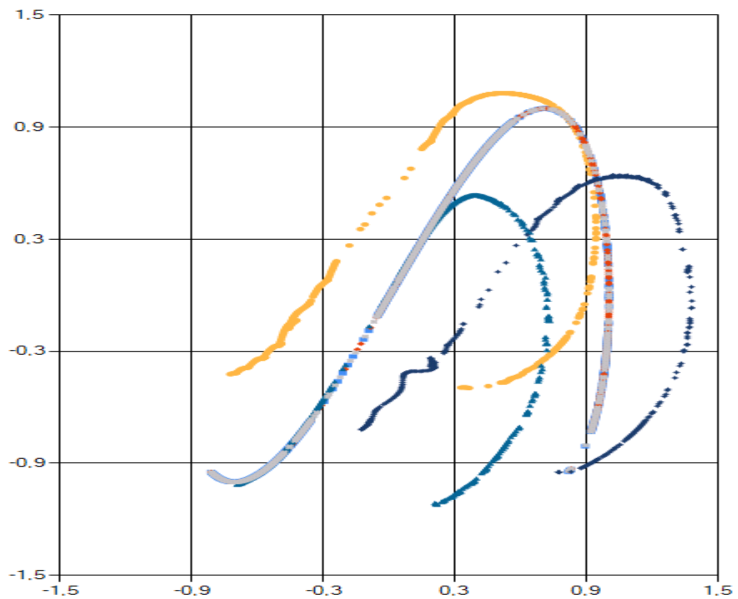


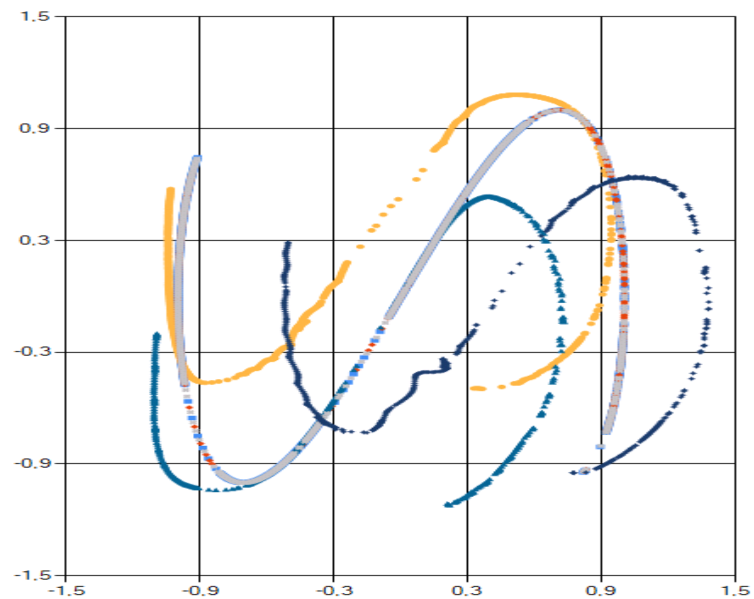
- Visualization,
- Task selection,
- Data storage for off-line analyzis (text files, SQL database).

Operator console









Communication

WiFi,

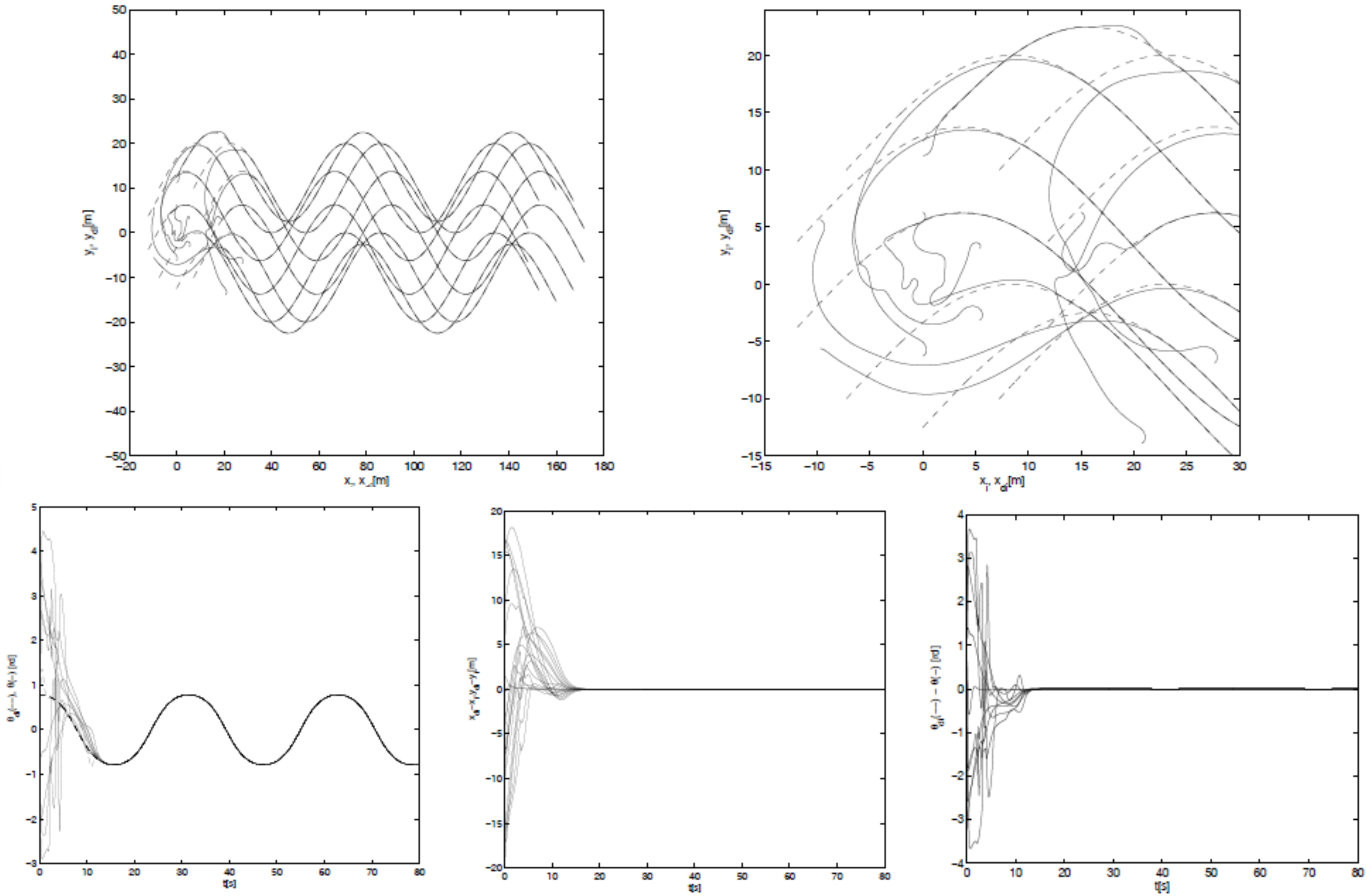
- UDP packets,
- LAN - infrastructure network mode (router is used),
- Localization system broadcasts packets containing information about the position and orientation of robots,
- Commands from the operator console are broadcasted or unicasted to robots (depending on the task).

cc2500 module

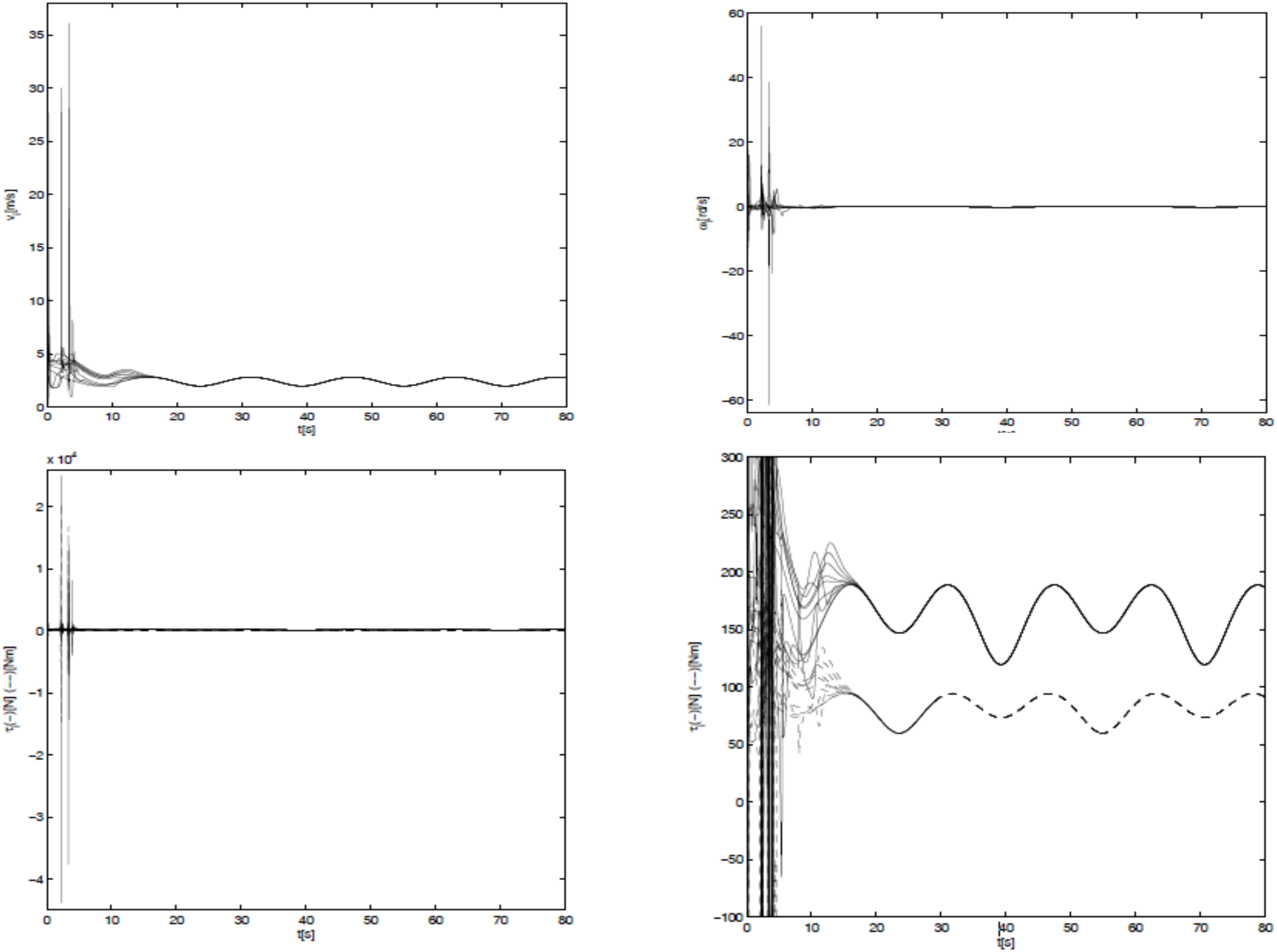
- Each block of the system requies specialized radio interface.

Multiple robots

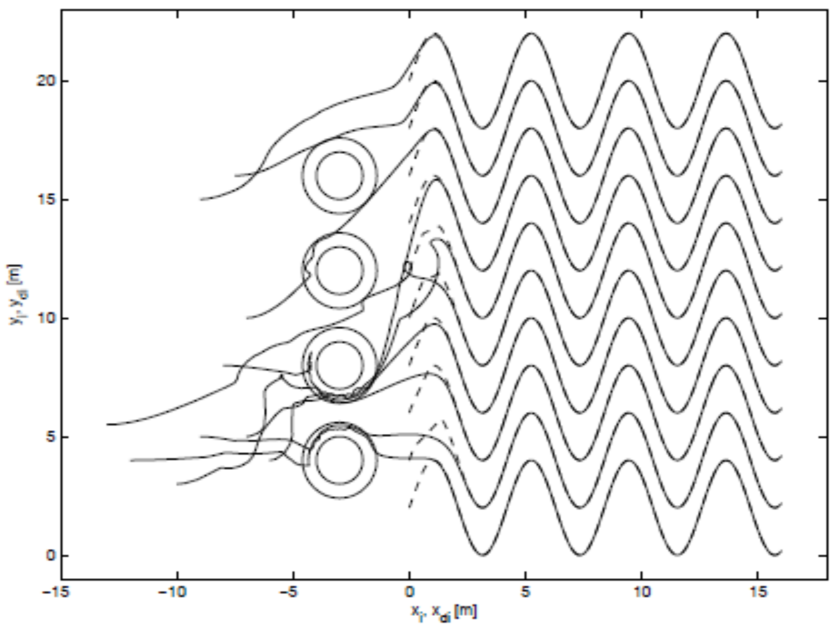
Simulation results – Do algorithm

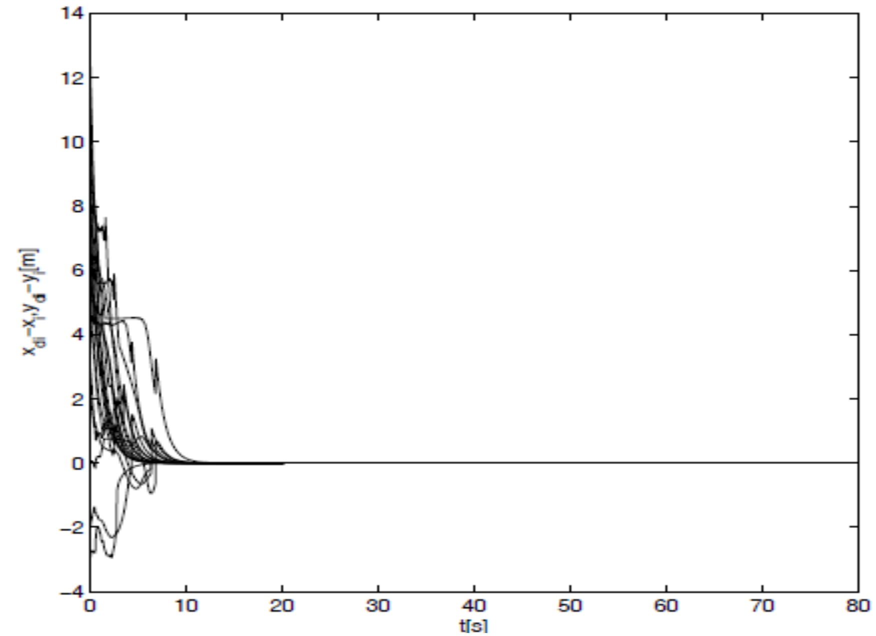


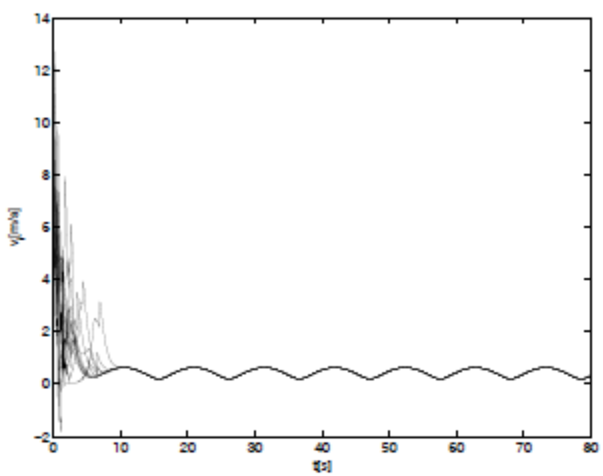
Simulation results – Do algorithm

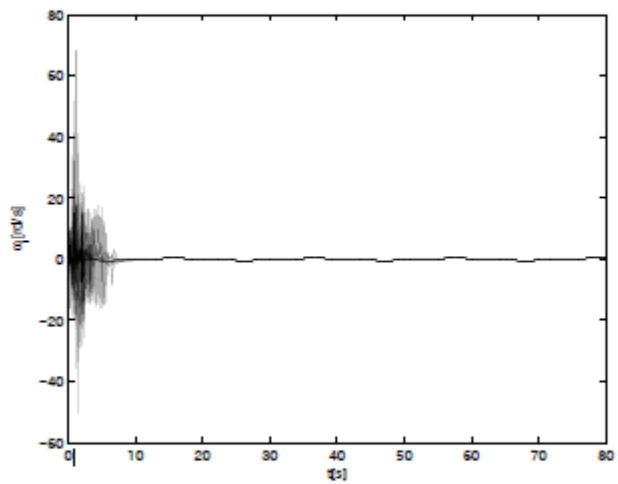


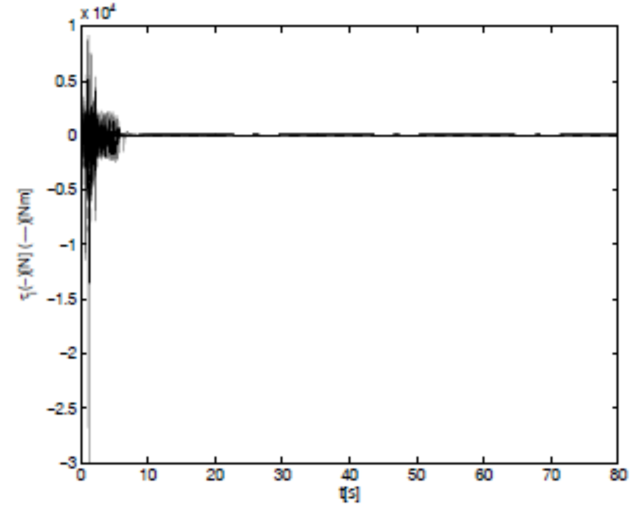
Simulation results – VFO algorithm



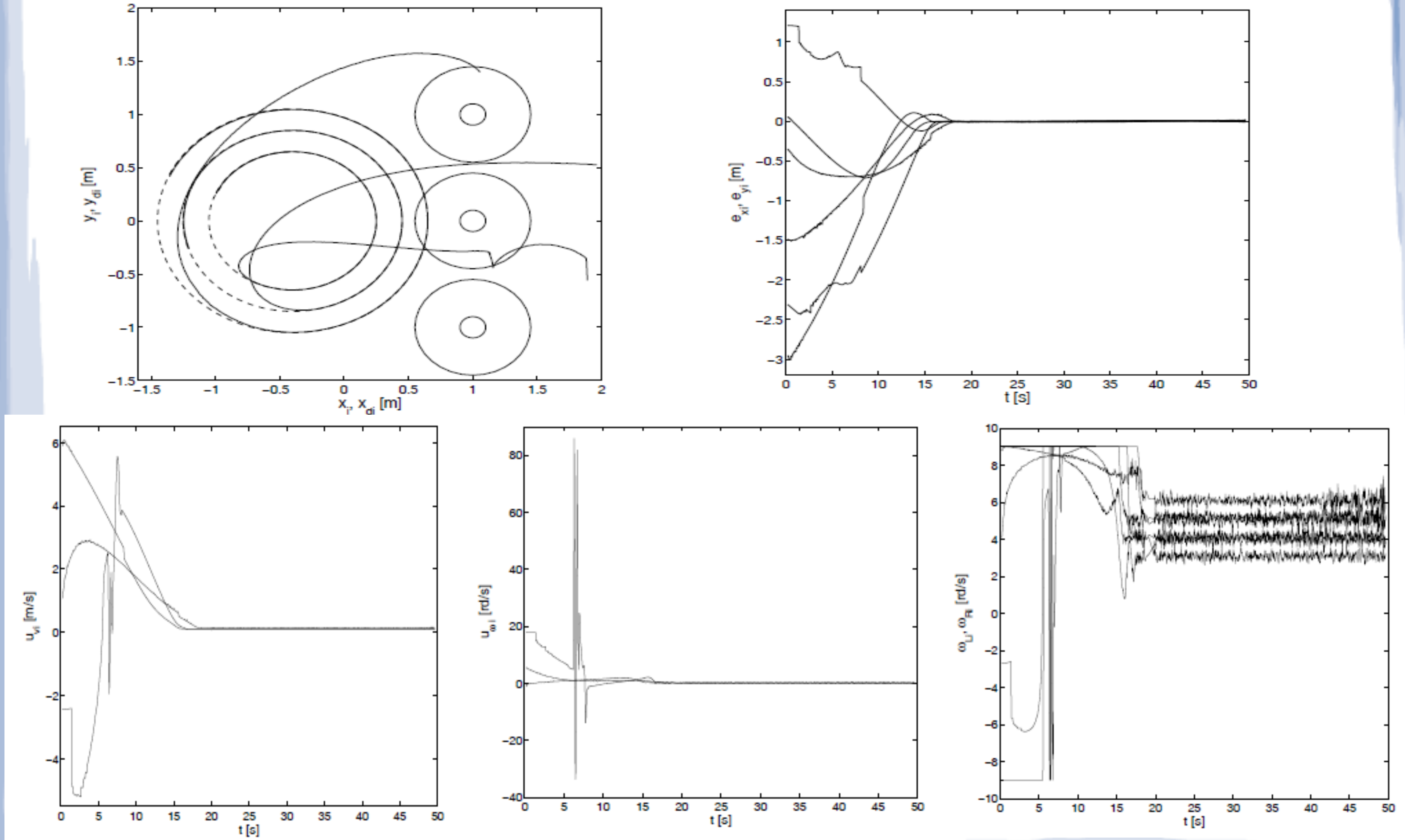




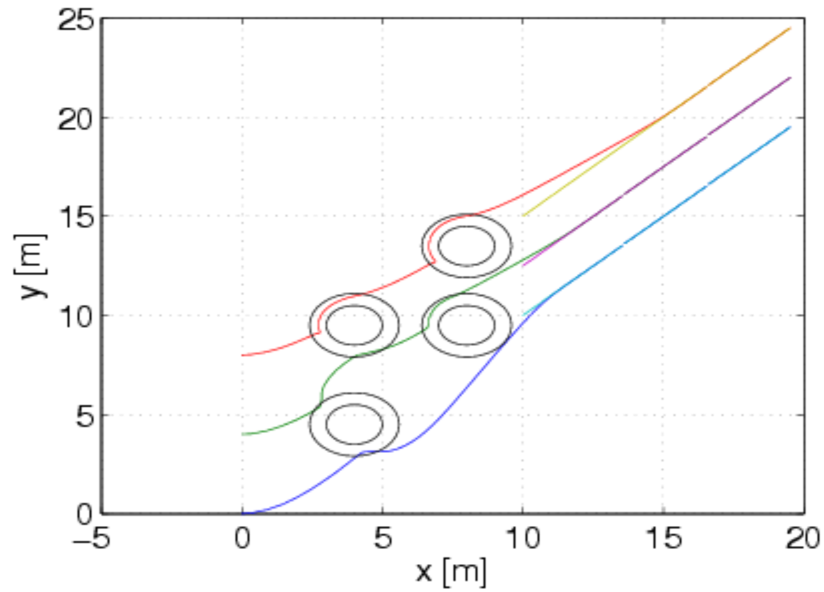


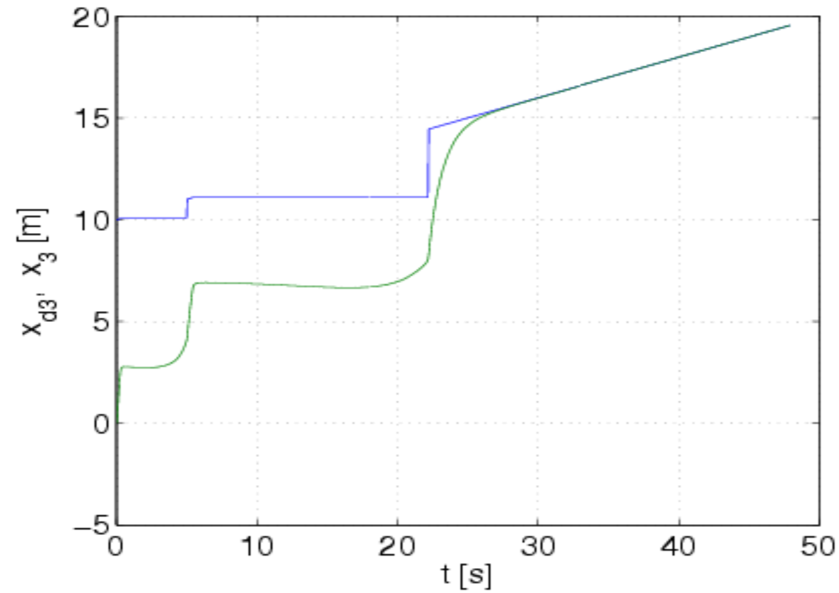


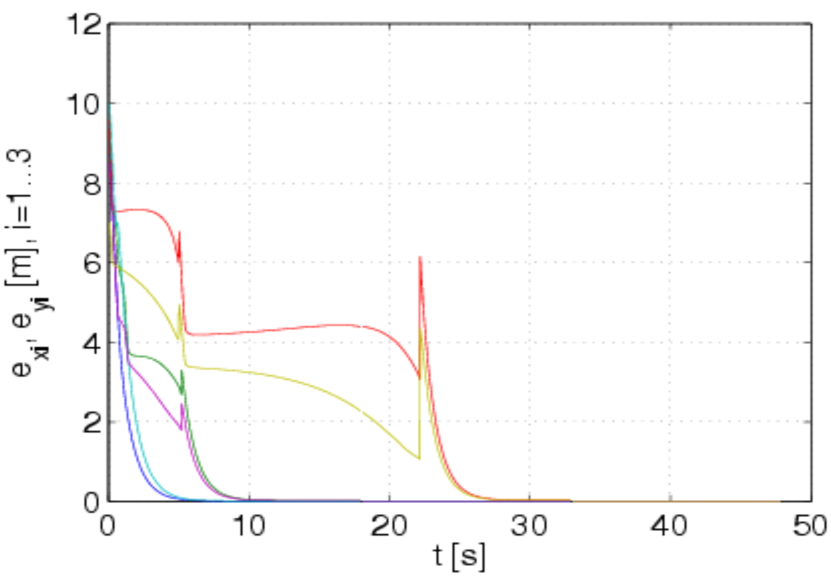
Simulation results – VFO algorithm

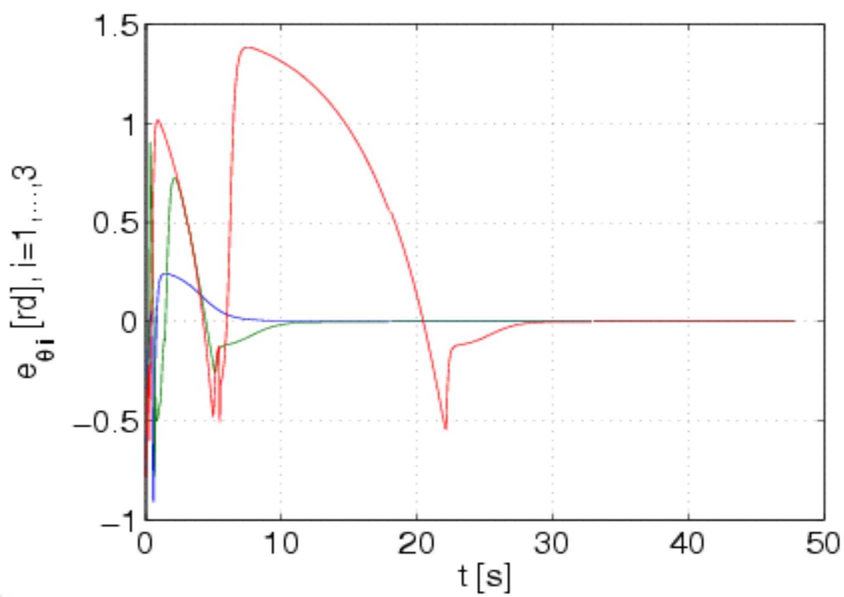


Simulation results

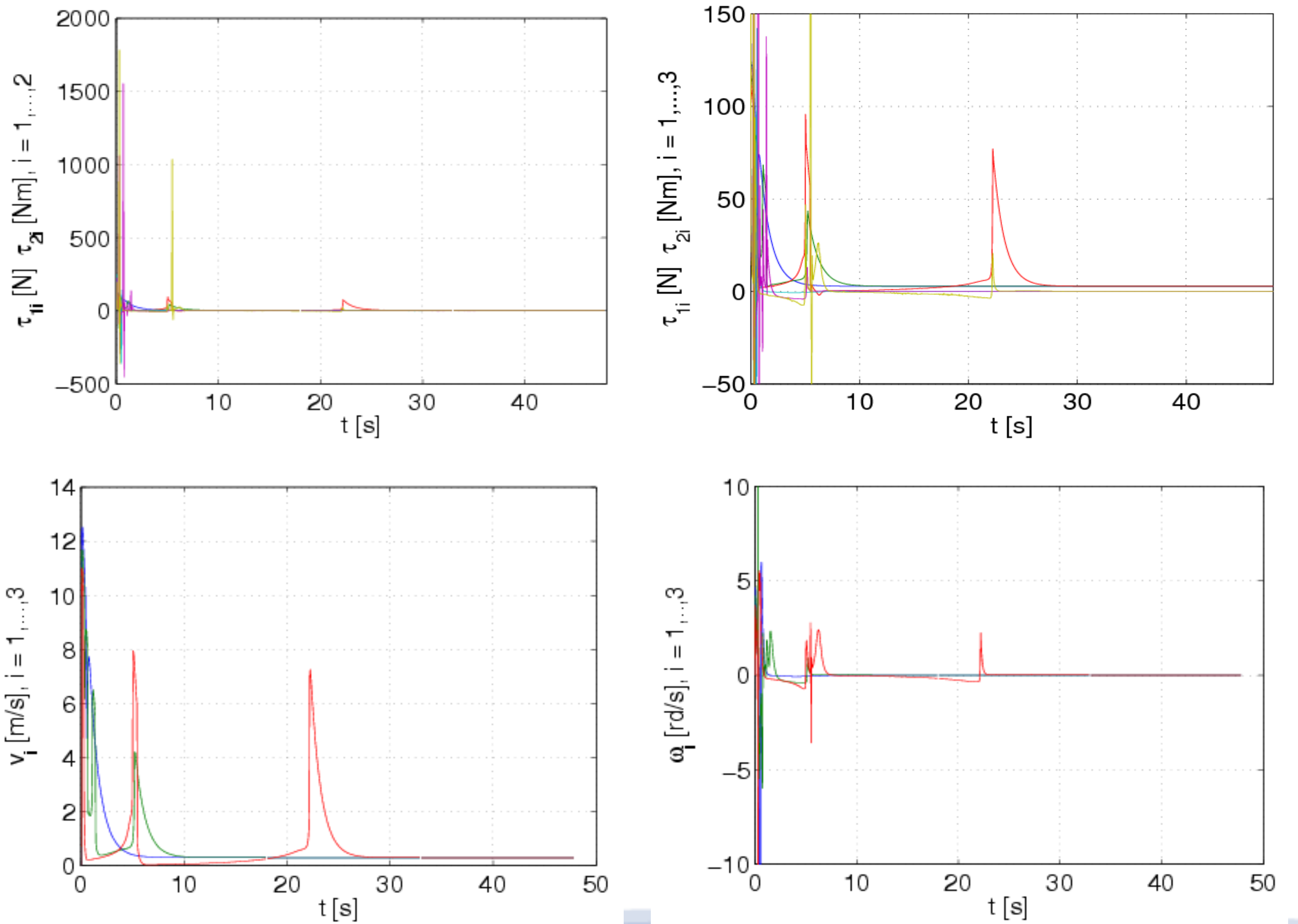




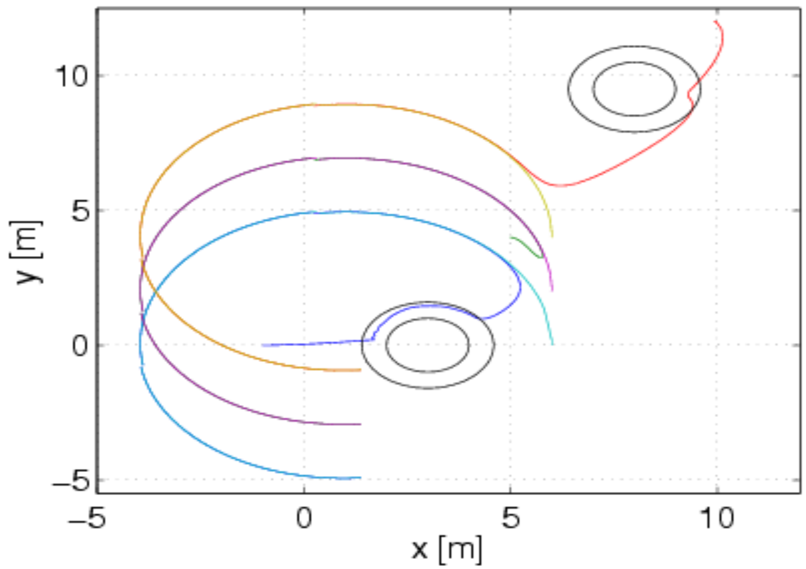


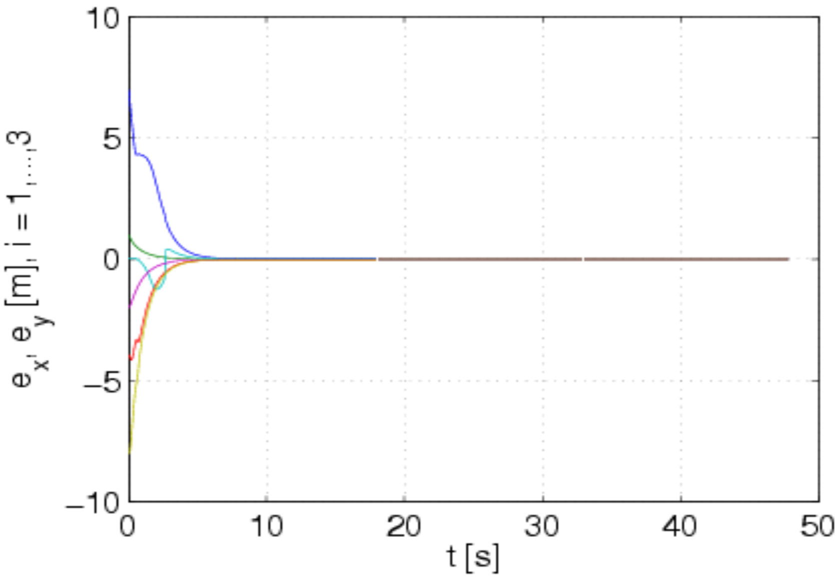


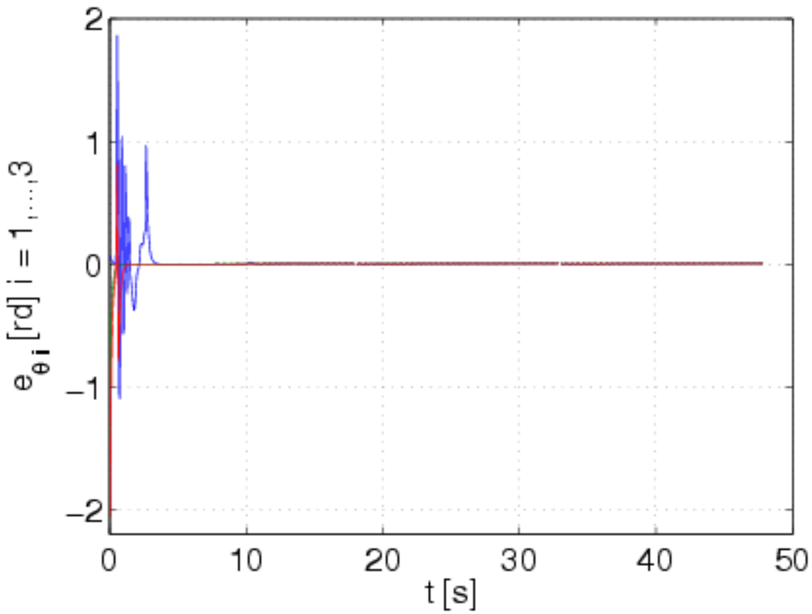
Simulation results (cont.)

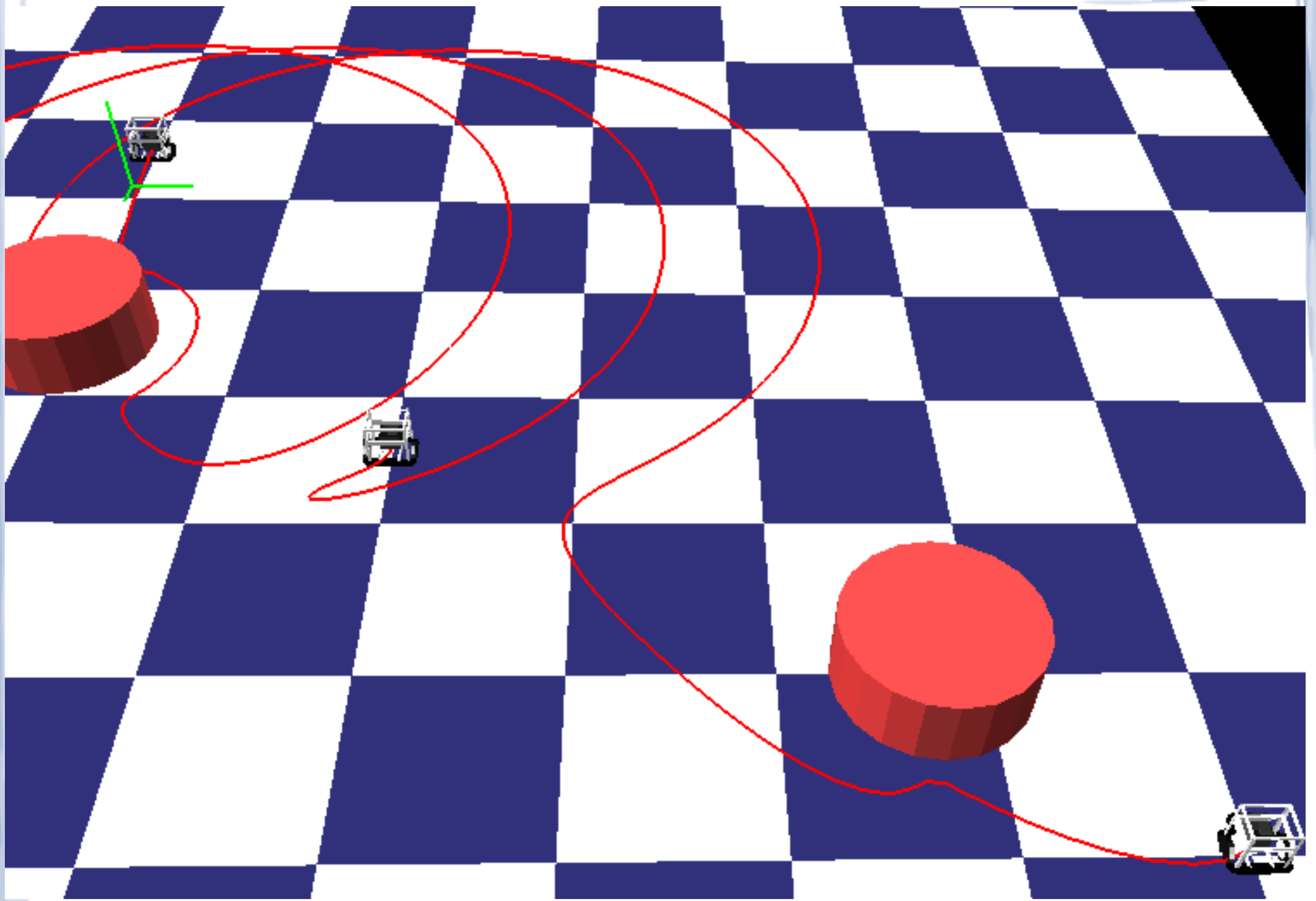


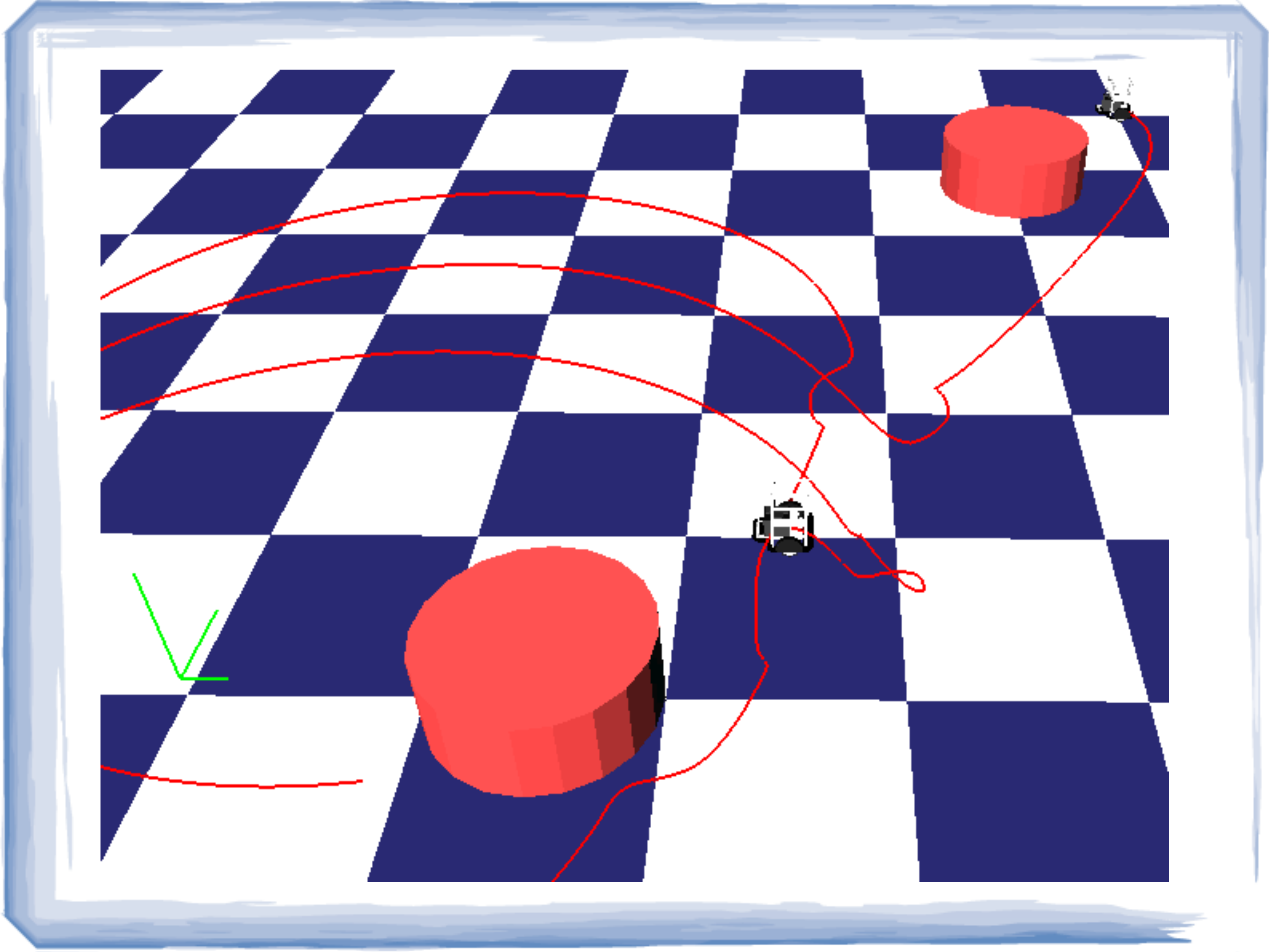
Simulation results (cont.)











Videos

Simulation results for kinematic model

Visualization for 2 robots:

sym_kin_2_rob.avi

Visualization for 8 robots:

sym_kin_8_rob.avi

Visualization for 16 robots:

sym_kin_16_rob.avi

Simulation results for dynamic model

Visualization for 3 robots – simple case:

sym_dyn_3_rob.avi

Visualization for 3 robots – bypassing:

sym_dyn_3_rob_omijanie.avi